CS 536 Announcements for Thursday, January 30, 2025

Course websites:

`pages.cs.wisc.edu/~hasti/cs536/epic`
`www.piazza.com/wisc/spring2025/compsci536`

Programming Assignment 1

- test code due Sunday, Feb. 2 by 11:59 pm
- other files due Thursday, Feb. 6 by 11:59 pm

Last Time

- intro to CS 536
- compiler overview
- start scanning
- finite state machines
  - formalizing finite state machines
  - coding finite state machines
  - deterministic vs non-deterministic FSMs

Today

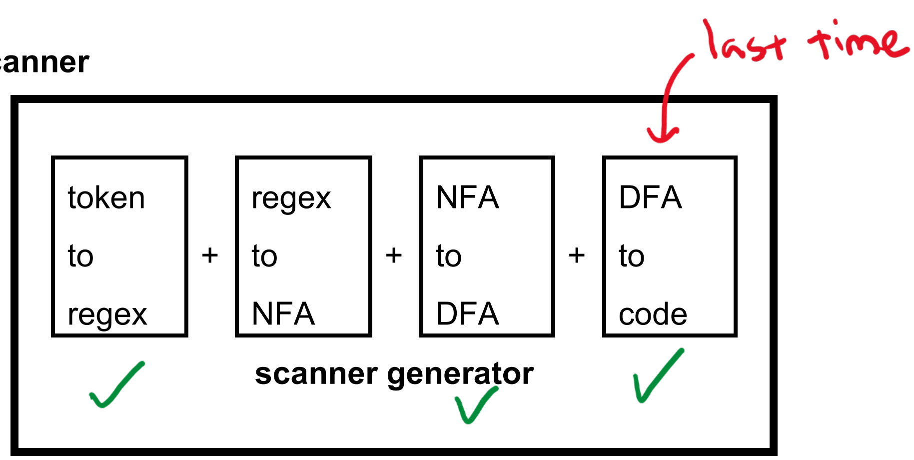
- non-deterministic FSMs
- equivalence of NFAs and DFAs
- regular languages
- regular expressions
- regular expressions → DFAs
- language recognition → tokenizers
- scanner generators
- JLex

Recall

- scanner : converts a sequence of characters to a sequence of tokens
- scanner implemented using FSMs
- FSMs can be DFA or NFA

Creating a scanner

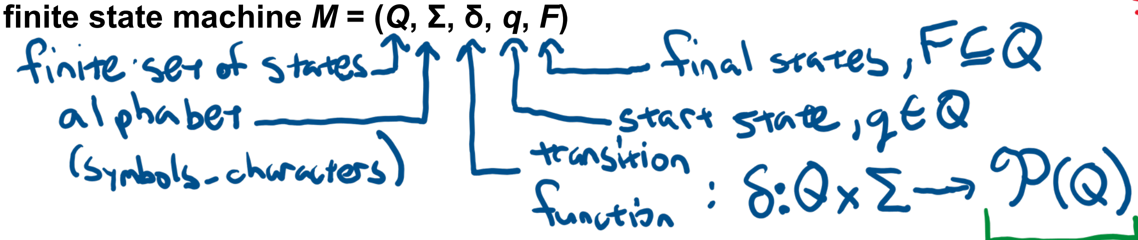
scanner =



𝒫(Q) = power set of Q = set of all subsets of Q

NFAs, formally

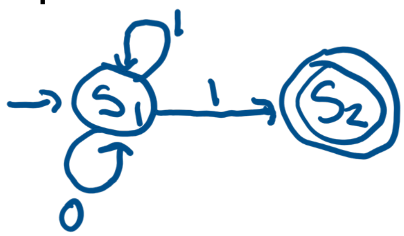
finite state machine *M* = (*Q*, Σ, δ, *q*, *F*)



↳ Q if DFA

L(*M*) = the language of FSM *M* = set of all strings *M* accepts

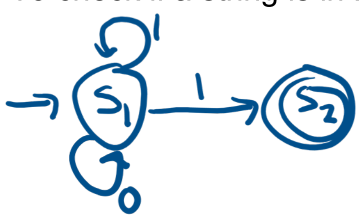
Example:

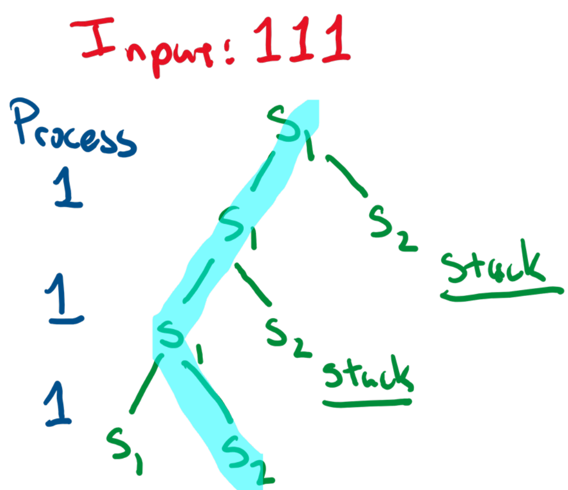


	0	1
s_1	$\{s_1\}$	$\{s_1, s_2\}$
s_2	$\{\}$	$\{\}$

"Running" an NFA

To check if a string is in L(*M*) of NFA *M*, simulate set of choices it could make.





The string is in L(*M*) iff there is at least one sequence of transitions that

- consumes all input (without getting stuck) and
- ends in one of the final states

NFA and DFA are equivalent

Two automata M and M^* are equivalent iff $L(M) = L(M^*)$

Lemmas to be proven:

Lemma 1: Given a DFA M, one can construct an NFA M^* that recognizes the same language as M, i.e., $L(M^*) = L(M)$

Lemma 2: Given an NFA M, one can construct a DFA M^* that recognizes the same language as M, i.e., $L(M^*) = L(M)$

Proving Lemma 2

Lemma 2: Given an NFA M, one can construct a DFA M^* that recognizes the same language as M, i.e., $L(M^*) = L(M)$

Part 1: Given an NFA M without ε-transitions, one can construct a DFA M^* that recognizes the same language as M

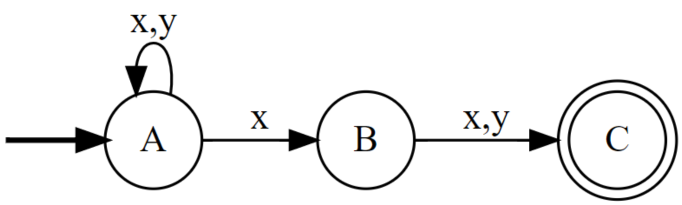
Part 2: Given an NFA M with ε-transitions, one can construct a NFA M^* without ε-transitions that recognizes the same language as M

NFA without ε-transitions to DFA

Observation: we can only be in finitely many subsets of states at any one time

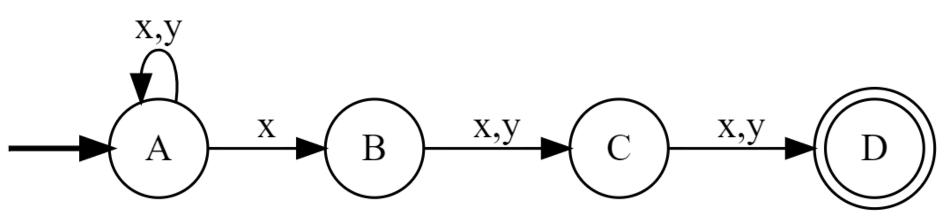
Idea: to do NFA *M* → DFA *M**, use a single state in *M** to simulate sets of states in *M*

Suppose *M* has |Q| states. Then *M** can have only up to $2^{|Q|}$ states.
Why?



A	B	C	
0	0	0	= { }
0	0	1	= {C}
0	1	0	= {B}
0	1	1	= {B,C}
1	0	0	= {A}
1	0	1	= {A,C}
1	1	0	= {A,B}
1	1	1	= {A,B,C}

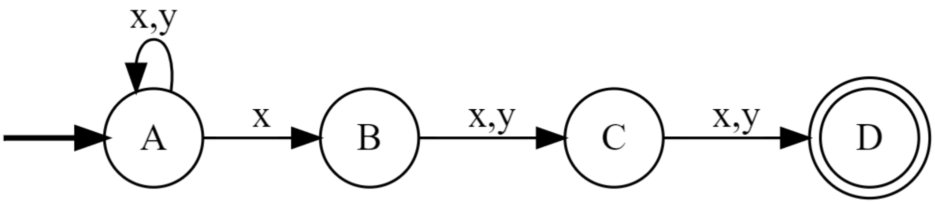
Example



	x	y
A	{A,B}	{A}
B	{C}	{C}
C	{D}	{D}
D	{ }	{ }

NFA without ε-transitions to DFA

Given NFA *M*:

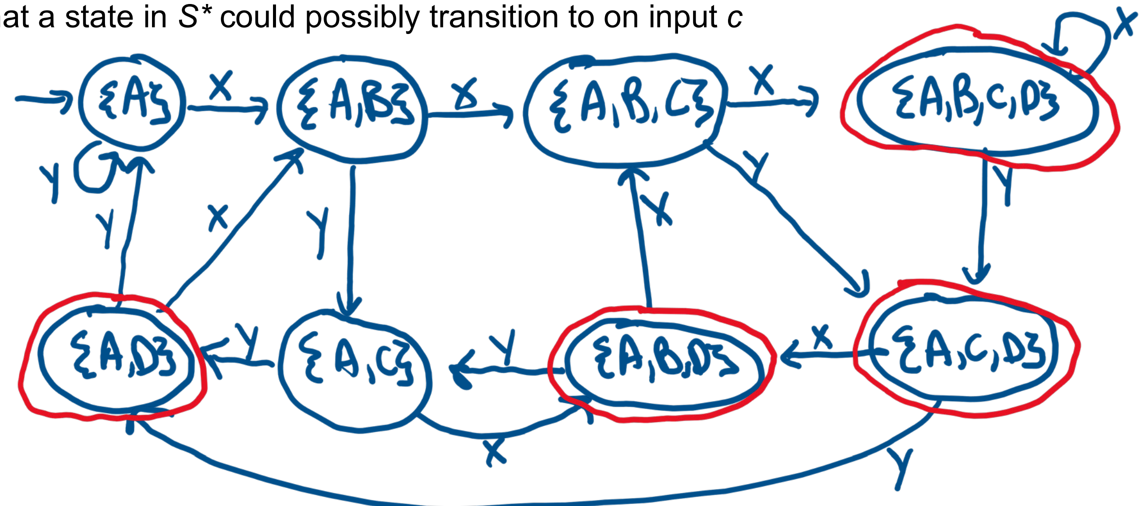


Build new DFA *M** where $Q^* \subseteq \mathcal{P}(Q)$ $|Q^*| \leq 2^{|Q|}$

To build DFA: Add an edge in *M** from state *S** on character *c* to state *T** if *T** represents the set of all states that a state in *S** could possibly transition to on input *c*

	x	y
A	A,B	A
B	C	C
C	D	D
D		

↑ final state in M

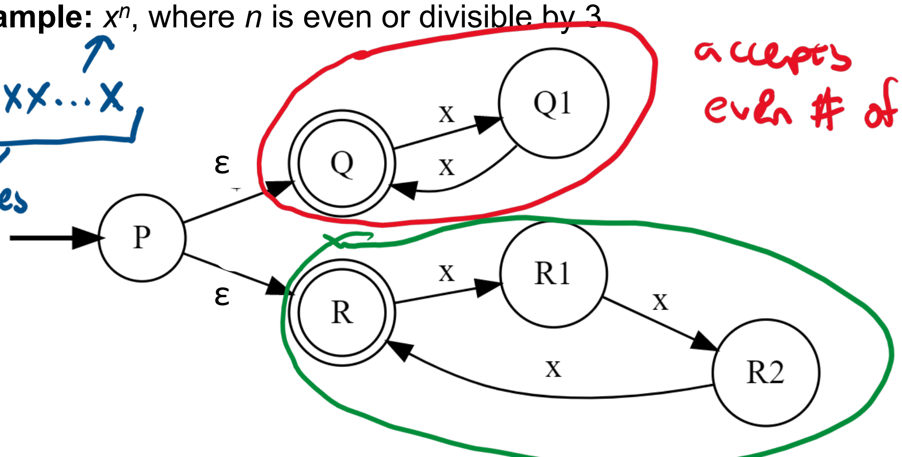


Any state in M* whose subset contains a final state of M is a final in M*

ε-transitions

Example: x^n, where n is even or divisible by 3

xxx…x (n times)



accepts even # of x's

accepts # of x's divisible by 3

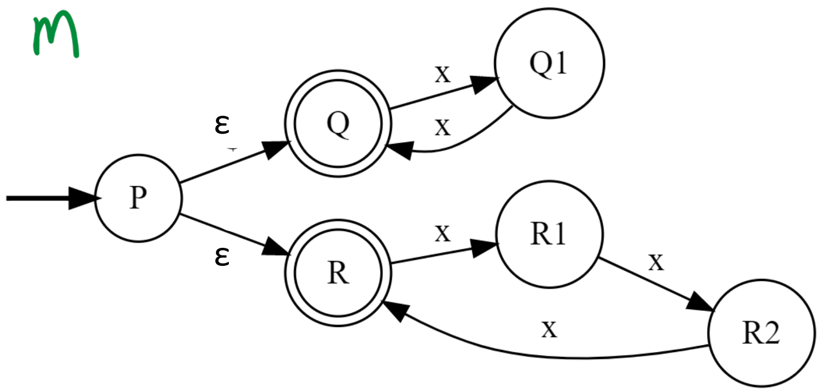
useful for taking union of 2 FSMs

Eliminating ε-transitions

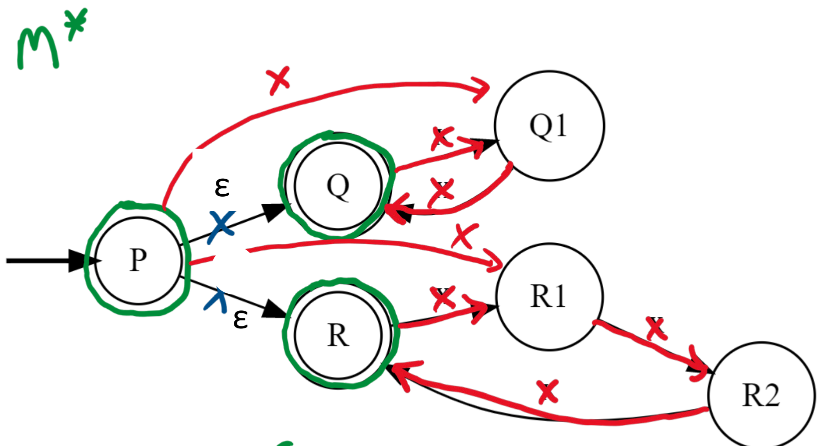
Goal: given NFA *M* with ε-transitions, construct an ε-free NFA *M** that is equivalent to *M*

Definition: *epsilon closure*

eclose(*S*) = set of all states reachable from *S* using 0 or more epsilon transitions



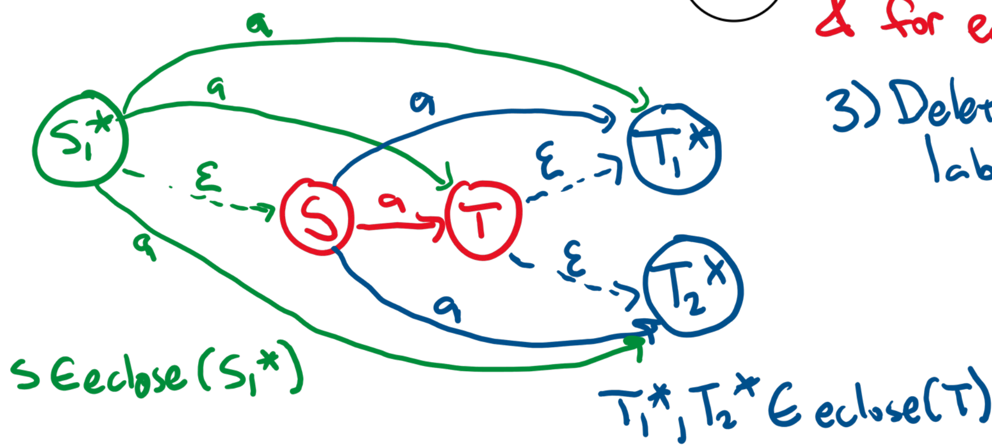
	eclose
P	{P, Q, R}
Q	{Q}
R	{R}
Q1	{Q1}
R1	{R1}
R2	{R2}



1) Make S a final state in M* iff eclose(S) contains an accepting state of M

2) For each edge S $\xrightarrow{a}$ T in M (a ≠ ε)
add edge to M*
S* $\xrightarrow{a}$ T*
for each S* s.t. S ∈ eclose(S*)
& for each T* s.t. T* ∈ eclose(T)

3) Delete all edges labelled with ε



Summary of FSMs

DFAs and NFAs are equivalent

- an NFA can be converted into a DFA, which can be implemented via the table-driven approach

ε-transitions do not add expressiveness to NFAs

- algorithm to remove ε-transitions

Regular Languages and Regular Expressions

Regular language

Any language recognized by an FSM is a *regular language*

Examples:

- single-line comments beginning with //
- hexadecimal integer literals in Java
- C/C++ identifiers
- {ε, ab, abab, ababab, abababab, …} ← try writing FSM

Regular expression (regex)

= a pattern that defines a regular language

regular language: (potentially infinite) set of strings

regular expression: represents a (potentially infinite) set of strings by a single pattern

Example: {$\boldsymbol{\varepsilon}$, ab, abab, ababab, abababab, …} ←→ (ab)*

Why do we need them?

- Each token in a programming language can be defined by a regular language
- Scanner-generator input = one regular expression for each token to be recognized by the scanner

→ regex's are inputs to scanner generator

Formal definition

A **regular expression** over an alphabet Σ is any of the following:

- ∅ (the empty regular expression)
- ε
- *a* (for any $a \in \Sigma$)

Moreover, if R_1 and R_2 are regular expressions over Σ, then so are: $R_1 \mid R_2$, $R_1 \cdot R_2$, $R_1{}^*$

Regular expressions (as an expression language)

regular expression = pattern describing a set of strings

operands: single characters, epsilon ε

operators:

Precedence: low → high

- alternation ("or"): a | b — match a, match b
- concatenation ("followed by"): a.b ab — match ab (catenation)
- iteration ("Kleene star"): a* — match 0 or more a's → ε, a, aa, aaa, … (Kleene closure, closure)

Conventions

	aa	is a.a
	a+	is aa*
L	letter	is a|b|c|d|…|y|z|A|B|…|Z
D	digit	is 0|1|2|…|9
	not(x)	is all characters except x

parentheses for grouping and overriding precedence, e.g., (ab)*

ab* ≡ a(b*)

Example: single-line comments beginning with //

// not('\n')* '\n'   ('\n' = newline)

Example: hexadecimal integer literals in Java

- must start 0x or 0X
- followed by at least one hexadecimal digit (hexdigit)
  - hexdigit = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f, A, B, C, D, E, F
- optionally can add long specifier (`l` or `L`) at end

0(x|X) hexdigit+ (ε | l | L)

Example: C/C++ identifiers (with one added restriction)

- sequence of letters/digits/underscores
- cannot begin with a digit
- cannot end with an underscore

(letter | _) (letter | digit | _)* (letter | digit) | letter

Resume at 7:45 pm

From regular expressions to NFAs

Overview of the process

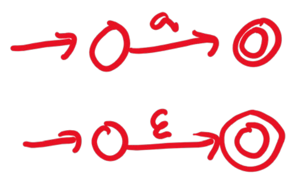
- Conversion of literals and epsilon → simple FAs
- Conversion of operators
  - convert operands to NFAs
  - join NFAs

Regex to NFA rules

Rules for operands

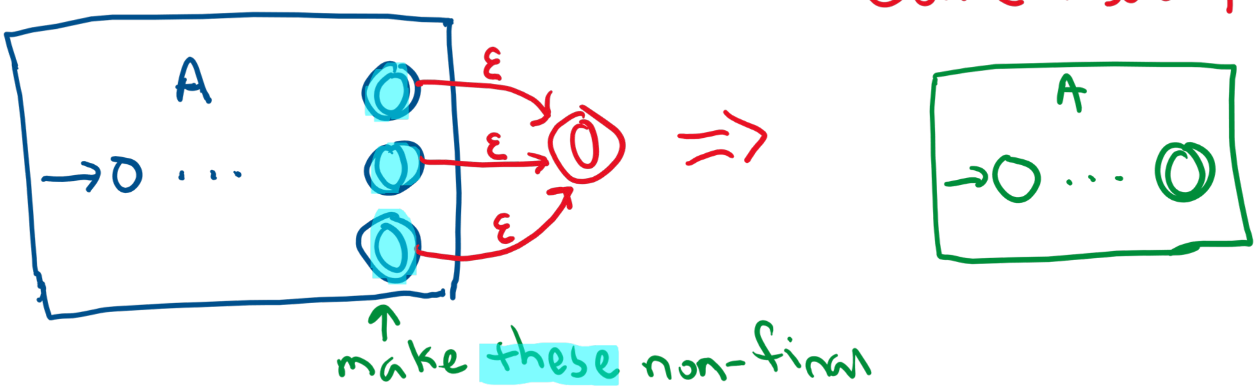
literal 'a'

epsilon ε

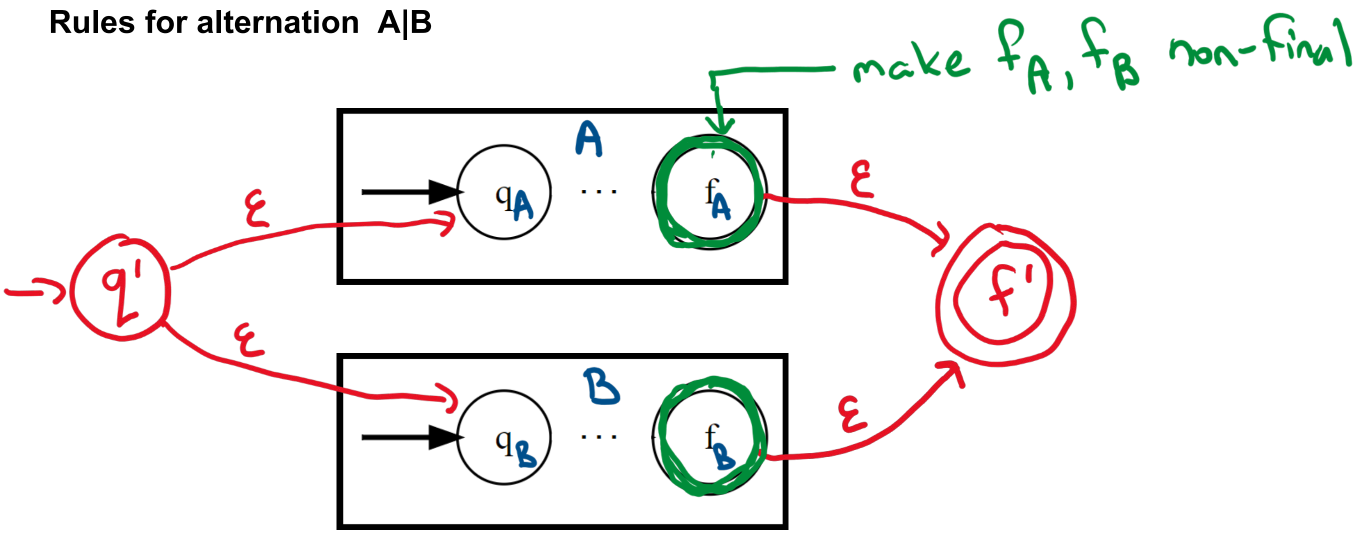


Suppose A is a regex with NFA:

Convert so only 1 final state

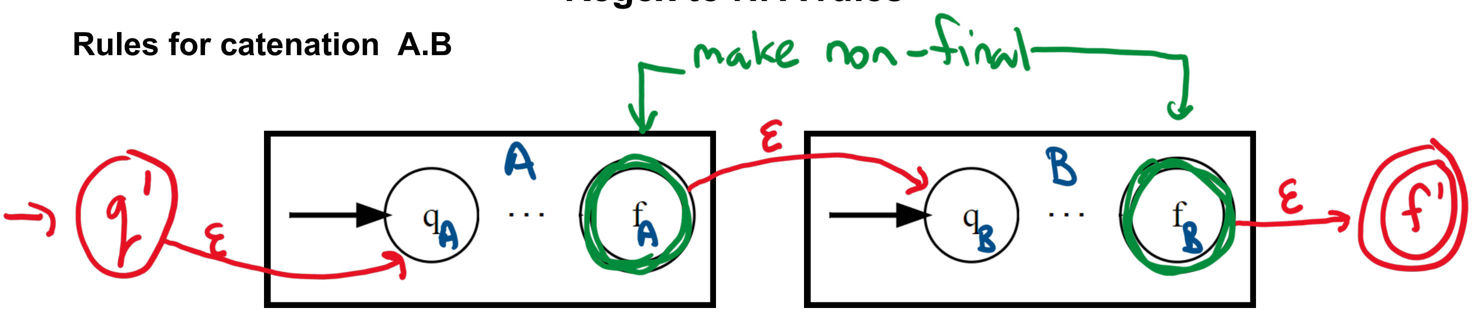


Rules for alternation A|B

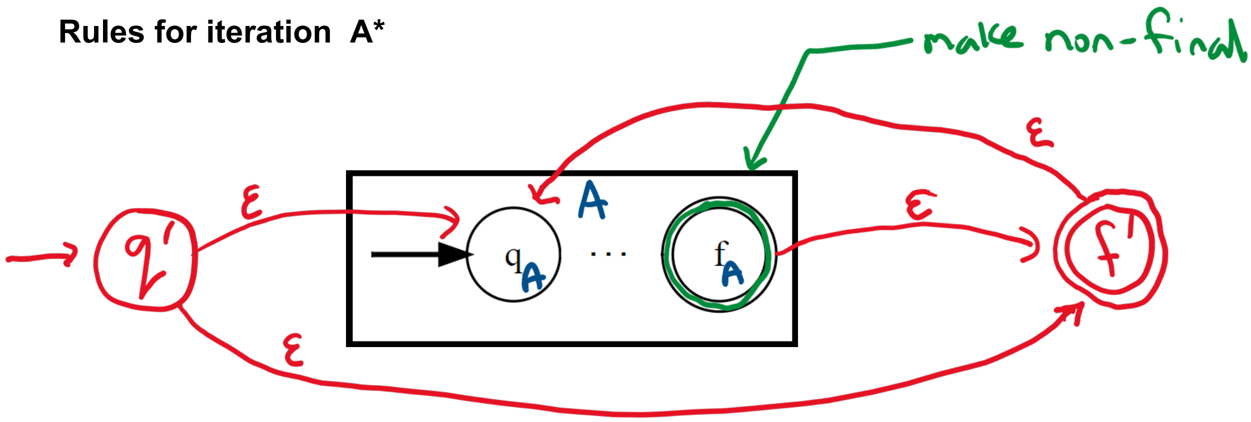


Regex to NFA rules

Rules for catenation A.B

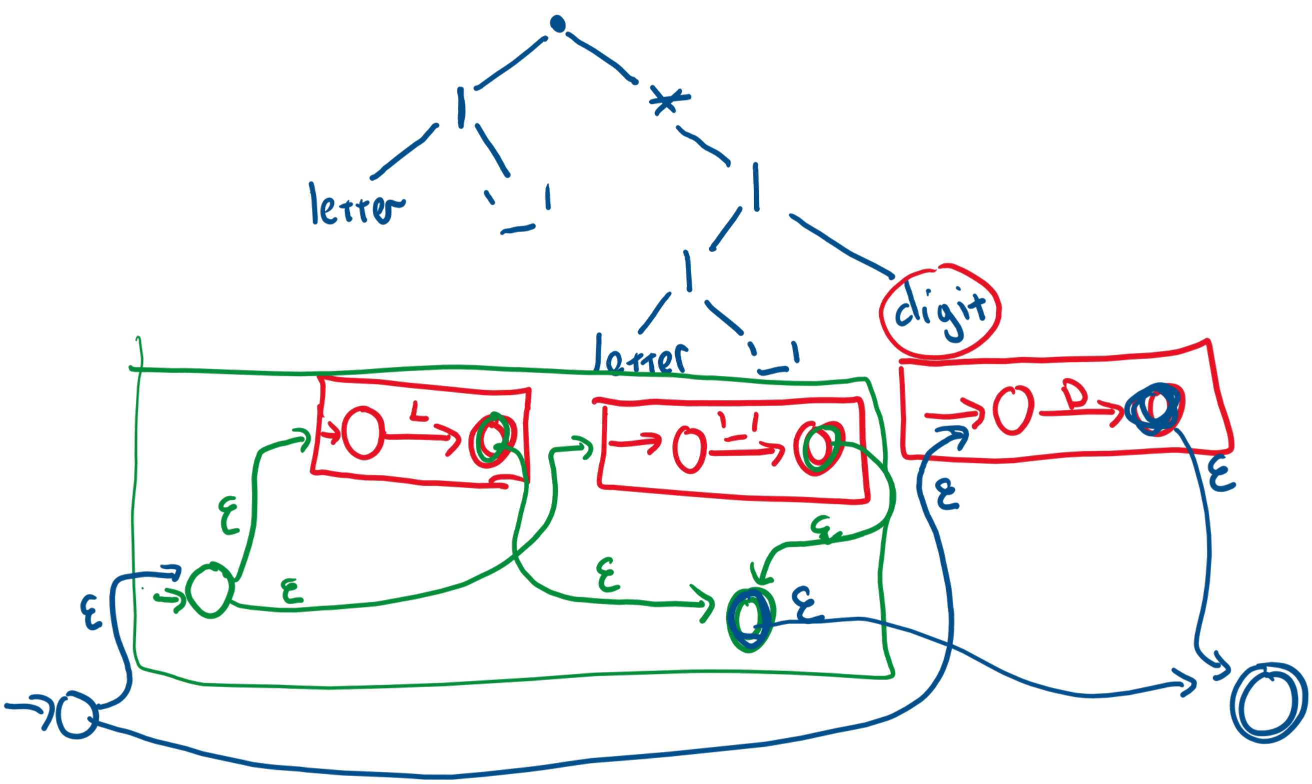


Rules for iteration A*



Tree representation of a regex

Consider regex: (letter | '_') (letter | '_' | digit)*



Regex to DFA

We now can do:

regex → NFA w/ε → NFA w/o ε → DFA

We can add one more step: **optimize DFA** → DFA

Theorem: For every DFA *M*, there exists a unique equivalent smallest DFA *M** that recognizes the same language as *M*.

↳ fewest # of states

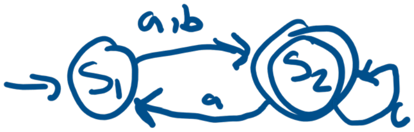
To optimize:

- remove unreachable states → can't get to from start state
- remove dead states → can't get to a final state from it
- merge equivalent states → same transitions (out) w/ same labels

But what's so great about DFAs?

Recall: state-transition function (δ) can be expressed as a table

➔ very efficient array representation



	a	b	c
s_1	s_2	s_2	
s_2	s_1		s_2

➔ efficient algorithm for running (any) DFA

```
s = start state
while (more input){
    c = read next char
    s = table[s][c]
}
if s is final, accept
else reject
```

What else do we need?

FSMs – only check for language membership of a string

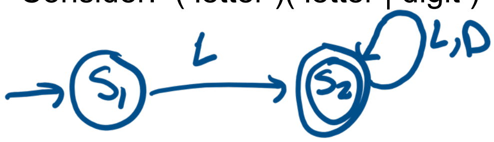
scanner needs to

- recognize a stream of many different tokens using the longest match
- know what was matched

Table-driven DFA → tokenizer

Idea: augment states with actions that will be executed when state is reached

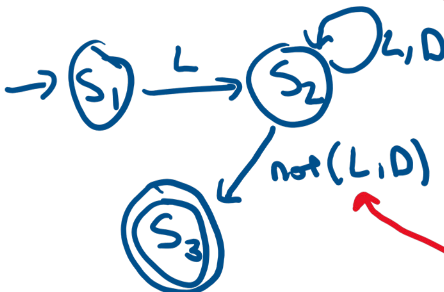
Consider: (letter)(letter | digit)*



State	action
S2	return ID ← token

Problem: Don't get longest match

avg=7



State	action
S3	return ID

avg=7

Problem: maybe we need this char

Actions needed:
- return a token
- put back a character
- report an error

Also add EOF token, EOF symbol to alphabet Σ

Scanner Generator Example

Language description:
consider a language consisting of two statements

- assignment statements: `ID = expr`
- increment statements: `ID += expr`

where `expr` is of the form:

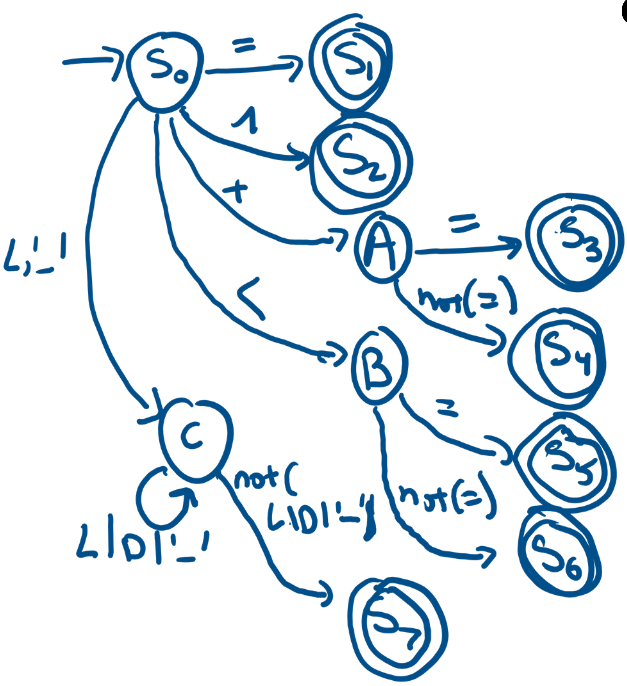
- `ID + ID`
- `ID ^ ID`
- `ID < ID`
- `ID <= ID`

and `ID` are identifiers following C/C++ rules (can contain only letters, digits, and underscores; can't start with a digit)

Tokens:

Token	Regular expression
ASSIGN	"="
INCR	"+="
PLUS	"+"
EXP	"^"
LESSTHAN	"<"
LEQ	"<="
ID	(letter|'_')(letter|'_'|digit)*

Combined DFA



Actions

S1: return ASSIGN

S2: return EXP

S3: return INC

S4: put 1 back, return PLUS

S5: return LEQ

S6: put 1 back, return LESSTHAN

S7: put 1 back, return ID

State-transition table

	=	+	^	<	_	letter	digit	EOF	none of these
S0	ret ASSIGN	A	ret EXP	B	C	C		ret EOF	
A	ret INC	put 1 back, ret PLUS							→
B	ret LEQ	put 1 back, ret LESSTHAN							→
C	put 1 back, ret ID			→	C	C	C	put 1 back, ret ID	→

```
do {
    read char
    perform action / update state
    if (action was to return a token)
       start again in start state
} while not(EOF or stuck)
```

Lexical analyzer generators
(aka scanner generators)

Formally define transformation from regex to scanner

Tools written to synthesize a lexer automatically

- Lex : UNIX scanner generator, builds scanner in C
- Flex : faster version of Lex
- JLex : Java version of Lex

JLex

Declarative specification (non-procedural)

- you don't tell JLex how to scan / how to match tokens
- you tell JLex what you want scanned (tokens) & what to do when a token is matched

Input: set of regular expressions + associated actions

JLex specification → .jlex eg xyz.jlex

Output: Java source code for a scanner

↳ xyz.jlex.java → compile to get Yylex.class

- ctor : takes input stream as arg
- next_token : return next token of input

Format of JLex specification

3 sections separated by %%

- user code section
- directives
- regular expression rules

Example

```
// User Code section:  For right now, we will not use it.
%%
DIGIT=        [0-9]
LETTER=       [a-zA-Z]
WHITESPACE=   [\040\t\n]
```

Directives — macro definitions; format: name = regular expression; [\040\t\n] → space, tab, newline

```
%state SPECIALINTSTATE
```

← state declaration

```
%implements java_cup.runtime.Scanner
%function next_token
%type java_cup.runtime.Symbol
```

} needed to use generated scanner with Java Cup

```
%eofval{
System.out.println("All done");
return null;
%eofval}
```

} tell JLex what to do on an EOF

```
%line
```

← turn on line counting (starts at 0)

```
%%  Regex rules
({LETTER}|"_")({DIGIT}|{LETTER}|"_")* {
                          System.out.println(yyline+1 + ": ID "
                                     + yytext()); }

"="             { System.out.println(yyline+1 + ": ASSIGN"); }
"+"             { System.out.println(yyline+1 + ": PLUS"); }
"^"             { System.out.println(yyline+1 + ": EXP"); }
"<"             { System.out.println(yyline+1 + ": LESSTHAN"); }
"+="            { System.out.println(yyline+1 + ": INCR"); }
"<="            { System.out.println(yyline+1 + ": LEQ"); }
{WHITESPACE}*   { }
.               { System.out.println(yyline+1 + ": bad char"); }
```

Regular expression rules section

Format: `<regex>{code}` where `<regex>` is a regular expression for a single token

- can use macros from Directives section – surround with curly braces `{ }`
- characters represent themselves (except special characters)
- characters inside `" "` represent themselves (except `\"`)
- `.` matches anything

Regular expression operators: `|` `*` `+` `?` `( )`

Character class operators: `-` `^` `\`

Using scanner generated by JLex in a program

```
// inFile is a FileReader initialized to read from the
// file to be scanned
Yylex scanner = new Yylex(inFile);
try {
    scanner.next_token();
} catch (IOException ex) {
    System.err.println(
              "unexpected IOException thrown by the scanner");
    System.exit(-1);
}
```