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**Truth or Lie?**

Which of the following statements are TRUE and which of them are FALSE?

- |  |   |   |   |
|--|---|---|---|
| 1. A graph is bipartite iff it contains no cycles of odd length            | T | — | F |
| 2. A complete graph of $n$ nodes has $n(n - 1)$ edges.                     | T | — | F |
| 3. Every DAG has exactly one topological ordering.                         | T | — | F |
| 4. In a two colorable graph of 8 nodes, the maximum number of edges is 15. | T | — | F |

**Uniqueness of path in a tree**

Prove the following statement.

If  $T = (V, E)$  is a tree, then for all pairs of nodes  $v, w \in V$ ,  $\exists$  unique simple path  $p$  between them.

## Money-making sequence of cryptocurrency exchanges

Let  $r_{vw}$  be the amount of cryptocurrency  $w$  that can be purchased with one unit of cryptocurrency  $v$ .

A sequence  $v_0, v_1, \dots, v_k$  (where  $v_k = v_0$ ) of cryptocurrency exchanges is *money-making* iff:

$$\prod_{i=1}^k r_{v_{i-1}v_i} > 1$$

Assume that you have an efficient algorithm, called Bellman-Ford, that finds a negative cost cycle in a graph  $G = (V, E)$ .

Given all cryptocurrency exchange rates  $r_{vw}$ , design an algorithm that finds a money-making sequence.