Speeding up Permutation Testing in Neuroimaging

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http://pages.cs.wisc.edu/~vamsi/pt_fast.html

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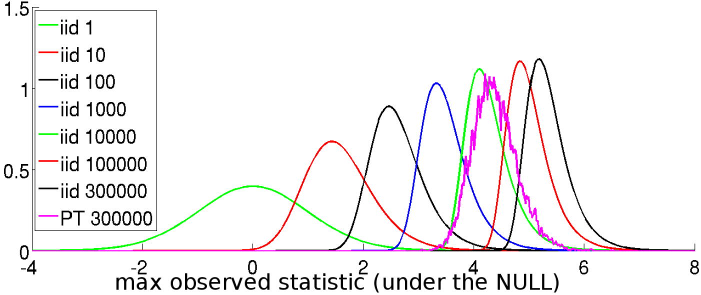
Permutation Testing

Setting

- **High-dimensional** measurements;
- **Highly correlated** covariates;
- Comparing **distinct** phenotype populations, statistically.

Under the **Global (Joint) Null Hypothesis**, the **max** observed test statistic is distributed as a function of the # of covariates:



Permutation Testing is an unbiased way of estimating this distribution from the sampled data.

Modelling Assumptions and Approach

Low-rank matrix completion

$$P = UW + S; \qquad P, UW, S \in \mathbb{R}^{v \times t}$$

$$S_{i,j} \sim \mathcal{N}(0, \sigma^2).$$

P: Permutation test matrix; v: voxels; t: tests

UW: Low-rank component;

$U \in \mathbb{R}^{v \times \mathbf{r}}$, $W \in \mathbb{R}^{\mathbf{r} \times t}$; **r is small**

S: Approx. iid Normal residual

Optimization

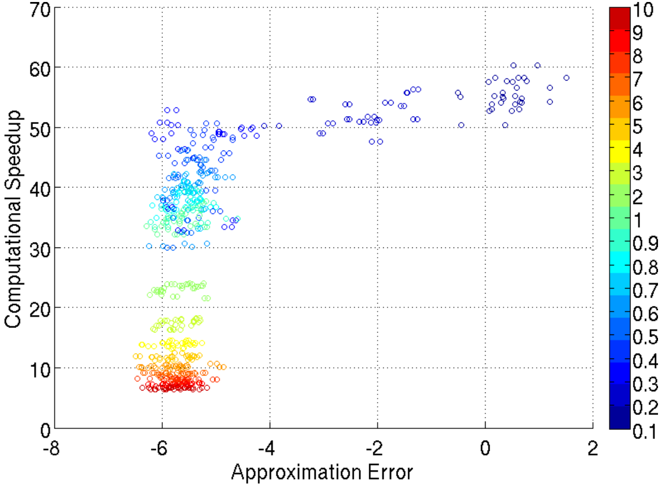
$$\min_{\tilde{\mathbf{P}}, U, W} \|\mathbf{P}_\Omega - \tilde{\mathbf{P}}_\Omega\|_F^2 \qquad \text{s.t. } \tilde{\mathbf{P}} = UW; \qquad U \text{ is column-wise orthogonal}$$

Theoretical guarantees

- Under realistic assumptions, we can model PP^T as a **low-rank perturbation** of a Wishart matrix, SS^T.
- The desired sample Null max distribution can be recovered with **bounded error**.

Trade-off

Thresholds can be recovered with **high fidelity** with a **50× speedup**.



Look for us at poster **Sun34**, and on the web at

`http://pages.cs.wisc.edu/~vamsi/pt_fast.html`