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## Truth or Lie?

Which of the following statements are TRUE and which of them are FALSE?

1. If a graph with  $n$  vertices is connected, then it must have at least  $n - 1$  edges. T — F
2. If a graph with  $n$  vertices has at least  $n$  edges, then it must be connected. T — F
3. If the max degree of vertices is  $k$ , then the graph's chromatic number  $\geq k + 1$ . T — F

## Detect Cycle Algorithm

Give an efficient algorithm to detect whether a given undirected graph contains a cycle. If the graph contains a cycle, then your algorithm should output one. It should output one cycle, not all of them.

## On Binary Trees

A binary tree is a rooted tree in which each node has at most two children. Show by induction that in any binary tree the number of nodes with two children is exactly one less than the number of leaves.

## Handshaking lemma

Prove that for all simple graphs, there is an even amount of nodes with odd degrees.

## Single Negative Weight Edge

Let  $G = (V, E)$  be a weighted, directed graph with exactly one known negative weight edge  $(u, v)$  and no negative-weight cycles. Give an algorithm to find the shortest distance from  $s$  to all other vertices in  $V$  that has the same running time as Dijkstra's algorithm.