Name: _____   Wisc id: _____

The Expedition Beyond the Wall

In a daring effort to understand the mysteries and threats beyond the Wall, a team of Night's Watch rangers plans an extensive expedition. Their goal is to chart the longest possible path through the treacherous lands beyond the Wall, mapping out territories, Wildling settlements, and any signs of the White Walkers.

The lands beyond the Wall are vast and uncharted, with numerous dangers lurking. Each significant landmark (like a mountain pass, an abandoned fortress, or a known Wildling camp) represents a node in the graph. The rangers must start at Castle Black and find a route that passes through at least k of these landmarks as possible without retracing their steps, maximizing their reconnaissance while avoiding repeated encounters with danger.

This expedition, representing a variant of the *Longest Path Problem*, is critical for gathering intelligence and preparing for potential threats. However, this decision problem is believed to be an NP-complete problem, reflecting the perilous and unpredictable nature of the lands beyond the Wall. Prove that it is NP-complete.

The Siege of Winterfell: Strategizing the Defense

As the long night approaches and the armies amass outside the walls of Winterfell, the commanders face a daunting task. The defense of Winterfell involves n critical tasks, each requiring a specific amount of time to complete, labeled as $a_1, a_2, \ldots, a_n$. There are p available units of the Northern army, each capable of executing one task at a time to its completion.

The stark challenge before you, as a strategist in Jon Snow's council, is to determine if it is possible to assign these tasks to your units such that the entire defense is organized and executed within a critical timeframe t. This strategy must ensure that no unit is overwhelmed and that all defensive measures are completed in the allotted time, preparing Winterfell for the impending siege.

This problem, known as the *Multiprocessor Scheduling* problem, is as formidable as the foes beyond the Wall and is known to be NP-complete, representing a true test of your strategic abilities in times of war. Prove such the claim to Jon Snow.