Propositions and Propositional Logic

Propositions

Goal: reason about discrete structures in a rigorous manner

Proof: chain of logical deductions starting from some axioms & ending in a proposition

Axiom: basic fact that we take for granted; underlying assumptions

Proposition:

Which of the following statements are propositions?

1. Mark Hill is the chair of the CS Department.
2. Today is not Thursday.
3. Serena Williams is the greatest women's tennis player ever.
4. It is snowing.
5. This sentence is false.

Operations on propositions

negation: ¬

conjunction: ∧

disjunction: ∨

implication: ⇒

equivalence (or biconditional): ⇔

Precedence

Propositional formulas

Properties of operators

Some of the most useful:

- associative
- commutative
- distributive
- DeMorgan's Laws
- conditional identities

Examples

1. $(\neg A) \vee B$
2. $(A \Rightarrow B) \wedge (B \Rightarrow A)$
3. $(A \wedge (A \Rightarrow B)) \Rightarrow B$
4. $(A \vee B) \wedge (A \vee \neg B) \wedge \neg A$

Logical equivalence

Show $A \Rightarrow B$ is logically equivalent to $\neg B \Rightarrow \neg A$

Show $A \Rightarrow B$ is ***not*** logically equivalent to $B \Rightarrow A$

Show $(A \vee B) \Rightarrow C$ is logically equivalent to $(A \Rightarrow C) \wedge (B \Rightarrow C)$