Name: _____   Wisc id: _____
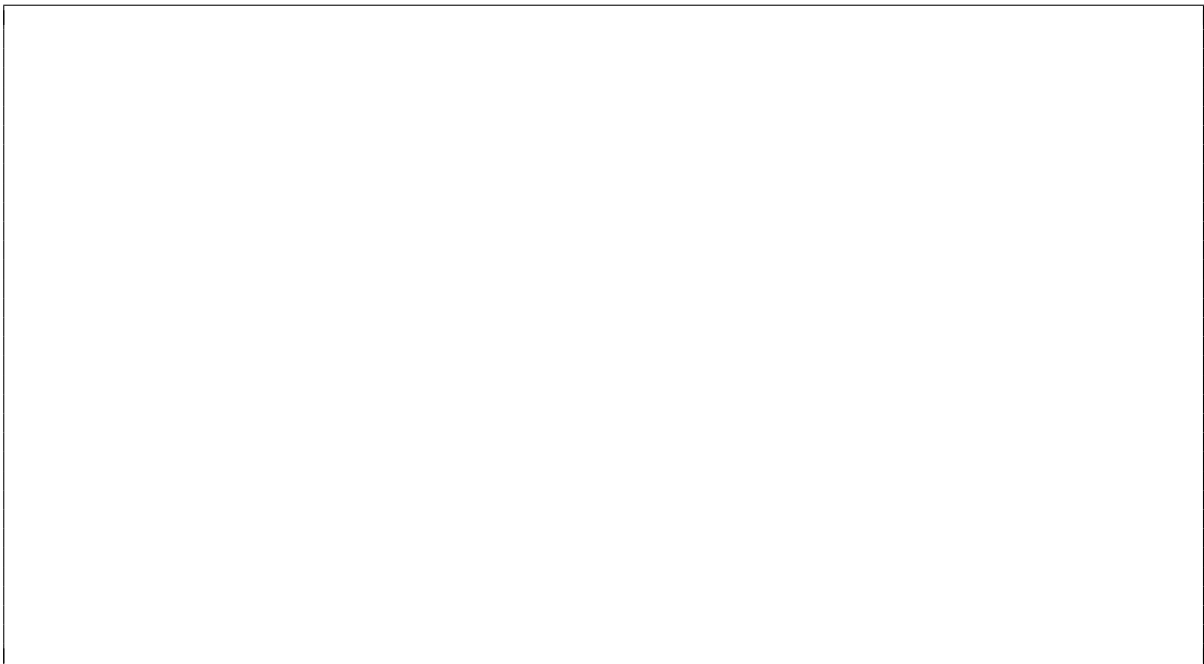
Problems

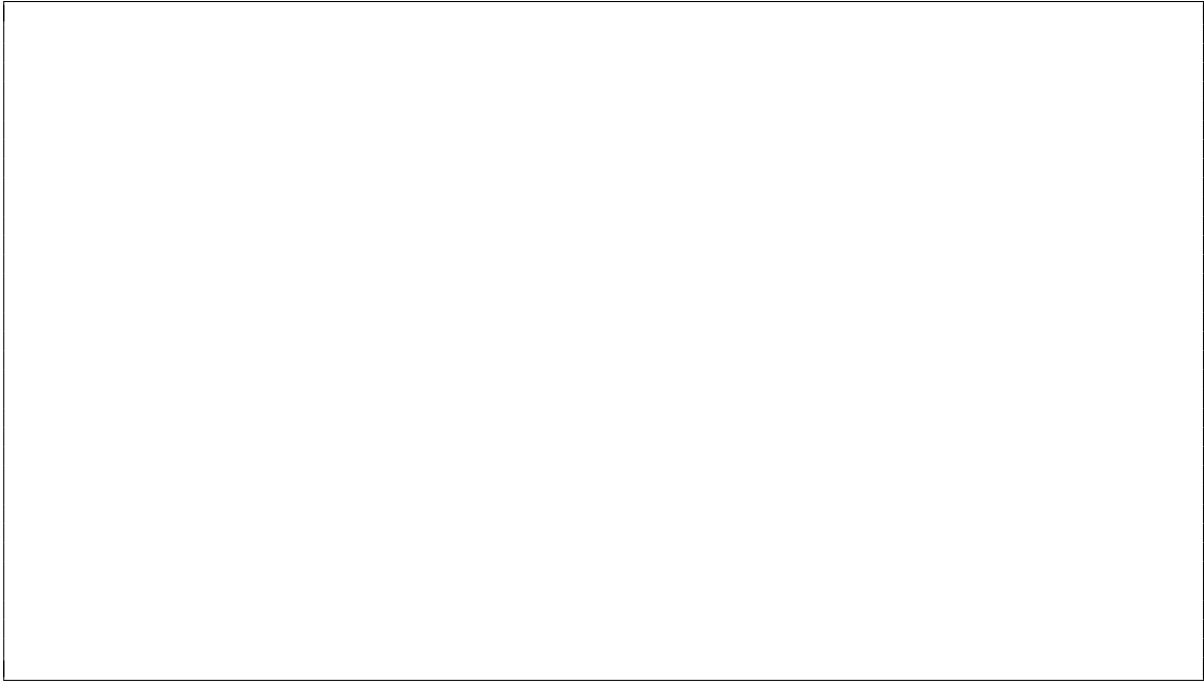Proof the correctness of each statement using induction.

1. $3 + 11 + \ldots + (8n - 5) = 4n^2 - n$

2. Let $a_1 = 11$, $a_2 = 21$, $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 3$. Prove that $a_n = 5 \cdot 2^n + 1$, $\forall n \geq 1$

3. Prove that $w^4 - 1$ is divisible by 16 for all odd positive integers w.

4. The Fibonacci sequence is defined as $F_n = F_{n-1} + F_{n-2}$ with base cases $F_1 = 1$ and $F_2 = 1$. The following is a closed-form expression for the n-th value in the sequence. Prove its correctness.

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right)$$