CS540 Introduction to Artificial Intelligence

Deep Learning I: Convolutional Neural Networks

University of Wisconsin-Madison

Fall 2024

Announcements

- **Homeworks**:
  - HW6 was due today
  - HW7 is released and will be due on **Thursday Apr. 4 at 11 AM**

- Class roadmap:

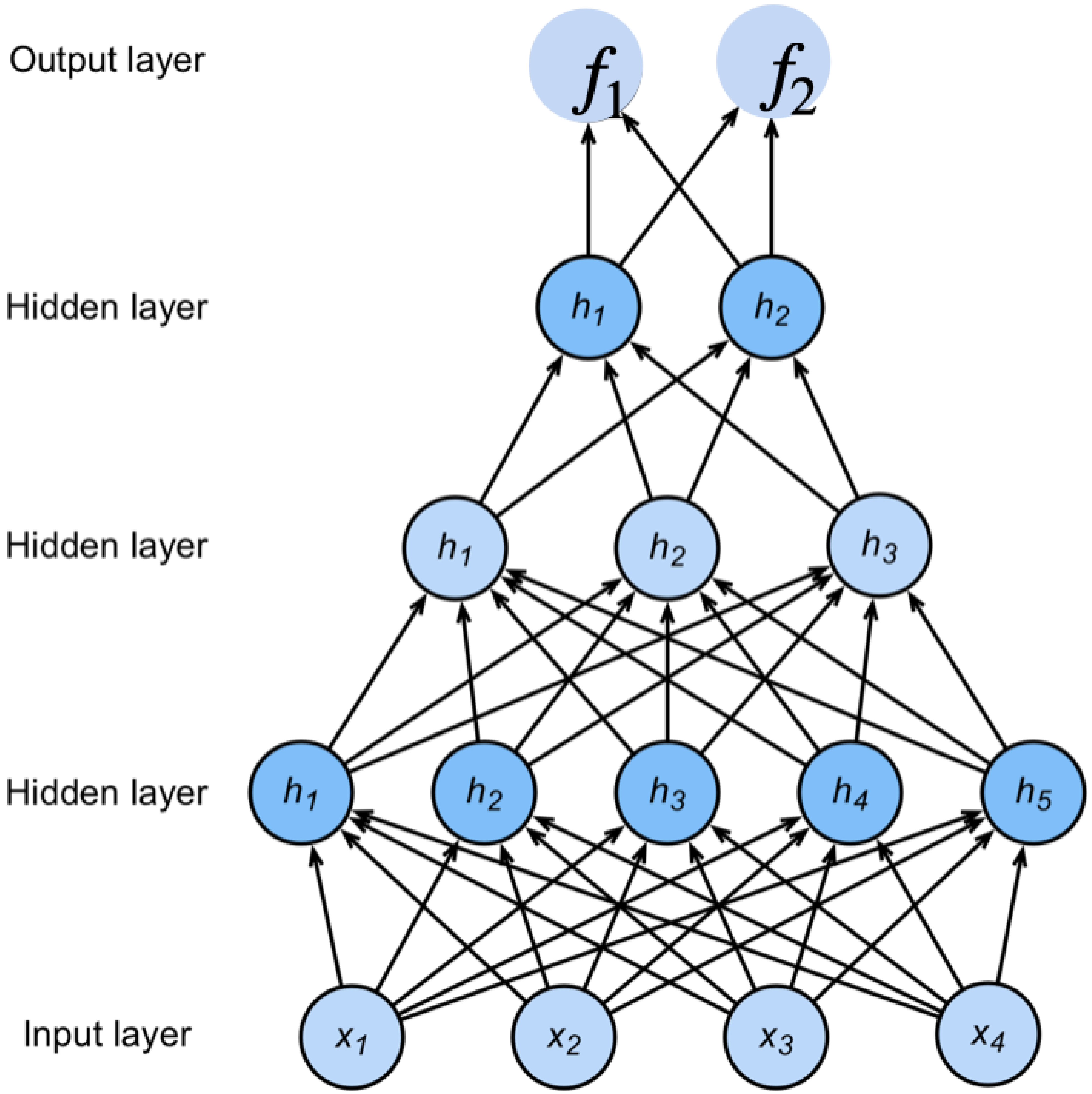
Thursday, Mar. 14	Machine Learning: Deep Learning I
Tuesday, Mar. 19	Machine Learning: Deep Learning II
Thursday, Mar. 21	Machine Learning: Deep Learning II

Deep Learning

Today’s Goals

- Build an understanding of convolutional neural networks.
- Why do we want convolutional layers?
- What are convolutional neural networks?
  - 2D vs 3D convolutional networks.
  - Padding and stride.
  - Multiple input and output channels
  - Pooling

Review: Multi-Layer Neural Networks



$$\mathbf{h}_1 = \sigma(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)})$$

$$\mathbf{h}_2 = \sigma(\mathbf{W}^{(2)}\mathbf{h}_1 + \mathbf{b}^{(2)})$$

$$\mathbf{h}_3 = \sigma(\mathbf{W}^{(3)}\mathbf{h}_2 + \mathbf{b}^{(3)})$$

$$\mathbf{f} = \mathbf{W}^{(4)}\mathbf{h}_3 + \mathbf{b}^{(4)}$$

$$\mathbf{p} = \text{softmax}(\mathbf{f})$$

NNs are composition of nonlinear functions

How to classify

Cats vs. dogs?

Dual

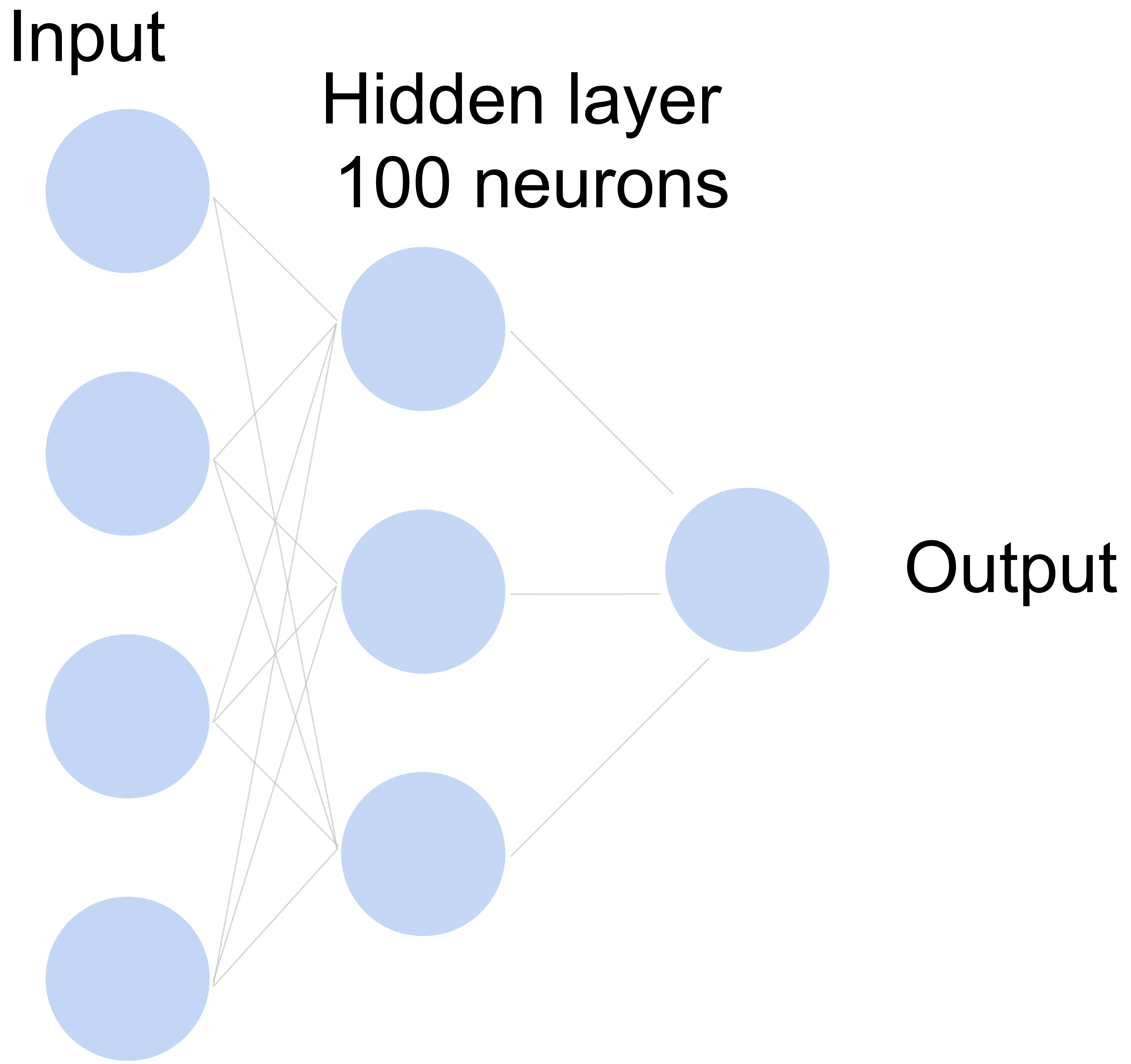
12MP

wide-angle and
telephoto cameras

36M floats in a RGB image!

Fully Connected Networks

Cats vs. dogs?



~ 36M elements x 100 = **~3.6B** parameters!

Convolutions come to rescue!

Where is Waldo?

Why Convolution?

- Translation Invariance
- Locality

2-D Convolution

$$\overset{\text{Input}}{\begin{bmatrix}0 & 1 & 2\\ 3 & 4 & 5\\ 6 & 7 & 8\end{bmatrix}} * \overset{\text{Kernel}}{\begin{bmatrix}0 & 1\\ 2 & 3\end{bmatrix}} = \overset{\text{Output}}{\begin{bmatrix}19 & 25\\ 37 & 43\end{bmatrix}}$$

$$0\text{x}0 + 1\text{x}1 + 3\text{x}2 + 4\text{x}3 = 19$$

2-D Convolution

Input

0	1	2
3	4	5
6	7	8

*

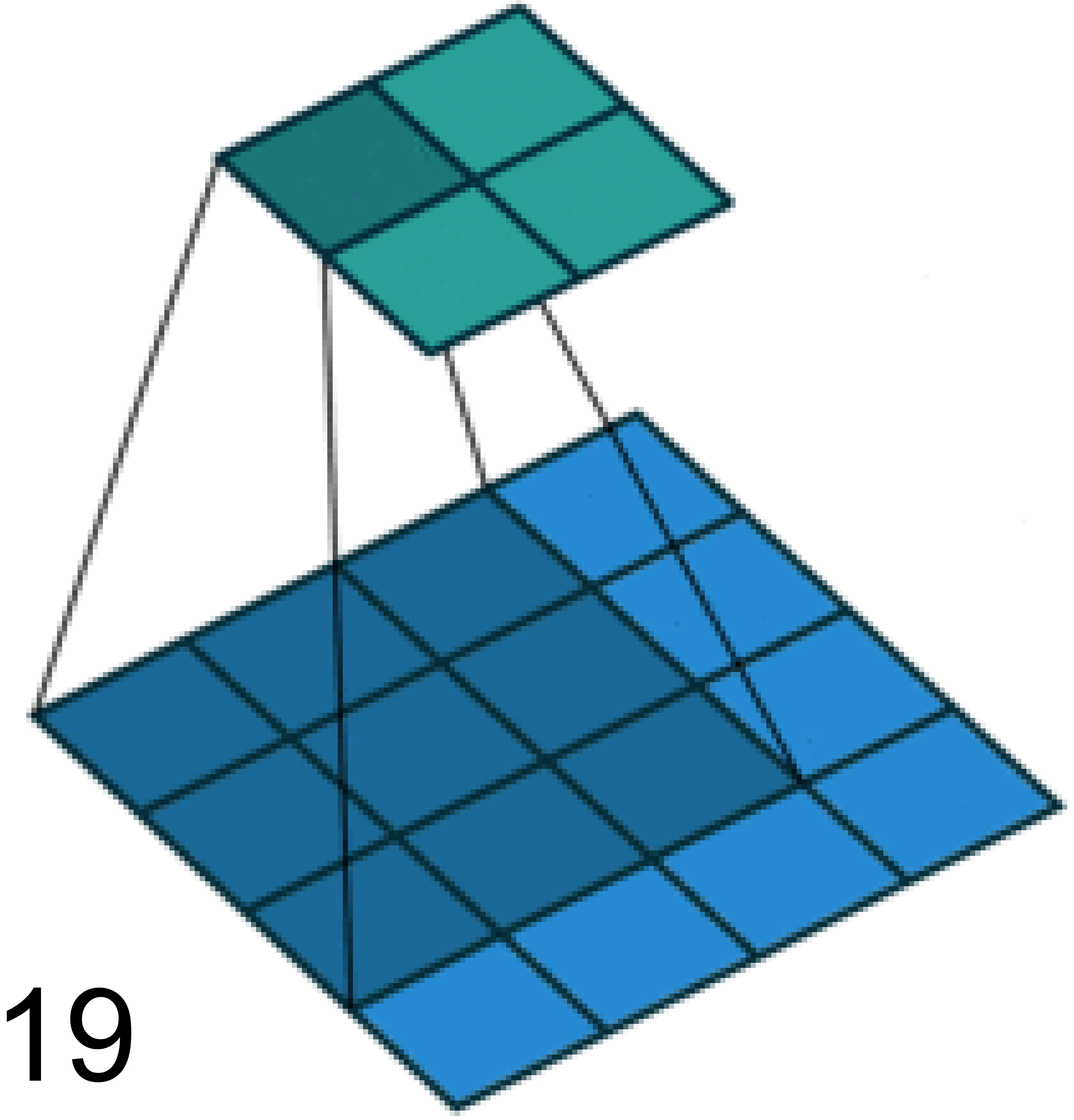
Kernel

0	1
2	3

=

Output

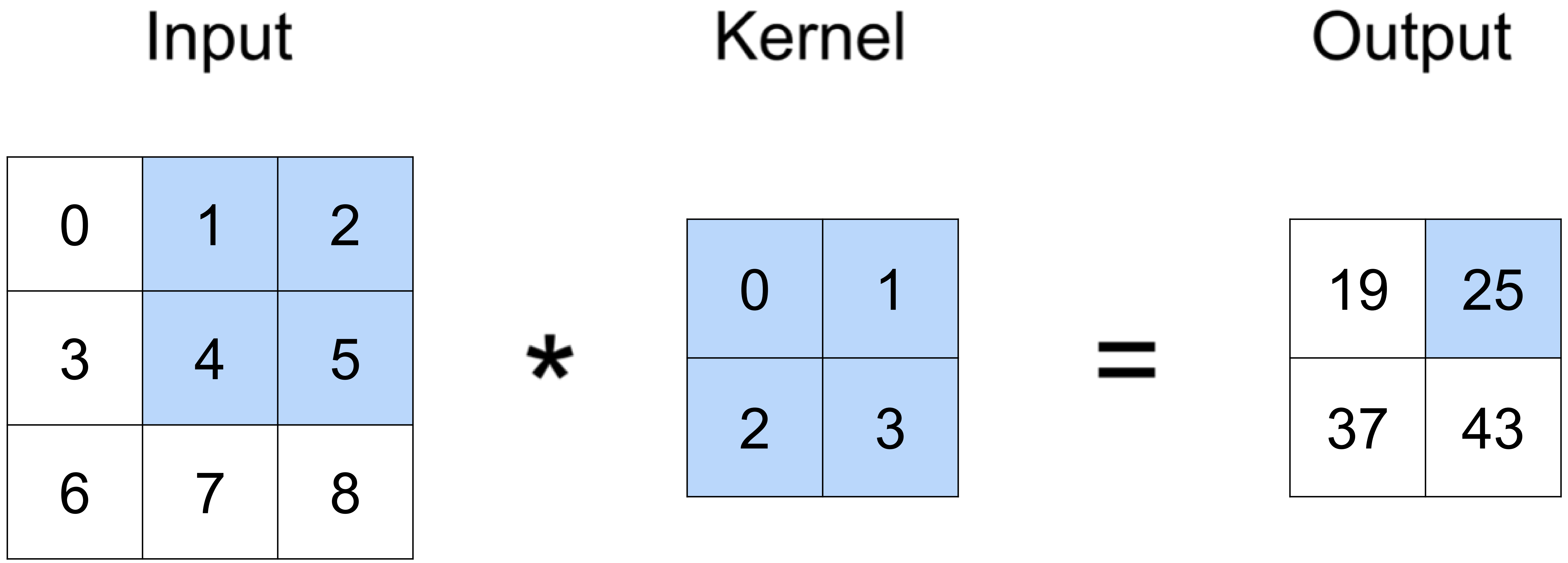
19	25
37	43



$$0\text{x}0 + 1\text{x}1 + 3\text{x}2 + 4\text{x}3 = 19$$

(vdumoulin@ Github)

2-D Convolution



$$1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25$$

2-D Convolution

$$\underset{\text{Input}}{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 19 & 25 \\ 37 & 43 \end{bmatrix}}$$

$$3x0 + 4x1 + 6x2 + 7x3 = 37$$

2-D Convolution

$$\underset{\text{Input}}{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 19 & 25 \\ 37 & 43 \end{bmatrix}}$$

$$4 \times 0 + 5 \times 1 + 7 \times 2 + 8 \times 3 = 43$$

2-D Convolution Layer

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 19 & 25 \\ 37 & 43 \end{bmatrix}$$

- **X**: n_h x n_w input matrix
- **W**: k_h x k_w kernel matrix
- **Y**: $(n_h - k_h + 1) \times (n_w - k_w + 1)$ output matrix

Convolution operator not multiplication

$$\mathbf{Y} = \mathbf{X} * \mathbf{W}$$

2-D Convolution Layer

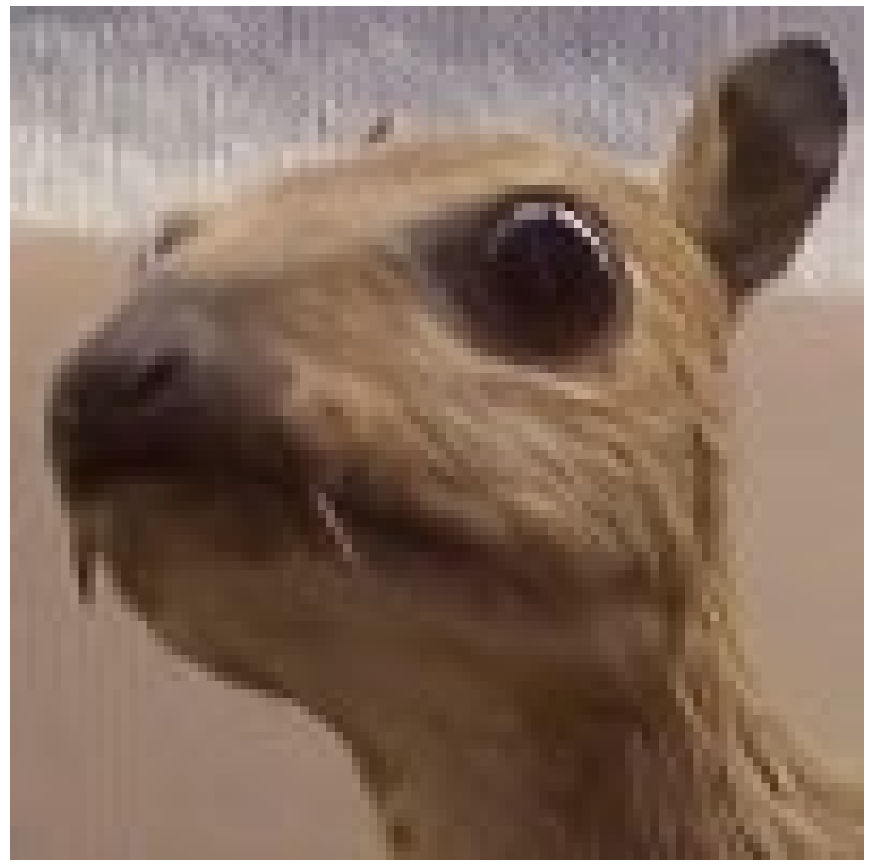
$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} + 1 = \begin{bmatrix} 20 & 26 \\ 38 & 44 \end{bmatrix}$$

- **X**: n_h x n_w input matrix
- **W**: k_h x k_w kernel matrix
- b: scalar bias
- **Y**: $(n_h - k_h + 1) \times (n_w - k_w + 1)$ output matrix

$$\mathbf{Y = X * W + b}$$

- **W** and *b* are learnable parameters

Examples

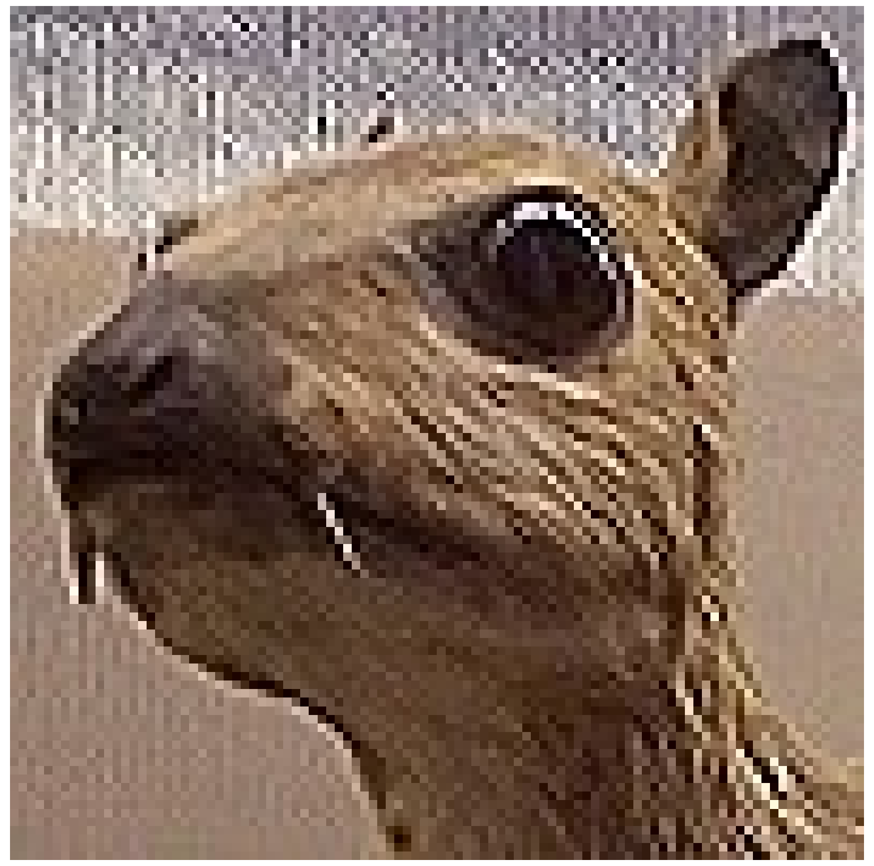


(wikipedia)

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

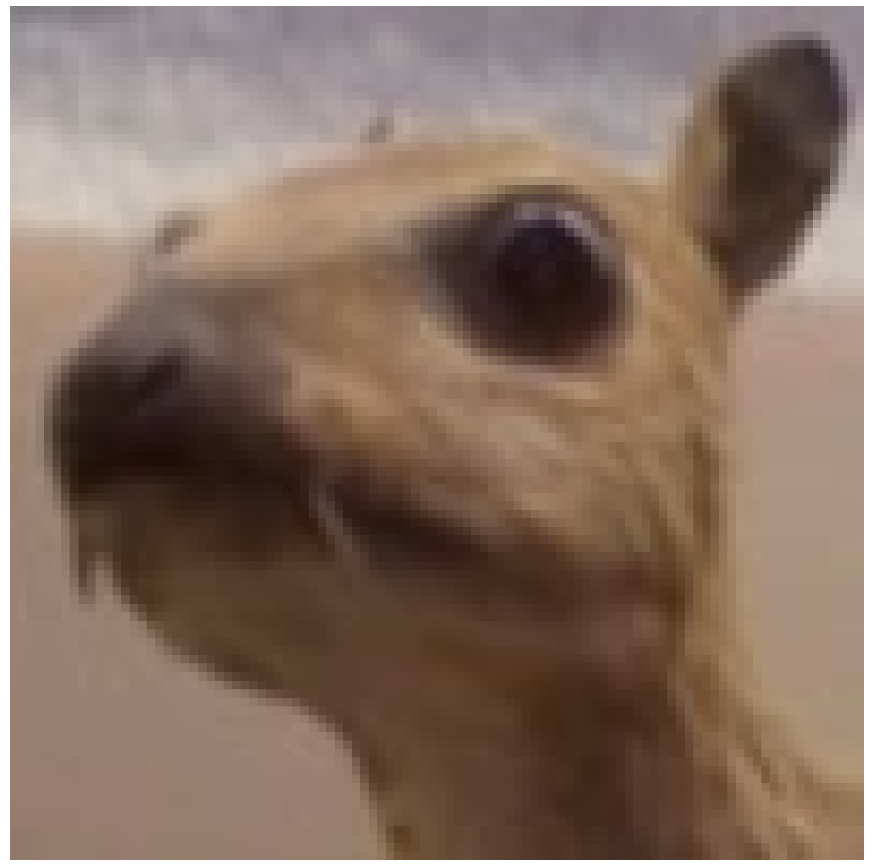
Edge Detection

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$



Sharpen

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$



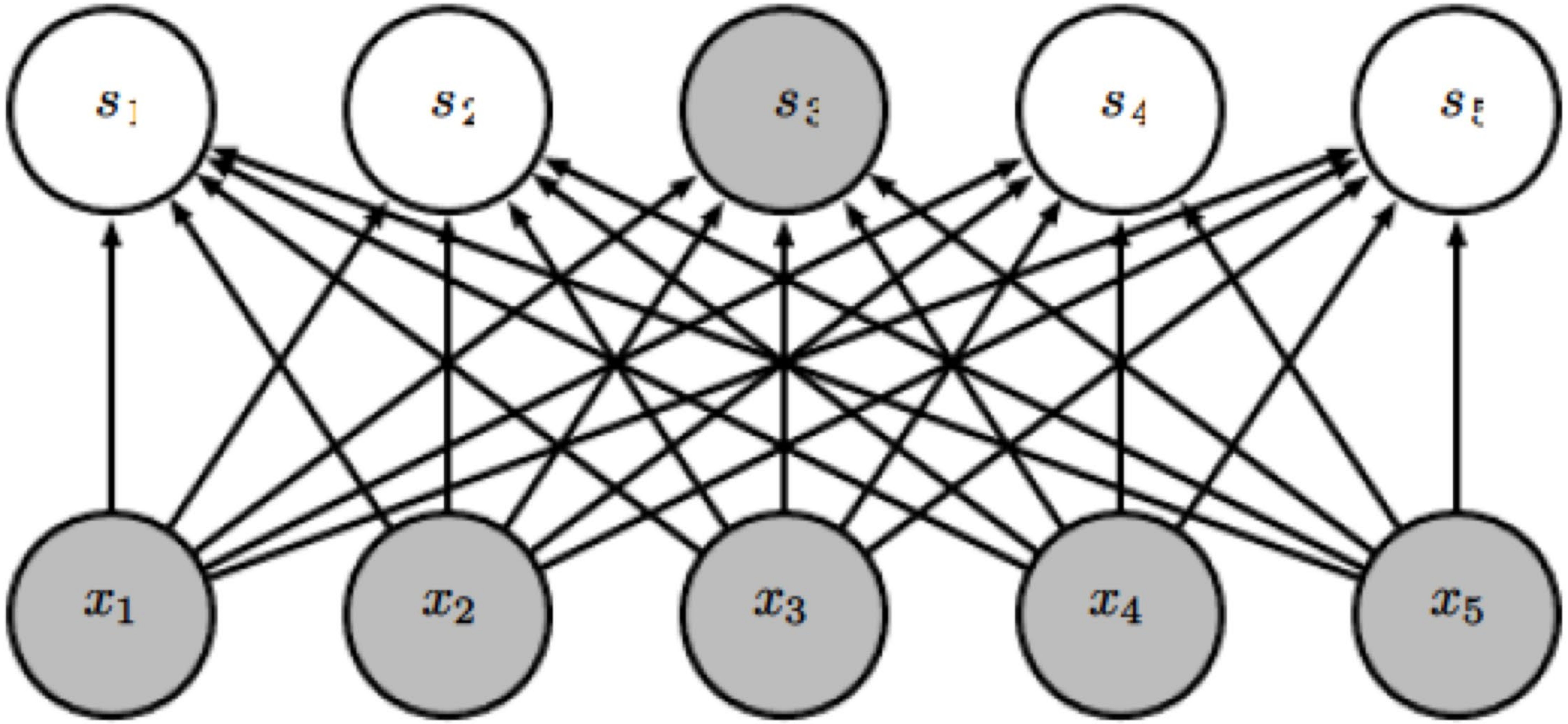
Gaussian Blur

Convolutional Neural Networks

- Convolutional networks: neural networks that use convolution in place of general matrix multiplication in at least one of their layers
- Strong empirical performance in applications – particularly computer vision.
- Examples: image classification, object detection.

Advantage: sparse interaction

Fully connected layer, $m \times n$ edges



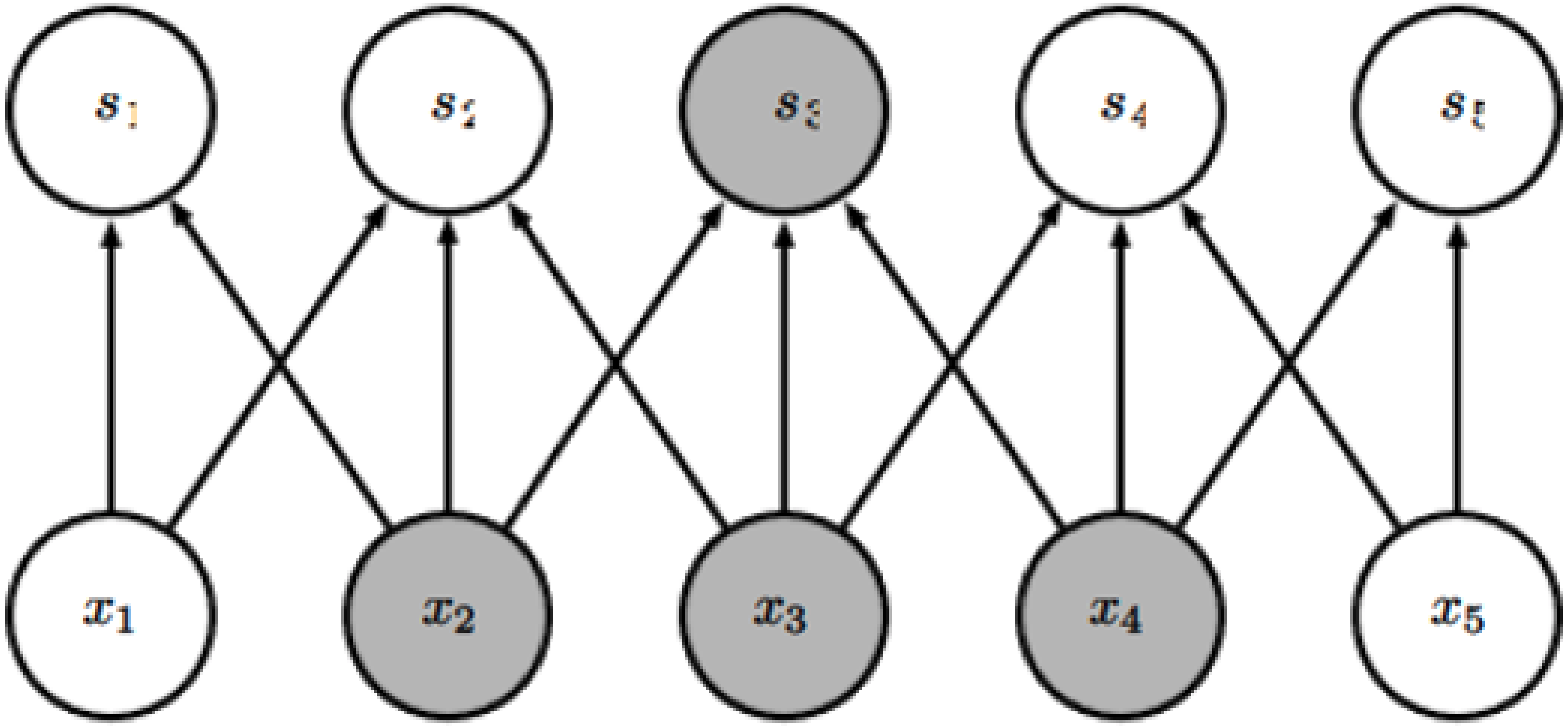
m output nodes

n input nodes

Figure from *Deep Learning,* by Goodfellow, Bengio, and Courville

Advantage: sparse interaction

Convolutional layer, $\leq m \times k$ edges



m output nodes

k kernel size

n input nodes

Figure from *Deep Learning,* by Goodfellow, Bengio, and Courville

Q1. Suppose we want to perform convolution as follows. What’s the output?

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} + 1 = ?$$

A. $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$

Q1. Suppose we want to perform convolution as follows. What's the output?

$$\begin{bmatrix}0 & 1 & 2\\ 3 & 4 & 5\\ 6 & 7 & 8\end{bmatrix} * \begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix} \boxed{+1} = \begin{bmatrix}1 & 2\\ 4 & 5\end{bmatrix}$$

$$0\times0 + 1\times1 + 3\times1 + 4\times(-1) + 1 = 1$$
$$1\times0 + 2\times1 + 4\times1 + 5\times(-1) + 1 = 2$$
$$3\times0 + 4\times1 + 6\times1 + 7\times(-1) + 1 = 4$$
$$4\times0 + 5\times1 + 7\times1 + 8\times(-1) + 1 = 5$$

A. $\begin{bmatrix}1 & 2\\ 4 & 5\end{bmatrix}$

B. $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

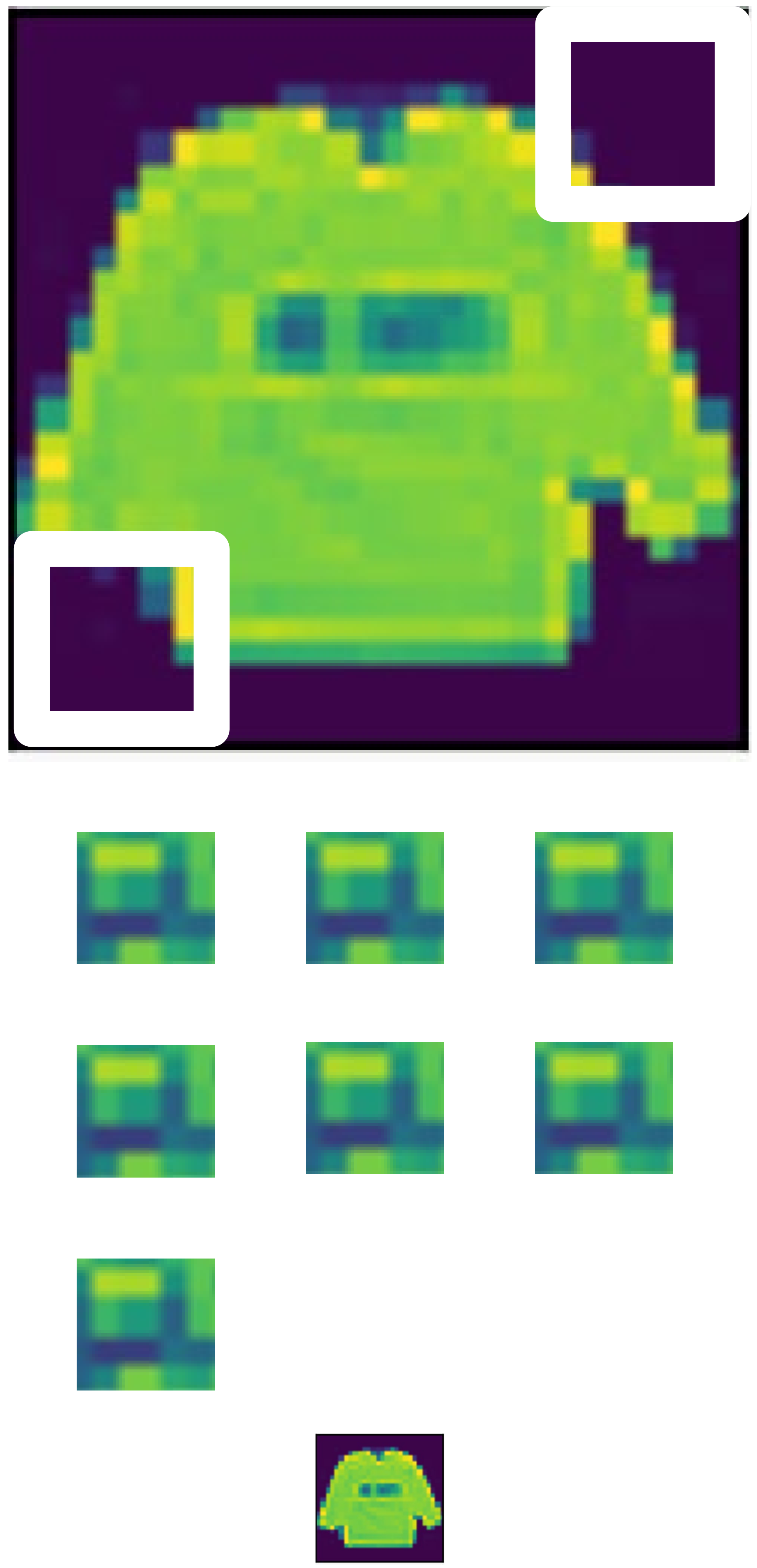
$\begin{bmatrix}1 & 3\\ 3 & 5\end{bmatrix}$

B. $\begin{bmatrix}0 & 1\\ 3 & 4\end{bmatrix}$

Padding and Stride

Padding

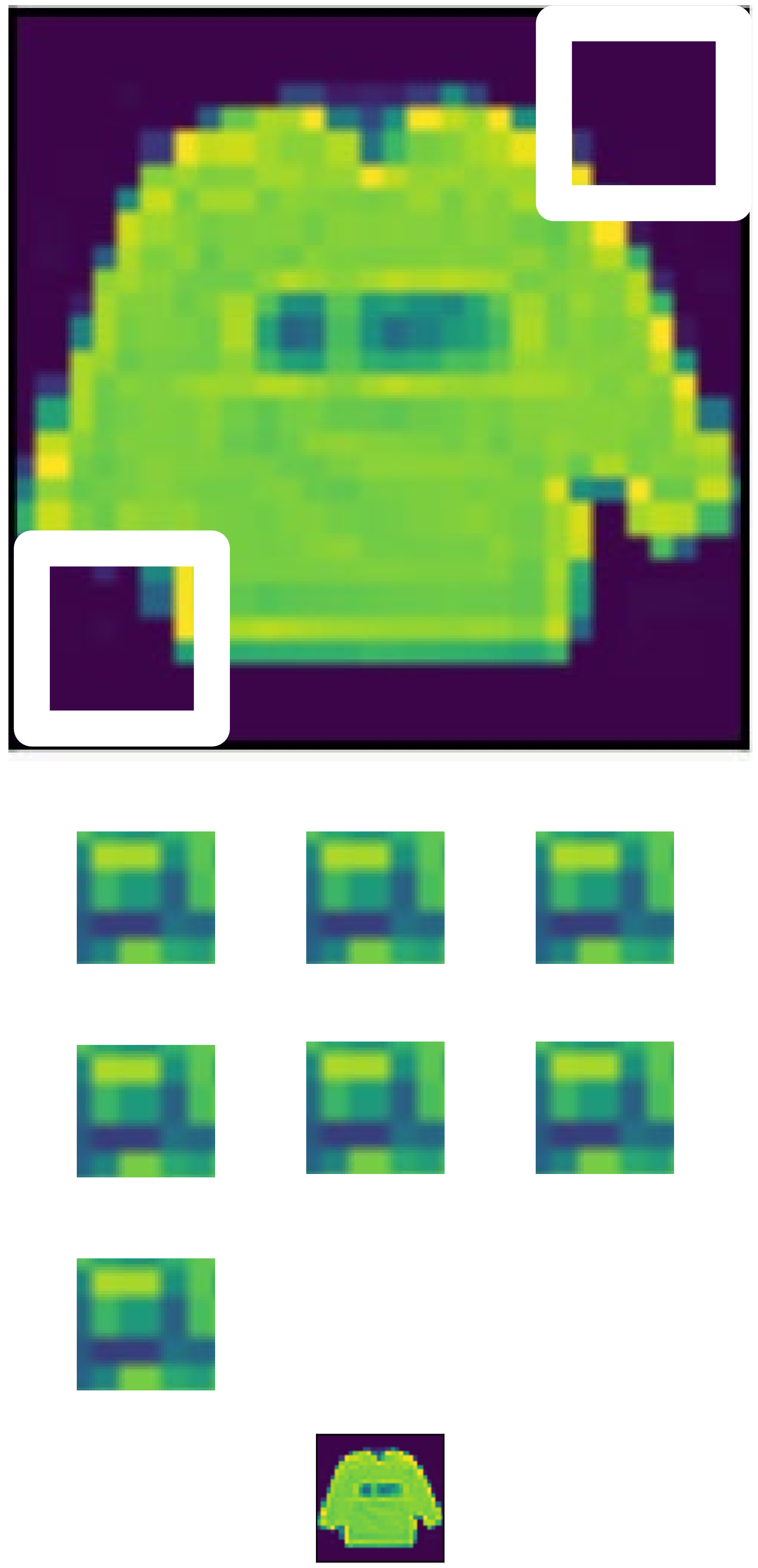
- Given a 32 x 32 input image
- Apply convolution with 5 x 5 kernel
  - 28 x 28 output with 1 layer
  - 4 x 4 output with 7 layers



Padding

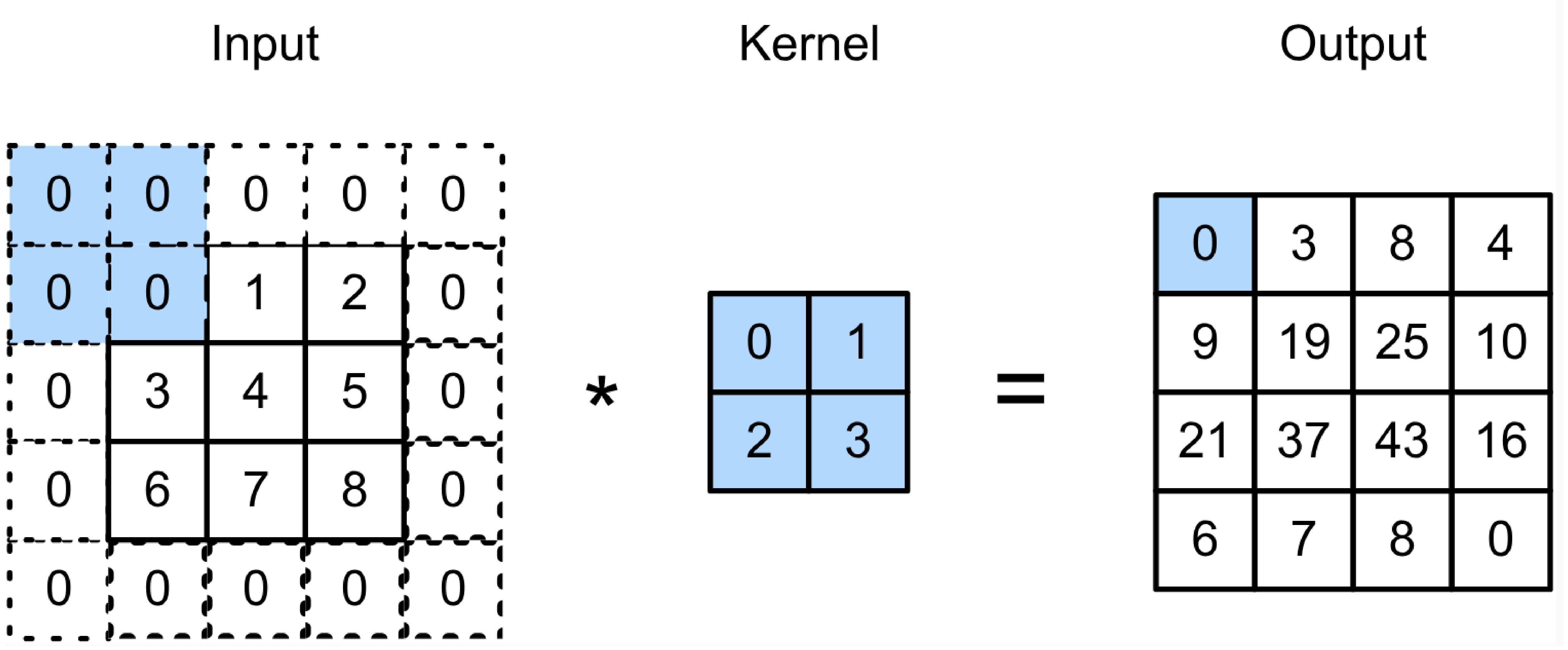
- Given a 32 x 32 input image
- Apply convolution with 5 x 5 kernel
  - 28 x 28 output with 1 layer
  - 4 x 4 output with 7 layers
- Shape decreases faster with larger kernels
  - Shape reduces from $n_h \times n_w$ to

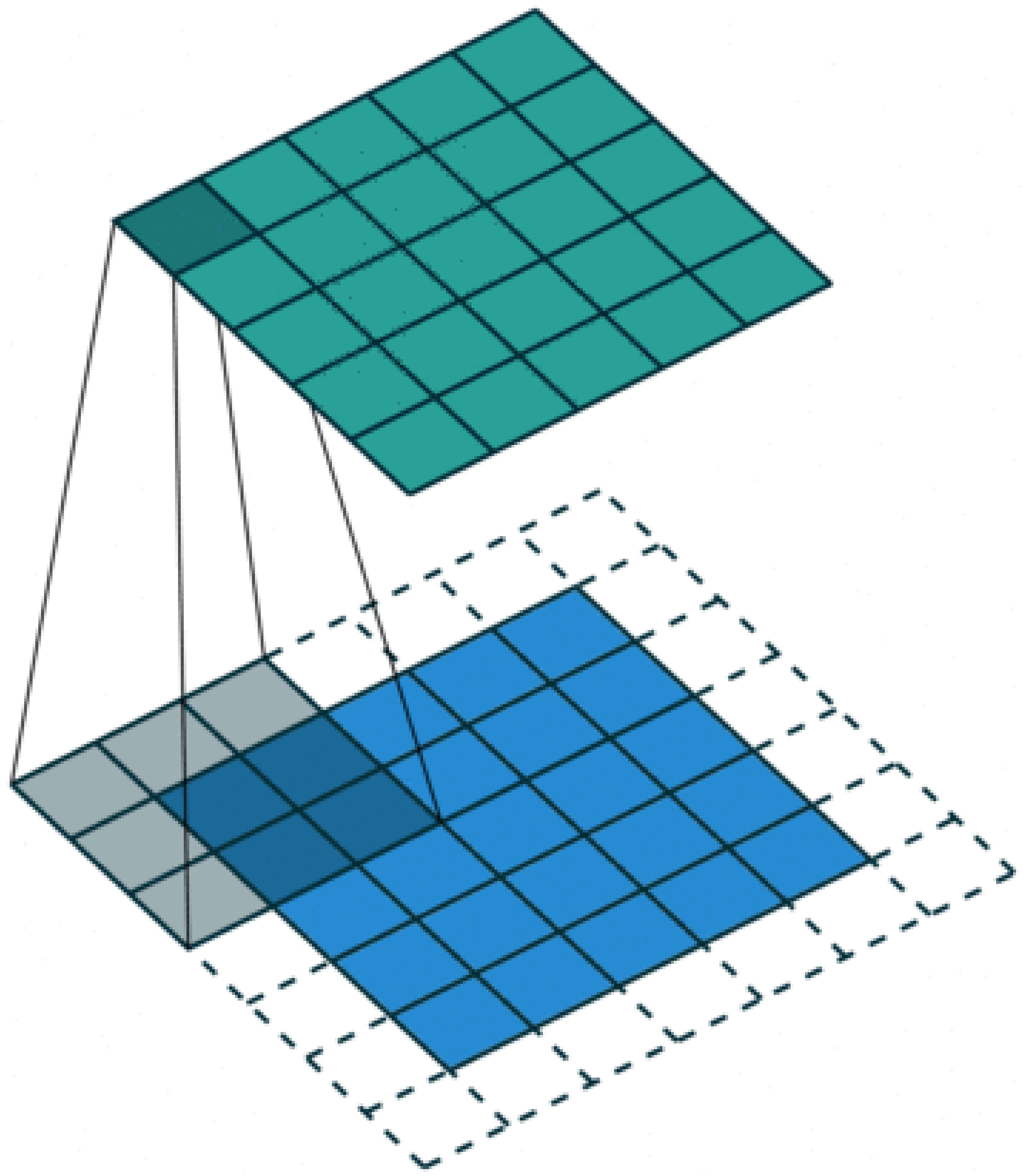
$$(n_h - k_h + 1) \times (n_w - k_w + 1)$$



Convolutional Layers: Padding

Padding adds rows/columns around input



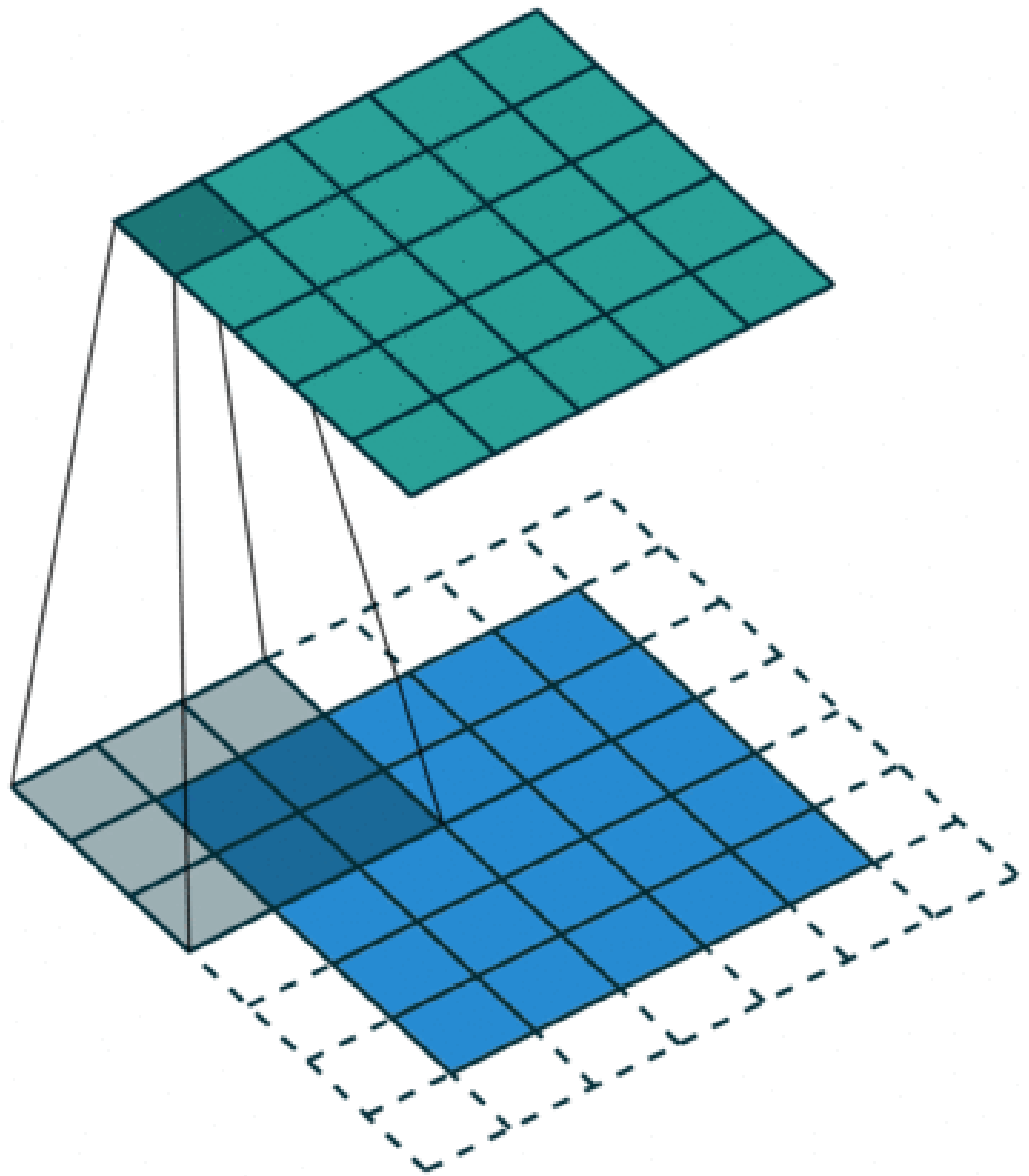


Convolutional Layers: Padding

Padding adds rows/columns around input

- Why?

1. Keeps **edge information**
2. Preserves sizes / allows deep networks
   - ie, for a 32x32 input image, 5x5 kernel, after 1 layer, get 28x28, after 7 layers, **only 4x4**
3. Can combine different filter sizes



Convolutional Layers: Padding

- Padding p_h rows and p_w columns, output shape is

$$(n_h\text{-}k_h+p_h+1) \times (n_w\text{-}k_w+p_w+1)$$

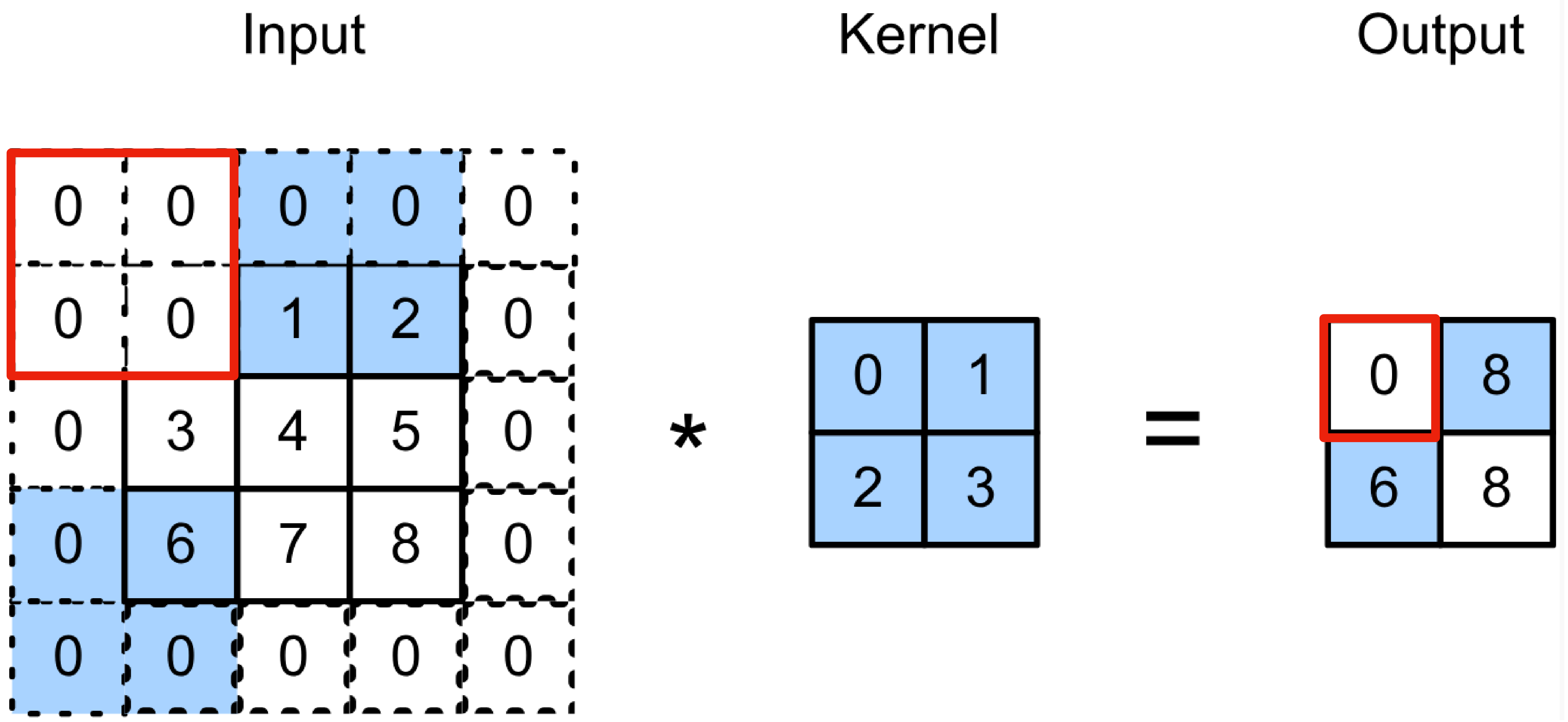
- Common choice is $p_h = k_h\text{-}1$ and $p_w=k_w\text{-}1$

  - Odd k_h: pad $p_h/2$ on both top and bottom
  - Even k_h: pad ceil($p_h/2$) on top, floor($p_h/2$) on bottom

Stride

- Stride is the #rows / #columns per slide

  Example: strides of 3 and 2 for height and width

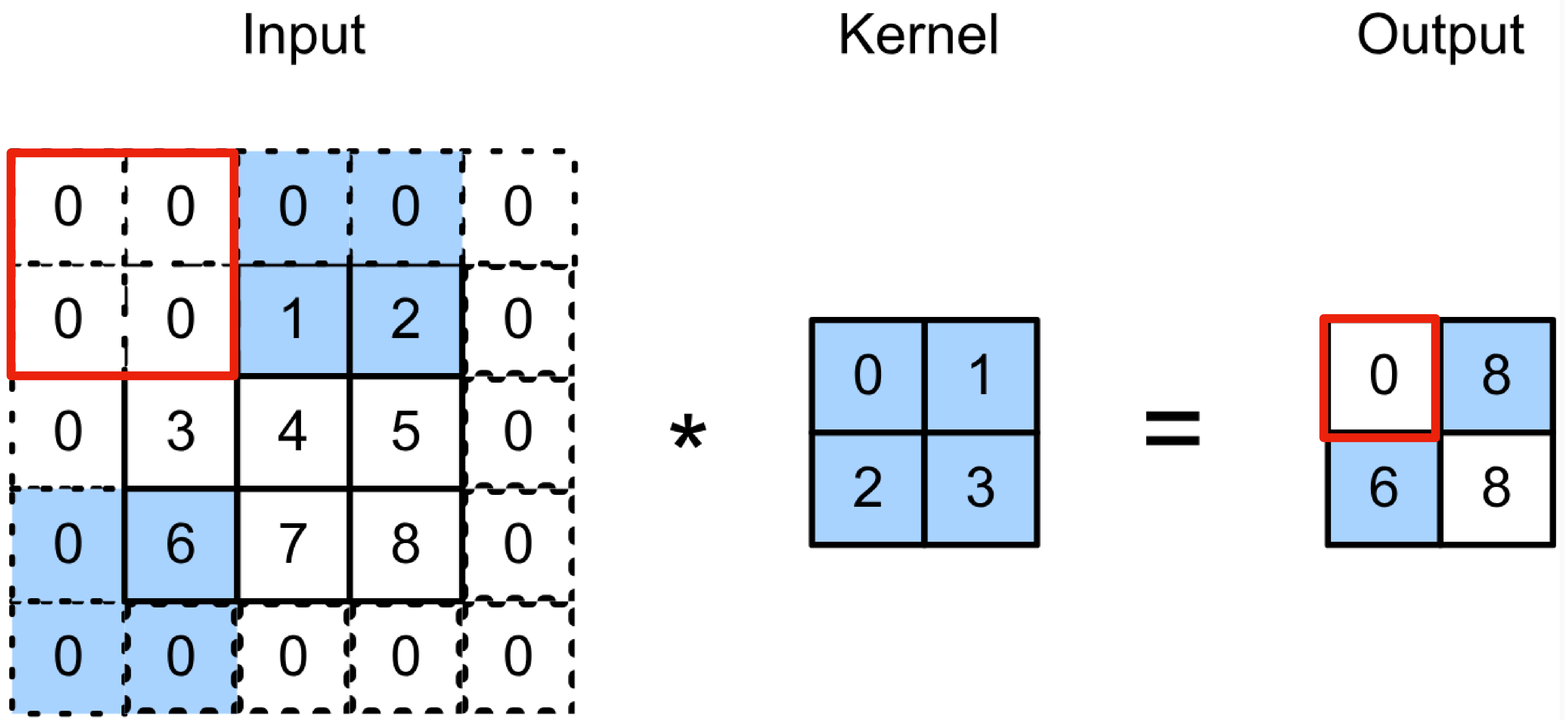


$$0\times 0 + 0\times 1 + 1\times 2 + 2\times 3 = 8$$
$$0\times 0 + 6\times 1 + 0\times 2 + 0\times 3 = 6$$

Stride

- Stride is the #rows / #columns per slide

Example: strides of 3 and 2 for height and width



$$0\times0 + 0\times1 + 1\times2 + 2\times3 = 8$$
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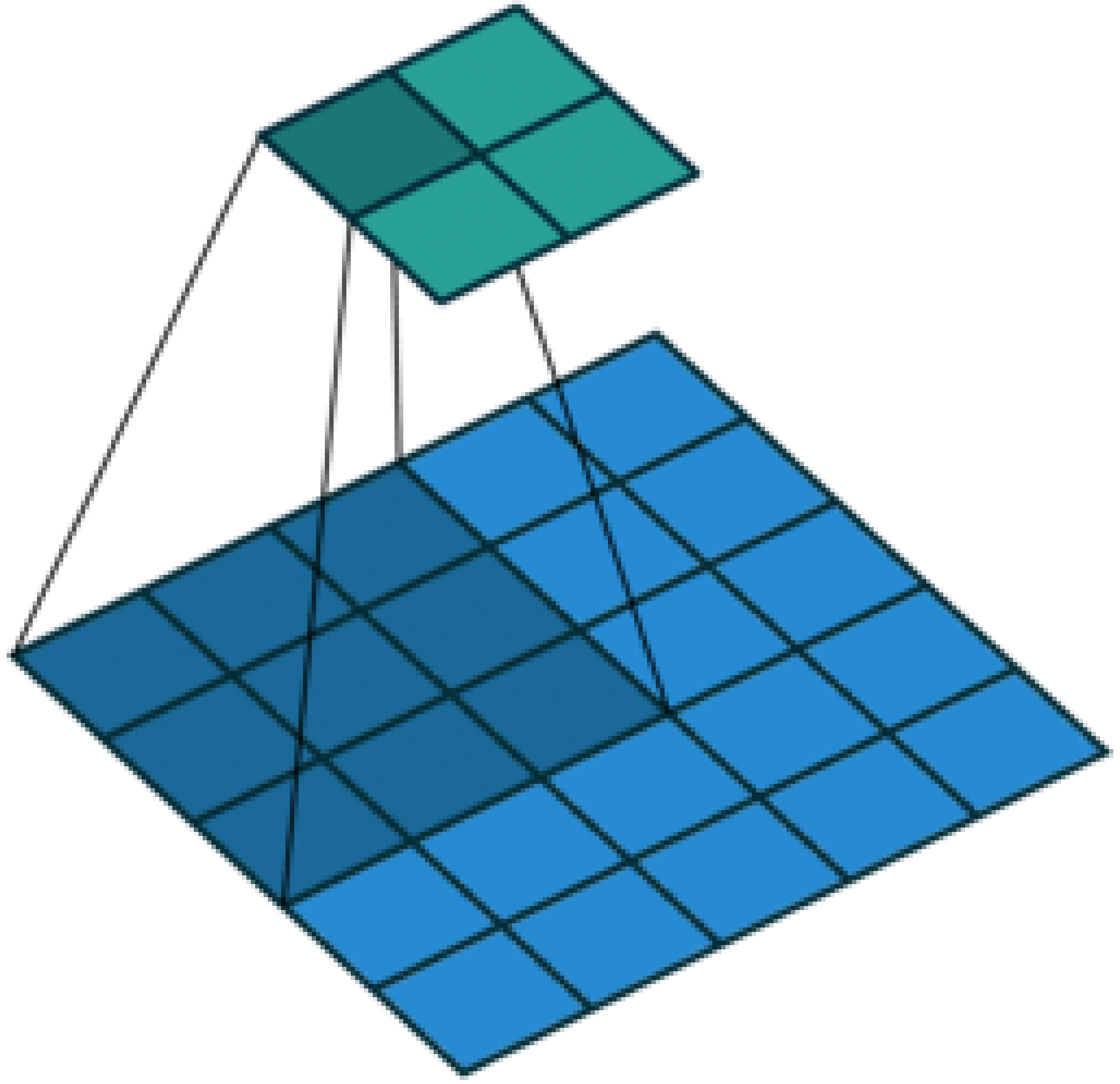


Stride 2,2

Convolutional Layers: Stride

- Given stride s_h for the height and stride s_w for the width, the output shape is

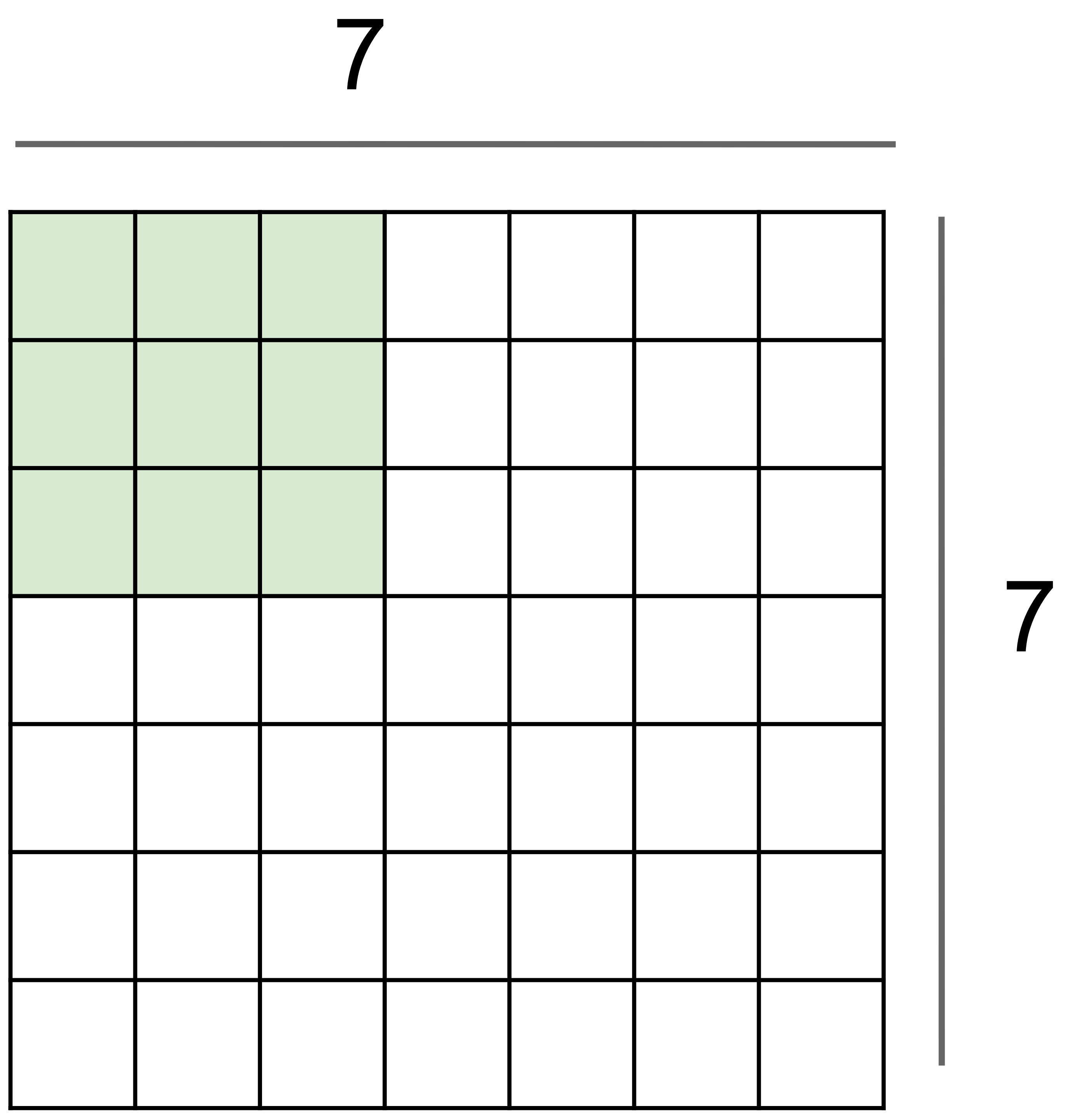
$$\lfloor(n_h-k_h+p_h+s_h)/s_h\rfloor \times \lfloor(n_w-k_w+p_w+s_w)/s_w\rfloor$$

- Set $p_h = k_h-1$, $p_w = k_w-1$, then get

$$\lfloor(n_h+s_h-1)/s_h\rfloor \times \lfloor(n_w+s_w-1)/s_w\rfloor$$

Q2. Suppose we want to perform convolution on a single channel image of size 7x7 (no padding) with a kernel of size 3x3, and stride = 2. What is the dimension of the output?

A. 3x3
B. 7x7
C. 5x5
D. 2x2



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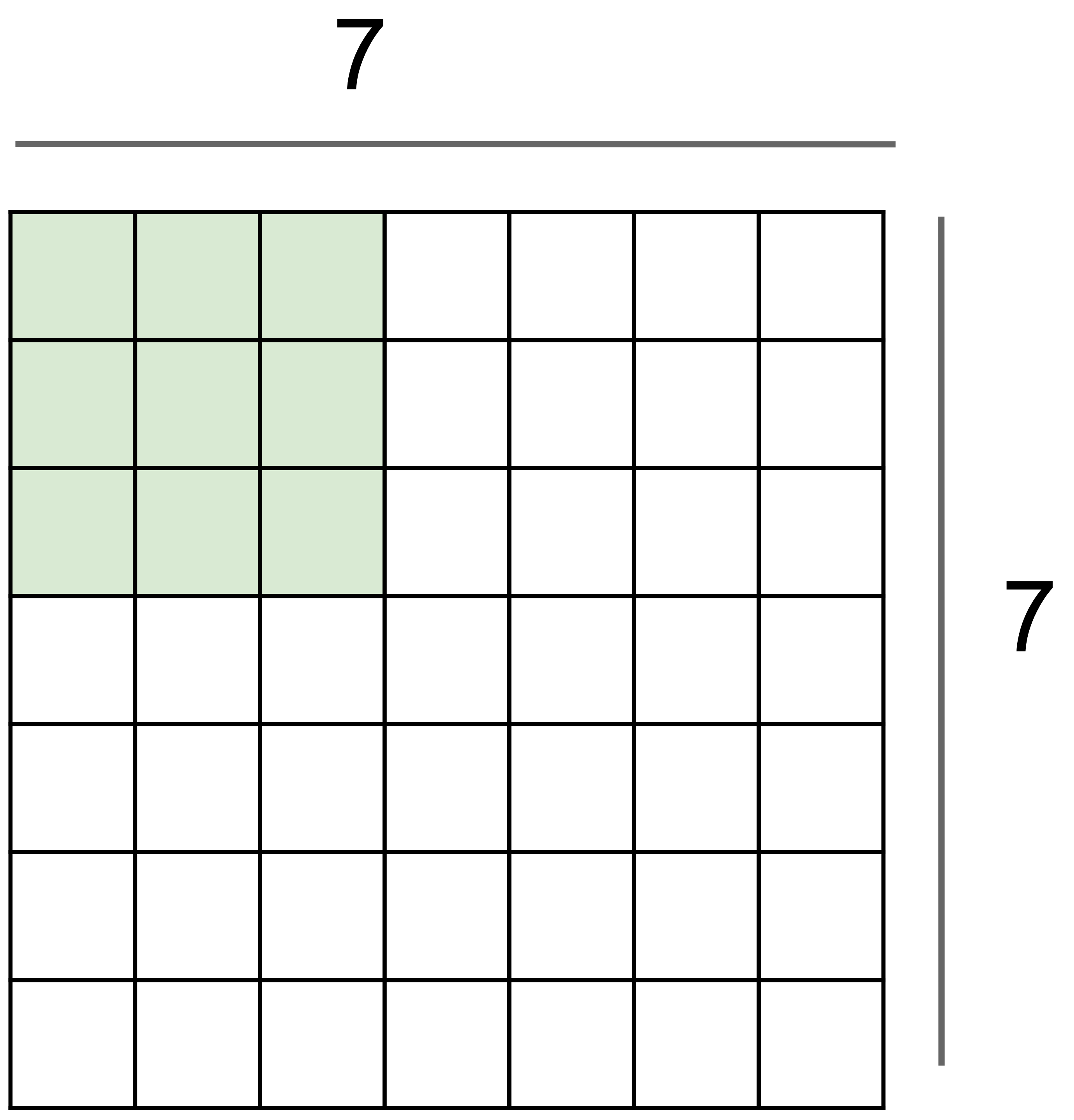
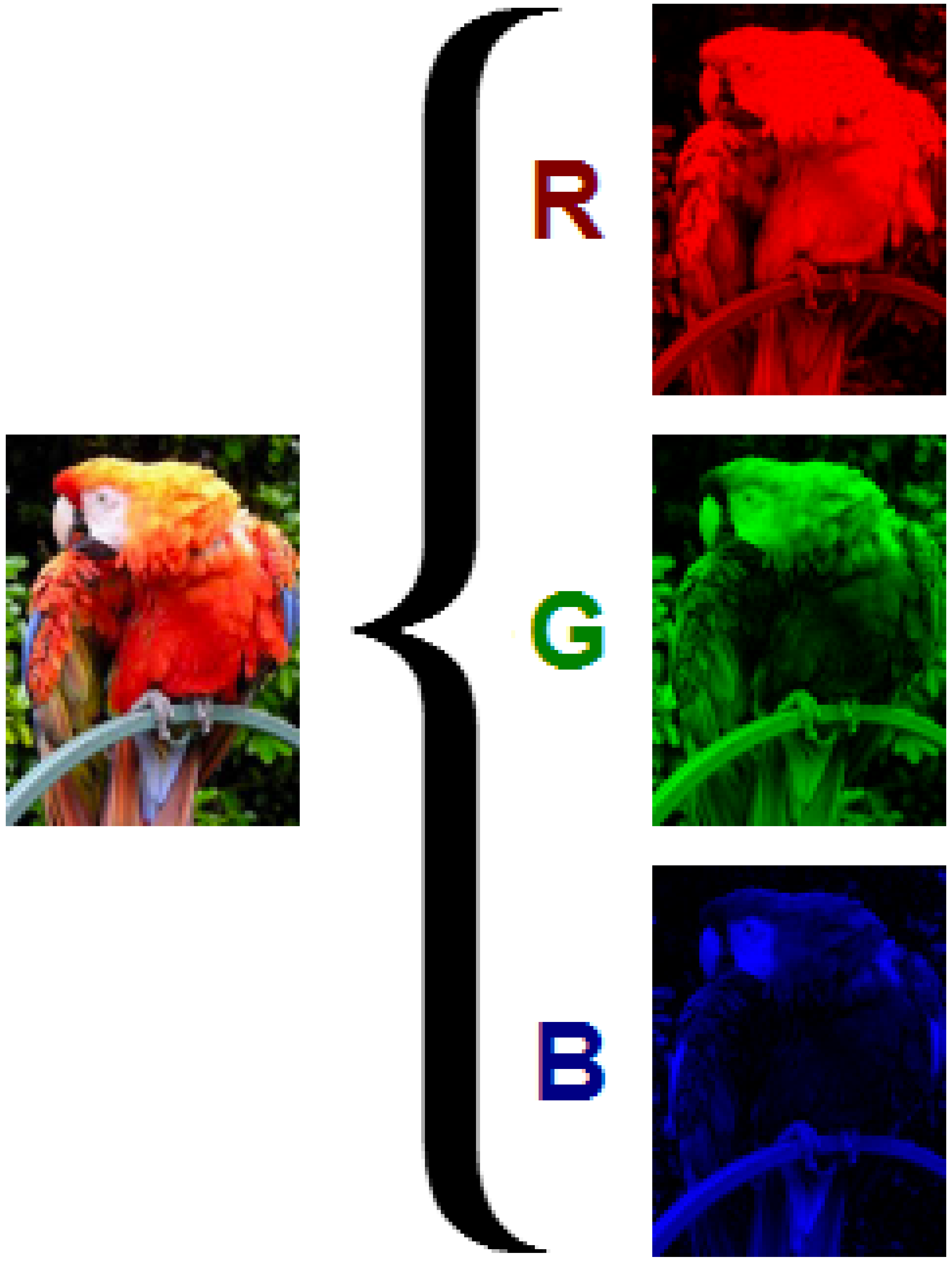


$$\lfloor (n_h - k_h + p_h + s_h)/s_h \rfloor \times \lfloor (n_w - k_w + p_w + s_w)/s_w \rfloor$$

Multiple Input and Output Channels

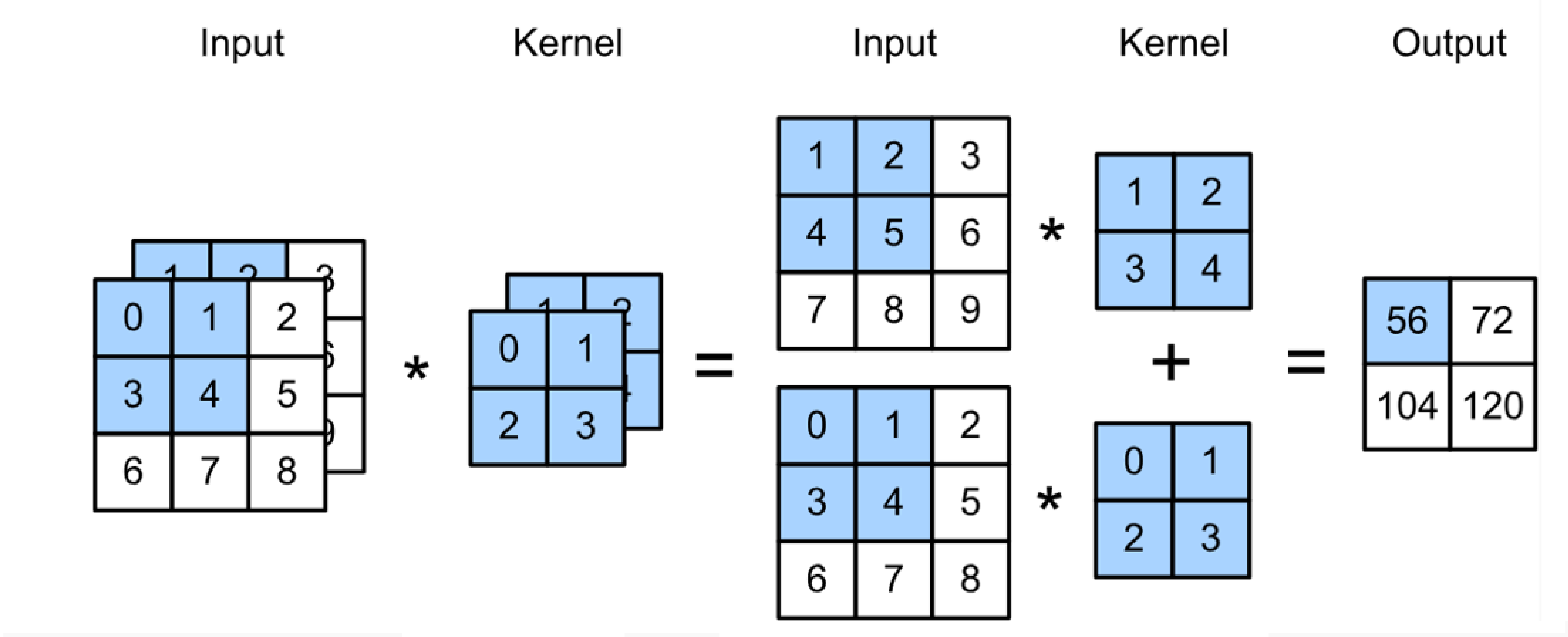
Multiple Input Channels

- Color image may have three RGB channels
- Converting to grayscale loses information



Multiple Input Channels

- Have a kernel matrix for each channel, and then sum results over channels



$$(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4)$$
$$+(0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3)$$
$$= 56$$

Convolutional Layers: Channels

- How to integrate multiple channels?
  - Have a kernel for each channel, and then sum results over channels

$$\mathbf{X} : c_i \times n_h \times n_w$$

$$\mathbf{W} : c_i \times k_h \times k_w$$

$$\mathbf{Y} : m_h \times m_w$$

$$\mathbf{Y} = \sum_{i=0}^{c_i} \mathbf{X}_{i,:,:} \star \mathbf{W}_{i,:,:}$$

"Slices" of tensors

Tensor: generalization of matrix to higher dimensions

Multiple Output Channels

- No matter how many inputs channels, so far we always get single output channel
- We can have **multiple 3-D kernels**, each one generates an output channel

Multiple Output Channels

- No matter how many inputs channels, so far we always get single output channel
- We can have **multiple 3-D kernels**, each one generates an output channel
- Input $\mathbf{X} : c_i \times n_h \times n_w$
- Kernels $\mathbf{W} : c_o \times c_i \times k_h \times k_w$
- Output $\mathbf{Y} : c_o \times m_h \times m_w$

$$\mathbf{Y}_{i,:,:} = \mathbf{X} \star \mathbf{W}_{i,:,:,:}$$
$$\text{for } i = 1, \ldots, c_o$$

Multiple Input/Output Channels

- Each 3-D kernel may recognize a particular pattern

(Gabor filters)

Q3. Suppose we want to perform convolution on an RGB image of size 224x224 (no padding) with 64 kernels, each with height 3 and width 3. Stride = 1. Which is a reasonable estimate of the total number of scalar multiplications involved in this operation (without considering any optimization in matrix multiplication)?

A. 64 x 3 x 3 x 222 x 222

B. 64 x 3 x 3 x 222

C. 3 x 3 x 222 x 222

D. 64 x 3 x 3 x 3 x 222 x 222

Q3. Suppose we want to perform convolution on an RGB image of size 224x224 (no padding) with 64 kernels, each with height 3 and width 3. Stride = 1. Which is a reasonable estimate of the total number of scalar multiplications involved in this operation (without considering any optimization in matrix multiplication)?

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A. 64 x 3 x 3 x 222 x 222

B. 64 x 3 x 3 x 222

C. 3 x 3 x 222 x 222

D. 64 x 3 x 3 x 3 x 222 x 222

For each kernel, we slide the window to 222 x 222 different locations. For each location, the number of multiplication is 3x3x3. So in total 64x3x3x3x222x222

Q4. Suppose we want to perform convolution on a RGB image of size 224 x 224 (no padding) with 64 kernels, each with height 3 and width 3. Stride = 1. The convolution layer has bias parameters. Which is a reasonable estimate of the total number of learnable parameters?

A. 64 x 222 x 222

B. 64 x 3 x 3 x 222

C. 3 x 3 x 3 x 64

D. (3 x 3 x 3 + 1) x 64

Q4. Suppose we want to perform convolution on a RGB image of size 224 x 224 (no padding) with 64 kernels, each with height 3 and width 3. Stride = 1. The convolution layer has bias parameters. Which is a reasonable estimate of the total number of learnable parameters?

A. 64 x 222 x 222

B. 64 x 3 x 3 x 222

C. 3 x 3 x 3 x 64

D. (3 x 3 x 3 + 1) x 64

Q4. Suppose we want to perform convolution on a RGB image of size 224 x 224 (no padding) with 64 kernels, each with height 3 and width 3. Stride = 1. The convolution layer has bias parameters. Which is a reasonable estimate of the total number of learnable parameters?

A. 64 x 222 x 222

B. 64 x 3 x 3 x 222

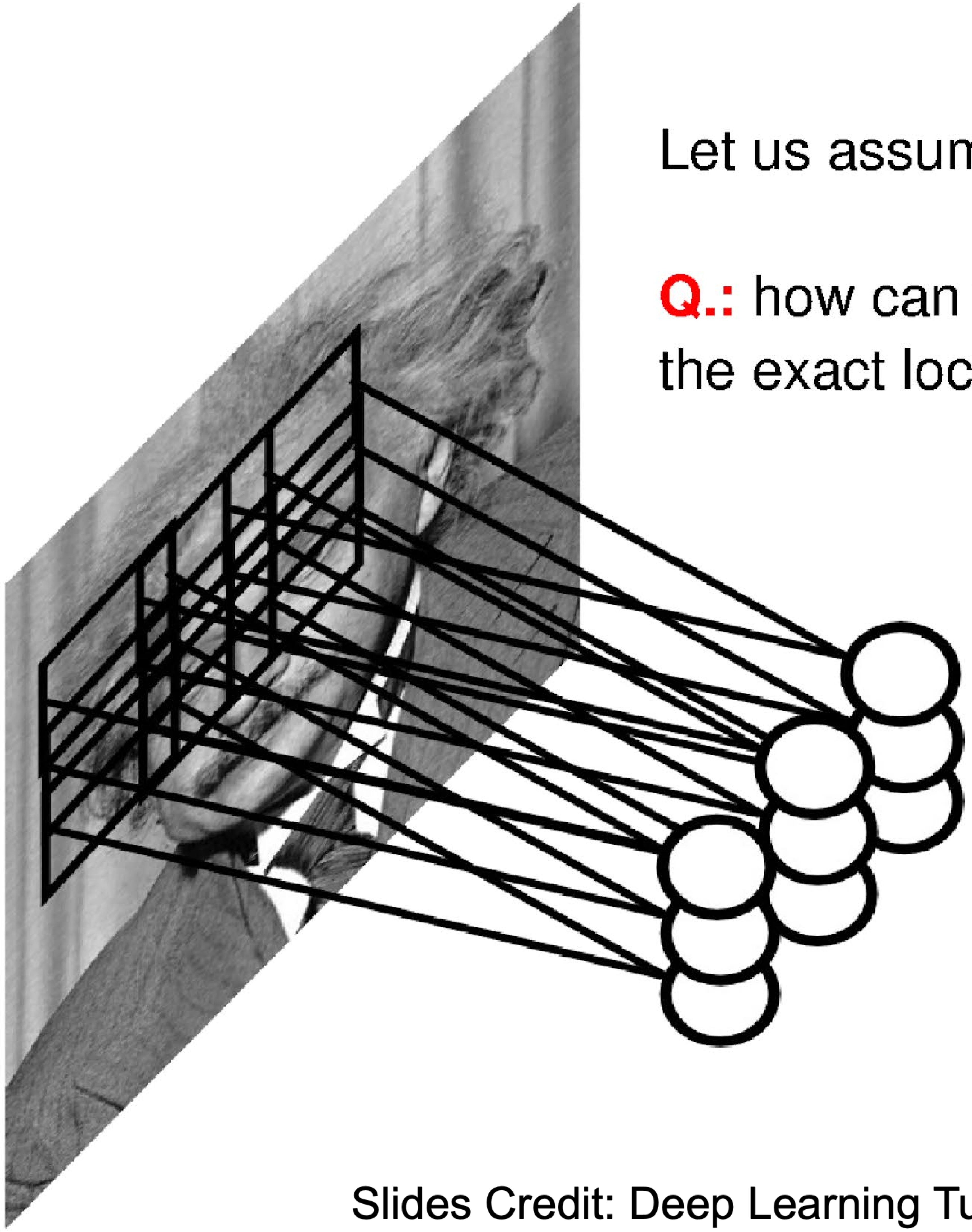
C. 3 x 3 x 3 x 64

D. (3 x 3 x 3 + 1) x 64

Each kernel is 3D kernel across 3 input channels, so has 3x3x3 parameters. Each kernel has 1 bias parameter. So in total (3x3x3+1)x64

Pooling Layer

Pooling

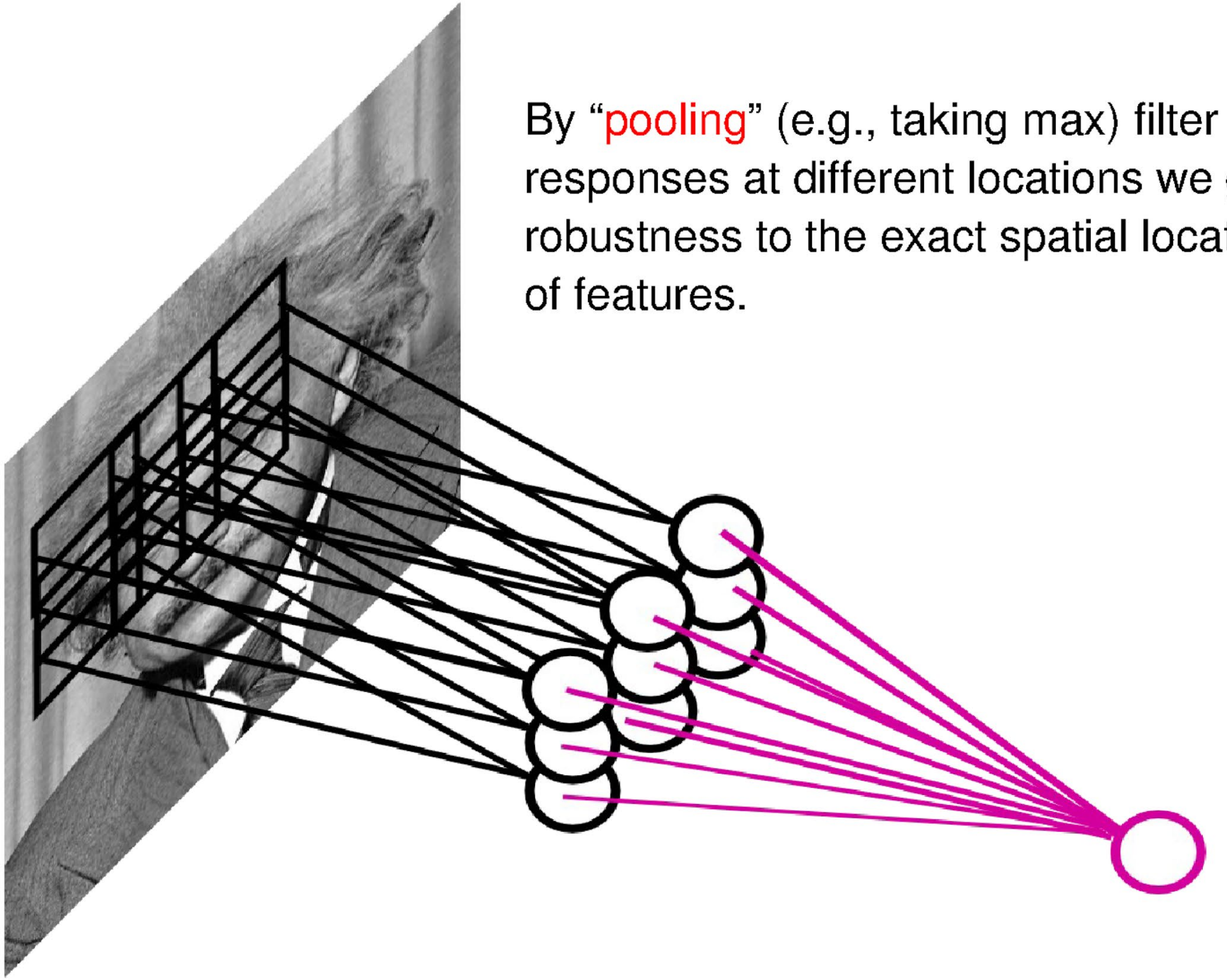


Let us assume filter is an “eye” detector.

Q.: how can we make the detection robust to the exact location of the eye?

Slides Credit: Deep Learning Tutorial by Marc’Aurelio Ranzato

Pooling

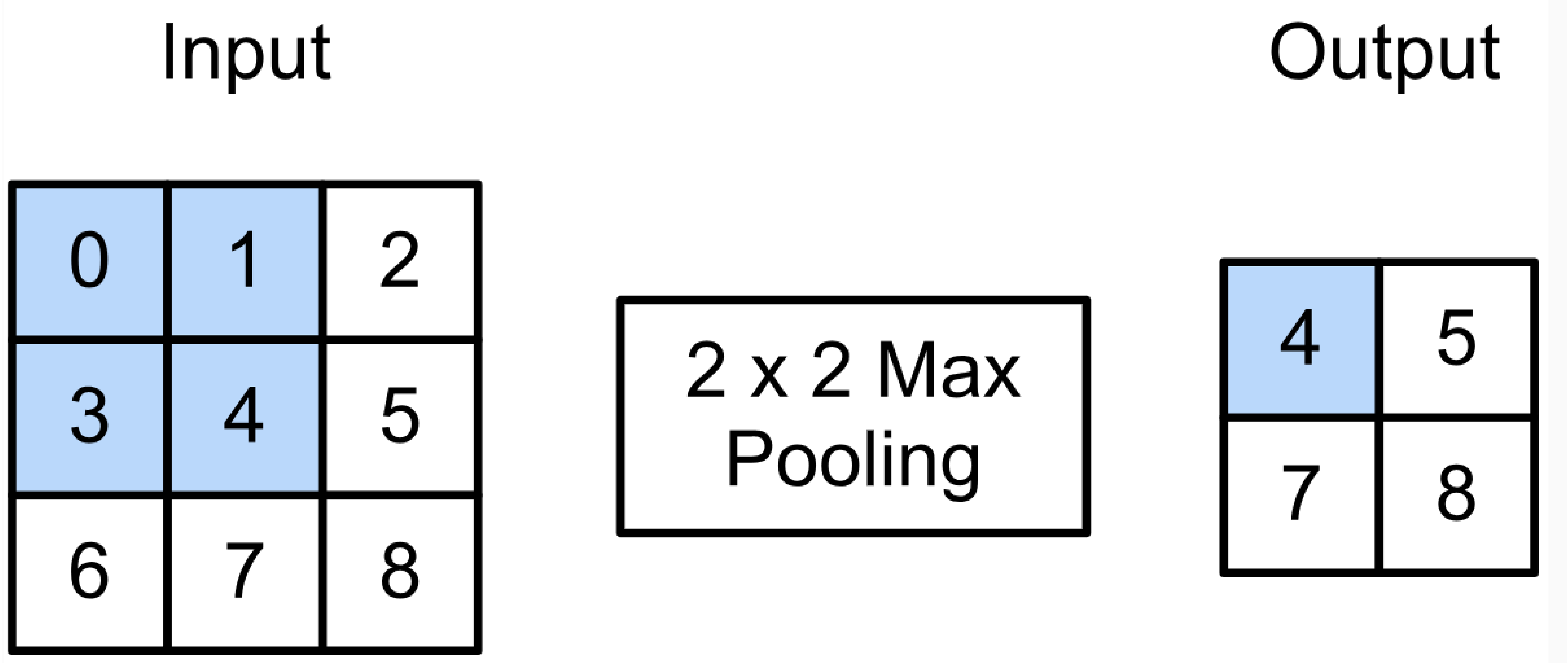


By "pooling" (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

Slides Credit: Deep Learning Tutorial by Marc'Aurelio Ranzato

2-D Max Pooling

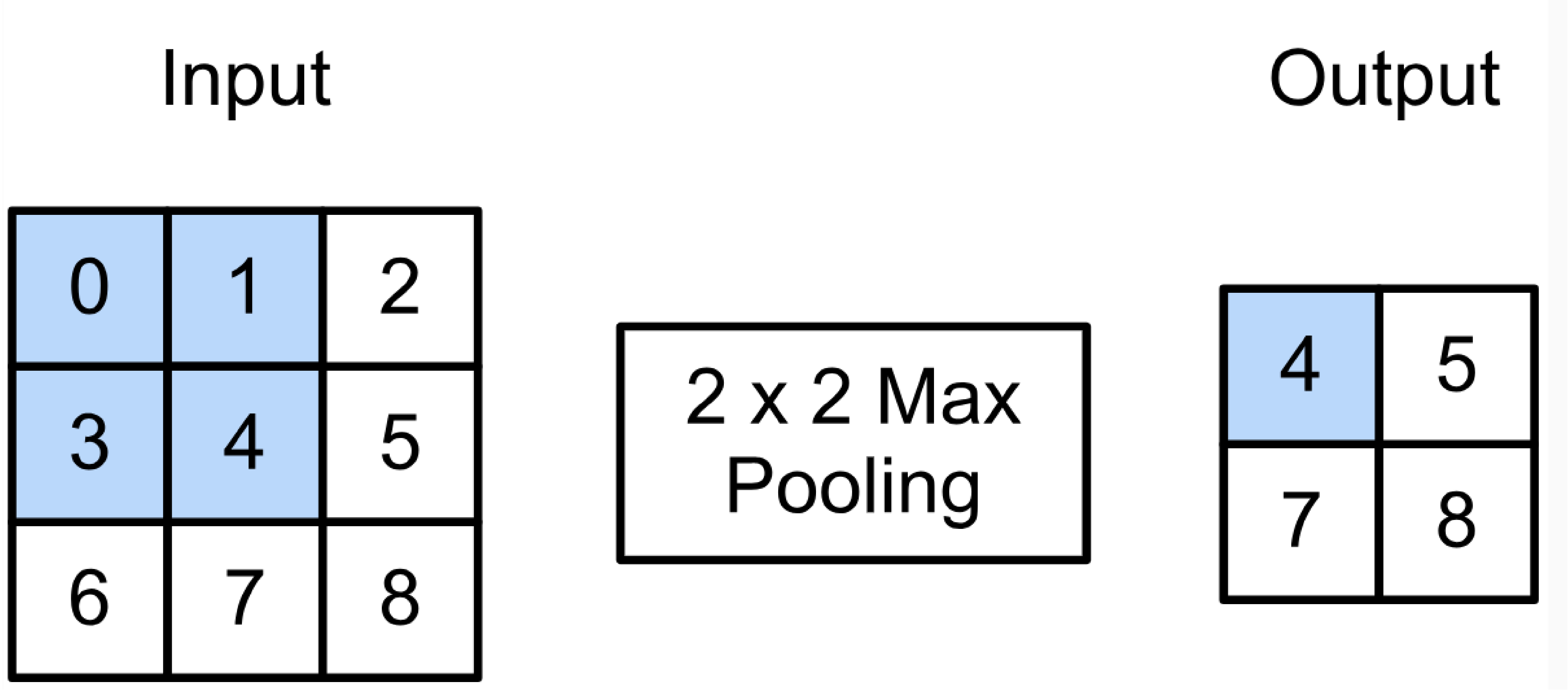
- Returns the maximal value in the sliding window



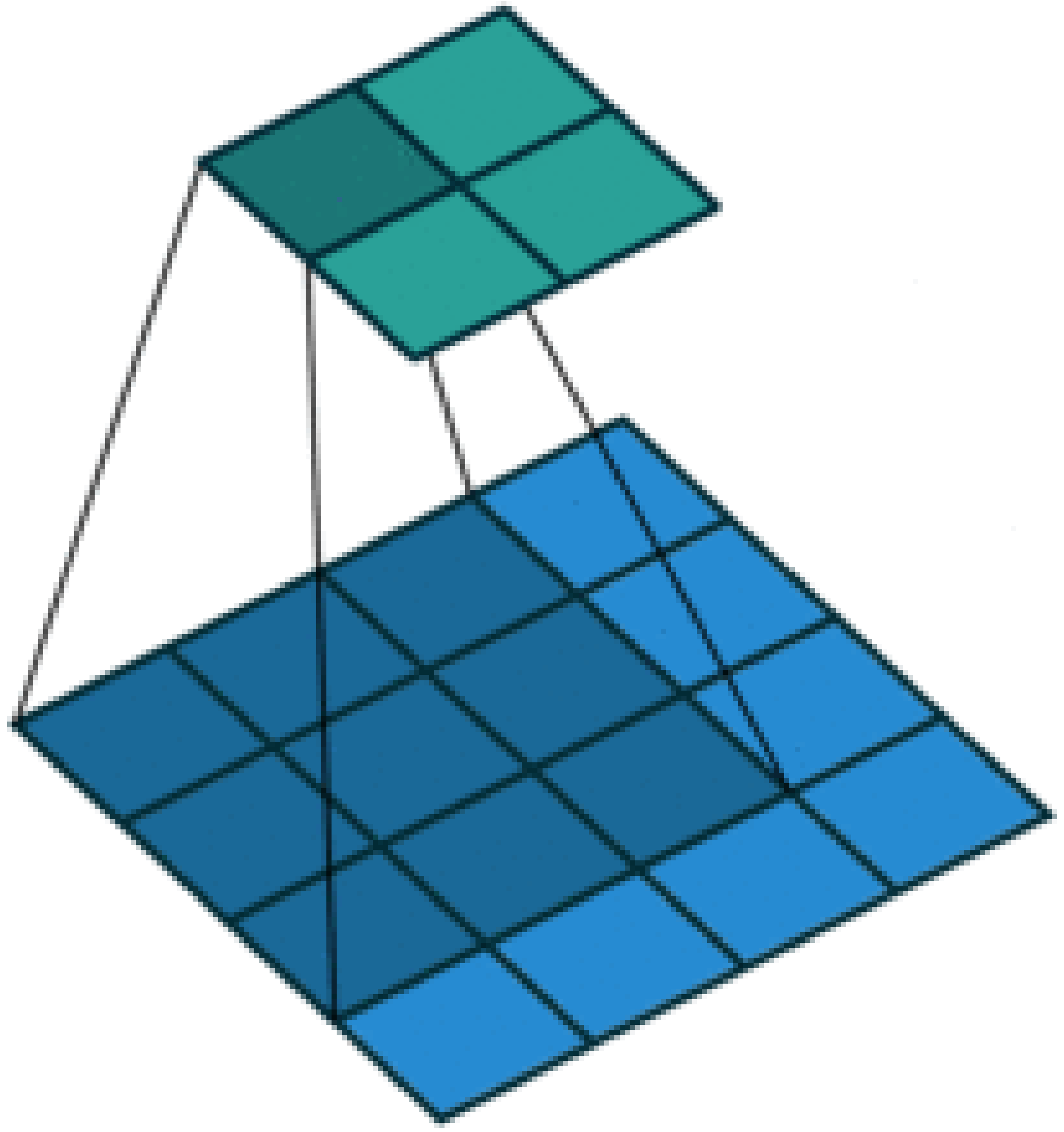
max(0,1,3,4) = 4

2-D Max Pooling

- Returns the maximal value in the sliding window



$$\max(0,1,3,4) = 4$$



Padding, Stride, and Multiple Channels

- Pooling layers have similar padding and stride as convolutional layers
- No learnable parameters
- Apply pooling for each input channel to obtain the corresponding output channel

  #output channels = #input channels

Padding, Stride, and Multiple Channels

- Pooling layers have similar padding and stride as convolutional layers
- No learnable parameters
- Apply pooling for each input channel to obtain the corresponding output channel

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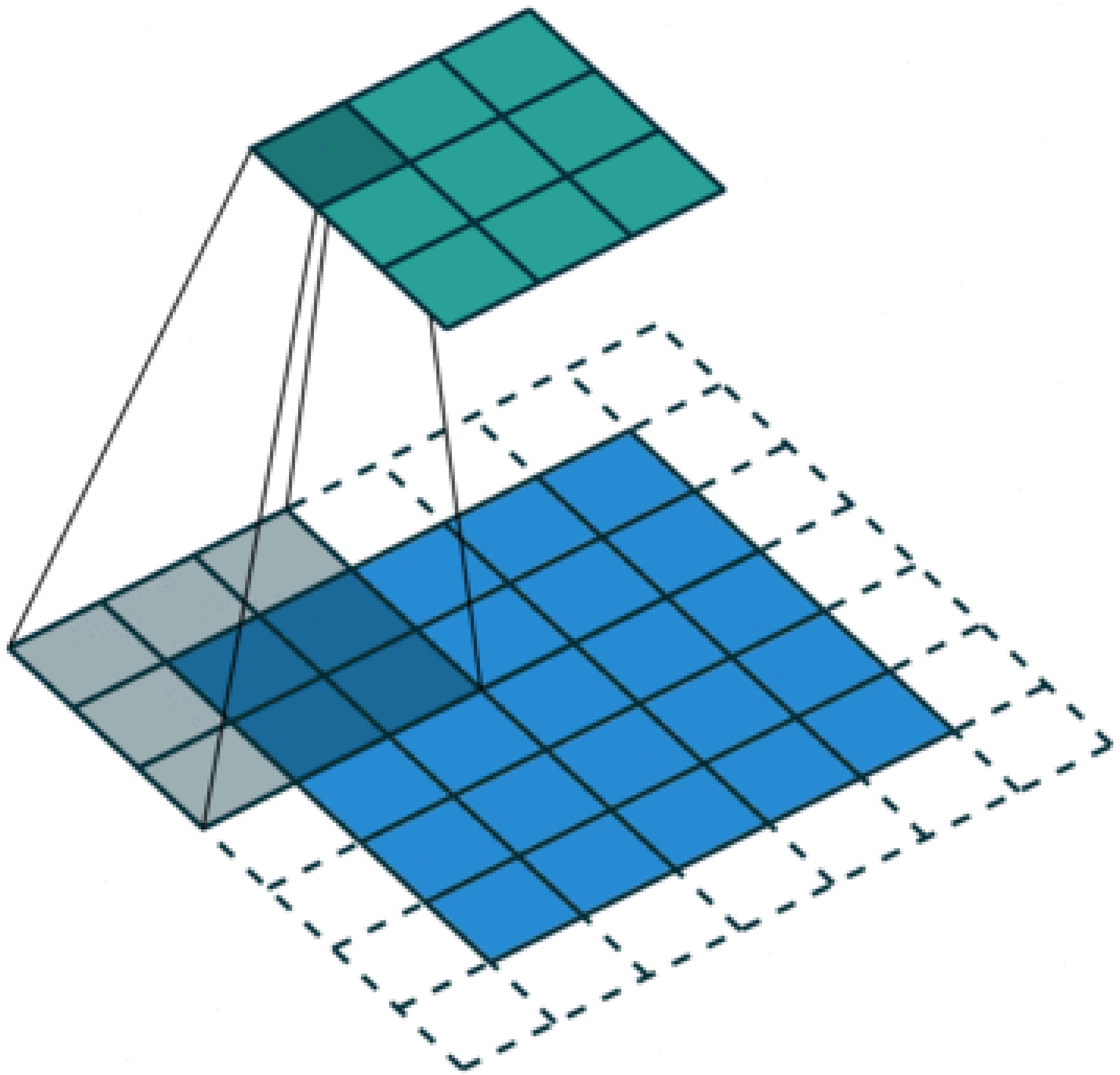
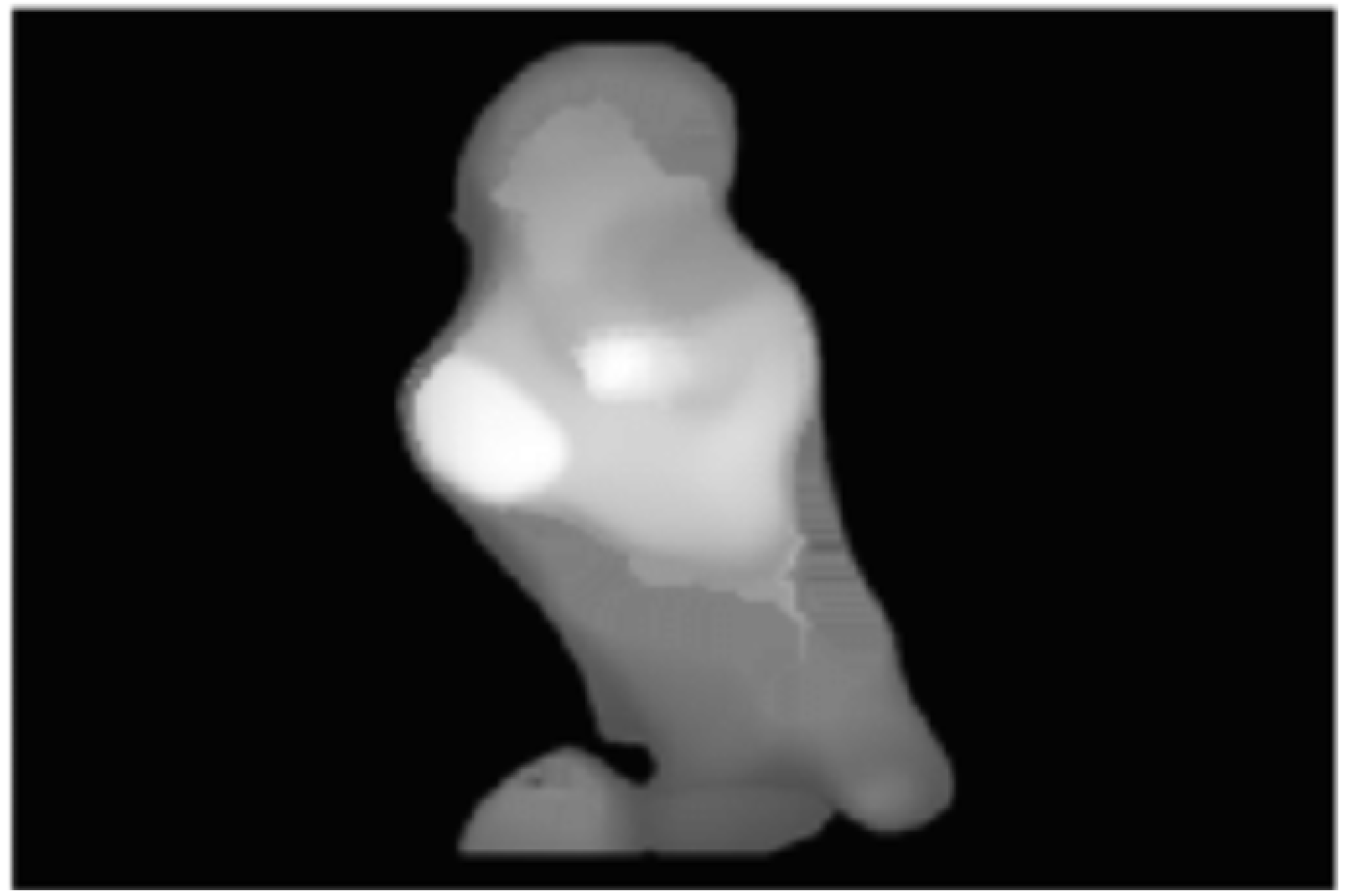


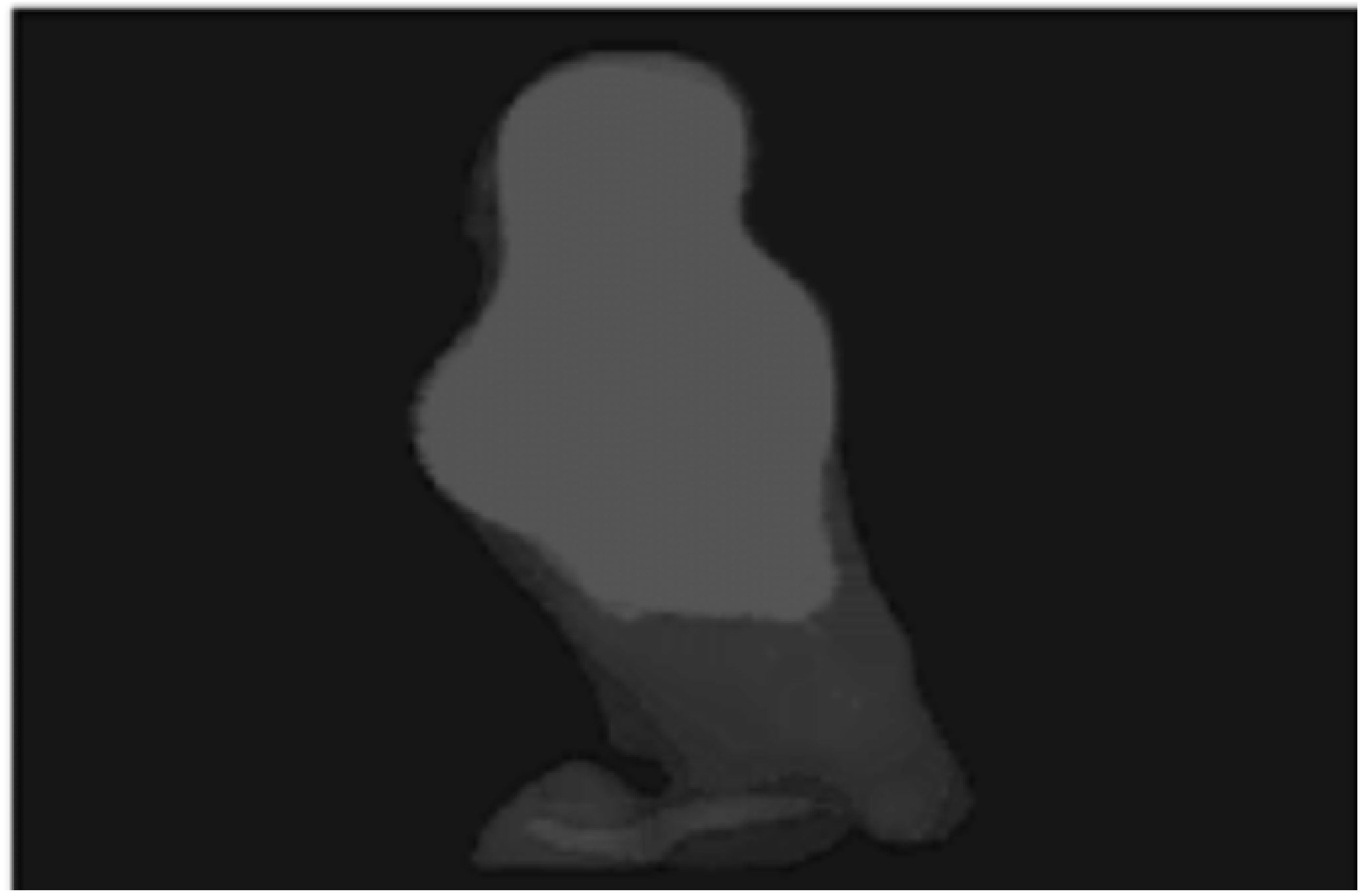
Average Pooling

- Max pooling: the strongest pattern signal in a window
- Average pooling: replace max with mean in max pooling
  - The average signal strength in a window

Max pooling



Average pooling



Q5. Suppose we want to perform 2x2 average pooling on the following single channel feature map of size 4x4 (no padding), and stride = 2. What is the output?

12	**20**	**30**	**0**
20	12	2	0
0	70	5	2
8	2	90	3

A.

20	**30**
70	90

B.

16	**8**
20	25

C.

20	**30**
20	25

D.

12	**2**
70	5

Q5. Suppose we want to perform 2x2 average pooling on the following single channel feature map of size 4x4 (no padding), and stride = 2. What is the output?

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A.

20	**30**
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B.

16	**8**
20	25

C.

20	**30**
20	25

D.

12	**2**
70	5

Q6. What is the output if we replace average pooling with 2 x 2 max pooling (other settings are the same)?

12	**20**	**30**	**0**
20	12	2	0
0	70	5	2
8	2	90	3

A.

20	**30**
70	90

B.

16	**8**
20	25

C.

20	**30**
20	25

D.

12	**2**
70	5

Q6. What is the output if we replace average pooling with 2 x 2 max pooling (other settings are the same)?

12	**20**	**30**	**0**
20	12	2	0
0	70	5	2
8	2	90	3

A.

20	**30**
70	90

B.

16	**8**
20	25

C.

20	**30**
20	25

D.

12	**2**
70	5

Summary

- Intro of convolutional computations
    - 2D convolution
    - Padding, stride
    - Multiple input and output channels
    - Pooling

Acknowledgement:

Some of the slides in these lectures have been adapted from materials developed by Alex Smola and Mu Li:

https://courses.d2l.ai/berkeley-stat-157/index.html