Problems

Proof the correctness of each statement.

1. \(1 + 4 + 7 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1)\)

2. \(\forall n \in \mathbb{N}, 5 \mid (6^n - 1)\)
3. The Fibonacci sequence is defined as \( F_n = F_{n-1} + F_{n-2} \) with base cases \( F_1 = 1 \) and \( F_2 = 1 \). The following is a closed-form expression for the \( n \)-th value in the sequence. Prove its correctness.

\[
F_n = \frac{1}{\sqrt{5}} \left( \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n \right)
\]