

Name: _____ Wisc id: _____

Minimum Spanning Trees: Truth or Lie?

Which of the following statements are TRUE and which of them are FALSE?

1. Prim's and Kruskal's will always return the same MST. T — F
2. A MST for a graph has $|V| - 1$ edges. T — F
3. There is a MST that can be used to find the shortest path between 2 vertices. T — F

Merge K Sorted Linked Lists

You are given K linked lists, each containing N elements sorted in ascending order. Your task is to merge these K linked lists into a single sorted linked list. Describe an algorithm to achieve this and analyze its time and space complexity.

Give Me a Classroom

You are given a set of lectures, each with a distinct starting and finishing time (s_i, f_i) . Each lecture i starts at time s_i and finishes at time f_i . Your task is to design an algorithm to find the minimum number of classrooms required to schedule all lectures such that no two lectures occur at the same time in the same classroom. Furthermore, provide a proof demonstrating the optimality of your algorithm.

Minimum Deci-Binary Summation

A decimal number is termed *deci-binary* if each of its digits is either a 0 or a 1, and it does not have any leading zeros. For instance, 101 and 1100 qualify as deci-binary numbers, while 112 and 3001 do not.

Given a number n that represents a positive decimal integer. Your task is to find and return the minimum number of positive deci-binary numbers that, when summed, yield the integer represented by n .

For example, $32 = 11 + 11 + 10$, so the answer would be 3. For $n = 52734$, answer would be 7.

Longest Wiggle Subsequence

A *subsequence* is a sequence that can be derived from another sequence by deleting elements (possibly none) without changing the order of the remaining elements.

A *wiggle sequence* is defined as a sequence where the differences between adjacent numbers strictly alternate between positive and negative. The first difference, if it exists, can be either positive or negative.

Given an integer array containing N integers, return the length of the longest wiggle subsequence.

Kruskal's algorithm

Algorithm: KRUSKAL'S

Input: G , a graph $G = (V, E)$

begin

 Let $T = \emptyset$

 Let $L = E$ and sort L

for each edge $e \in L$ in ascending order **do**

if $H = (V, T \cup \{e\})$ is an acyclic graph **then**

$T \leftarrow T \cup \{e\}$

return $H = (V, T)$

Prim's Algorithm

Algorithm: PRIM'S

Input: G , a graph $G = (V, E)$

begin

 Let $U = \{v\}$, where $v \in V$

 Let $T = \emptyset$

while $U \neq V$ **do**

 Find least-weight edge $e = (u, v) : u \in U \wedge v \in V \setminus U$

$T \leftarrow T \cup \{e\}$

$U \leftarrow U \cup \{v\}$

return $H = (U, T)$
