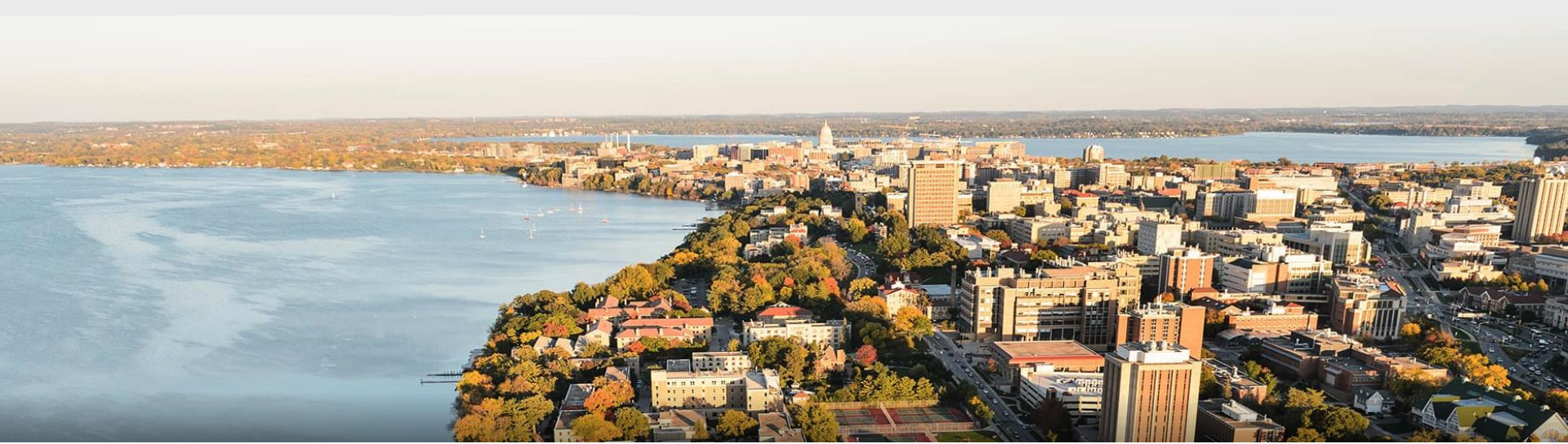


CS 540 Introduction to Artificial Intelligence

Search III: Optimization, continued

University of Wisconsin–Madison
December 3, 2025
Fall 2025

Announcements

- Homework:
  - HW10 due Dec 9 at 11:59 PM

- Final Exam
  - Saturday, Dec 13, 12:25-2:25 PM
  - CHEM S249

Advanced Search
Ethics and Trust in AI

HelioCampus Course Evaluations

- Please do them! Feedback used to design future versions of 540
- Final exam incentive
  - With >50% participation, instructors will release an “excluded topics” list for final exam
  - With >70%, more topics excluded
- Survey open until 12/10

Outline

- Review Optimization & Hill-climbing
  - More difficult problems, basics, local optima, variations
- Genetic Algorithms

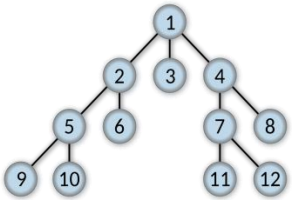
- Fill out course evaluations

REVIEW

Search vs. Optimization

Before: wanted a **path** from start state to goal state

- Uninformed search, informed search



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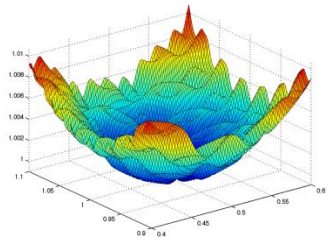
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New setting: optimization

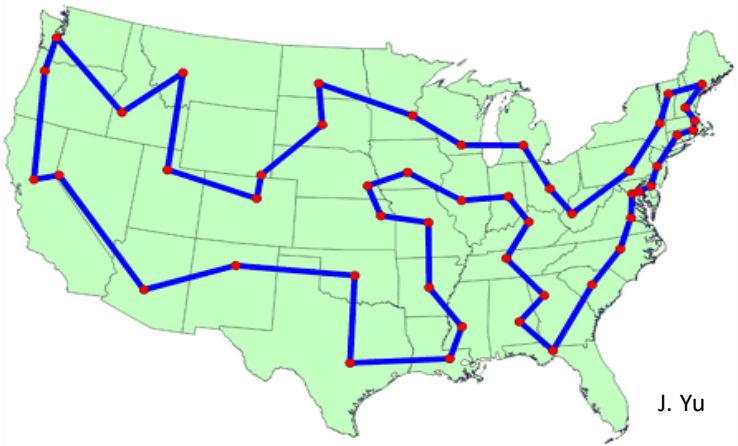
- States *s* have values ***f***(*s*)
- Want: Find *s* with optimal value ***f***(*s*) (i.e, **optimize** over states)
- Challenging settings: **too many states** for previous search approaches, but maybe not a differentiable function for gradient descent.

Examples: TSP

Famous graph theory problem.

- Get a graph G = (V,E). **Goal**: a path that visits each node exactly once and returns to the initial node (a **tour**).
  - State: a particular tour (i.e., ordered list of nodes)
  - $f(s)$: total weight of the tour

  (e.g., total miles traveled)



Examples: Satisfiability

Boolean satisfiability (e.g., 3-SAT)

- Recall our logic lecture. Conjunctive normal form

$$(A \vee \neg B \vee C) \wedge (\neg A \vee C \vee D) \wedge (B \vee D \vee \neg E) \wedge (\neg C \vee \neg D \vee \neg E) \wedge (\neg A \vee \neg C \vee E)$$

  - Goal: find if satisfactory assignment exists.
  - State: assignment to variables
  - ***f***(*s*): # satisfied clauses

$$R(x,a,d) \wedge R(y,b,d) \wedge R(a,b,e) \wedge R(c,d,f) \wedge R(z,c,0)$$

$$\begin{array}{l}
R(0,a,d) \wedge R(0,b,d) \wedge R(a,b,e) \wedge R(c,d,f) \wedge R(0,c,0) \\
R(0,a,d) \wedge R(0,b,d) \wedge R(a,b,e) \wedge R(c,d,f) \wedge R(1,c,0) \\
R(0,a,d) \wedge R(1,b,d) \wedge R(a,b,e) \wedge R(c,d,f) \wedge R(0,c,0) \\
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R(1,a,d) \wedge R(1,b,d) \wedge R(a,b,e) \wedge R(c,d,f) \wedge R(1,c,0)
\end{array}$$

$$R(\neg x,a,b) \wedge R(b,y,c) \wedge R(c,d,\neg z)$$

$$\begin{array}{l}
R(\ 1,a,b) \wedge R(b,0,c) \wedge R(c,d,\ 1) \\
R(\ 1,a,b) \wedge R(b,0,c) \wedge R(c,d,\ 0) \\
R(\ 1,a,b) \wedge R(b,1,c) \wedge R(c,d,\ 1) \\
R(\ 1,a,b) \wedge R(b,1,c) \wedge R(c,d,\ 0) \\
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R(\ 0,a,b) \wedge R(b,1,c) \wedge R(c,d,\ 0)
\end{array}$$

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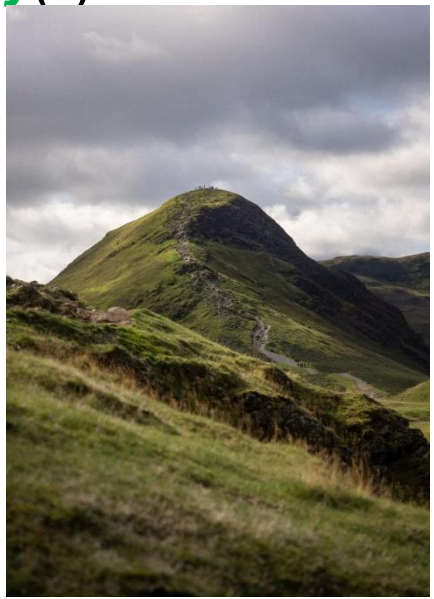
Hill Climbing

One approach to such optimization problems

- Basic idea: start at one state, move to a neighbor with a better $f(s)$ value, repeat until no neighbors have better $f(s)$ value.

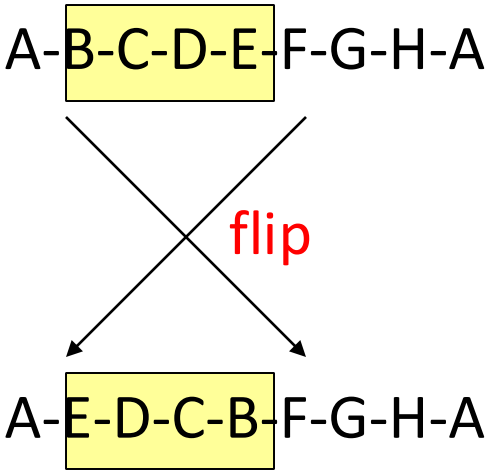
- **Q**: how do we define **neighbor**?
  - Not as obvious as our successors in search
  - Problem-specific
  - As we'll see, needs a careful choice

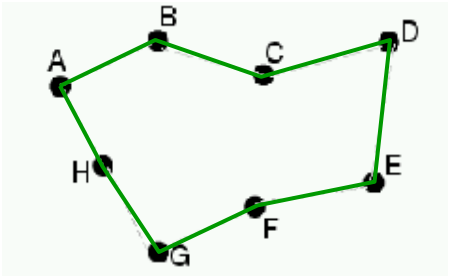


Defining Neighbors: TSP

Define neighbors by small changes

- Example: 2-change: A-E and B-F





Defining Neighbors: SAT

For Boolean satisfiability,

- Define neighbors by flipping one assignment of one variable

Starting state: (A=T, B=F, C=T, D=T, F=T)

(A=**F**, B=F, C=T, D=T, E=T)
(A=T, B=**T**, C=T, D=T, E=T)
(A=T, B=F, C=**F**, D=T, E=T)
(A=T, B=F, C=T, D=**F**, E=T)
(A=T, B=F, C=T, D=T, E=**F**)

$A \vee \neg B \vee C$
$\neg A \vee C \vee D$
$B \vee D \vee \neg E$
$\neg C \vee \neg D \vee \neg E$
$\neg A \vee \neg C \vee E$

Evolution & Choosing Neighbors

- Neighboring state ⇔ genetic mutation
- What happens if mutation rate is very high?
  - ie, neighbors very different from current state
- What happens if mutation rate is very low?
  - ie, neighbors very similar to current state

Hill Climbing Algorithm

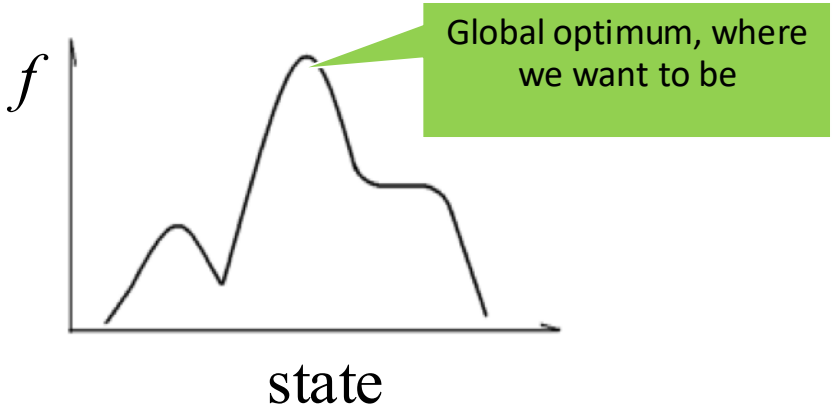
Pseudocode:

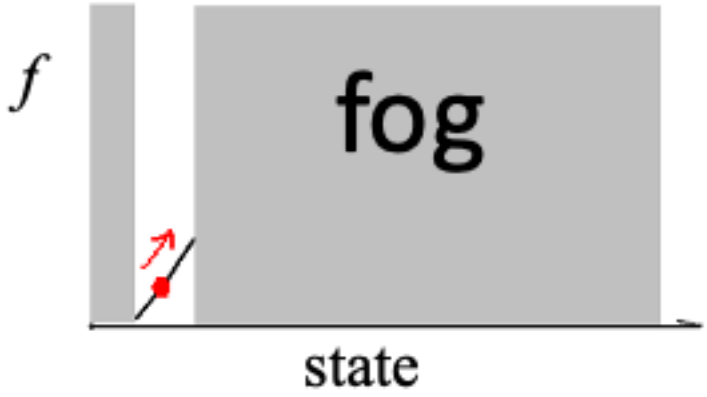
1. Pick initial state *s*
2. Pick *t* in **neighbors**(*s*) with the best ***f***(*t*)
3. if ***f***(*t*) is not better than ***f***(*s*) THEN stop, return *s*
4. *s* ← *t*. goto 2.

What could happen? **Local optima!**

Hill Climbing: Local Optima

Q: Why is it called hill climbing?





L: What's actually going on.

R: What we get to see.

Escaping Local Optima

Simple idea 1: random restarts

- Stuck: pick a random new starting point, re-run.
- Do *k* times, return best of the *k* runs.

Simple idea 2: reduce greed

- "Stochastic" hill climbing: randomly select between neighbors.
- Probability of selecting a neighbor should be proportional to the value of that neighbor.

Simulated Annealing

- Algorithm slowly increases greed over time

- Pseudocode

Pick initial state s; T=1
For k = 0 through K:
    $T \leftarrow T*0.99$ *(cool down)*
    Pick a random neighbor $t \leftarrow$ neighbor(s)
    If $f(t)$ better than $f(s)$, then $s \leftarrow t$
    Else with prob. $P(f(s), f(t), \boldsymbol{T})$ still do $s \leftarrow t$
Output: the best state ever seen

The interesting bit

Simulated Annealing: Picking Probability

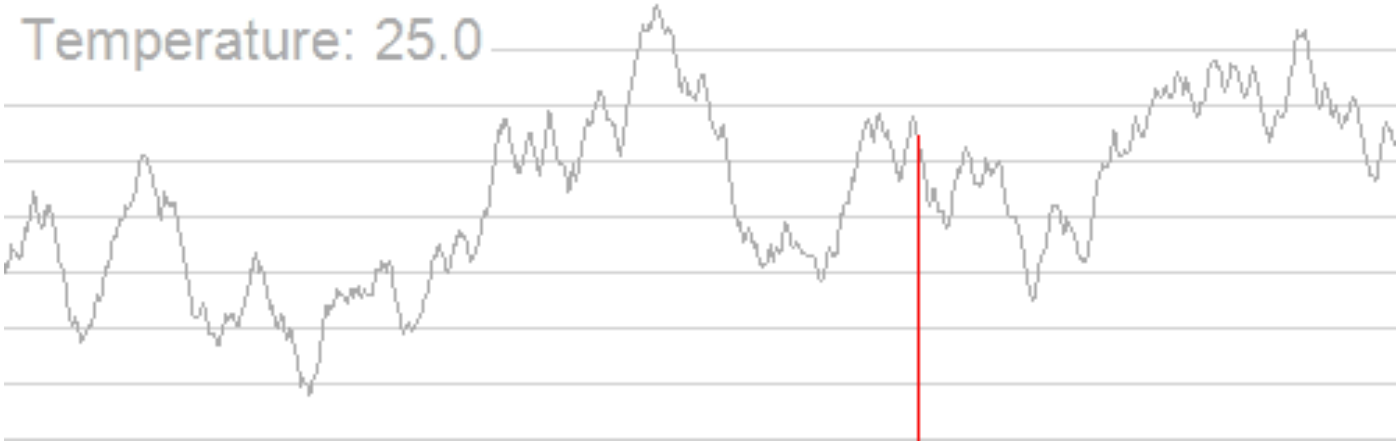
How do we pick probability P? Note 3 parameters.

- Decrease with time
- Decrease with gap $|f(s) - f(t)|$: $\exp\left(-\frac{|f(s)-f(t)|}{Temp}\right)$

- Temperature cools over time.
  - So: high temperature, accept any *t*
  - But, low temperature, behaves like hill-climbing
  - Still, $|f(s) - f(t)|$ plays a role: if big, replacement probability low.

Simulated Annealing: Visualization

What does it look like in practice?



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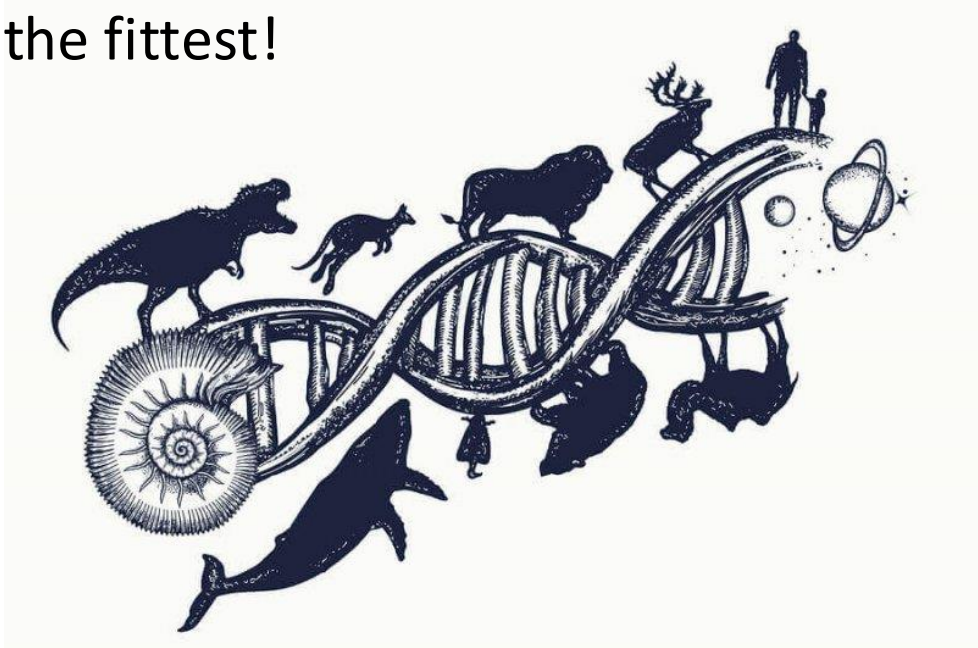
GENETIC ALGORITHMS

Genetic Algorithms

Optimization approach based on nature

- Survival of the fittest!

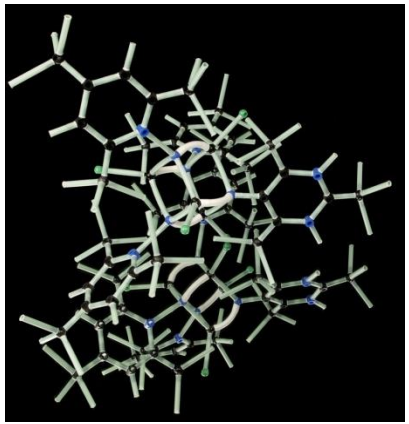


Evolution Review

Encode genetic information in DNA (four bases)

- A/C/T/G: nucleobases acting as symbols

- Two types of changes
  - Crossover: exchange between parents' codes
  - Mutation: rarer random process
    - Happens at individual level



Natural Selection

Competition for resources

- Organisms with better fitness ➔ better probability of reproducing
- Repeated process: fit become larger proportion of population

Goal: use these principles for optimization

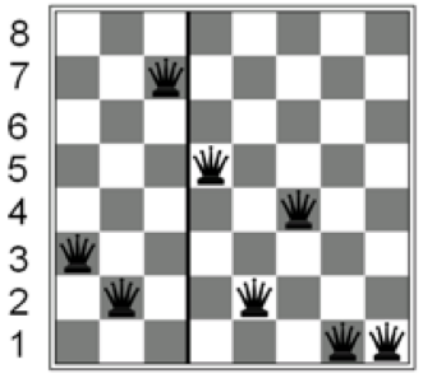
- New terminology: state is '**individual**'
- Value $f(s)$ is now the '**fitness**'

Genetic Algorithms Setup I

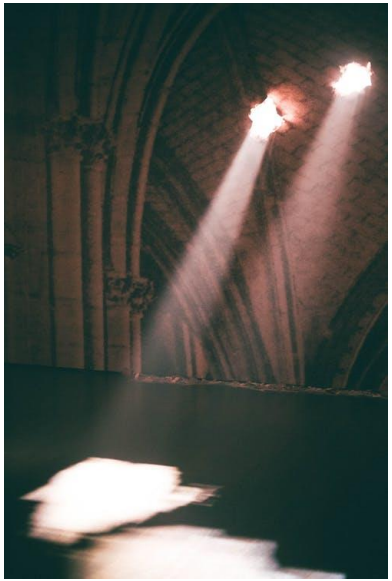
Keep around a fixed number of states/individuals

- Call this the **population**

For our n Queens game example, an individual:



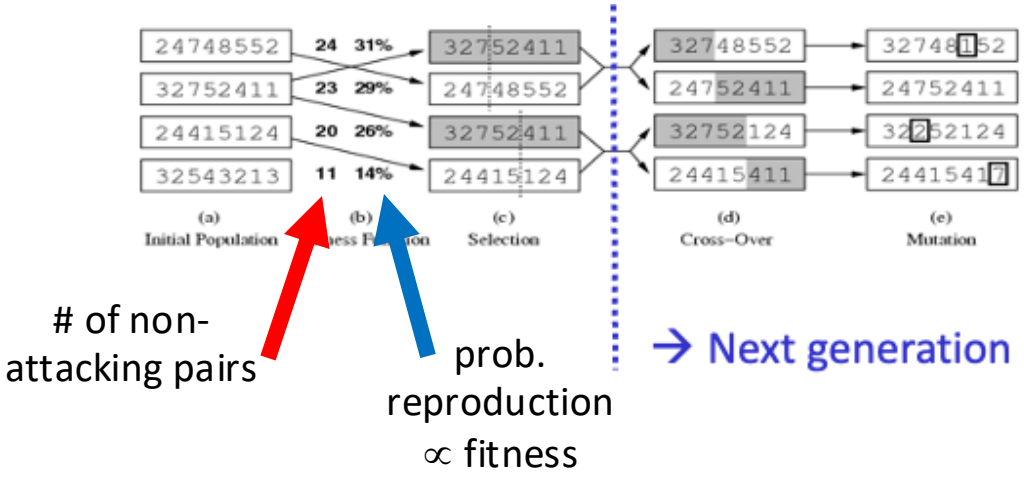
(3 2 7 5 2 4 1 1)



Genetic Algorithms Setup II

Goal of genetic algorithms: optimize using principles inspired by mechanism for evolution

- Analogous to **natural selection, cross-over**, and **mutation**



Genetic Algorithms Pseudocode

Just one variant:

1. Let $s_1, \ldots, s_N$ be the current population
2. Let $p_i = \boldsymbol{f}(s_i) / \Sigma_j \boldsymbol{f}(s_j)$ be the reproduction probability
3. for $k = 1$; $k<N$; k+=2
   - parent1 = randomly pick according to p
   - parent2 = randomly pick another
   - randomly select a crossover point, swap strings of parents 1, 2 to generate children $t[k]$, $t[k+1]$
4. for $k = 1$; $k<=N$; k++
   - Randomly mutate each position in $t[k]$ with a small probability (mutation rate)
5. The new generation replaces the old: $\{ s \} \leftarrow \{ t \}$. Repeat

Reproduction: Proportional Selection

Reproduction probability: $p_i = f(s_i) / \Sigma_j f(s_j)$

- **Example**: $\Sigma_j f(s_j) = 5+20+11+8+6=50$
- $p_1=5/50=10\%$

Individual	Fitness	Prob.
A	5	10%
B	20	40%
C	11	22%
D	8	16%
E	6	12%

Example: Scheduling Courses

Let's run through an example:

- **5 courses: A,B,C,D,E**
- *3 time slots:* Mon/Wed, Tue/Thu, Fri/Sat
- Students wish to enroll in three courses
- Goal: maximize student enrollment

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Let's run through an example:

- State: course assignment to time slot

M	M	F	T	M
A	B	C	D	E

= MMFTM

- Here:
  - Courses A, B, E scheduled Mon/Wed
  - Course D scheduled Tue/Thu
  - Course C scheduled Fri/Sat

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Value of a state? Say `MMFTM`

Courses	Students	Can enroll?
A B C	2	No
A B D	7	No
A D E	3	No
B C D	4	Yes
B D E	10	No
C D E	5	Yes

- Here 4+5=9 students can enroll in desired courses

Example: Scheduling Courses

First step:

- Randomly initialize and evaluate states

`MMFTM` = 9

`TTFMM` = 4

`FMTTF` = 19

`MTTTF` = 3

`MMFTM` = 26%

`TTFMM` = 11%

`FMTTF` = 54%

`MTTTF` = 9%

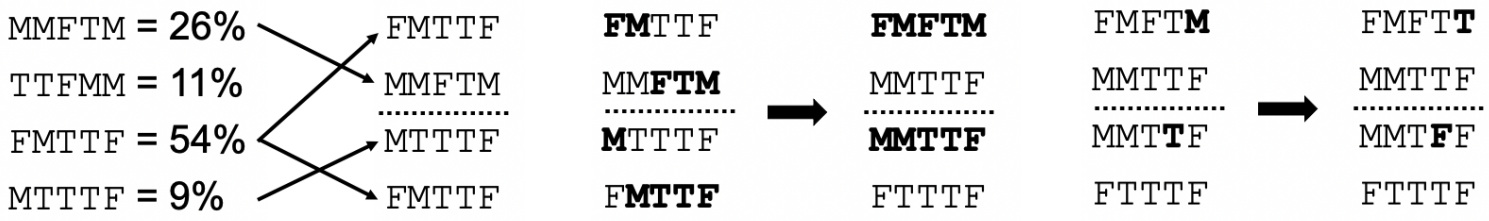
- Calculate reproduction probabilities

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Next steps:

- Select parents using reproduction probabilities
- Perform crossover
- Randomly mutate new children



Example: Scheduling Courses

Continue:

- Now, get our function values for updated population
- Calculate reproduction probabilities

FMFTT = 11	FMFTT = 39%
MMTTF = 13	MMTTF = 46%
MMTFF = 4	MMTFF = 14%
FTTTF = 0	FTTTF = 0%

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Variations & Concerns

Many **possibilities**:

- Parents survive to next generation
- Use ranking instead of exact value of $f(s)$ for reproduction probabilities (reduce influence of extreme f values)

Some **challenges**

- Formulating a good state encoding
- Lack of diversity: converge too soon
- Must pick a lot of parameters

Summary

- Challenging optimization problems
  - First, try hill climbing. Simplest solution
- Simulated annealing
  - More sophisticated approach; helps with local optima
- Genetic algorithms
  - Biology-inspired optimization routine

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