CS 540 Introduction to Artificial Intelligence
Reinforcement Learning I

University of Wisconsin-Madison
Spring 2024

Announcements

- **Homework**:
  - HW9 due on **Tuesday 23rd at 11 AM**
- Class roadmap:

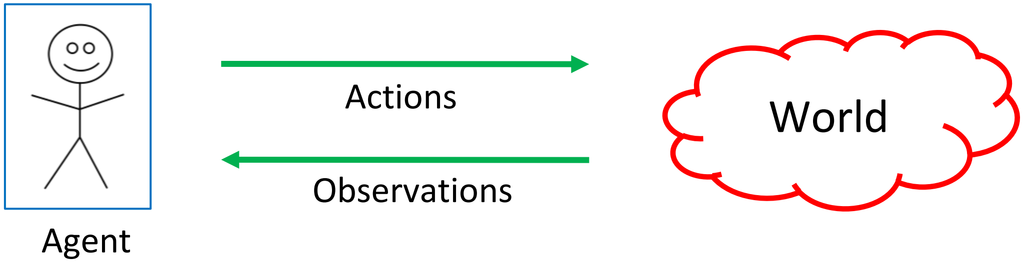
Thursday, Apr. 18th	Introduction to Reinforcement Learning
Tuesday, Apr. 23rd	Reinforcement Learning II
Thursday, Apr. 25th	Advanced Search

Outline

- Introduction to reinforcement learning
  - Basic concepts, mathematical formulation, MDPs, policies.
- Learning policies
  - Q-learning, action-values, exploration vs exploitation.

Back to Our General Model

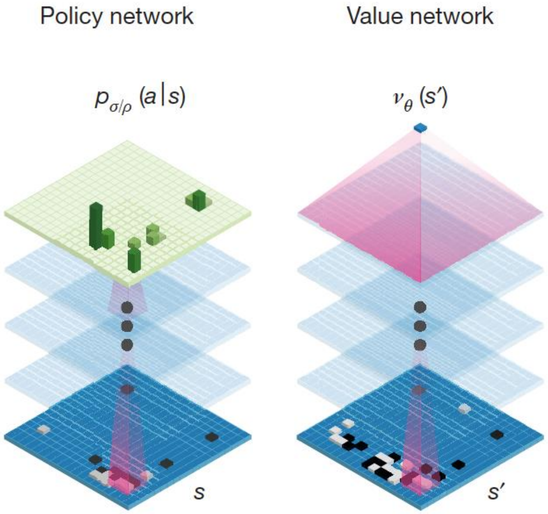
We have an **agent** **interacting** with the **world**



- Agent receives a reward based on state of the world
  - **Goal**: maximize reward / utility   **($$$)**
  - Note: **data** consists of actions & observations
    - Compare to unsupervised learning and supervised learning

Examples: Gameplay Agents

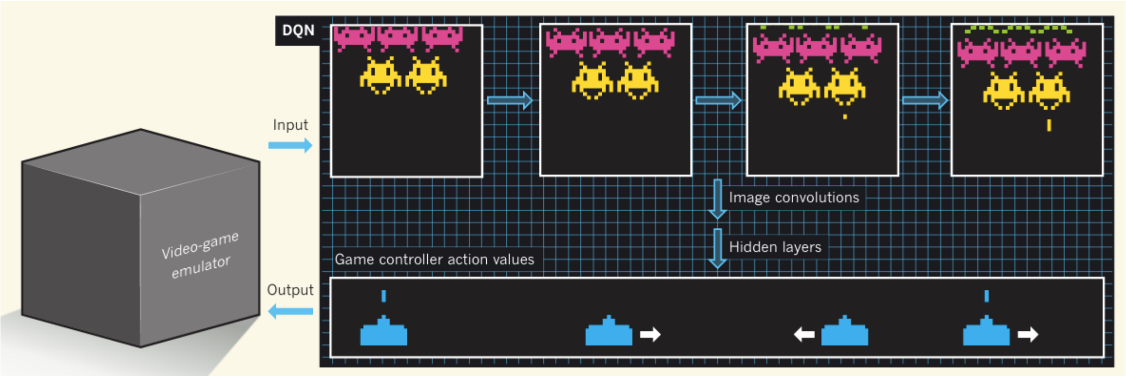
AlphaZero:



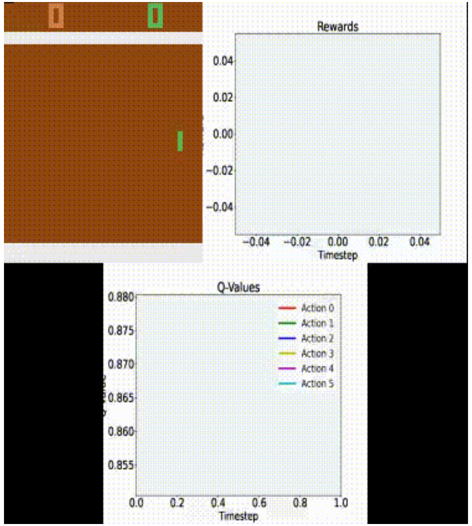
https://deepmind.com/research/alphago/

Examples: Video Game Agents

Pong, Atari



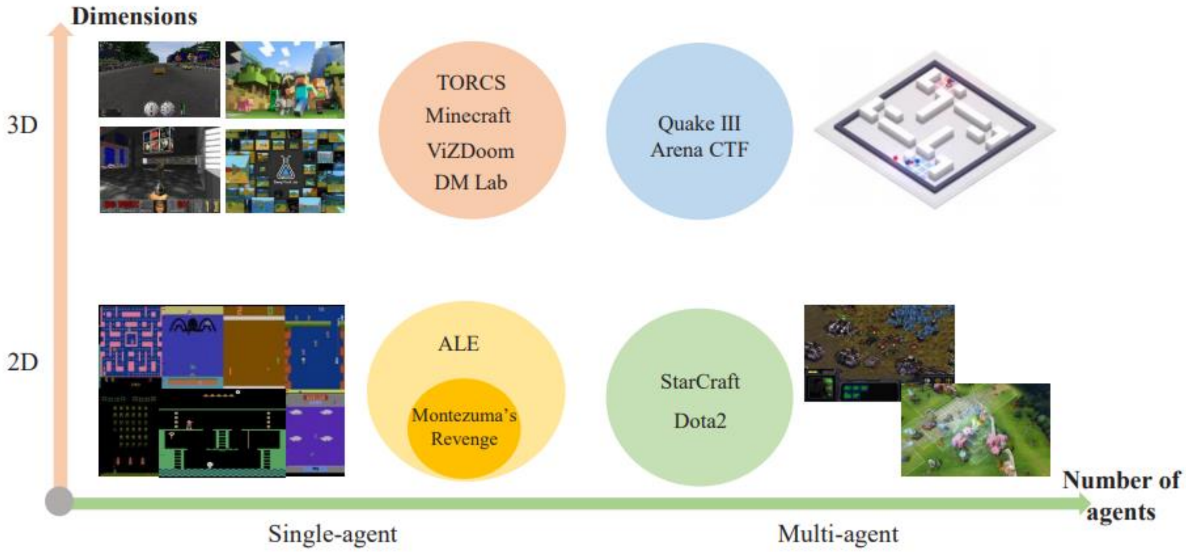
Mnih et al, “Human-level control through deep reinforcement learning”



A. Nielsen

Examples: Video Game Agents

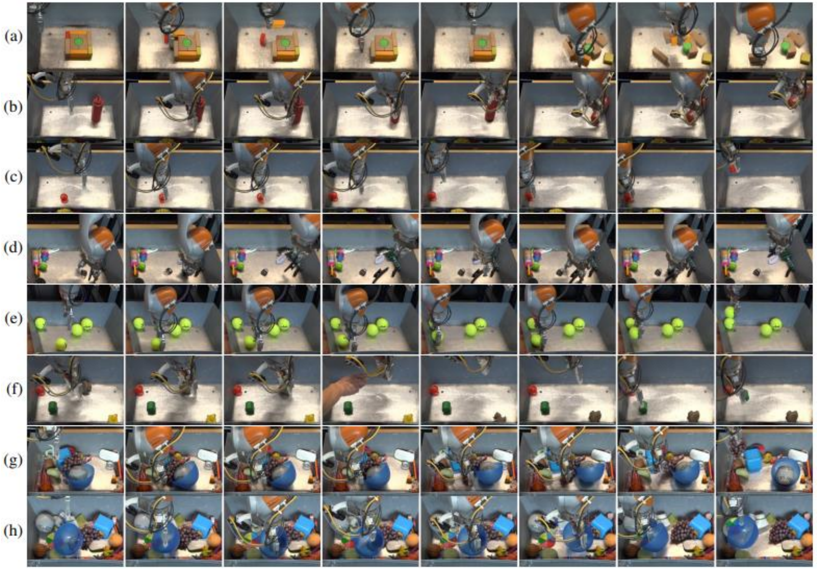
Minecraft, Quake, StarCraft, and more!



Shao et al, "A Survey of Deep Reinforcement Learning in Video Games"

Examples: Robotics

Training robots to perform tasks (e.g., grasp objects!)

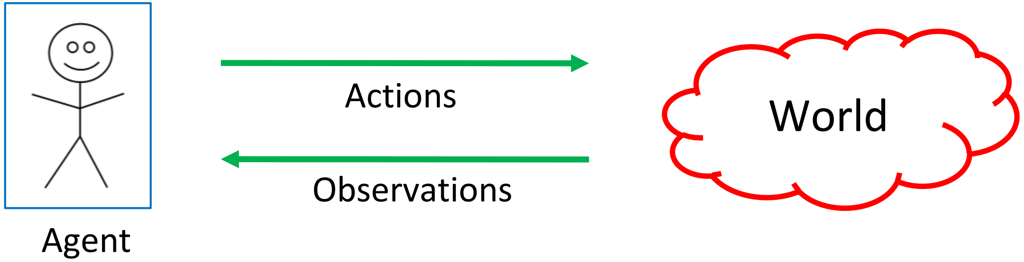


Ibarz et al, " How to Train Your Robot with Deep Reinforcement Learning – Lessons We’ve Learned "

Building The Theoretical Model

Basic setup:

- Set of states, S
- Set of actions, A



- Information: at time t, observe state $s_t \in S$. Get reward r_t
- Agent makes choice $a_t \in A$. State changes to s_{t+1}, continue

Goal: find a map from **states to actions** that maximize rewards.

A "policy"

Markov Decision Process (MDP)

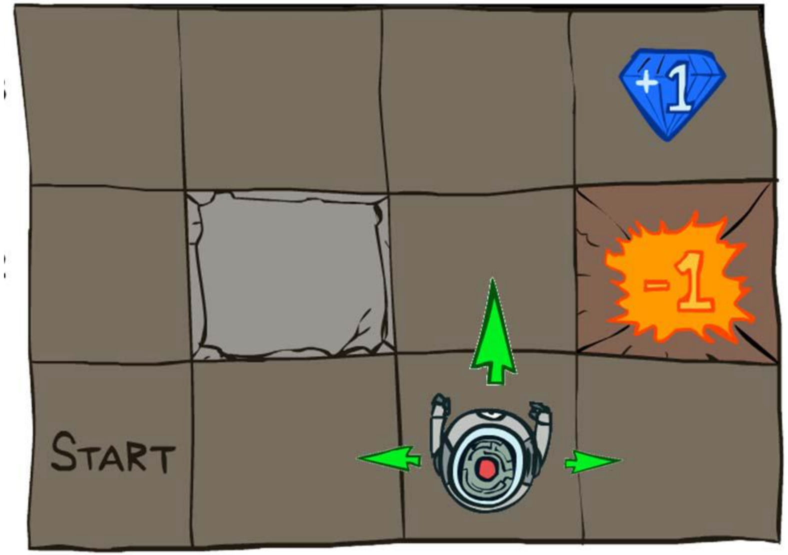
The formal mathematical model:

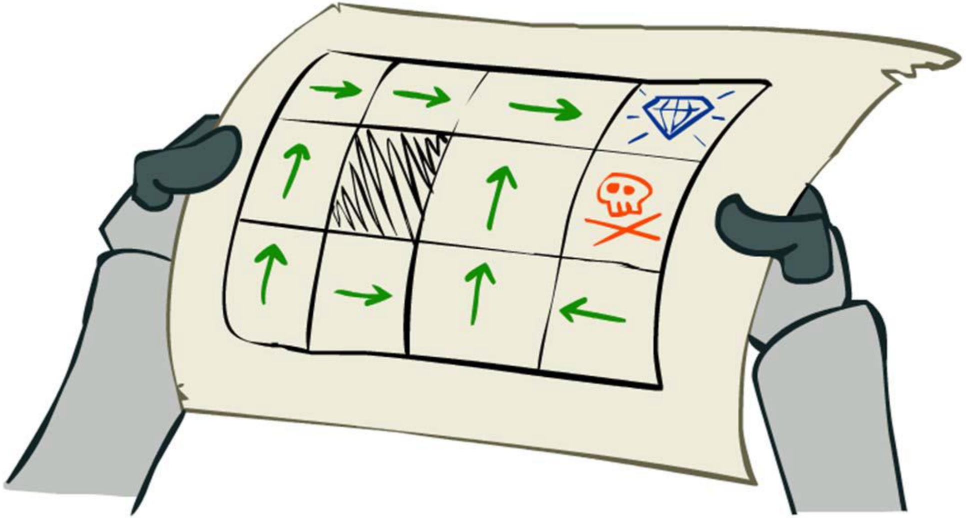
- **State set** S. Initial state $s_{0.}$ **Action set** A
- **Reward function:** $r(s_t)$
- **State transition model**: $P(s_{t+1}|s_t, a_t)$
  - Markov assumption: transition probability only depends on s_t and a_t, and not earlier history (previous actions or states)
- More generally: $r(s_t, a_t)$, potentially random
- **Policy**: $\pi(s) : S \rightarrow A$ action to take at a particular state

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots$$

Example of MDP: Grid World

Robot on a grid; goal: find the best policy

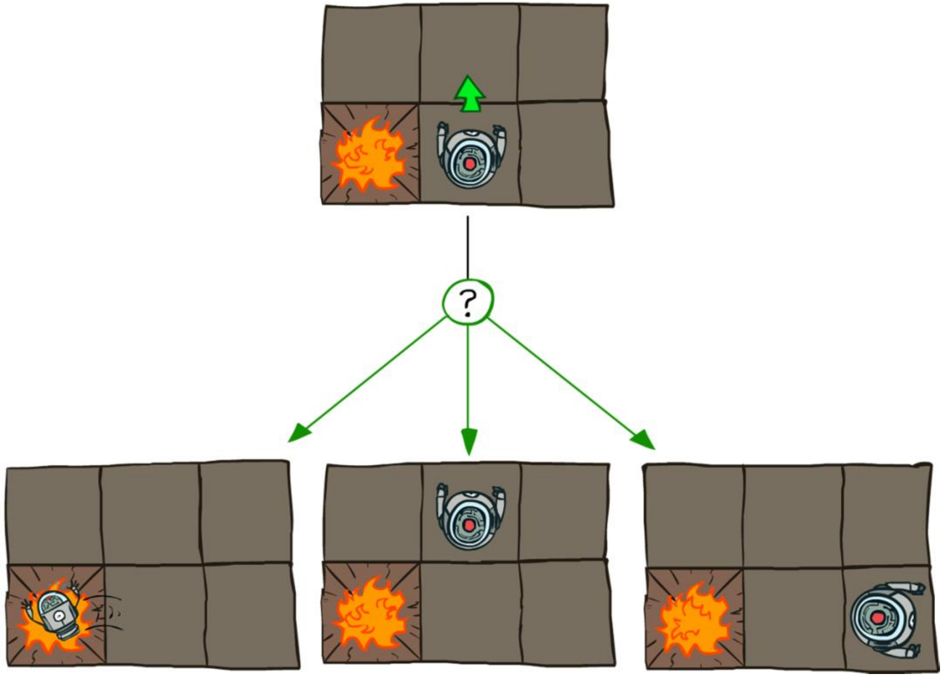


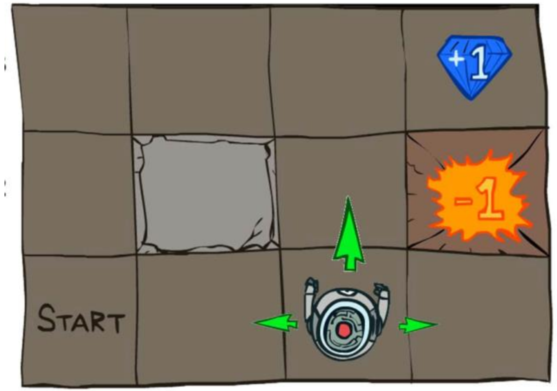


Source: P. Abbeel and D. Klein

Example of MDP: Grid World

Note: (i) Robot is unreliable (ii) Reach target fast

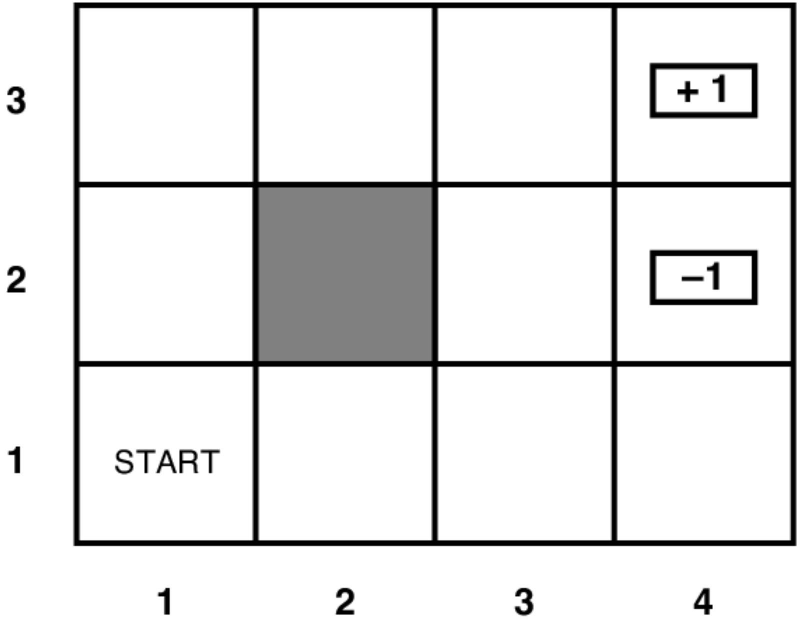


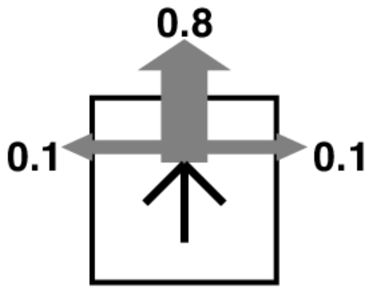


$\boldsymbol{r}(\boldsymbol{s}) = -0.04$ for every non-terminal state

Grid World Abstraction

Note: (i) Robot is unreliable (ii) Reach target fast

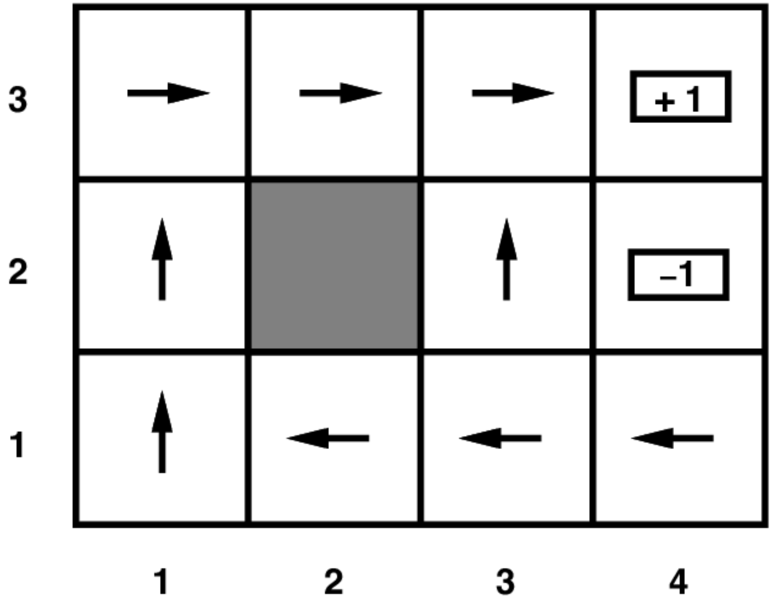


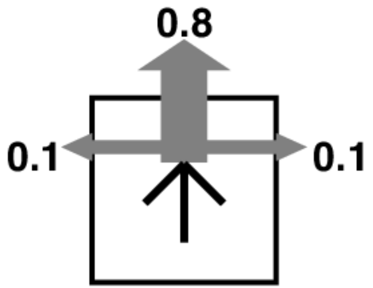


$r(s) = -0.04$ for every non-terminal state

Grid World Optimal Policy

Note: (i) Robot is unreliable (ii) Reach target fast





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Back to MDP Setup

The formal mathematical model:

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  - Markov assumption: transition probability only depends on s_t and a_t, and not previous actions or states.
- **Reward function:** $r(s_t)$
- **Policy:** $\pi(s) : S \rightarrow A$ action to take at a particular state.

How do we find the best policy?

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

Reinforcement Learning Challenges

Credit-assignment:

- May take many actions before reward is received. Which ones were most important?
- Example: You study 15 minutes a day all semester. The morning of the final exam, you eat a bowl of yogurt. You receive an A on the final. Was it the studying or the yogurt that led to the A?

Exploration vs. Exploitation:

- Transition probabilities and reward may be unknown to the learner.
- Should you keep trying actions that led to reward in the past or try new actions that might lead to even more reward?

Break & Quiz

Q 1.1 Which of the following statement about MDP is **not** true?

- A. The reward function must output a scalar value
- B. The policy maps states to actions
- C. The probability of next state can depend on current and previous states
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards

Break & Quiz

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- **C. The probability of next state can depend on current and previous states**
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards

Break & Quiz

Q 1.1 Which of the following statement about MDP is **not** true?

- A. The reward function must output a scalar value (**True: need to be able to compare**)
- B. The policy maps states to actions (**True: a policy tells you what action to take for each state**).
- **C. The probability of next state can depend on current and previous states** (**False: Markov assumption**).
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards (**True: want to maximize rewards overall**).

Defining the Optimal Policy

For policy π, **expected utility** over all possible state sequences from s_0 produced by following that policy:

Utility of sequence

$$V^{\pi}(s_0) = \sum_{\substack{\text{sequences} \\ \text{starting from } s_0}} P(\text{sequence})U(\text{sequence})$$

Probability of sequence when following π

Called the **value function** (for π, s_0)

Discounting Rewards

One issue: these are possibly infinite series. **Convergence**?

- Solution: discount future rewards.

$$U(s_0, s_1 \ldots) = r(s_0) + \gamma r(s_1) + \gamma^2 r(s_2) + \ldots = \sum_{t \geq 0} \gamma^t r(s_t)$$

- Discount factor γ between 0 and 1
  - Set according to how important **present** is versus **future**
  - Note: has to be less than 1 for convergence

From Value to Policy

Now that $V^{\pi}(s_0)$ is defined, what a should we take?

- First, let π^* be the **optimal** policy for $V^{\pi}(s_0)$, and $V^*(s_0)$ its expected utility.
- What's the expected utility following an action?
  - Specifically, action a in state s?

$$\sum_{s'} P(s'|s, a)V^*(s')$$

All the states we could go to

Transition probability

Expected rewards

Slight Problem...

Now we can get the optimal policy by doing

$$\pi^*(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

- So we need to know $V^*(s)$ (and P).
  - But it was defined in terms of the optimal policy!
  - So we need some other approach to get $V^*(s)$.
  - Instead, learn about the utility of actions directly.

Bellman Equation

Let's walk over one step for the value function:

$$V^*(s) = r(s) + \gamma \max_a \underbrace{\sum_{s'} P(s'|s,a)V^*(s')}_{\text{Discounted expected future rewards}}$$

Current state reward

Discounted expected future **rewards**

- Richard Bellman: inventor of dynamic programming

Bellman Equation

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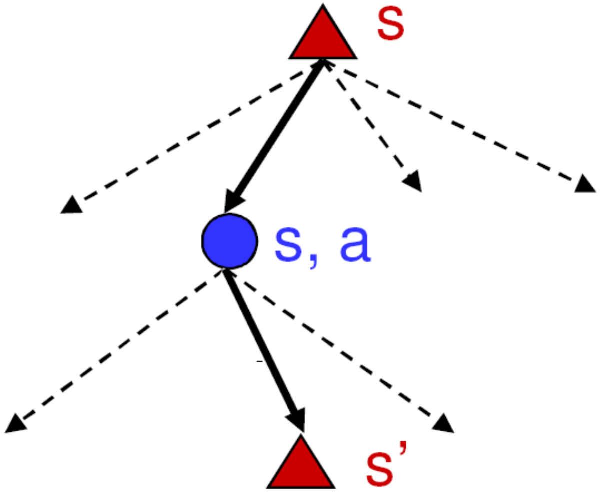


Credit L. Lazbenik

$$V^*(s) = r(s) + \gamma \max_a \underbrace{\sum_{s'} P(s'|s,a)V^*(s')}_{\text{Discounted expected future rewards}}$$

Current state reward

Discounted expected future **rewards**

Value Iteration

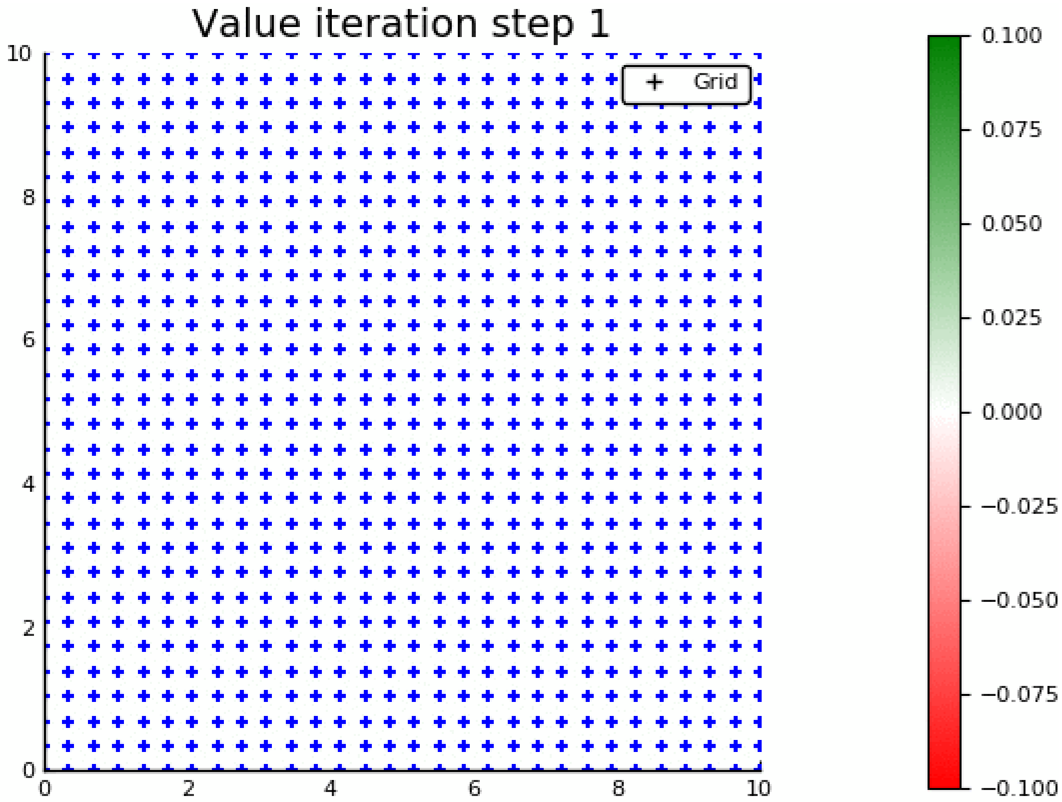
Q: how do we find $V^*(s)$?

- Why do we want it? Can use it to get the best policy
- Know: reward $r(s)$, transition probability $P(s'|s,a)$
  - Knowing r and P is the "planning" problem. In reality r and P must be estimated from interactions : "reinforcement learning"
- Also know $V^*(s)$ satisfies Bellman equation (recursion above)

A: Use the property. Start with $V_0(s)=0$. Then, update

$$V_{i+1}(s) = r(s) + \gamma \max_{a} \sum_{s'} P(s'|s,a) V_i(s')$$

Value Iteration: Demo



Source: POMDPBGallery Julia Package

Break & Quiz

Q 2.1 Consider an MDP with 2 states {*A, B*} and 2 actions: **"stay"** at current state and "**move**" to other state. Let **r** be the reward function such that **r**(*A*) = 1, **r**(*B*) = 0. Let γ be the discounting factor. Let π: π(*A*) = π(*B*) = **move** (i.e., an "always move" policy). What is the value function $V^{\pi}(A)$?

- A. 0
- B. 1 / (1 -γ)
- C. 1 / (1 -γ^2)
- D. 1

Break & Quiz

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- A. 0
- B. $1/(1-\gamma)$
- **C. $1/(1-\gamma^2)$**
- D. 1

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- A. 0
- B. 1/(1-γ)
- **C. 1/(1-γ^2)** **(**States: A,B,A,B,… rewards 1,0, γ^2,0, γ^4,0, …)
- D. 1

Q-Learning

- Our **next** reinforcement learning algorithm.
- Does not require knowing *r* or *P*. Learn from data of the form:$\{(s_t, a_t, r_t, s_{t+1})\}$.
- Learns an action-value function *Q**(*s*,*a*) that tells us the expected value of taking *a* in state *s*.
  - Note: $V^*(s) = \max_a Q^*(s, a)$.
- Optimal policy is formed as $\pi^*(s) = \mathrm{arg}\max_a Q^*(s, a)$

The Q*(s,a) function

- Starting from state s, perform (perhaps suboptimal) action a. THEN follow the optimal policy

$$Q^*(s,a) = r(s) + \gamma \sum_{s'} P(s'|s,a) V^*(s')$$

- Equivalent to

$$Q^*(s,a) = r(s) + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q^*(s',a')$$

Q-Learning Iteration

How do we get Q(*s*,*a*)?

- Iterative procedure

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r(s_t) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Learning rate

Idea: combine old value and new estimate of future value.

Note: We are using a policy to take actions; based on the estimated Q!

Q-Learning

Estimate Q*(s,a) from data $\{(s_t, a_t, r_t, s_{t+1})\}$:

1. Initialize Q(.,.) arbitrarily (eg all zeros)
    1. Except terminal states Q($s_{terminal}$,.)=0
2. Iterate over data until Q(.,.) converges:

$$Q(s_t, a_t) \leftarrow (1-\alpha)Q(s_t, a_t) + \alpha(r_t + \gamma \max_b Q(s_{t+1}, b))$$

Learning rate

Exploration Vs. Exploitation

General question!

- **Exploration:** take an action with unknown consequences
  - Pros:
    - Get a more accurate model of the environment
    - Discover higher-reward states than the ones found so far
  - Cons:
    - When exploring, not maximizing your utility
    - Something bad might happen
- **Exploitation:** go with the best strategy found so far
  - Pros:
    - Maximize reward as reflected in the current utility estimates
    - Avoid bad stuff
  - Cons:
    - Might prevent you from discovering the true optimal strategy

Q-Learning: ε-Greedy Behavior Policy

Getting data with both **exploration and exploitation**

- With probability ε, take a random action; else the action with the highest (current) Q(*s*,*a*) value.

$$a = \begin{cases} \text{argmax}_{a \in A} Q(s, a) & \text{uniform}(0,1) > \epsilon \\ \text{random } a \in A & \text{otherwise} \end{cases}$$

Q-learning Algorithm

Input: step size α, exploration probability ϵ

1. set $Q(s,a) = 0$ for all s, a.
2. For each episode:
3. Get initial state s.
4. While (s not a terminal state):
5. Perform a = ϵ-greedy(Q, s), receive r, s'
6. $Q(s,a) = (1-\alpha)Q(s,a) + \alpha(r + \gamma \max_{a'} Q(s',a'))$
7. $s \leftarrow s'$
8. End While
9. End For

Explore: take action to see what happens.

Update action-value based on result.

Break & Quiz

Q 2.1 For Q learning to converge to the true Q function, we must

- A. Visit every state and try every action
- B. Perform at least 20,000 iterations.
- C. Re-start with different random initial table values.
- D. Prioritize exploitation over exploration.

Break & Quiz

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Break & Quiz

Q 2.1 For Q learning to converge to the true Q function, we must

- **A. Visit every state and try every action**
- B. Perform at least 20,000 iterations. (No: this is dependent on the particular problem, not a general constant).
- C. Re-start with different random initial table values. (No: this is not necessary in general).
- D. Prioritize exploitation over exploration. (No: insufficient exploration means potentially unupdated state action pairs).

Summary

- Reinforcement learning setup
- Mathematical formulation: MDP
- Bellman Equation
- Value Iteration Algorithm
- The Q-learning Algorithm