

CS540 Introduction to Artificial Intelligence **Neural Networks: Review** University of Wisconsin-Madison

Spring 2023



Announcements

- Homeworks:
 - HW 7 due in one wee
- Midterms are being grade
- Final exam is May 12, 5:
- Class roadmap:
- Practice Questions on Canvas

eek Ided; solutions on 5:05 - 7:05 pm.	Canvas.
Tuesday, April 4	Neural Network Review
Thursday, April 6	Uninformed Search
Tuesday, April 11	Informed Search
Thursday, April 13	Advanced Search



How to classify

Cats vs. dogs?



- Typically, no activation on outputs, mean squared error loss function.





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Inspiration from neuroscience

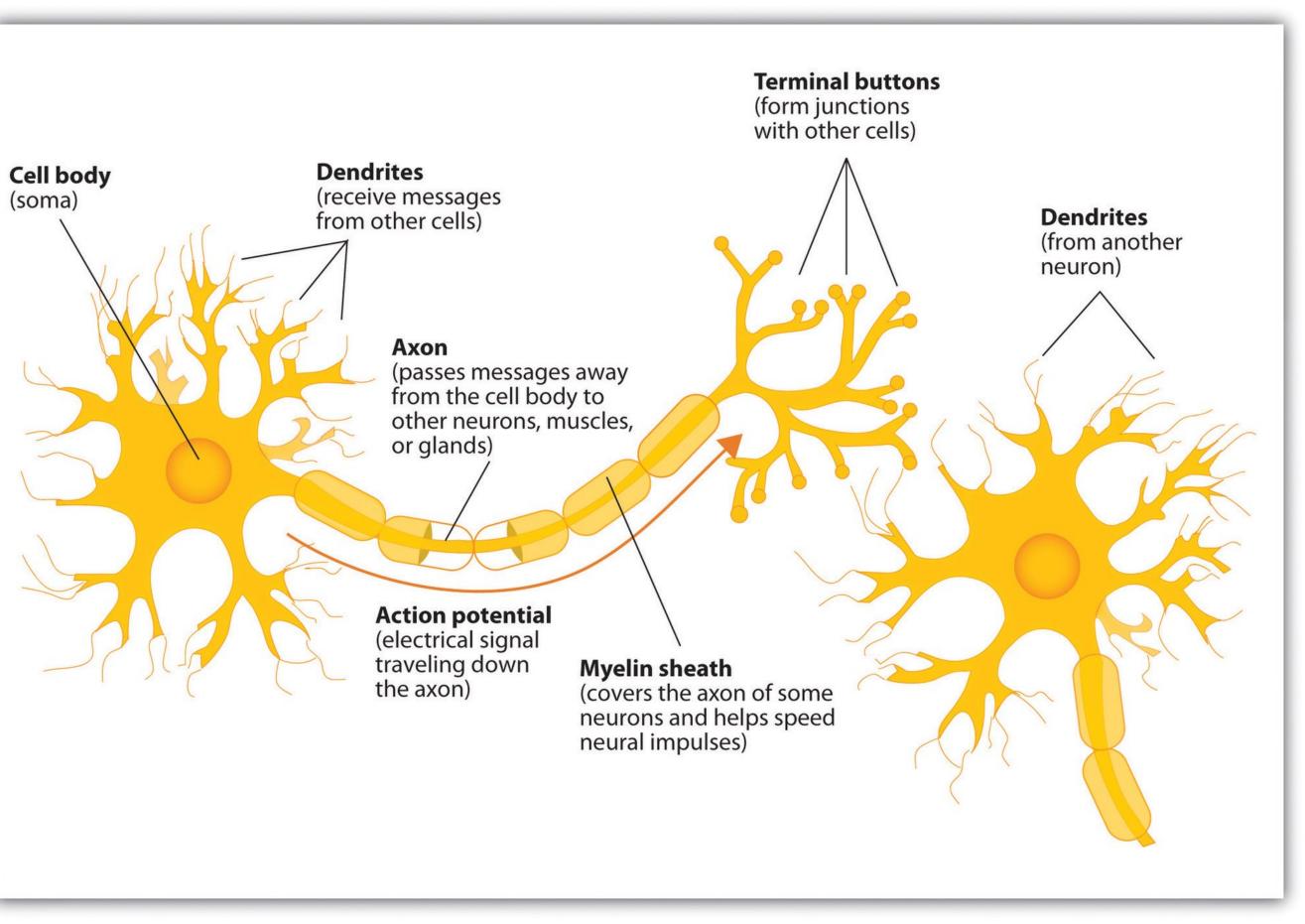
- Inspirations from human brains



(soma)

(wikipedia)

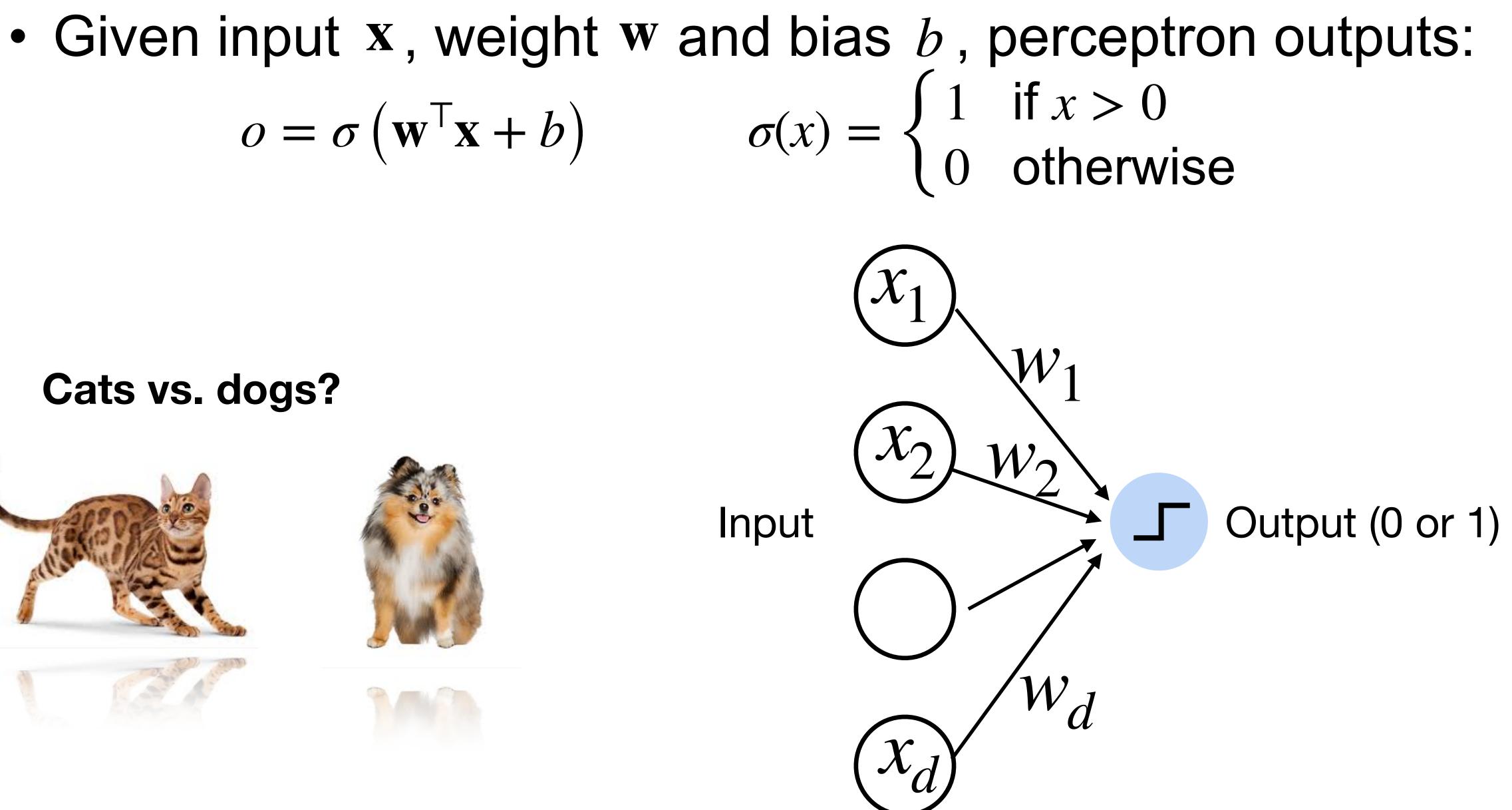
- Networks of simple and homogenous units (a.k.a neuron)



Perceptron

Cats vs. dogs?





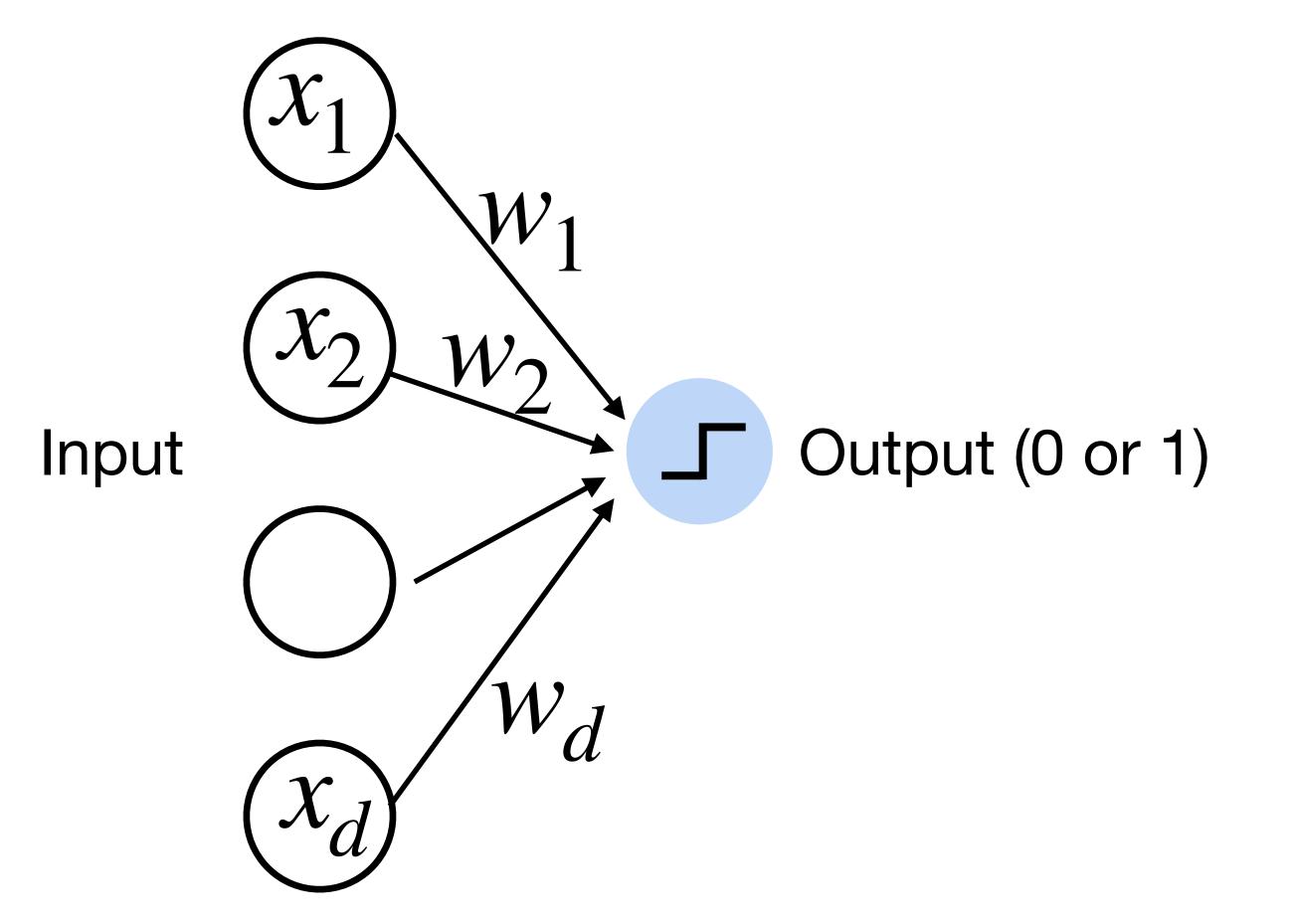
Perceptron

 $o = \sigma \left(\mathbf{w}^{\mathsf{T}} \mathbf{x} + b \right)$

Cats vs. dogs?



• Given input x, weight w and bias b, perceptron outputs: $\sigma(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$ Activation function



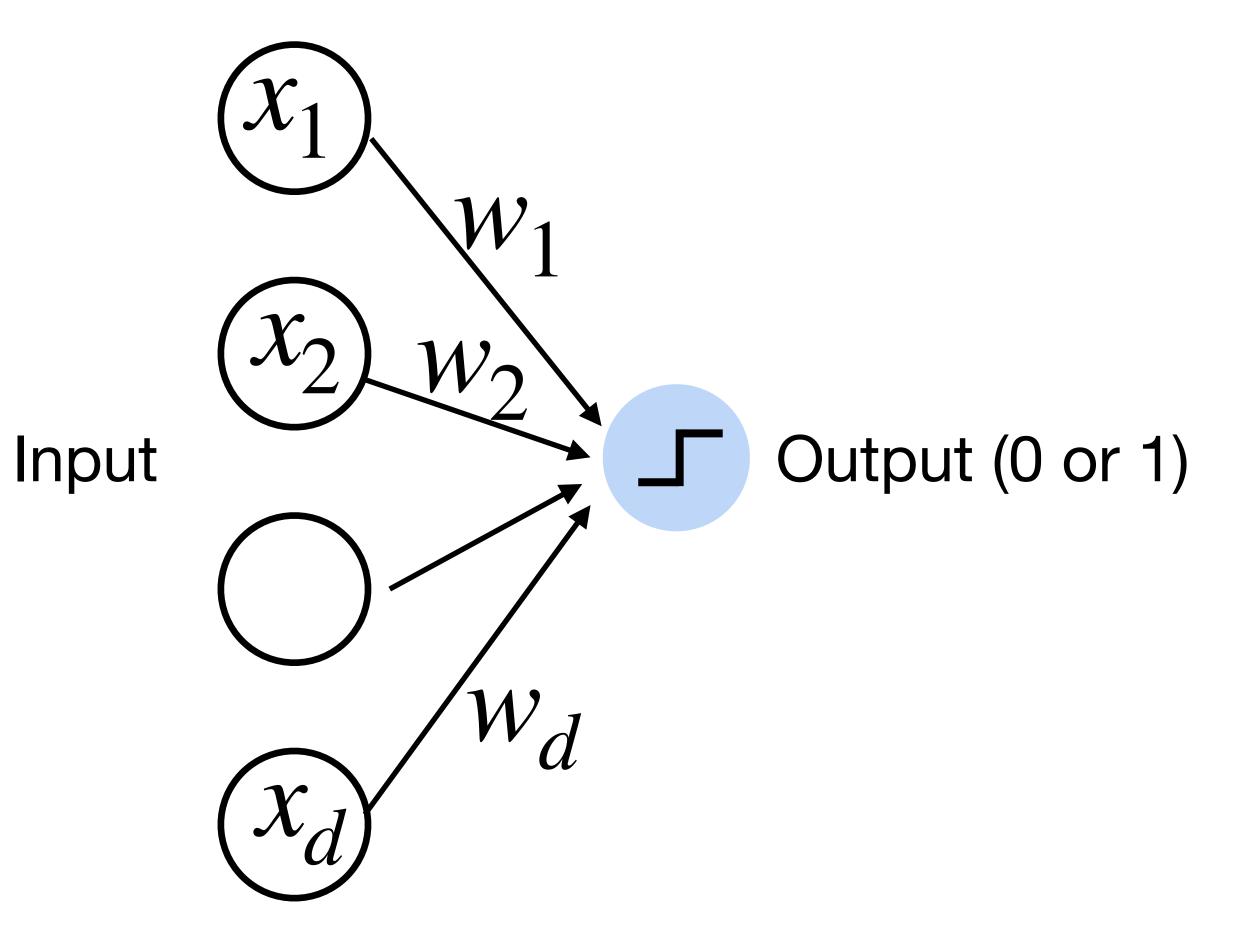


Perceptron

• Goal: learn parameters $\mathbf{W} = \{w_1, w_2, \dots, w_d\}$ and b to minimize the classification error

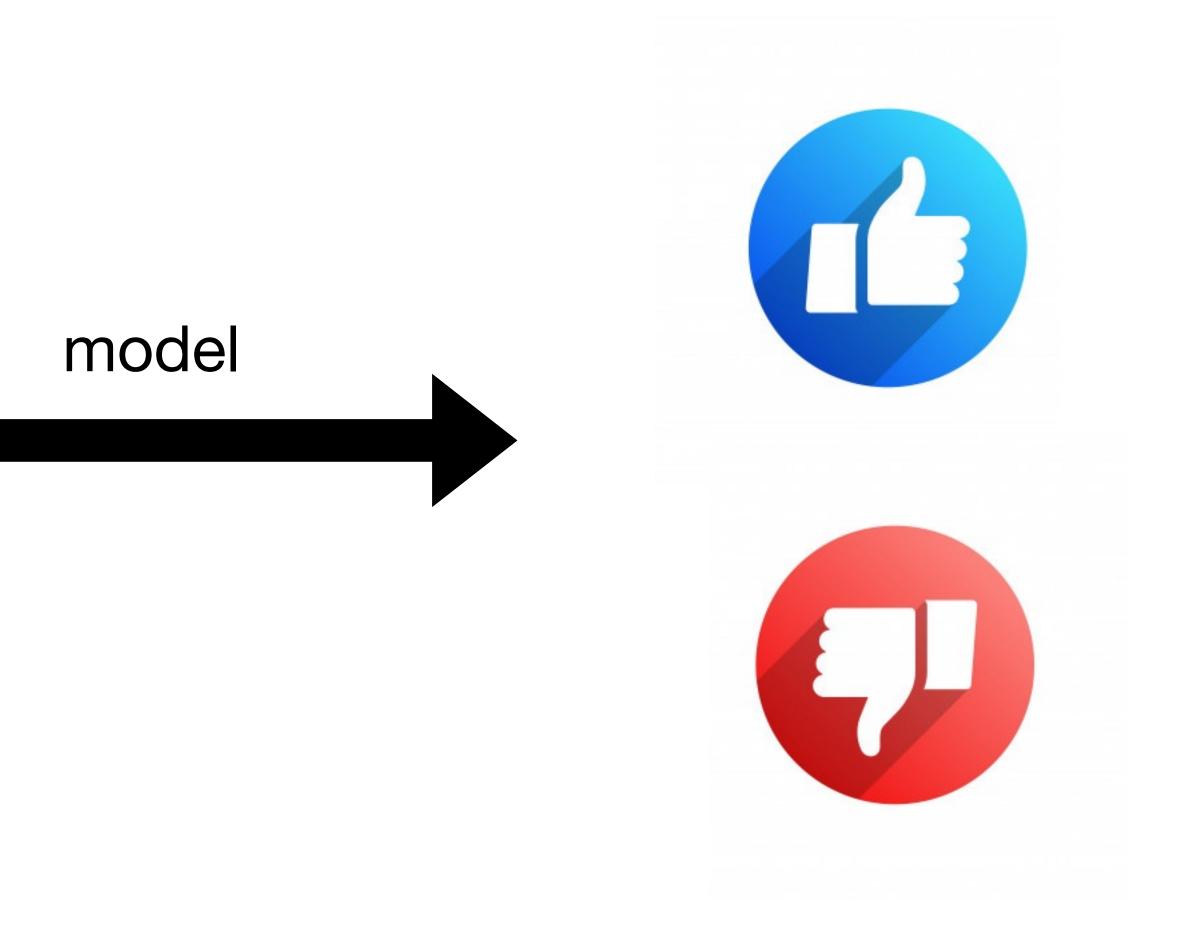
Cats vs. dogs?





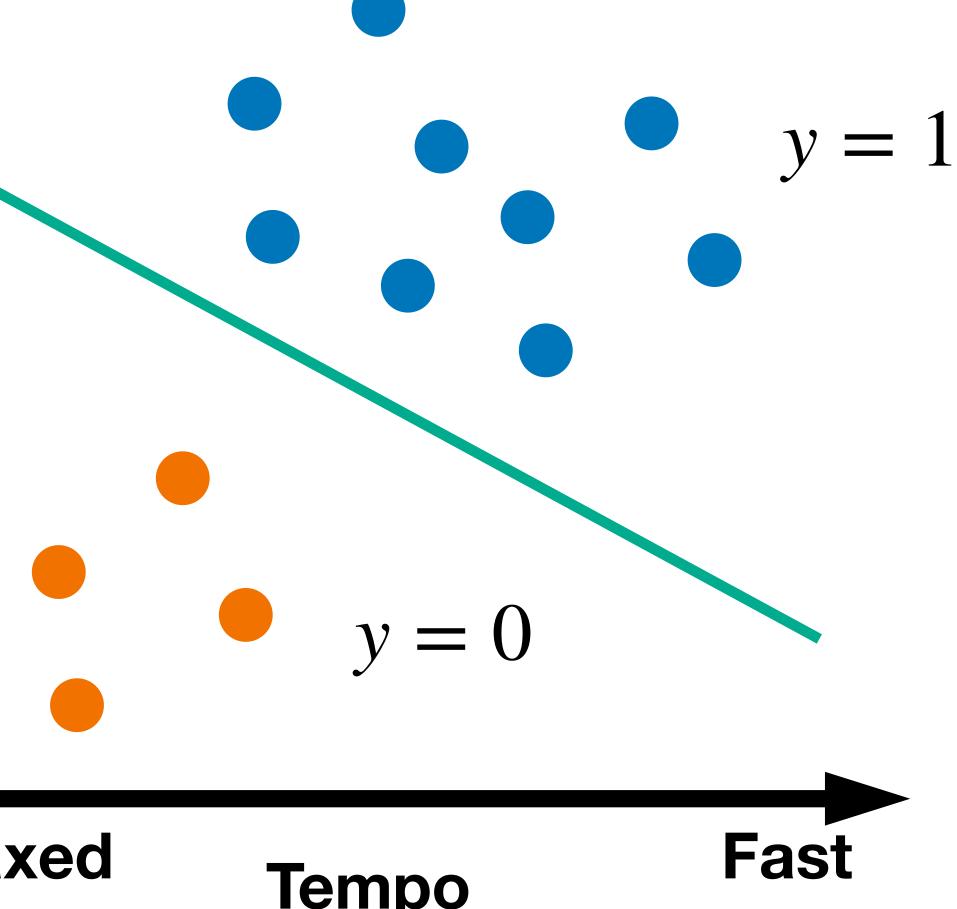
Example 2: Predict whether a user likes a song or not







Example 2: Predict whether a user likes a song or not Using Perceptron Intensity y = 1**User Sharon** DisLike Like Fast Relaxed Tempo



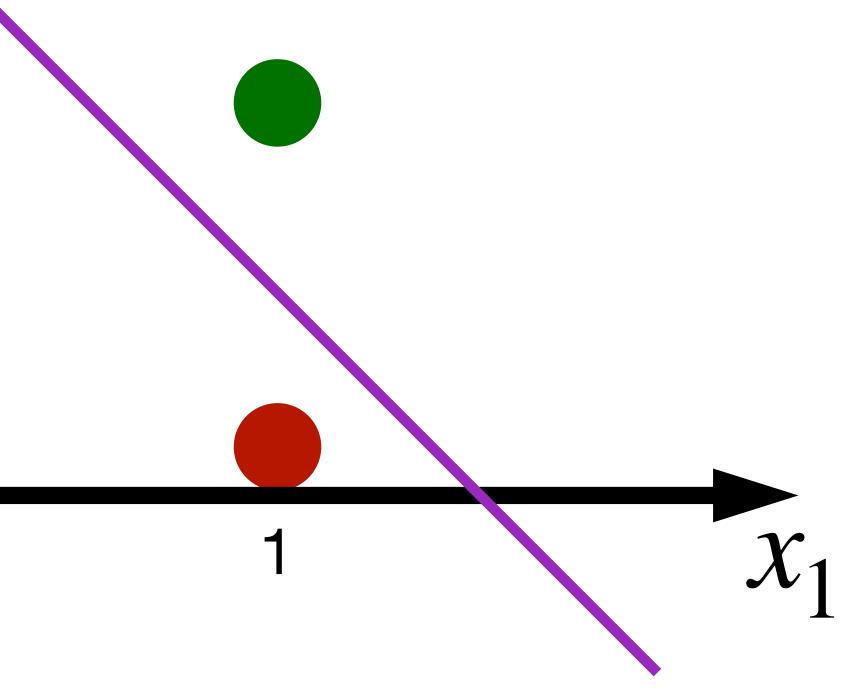


Learning logic functions using perceptron The perceptron can learn an AND function $x_1 = 1, x_2 = 1, y = 1$ $x_1 = 1, x_2 = 0, y = 0$ $x_1 = 0, x_2 = 1, y = 0$ $x_1 = 0, x_2 = 0, y = 0$

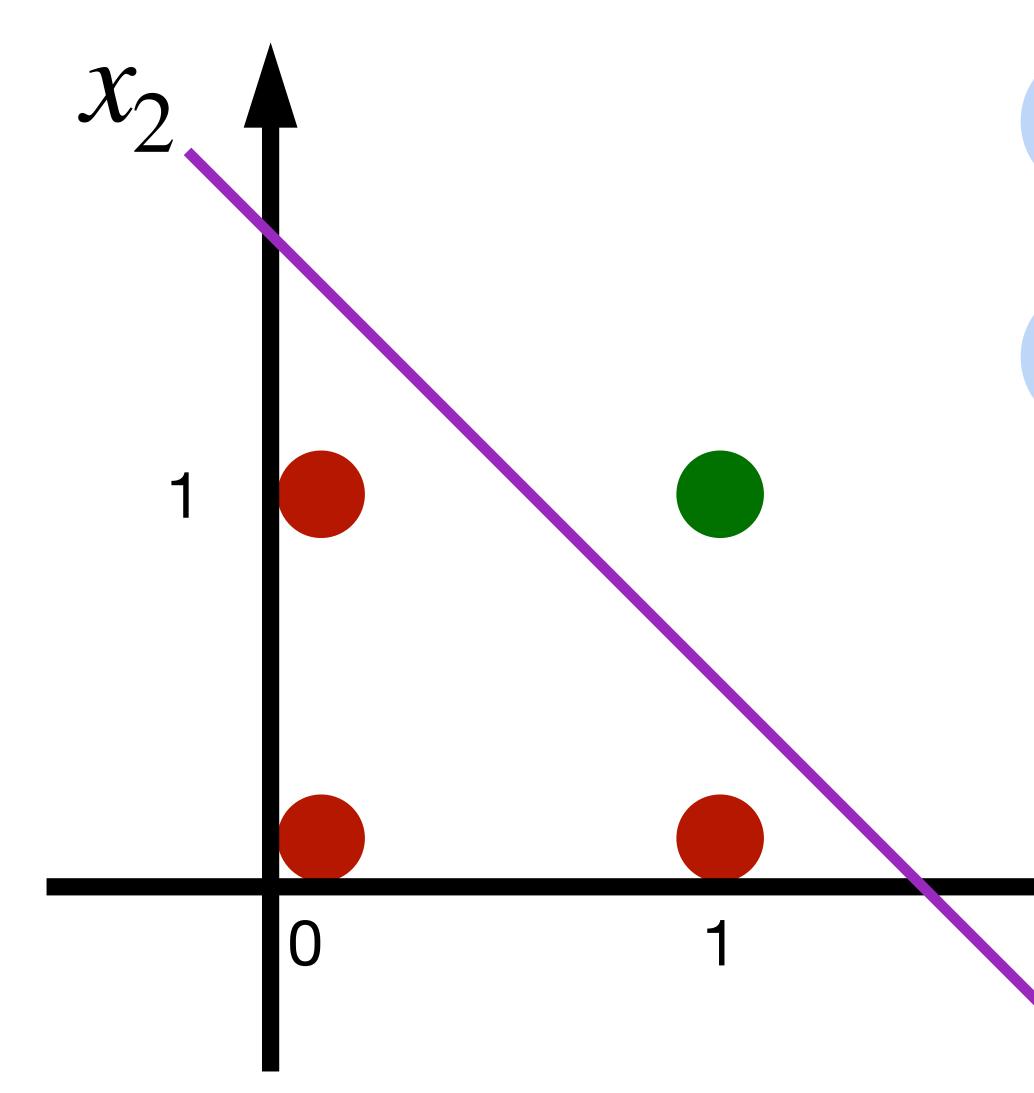


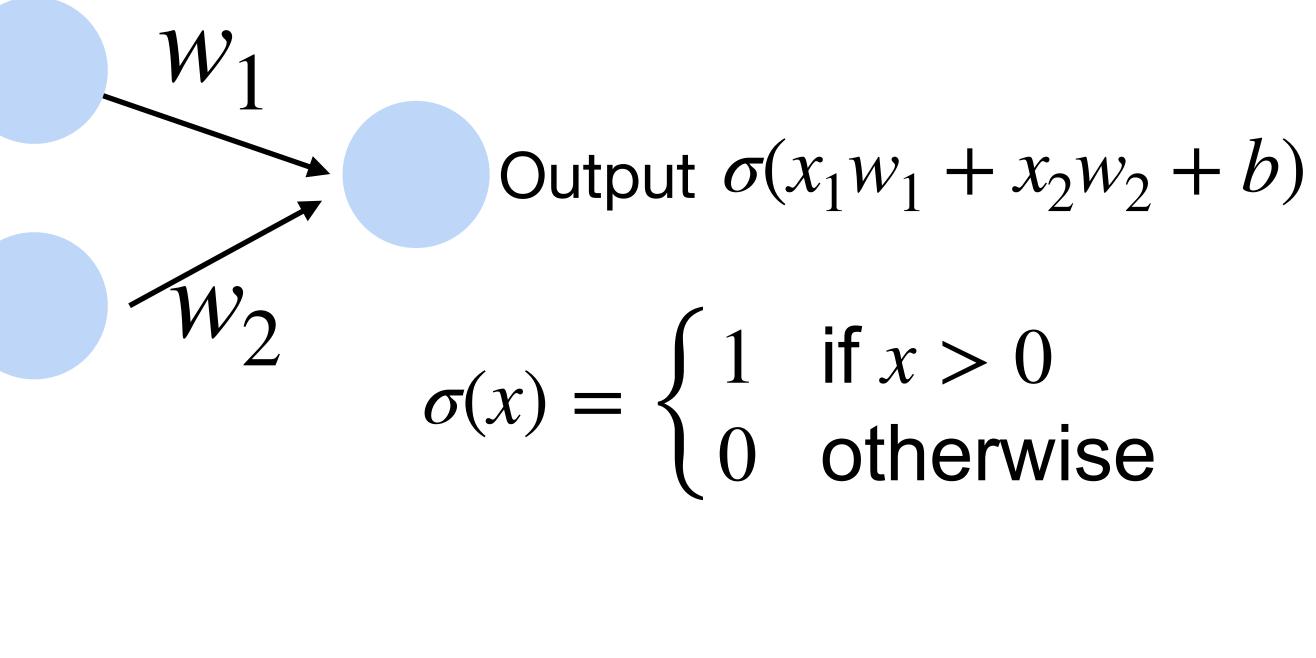


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Learning logic functions using perceptron The perceptron can learn an AND function

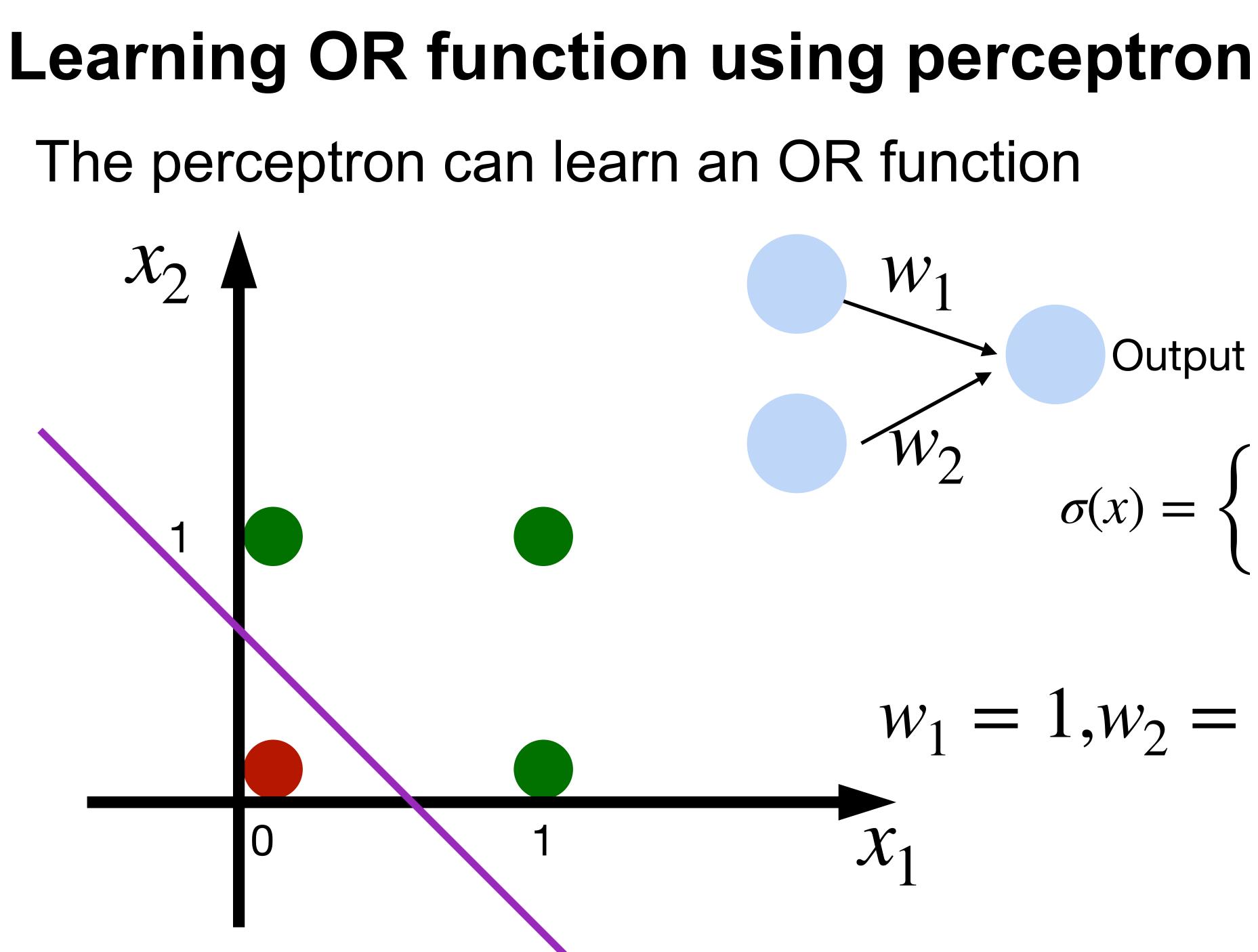


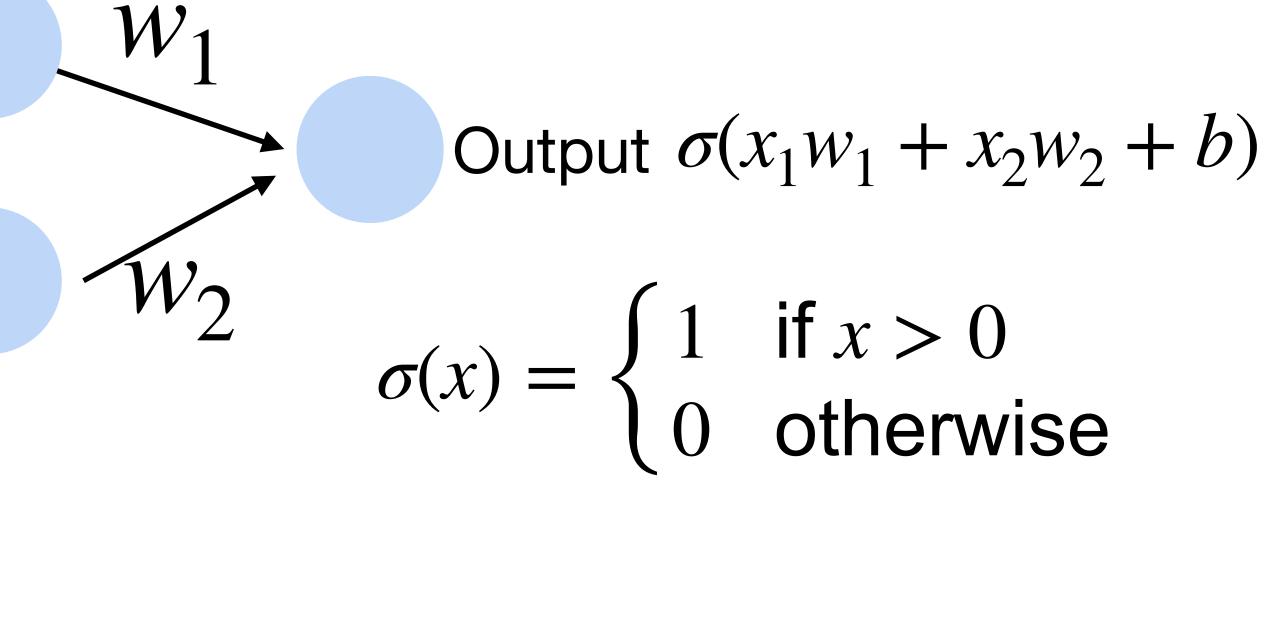


 $w_1 = 1, w_2 = 1, b = -1.5$









 $w_1 = 1, w_2 = 1, b = -0.5$

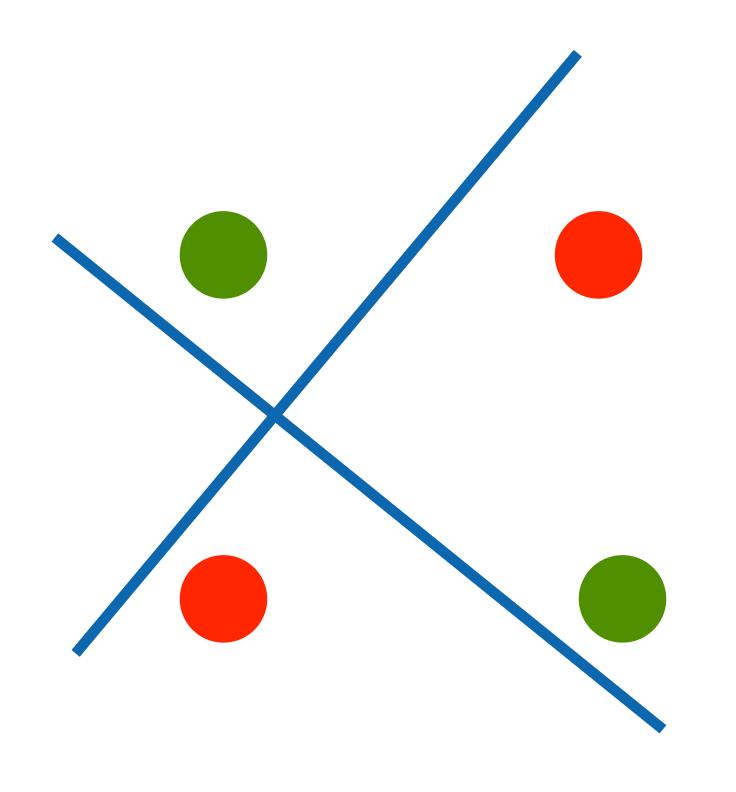




XOR Problem (Minsky & Papert, 1969)

The perceptron cannot learn an XOR function (neurons can only generate linear separators)

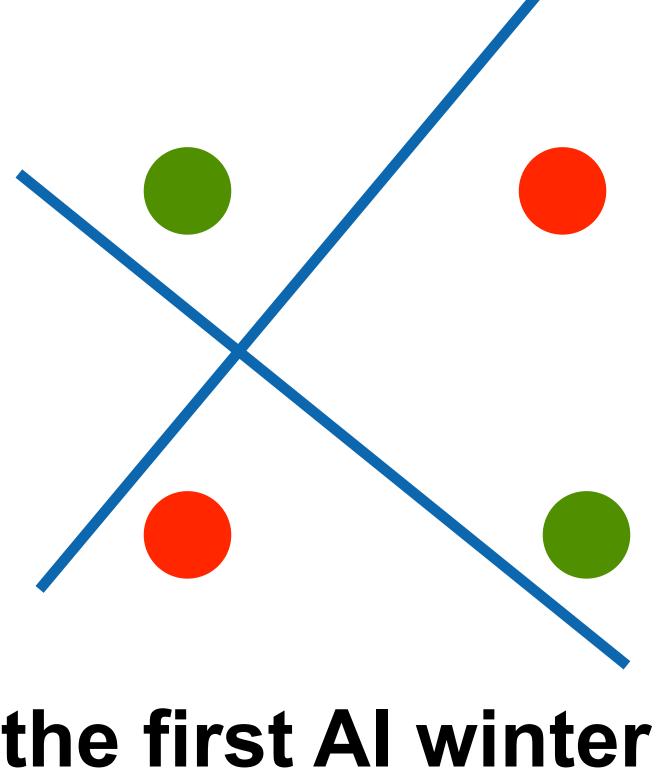
- $x_1 = 1, x_2 = 1, y = 0$ $x_1 = 1, x_2 = 0, y = 1$
- $x_1 = 0, x_2 = 1, y = 1$
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XOR Problem (Minsky & Papert, 1969)

The perceptron cannot learn an XOR function (neurons can only generate linear separators)

- $x_1 = 1, x_2 = 1, y = 0$
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This contributed to the first AI winter

Quiz break

Which one of the following is NOT true about perceptron?

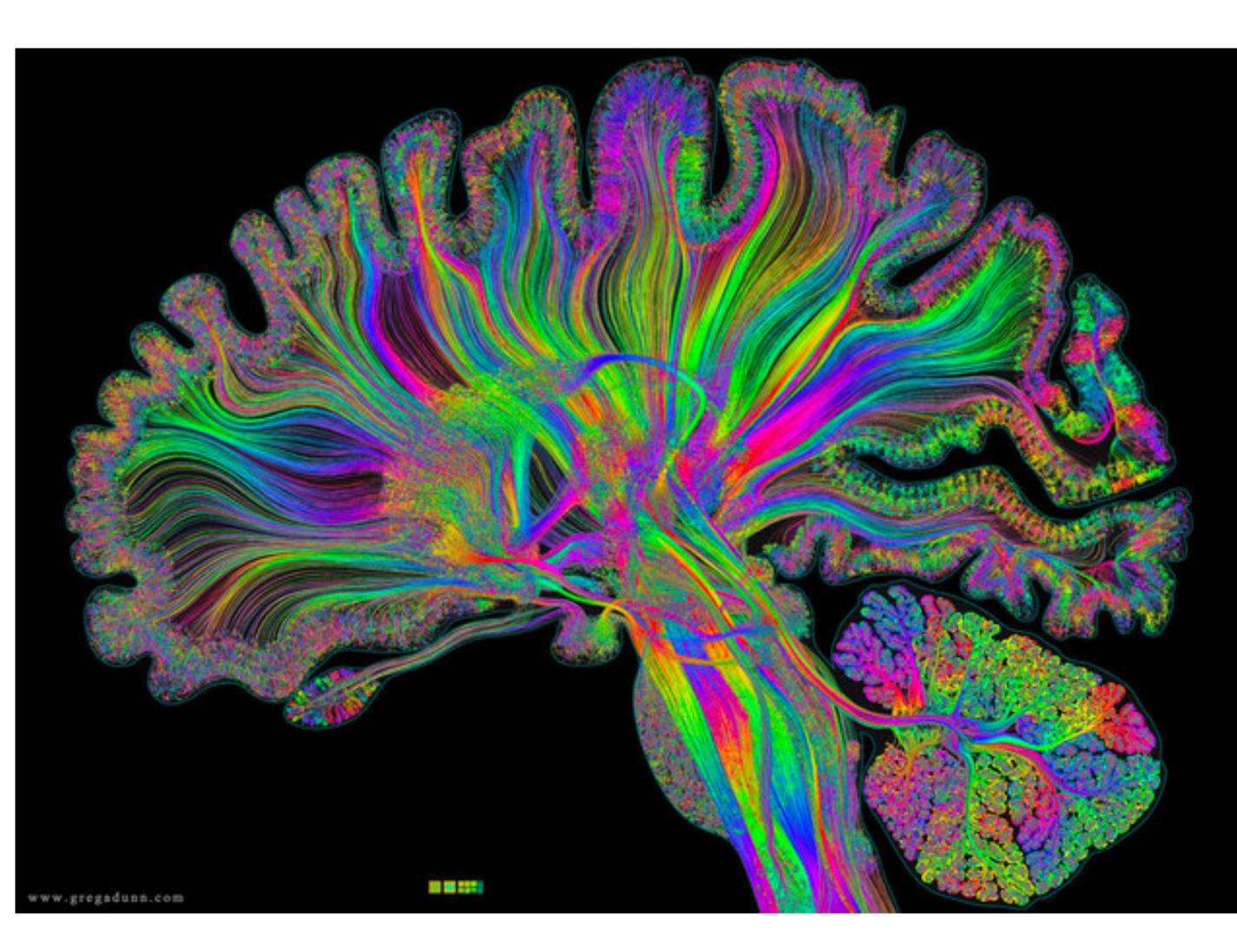
- A. Perceptron only works if the data is linearly separable.
- B. Perceptron can learn AND function
- C. Perceptron can learn XOR function
- D. Perceptron is a supervised learning algorithm

Quiz break

Which one of the following is NOT true about perceptron?

- A. Perceptron only works if the data is linearly separable.
- B. Perceptron can learn AND function
- C. Perceptron can learn XOR function
- D. Perceptron is a supervised learning algorithm

Multilayer Perceptron



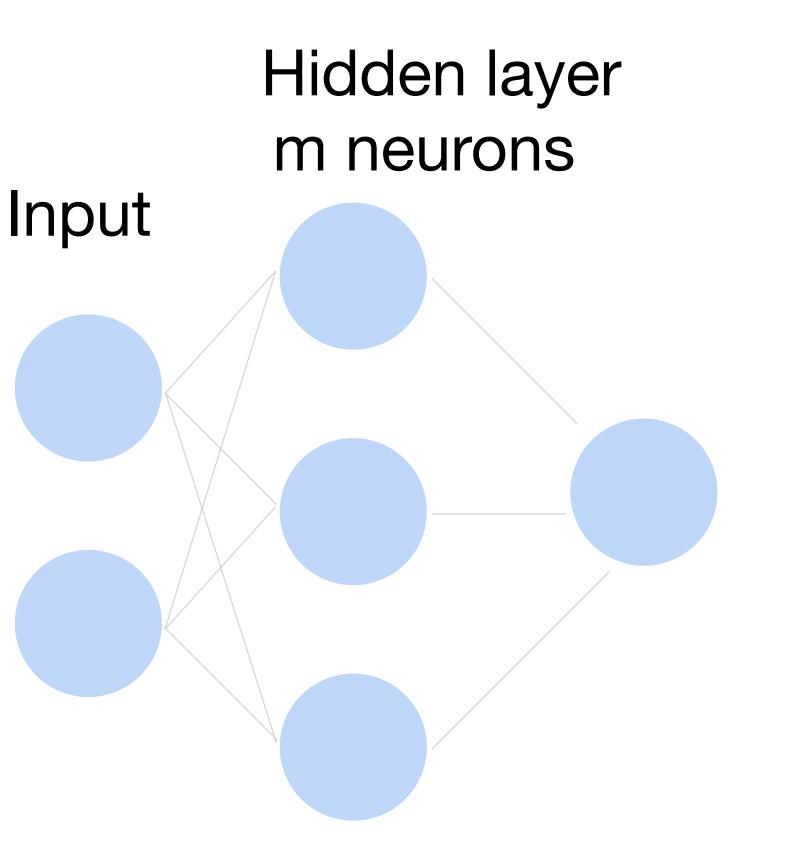
Single Hidden Layer

How to classify Cats vs. dogs?







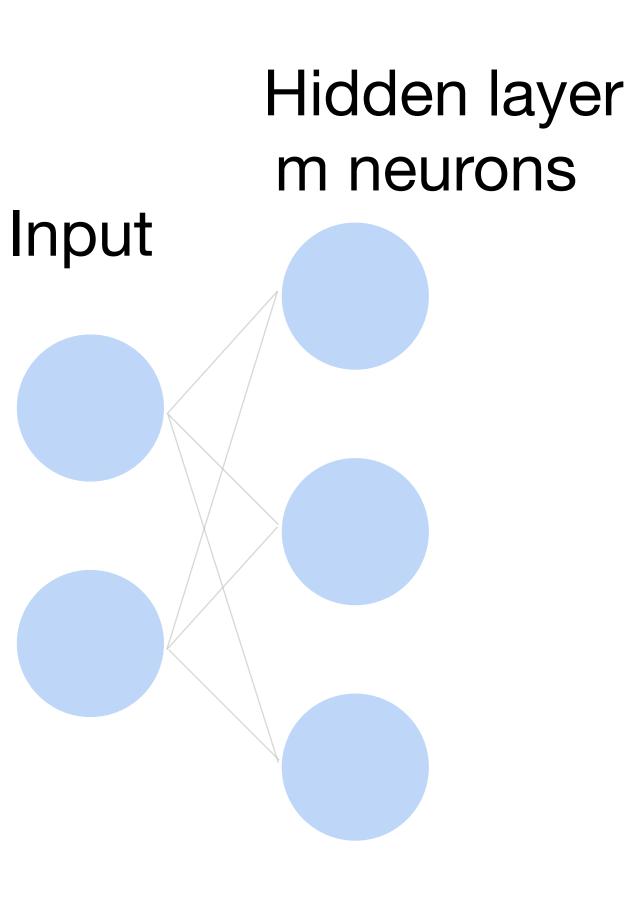


Single Hidden Layer

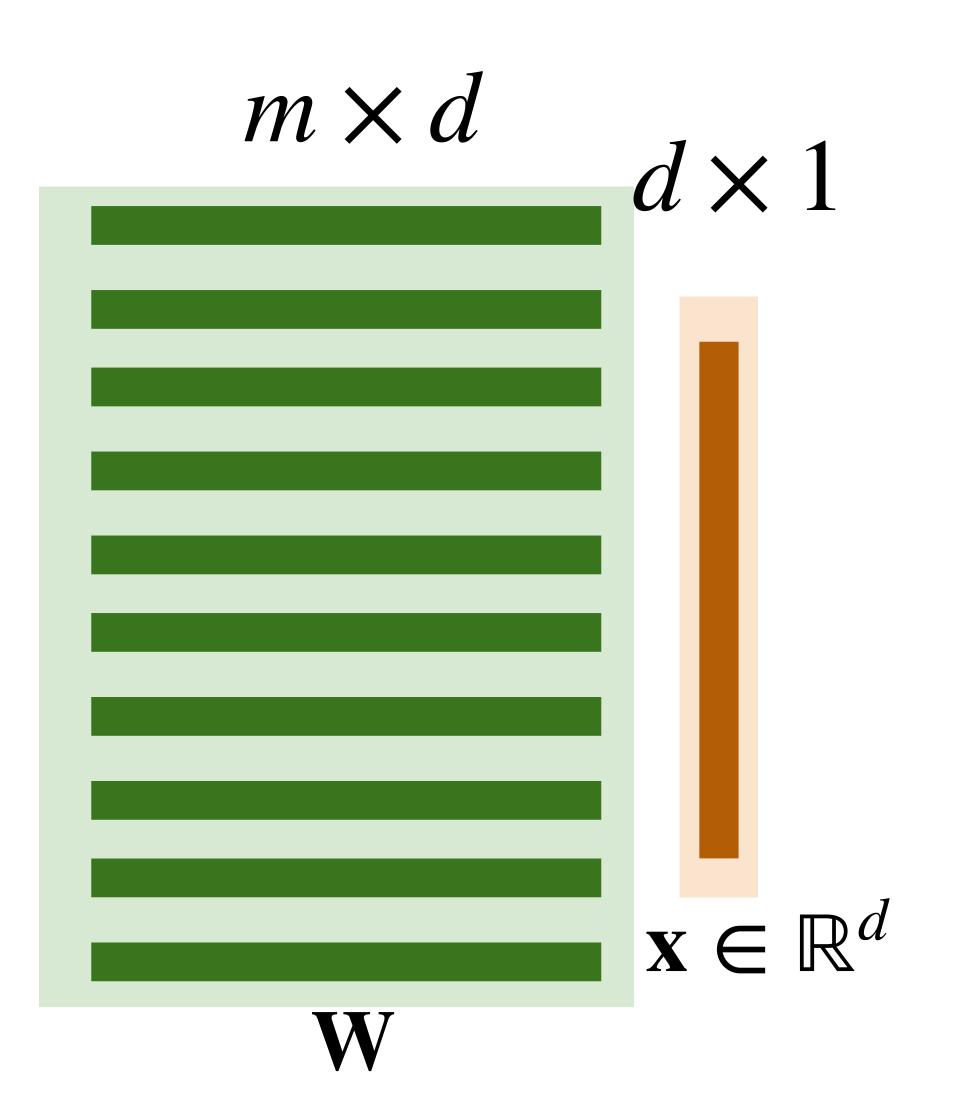
- Input $\mathbf{x} \in \mathbb{R}^d$
- Hidden $\mathbf{W} \in \mathbb{R}^{m \times d}, \mathbf{b} \in \mathbb{R}^m$
- Intermediate output

$\mathbf{h} = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$

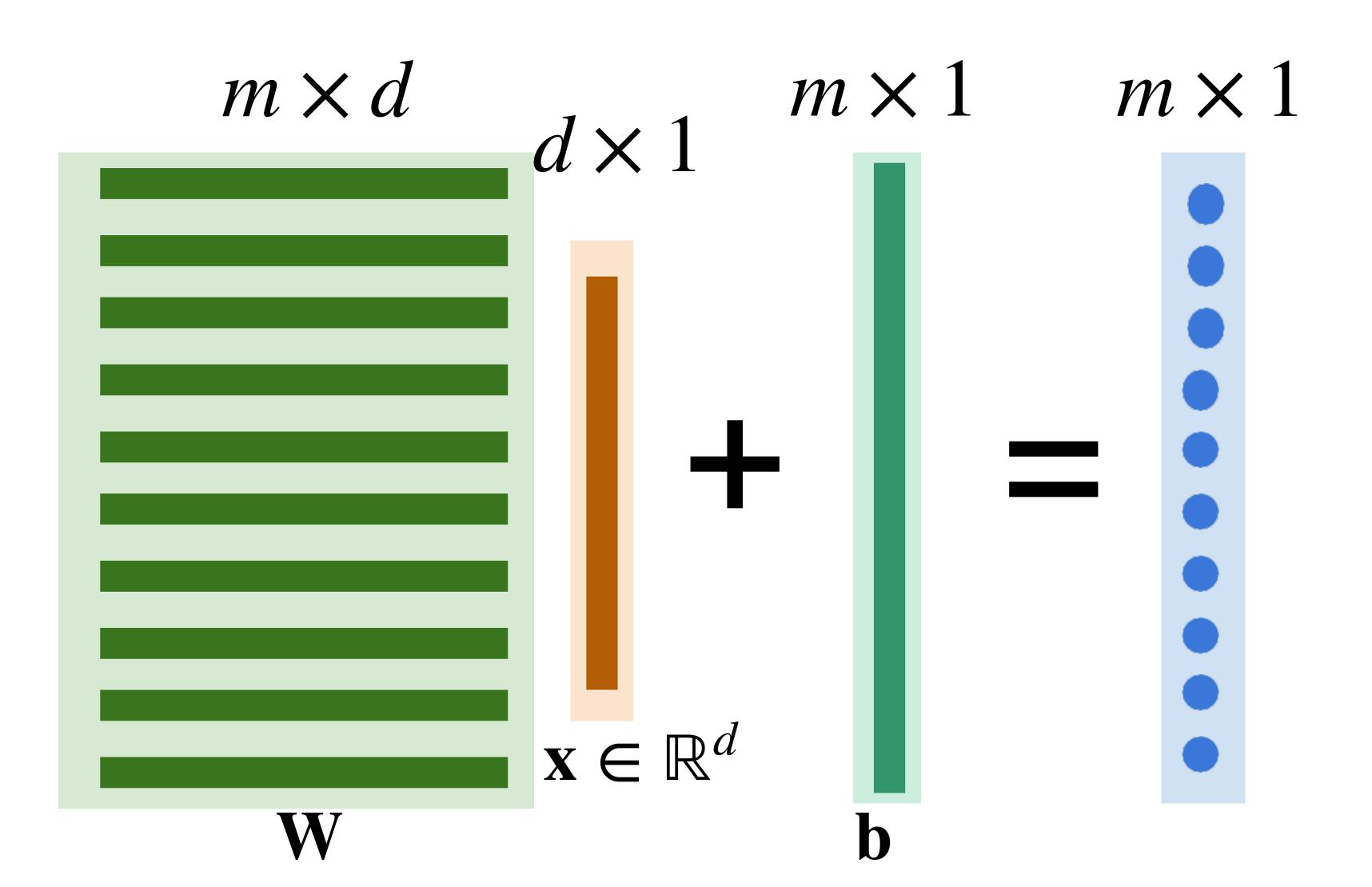
 σ is an element-wise activation function



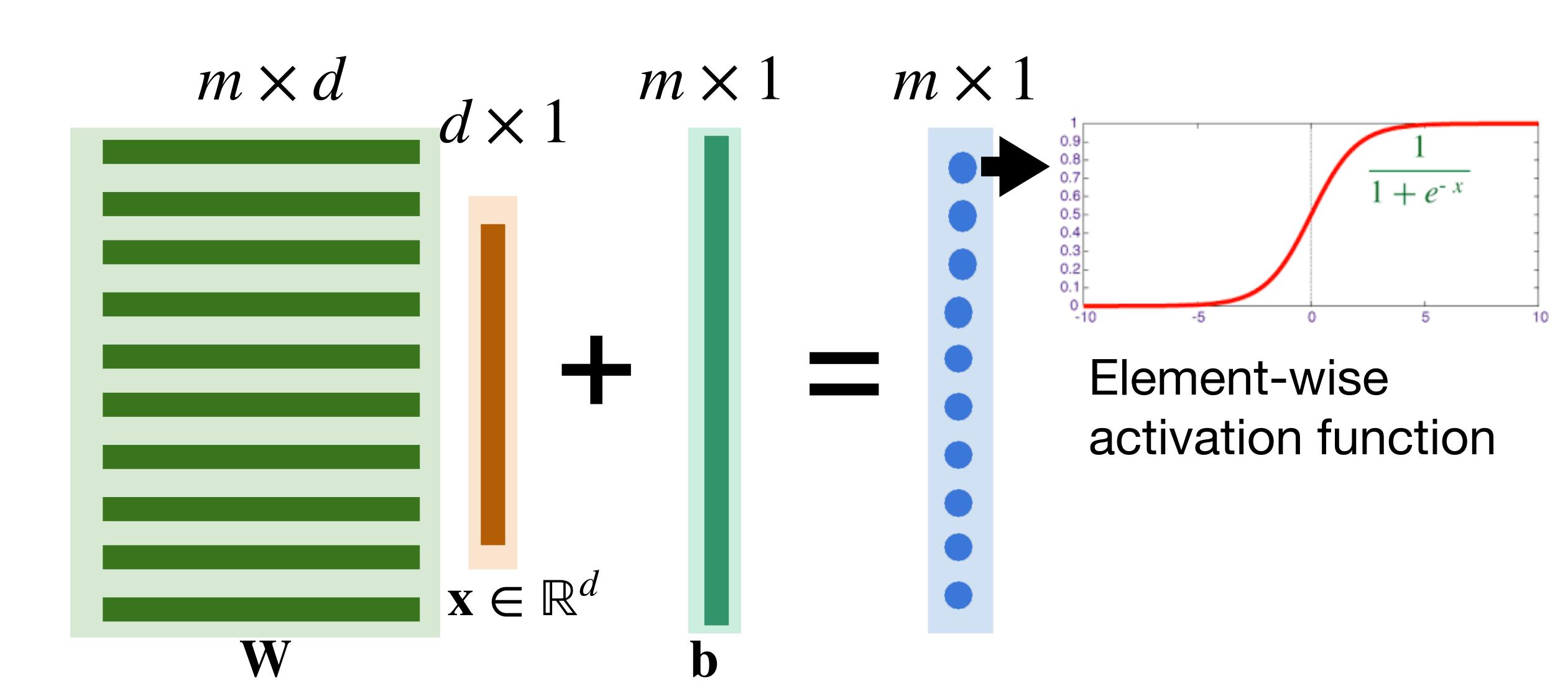
Neural networks with one hidden layer

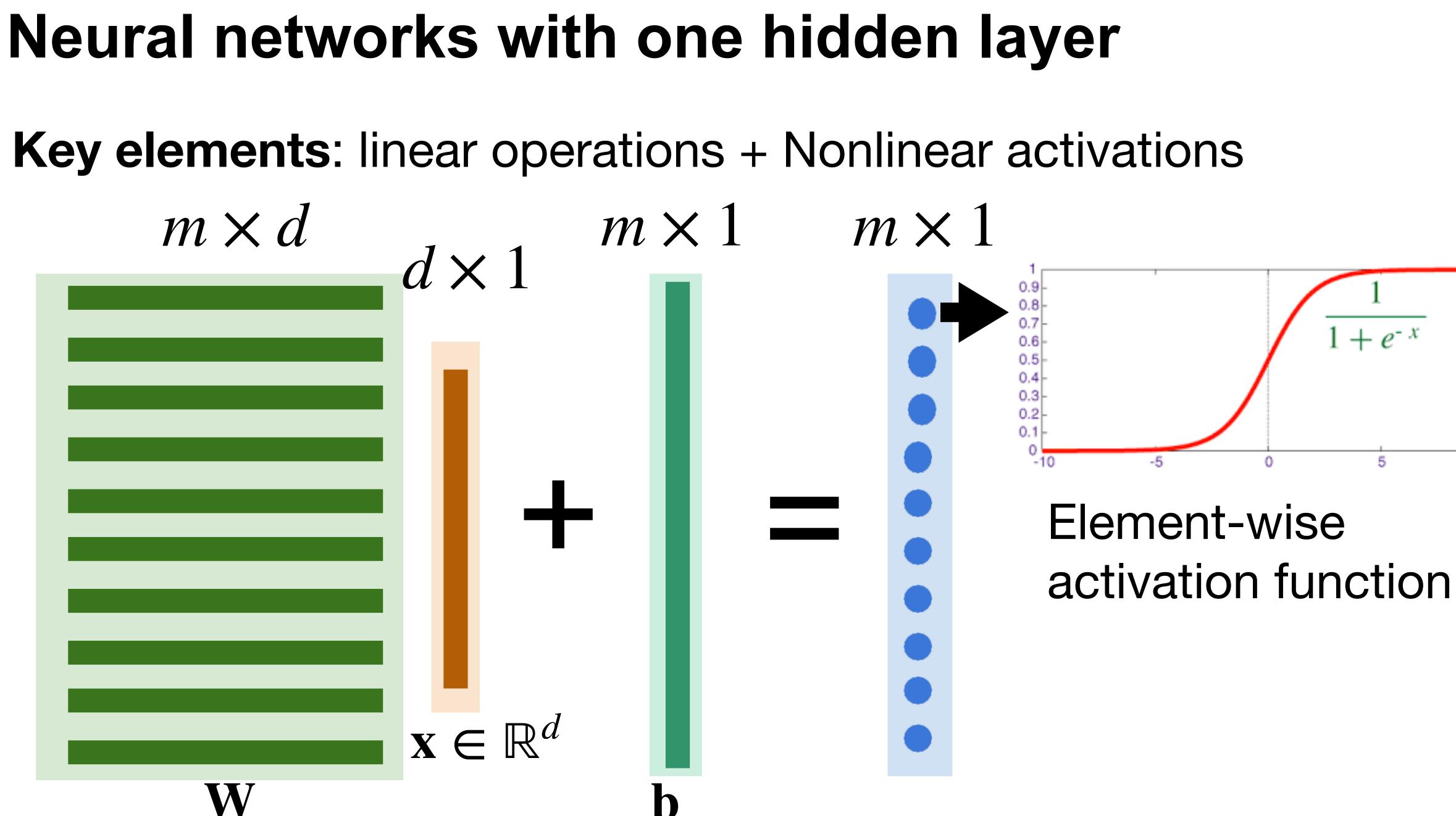


Neural networks with one hidden layer



Neural networks with one hidden layer





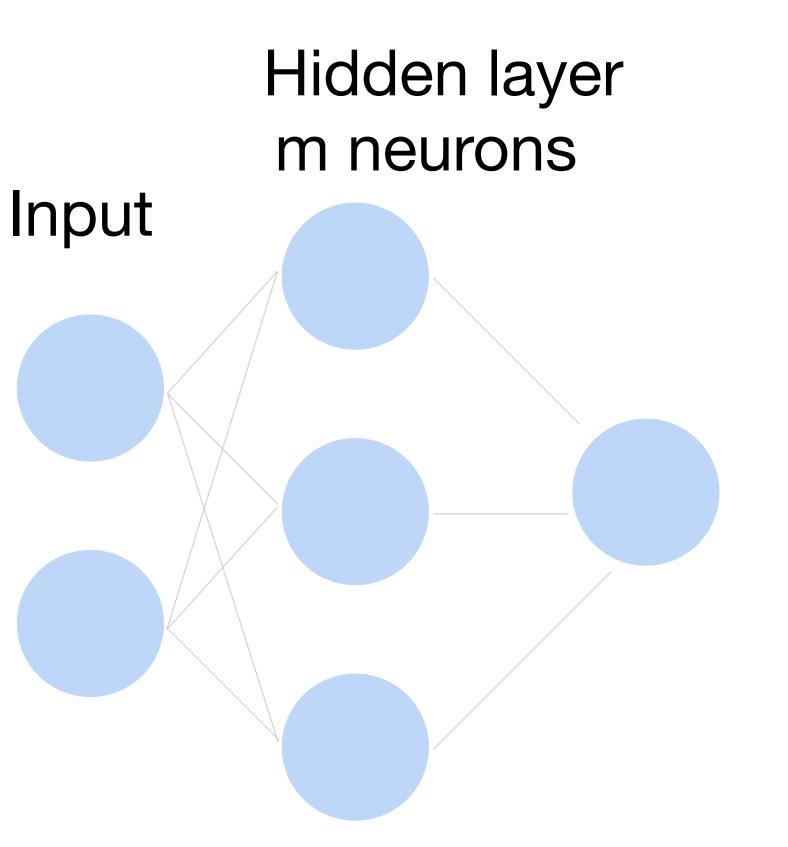




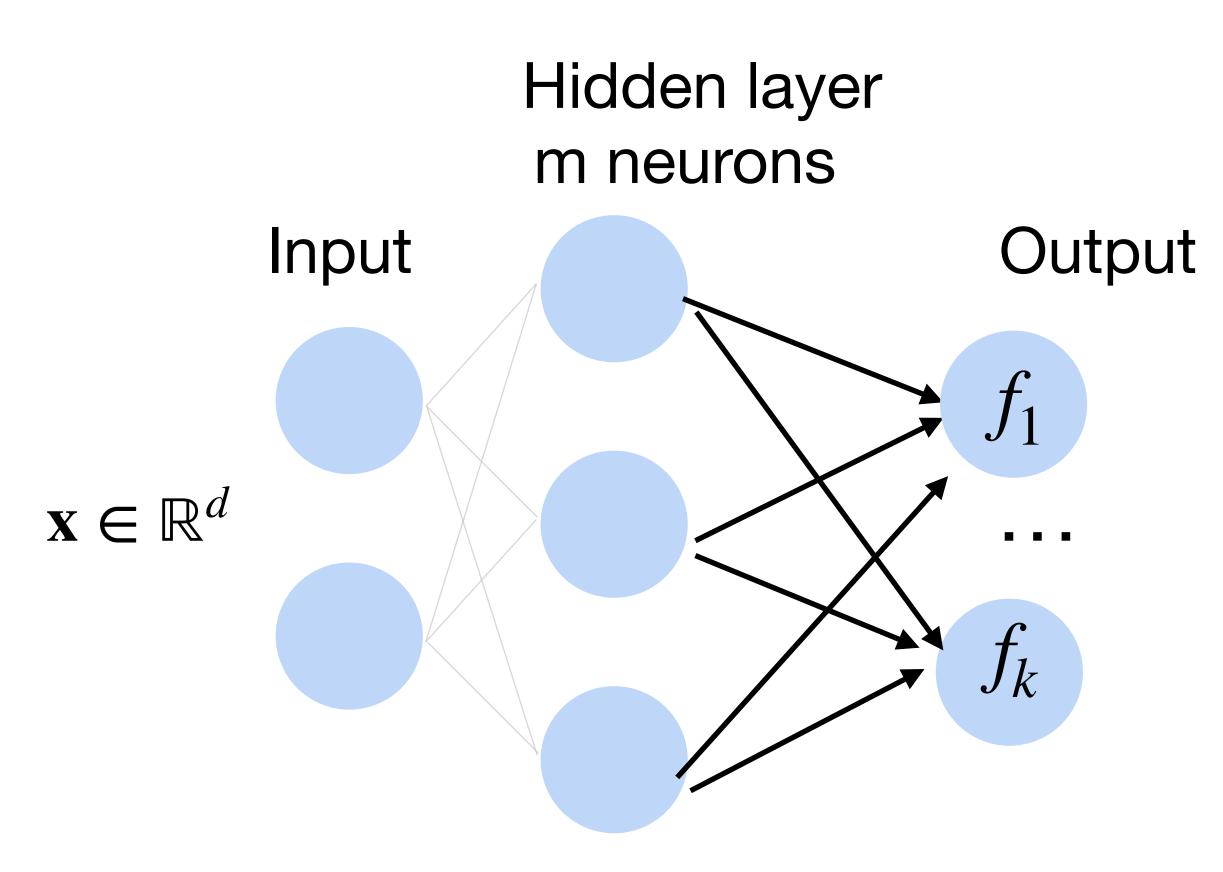
Single Hidden Layer

- Output $f = \mathbf{w}_2^{\mathsf{T}}\mathbf{h} + b_2$
- Normalize the output into probability using sigmoid

$$p(y = 1 | \mathbf{x}) = \frac{1}{1 + e^{-f}}$$



Multi-class classification

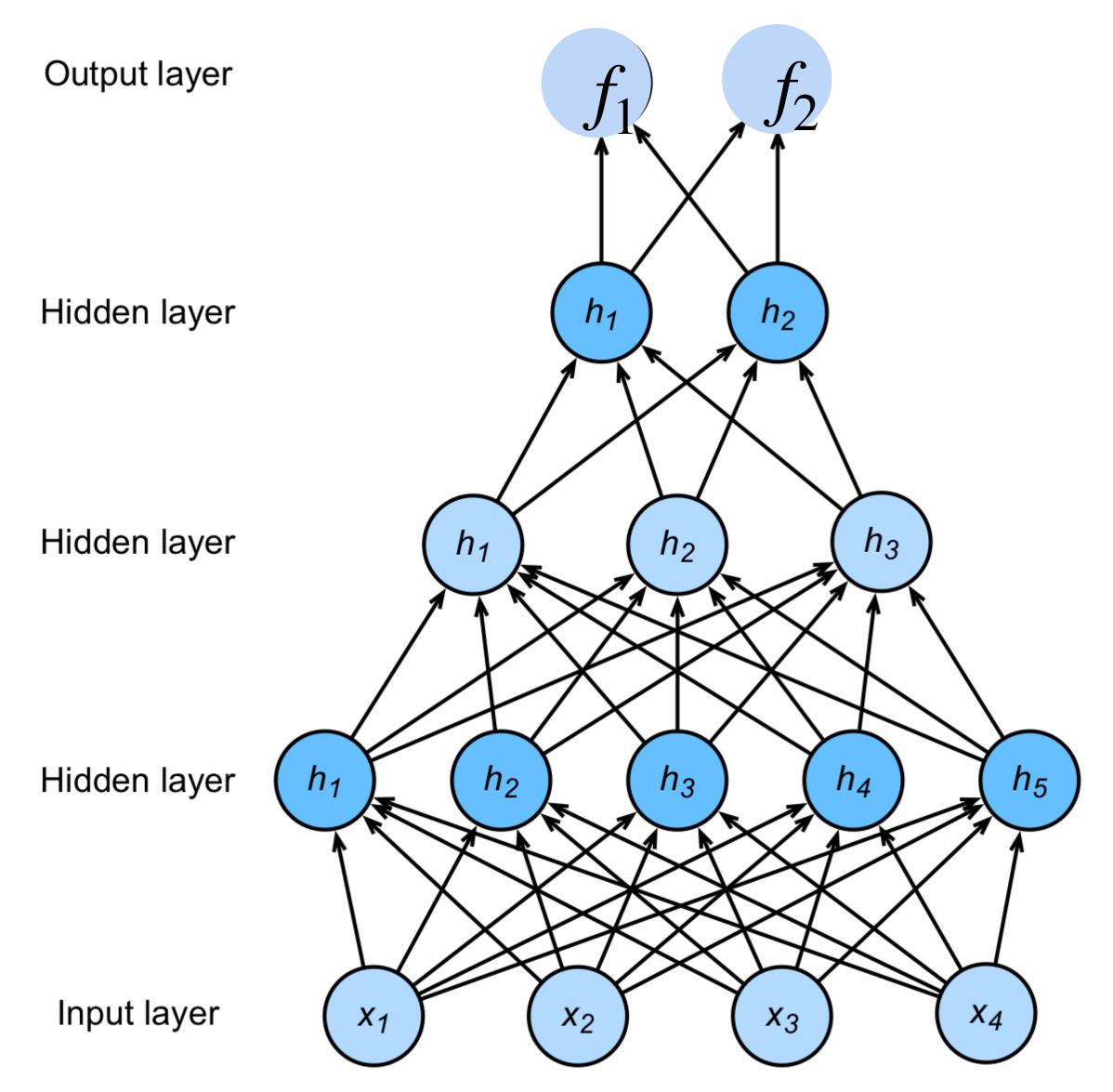


Turns outputs f into k probabilities (sum up to 1 across k classes)

$p(y | \mathbf{x}) = \text{softmax}(\mathbf{f})$ $\exp f_y(x)$ $\sum_{k}^{k} \exp f_{i}(x)$

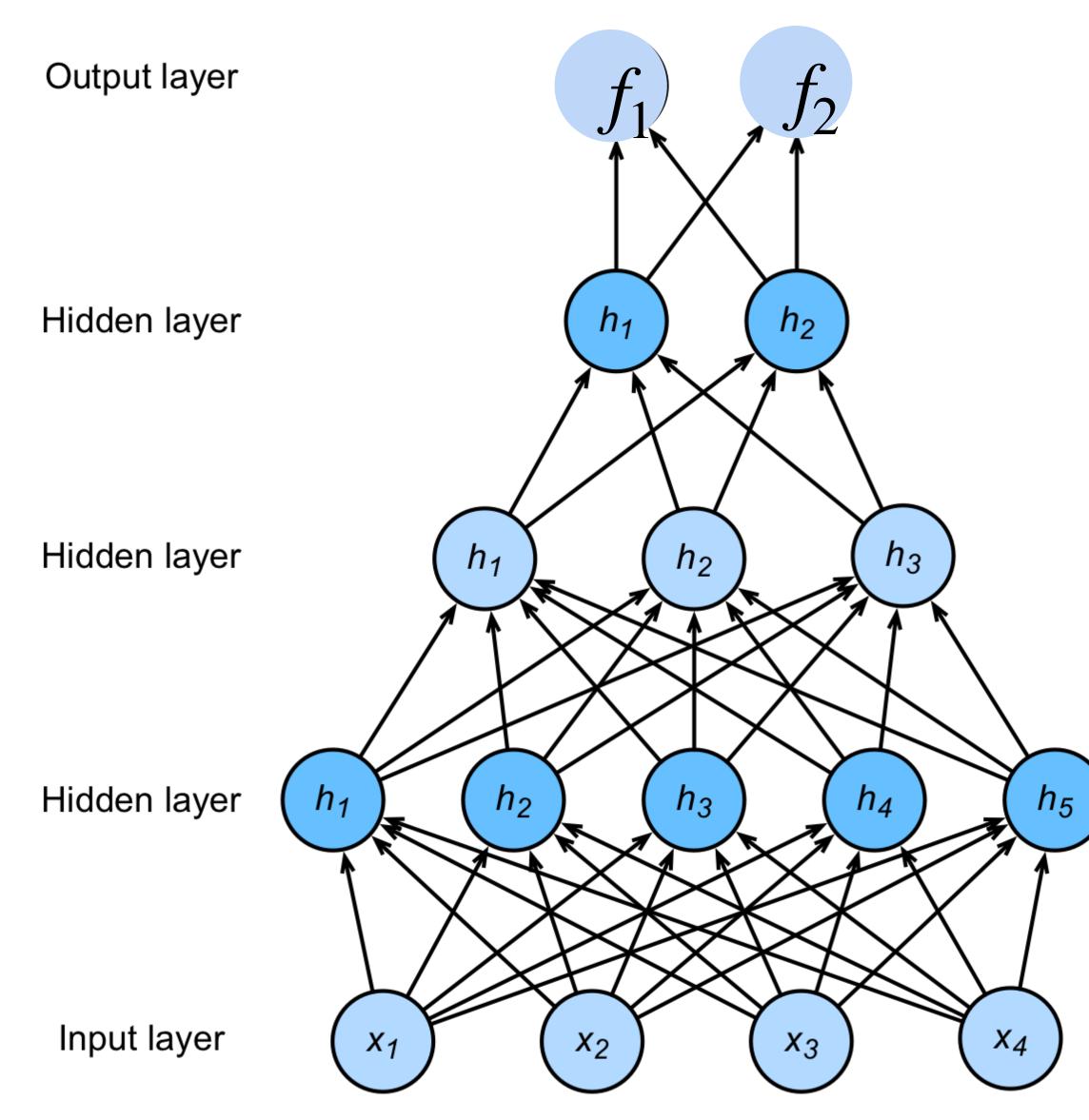


Deep neural networks (DNNs)



$\begin{aligned} \mathbf{h}_1 &= \sigma(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1) \\ \mathbf{h}_2 &= \sigma(\mathbf{W}_2 \mathbf{h}_1 + \mathbf{b}_2) \\ \mathbf{h}_3 &= \sigma(\mathbf{W}_3 \mathbf{h}_2 + \mathbf{b}_3) \\ \mathbf{f} &= \mathbf{W}_4 \mathbf{h}_3 + \mathbf{b}_4 \\ \mathbf{y} &= \mathrm{softmax}(\mathbf{f}) \end{aligned}$

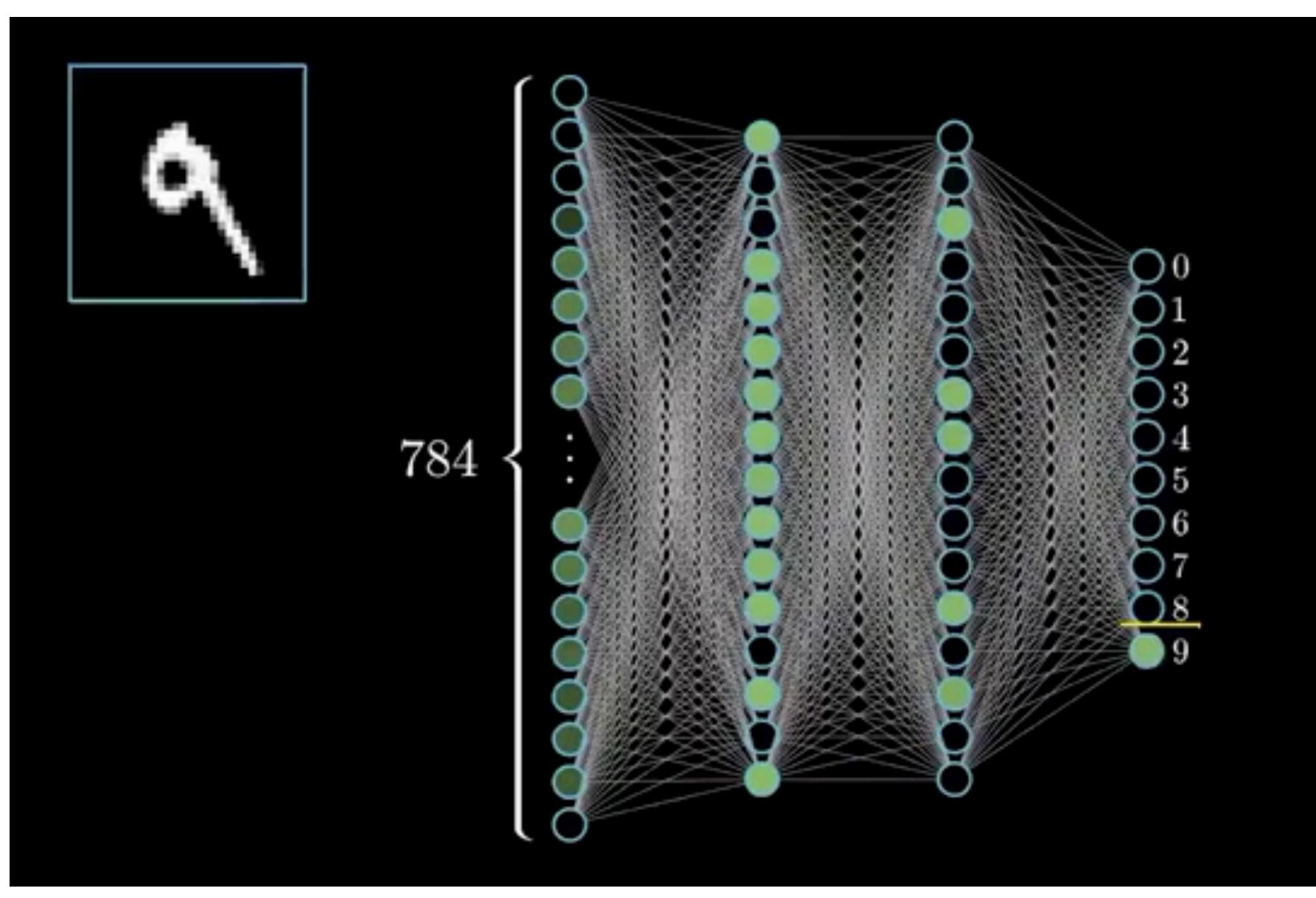
Deep neural networks (DNNs)



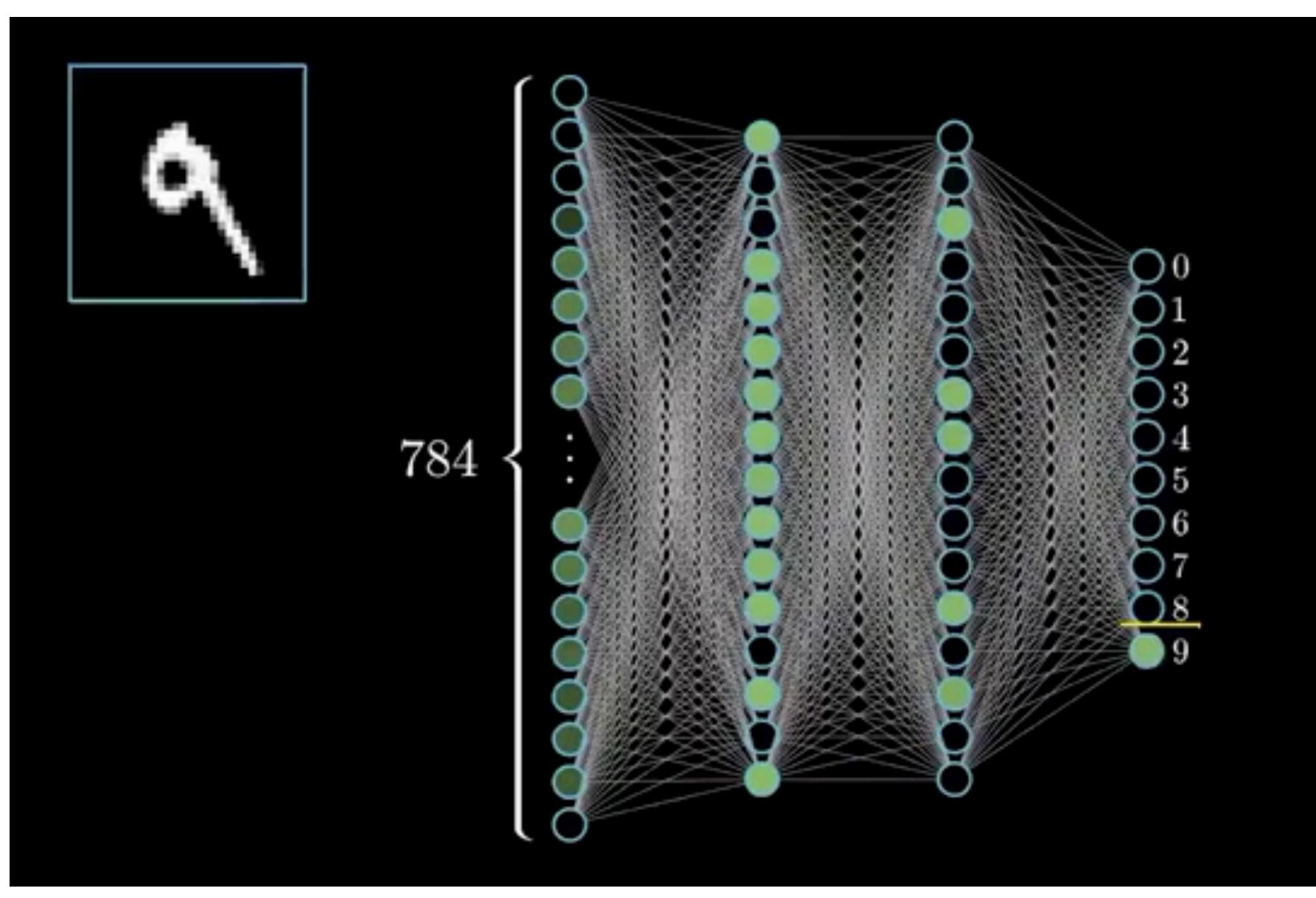
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NNs are composition of nonlinear functions

Classify MNIST handwritten digits



Classify MNIST handwritten digits



How to train a neural network? Loss function: $\frac{1}{|D|} \sum_{i} \ell(\mathbf{x}_{i}, y_{i})$

Hidden layer m neurons Input



How to train a neural network? Loss function: $\frac{1}{|D|} \sum_{i} \ell(\mathbf{x}_{i}, y_{i})$ **Per-sample loss:**

K $\ell(\mathbf{x}, y) = \sum_{j=1}^{\infty} -y_j \log p_j$ *j*=1

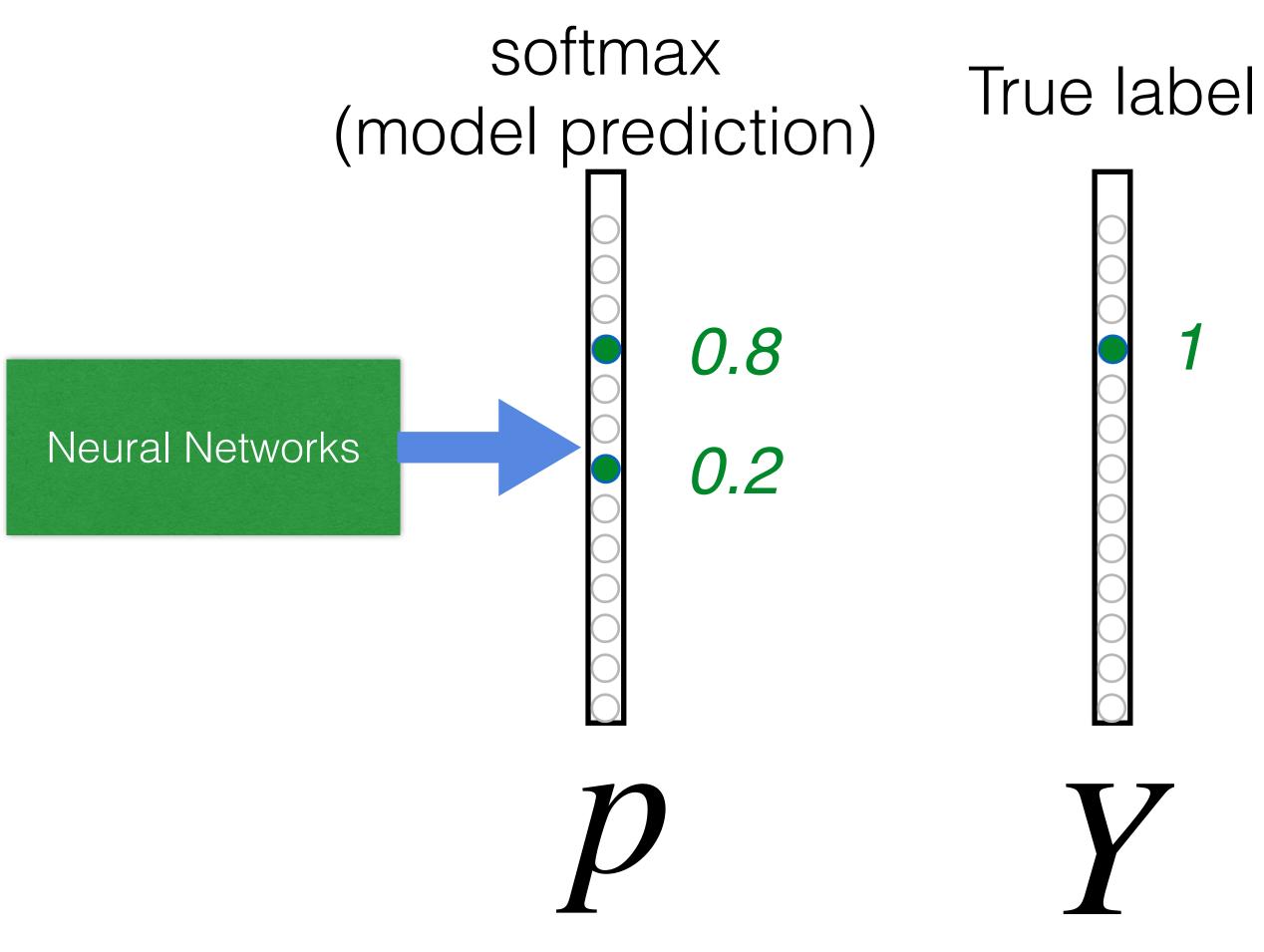
Hidden layer m neurons Input



How to train a neural network? **Loss function:** $\frac{1}{|D|} \sum_{i} \ell(\mathbf{x}_{i}, y_{i})$ Hidden layer **Per-sample loss:** m neurons Input K $\ell(\mathbf{x}, y) = \sum_{j=1}^{\infty} -y_j \log p_j$ *j*=1 Also known as cross-entropy loss or softmax loss



Cross-Entropy Loss



$L_{CE} = \sum_{j=1}^{\infty} -y_j \log(p_j)$ $= -\log(0.8)$

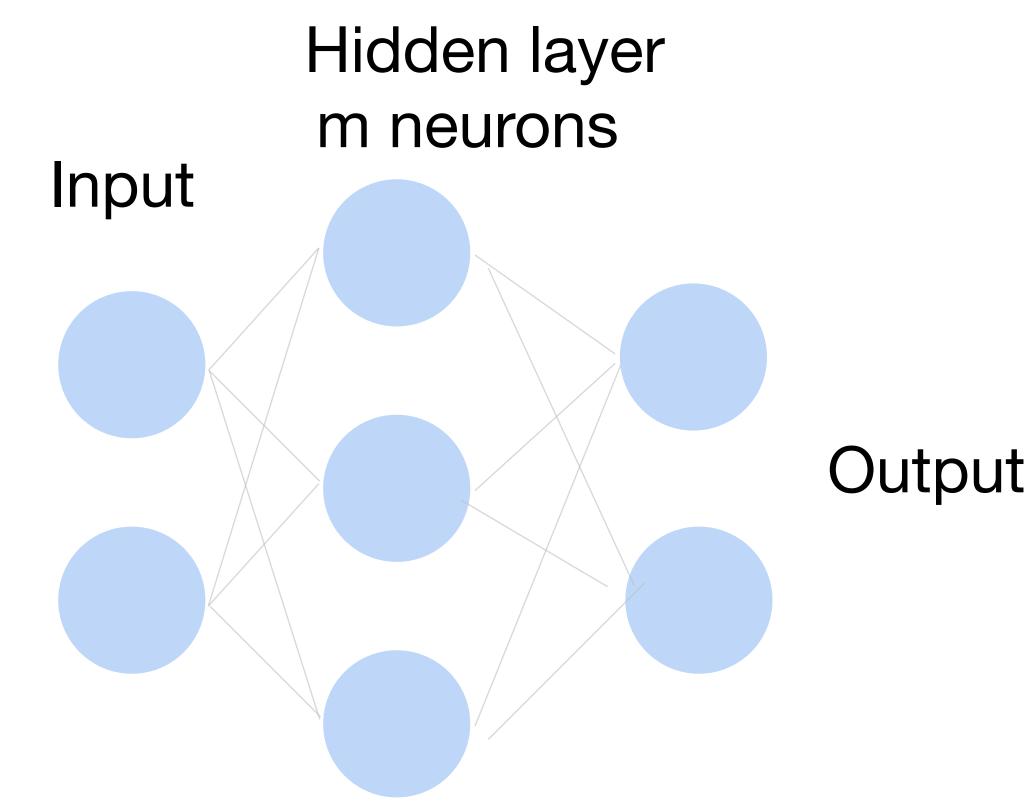
Goal: push **p** and **Y** to be identical

How to train a neural network?

Update the weights W to minimize the loss function

 $L = \frac{1}{|D|} \sum_{i} \ell(\mathbf{x}_{i}, y_{i})$

Use gradient descent!



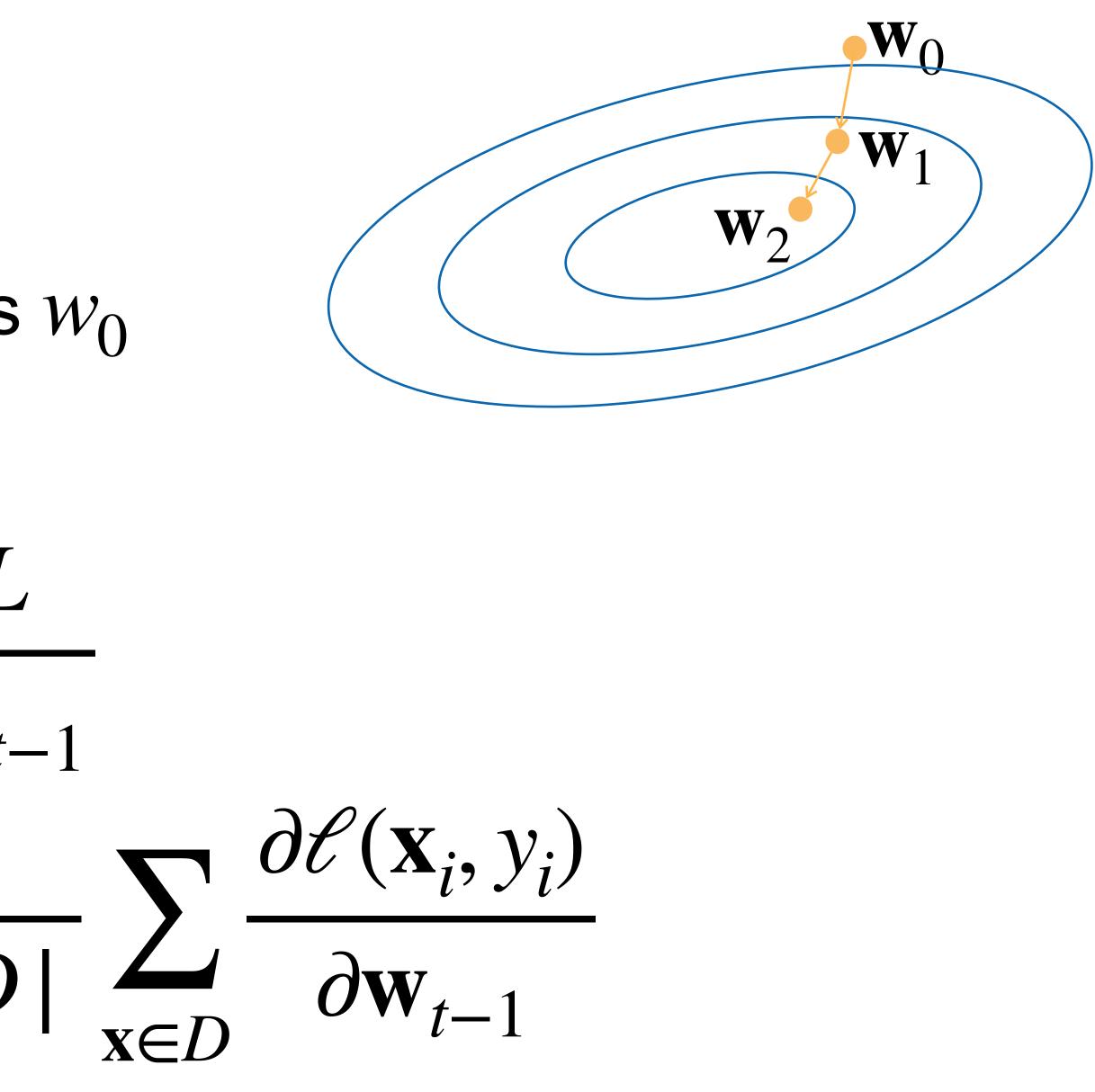


Gradient Descent

- Choose a learning rate $\alpha > 0$
- Initialize the model parameters w_0
- For t =1,2,...

• Update parameters: $\mathbf{w}_{t} = \mathbf{w}_{t-1} - \alpha \frac{\partial L}{\partial \mathbf{w}_{t-1}}$ $= \mathbf{w}_{t-1} - \alpha \frac{1}{|D|}$

Repeat until converges

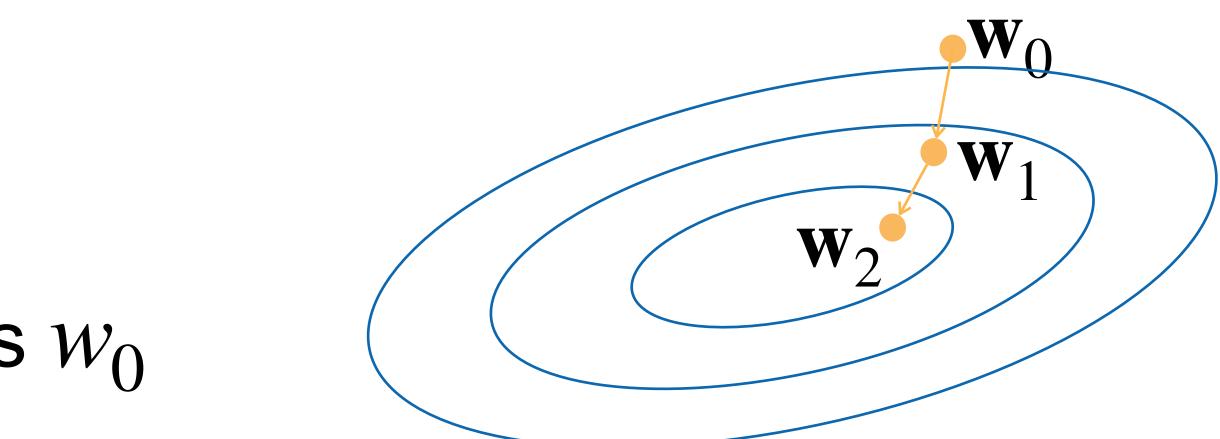


Gradient Descent

- Choose a learning rate $\alpha > 0$
- Initialize the model parameters w_0
- For t =1,2,...

• Update parameters: D can ∂L $\mathbf{w}_t = \mathbf{w}_{t-1} - \alpha \frac{\partial \mathbf{w}_{t-1}}{\partial \mathbf{w}_{t-1}}$ (\mathbf{X}_i, y_i) $= \mathbf{W}_{t-1}$ x∈D

Repeat until converges



be very large. Expensive

Minibatch Stochastic Gradient Descent

- Choose a learning rate $\alpha > 0$
- Initialize the model parameters W_0
- For t =1,2,...
 - Randomly sample a subset (mini-batch) $B \subset D$ Update parameters:

$$\mathbf{w}_t = \mathbf{w}_{t-1} - \alpha_{\parallel 1}$$

• Repeat

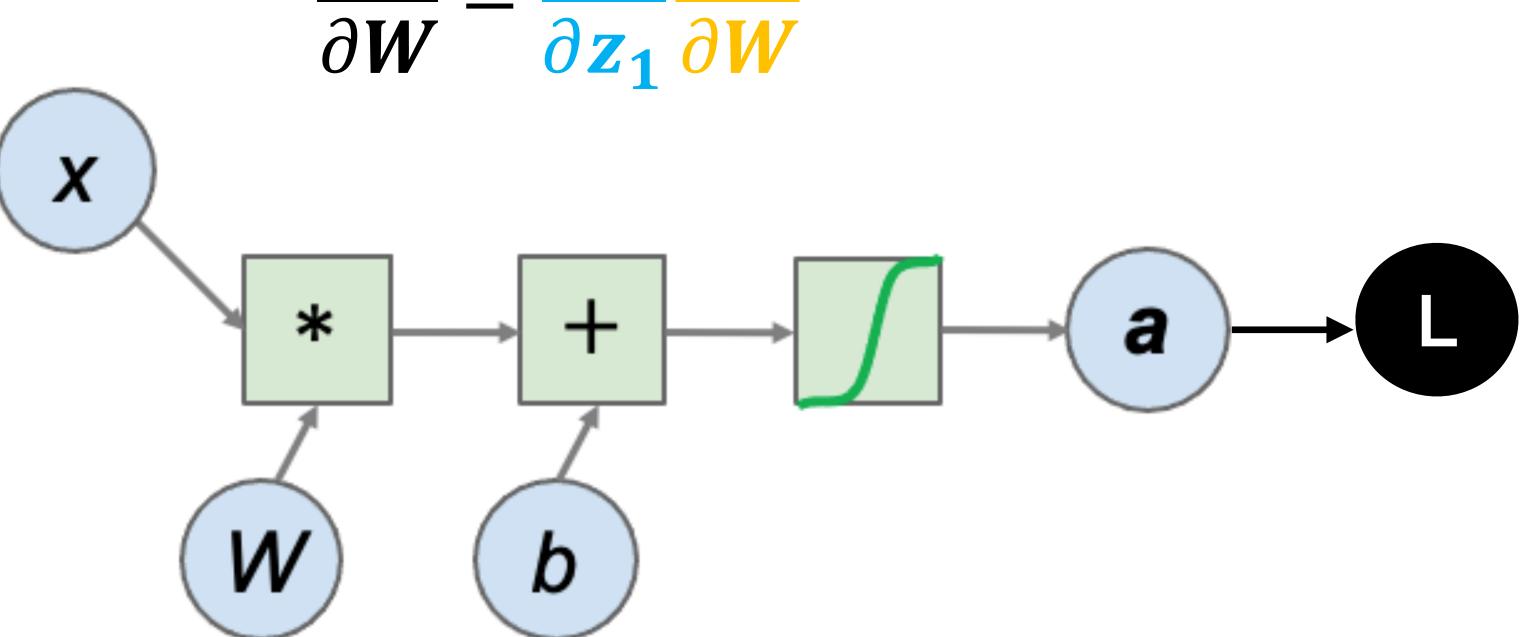
 $\frac{1}{B|} \sum_{\mathbf{x} \in B} \frac{\partial \ell(\mathbf{x}_i, y_i)}{\partial \mathbf{w}_{t-1}}$

weights and biases.

- weights and biases.
- Gradient to a variable =

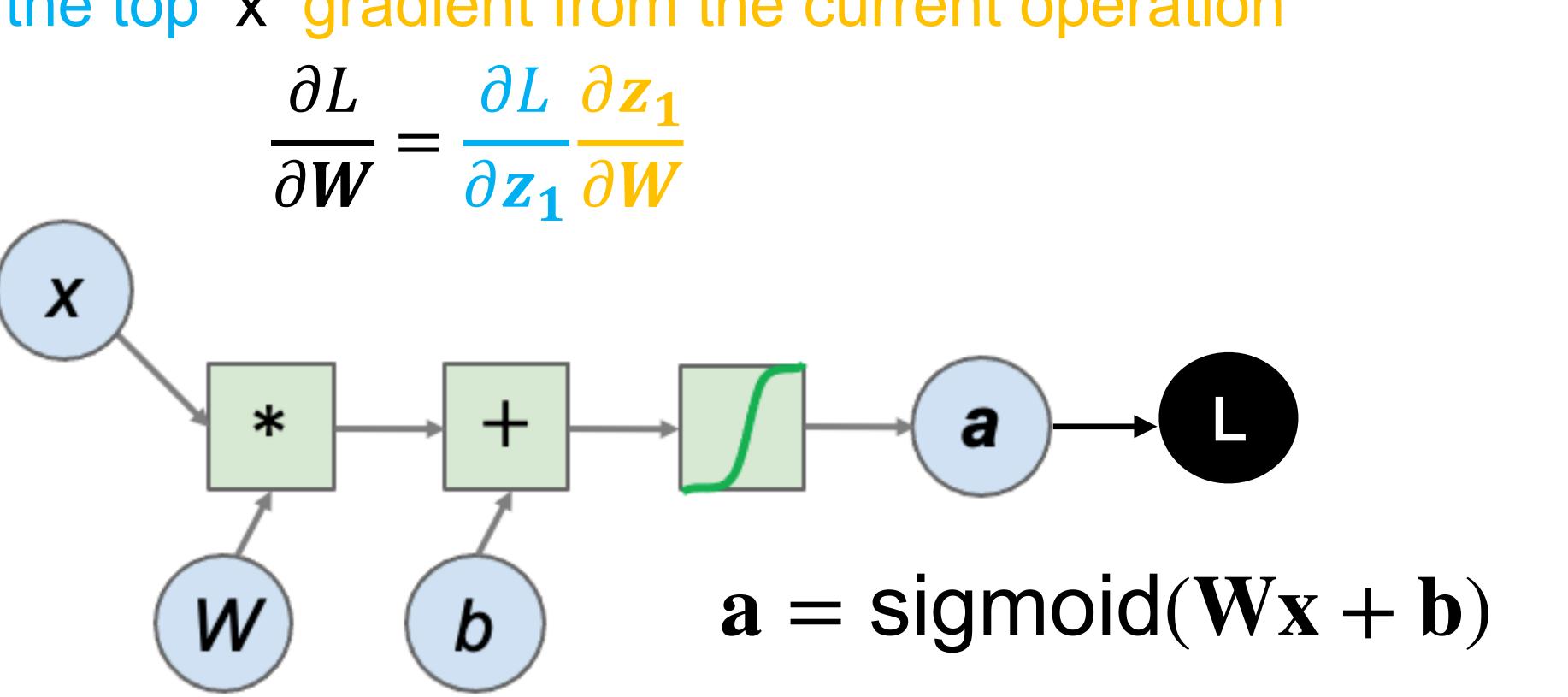
gradient on the top x gradient from the current operation $\frac{\partial L}{\partial W} = \frac{\partial L}{\partial z_1} \frac{\partial z_1}{\partial W}$

- weights and biases.
- Gradient to a variable =



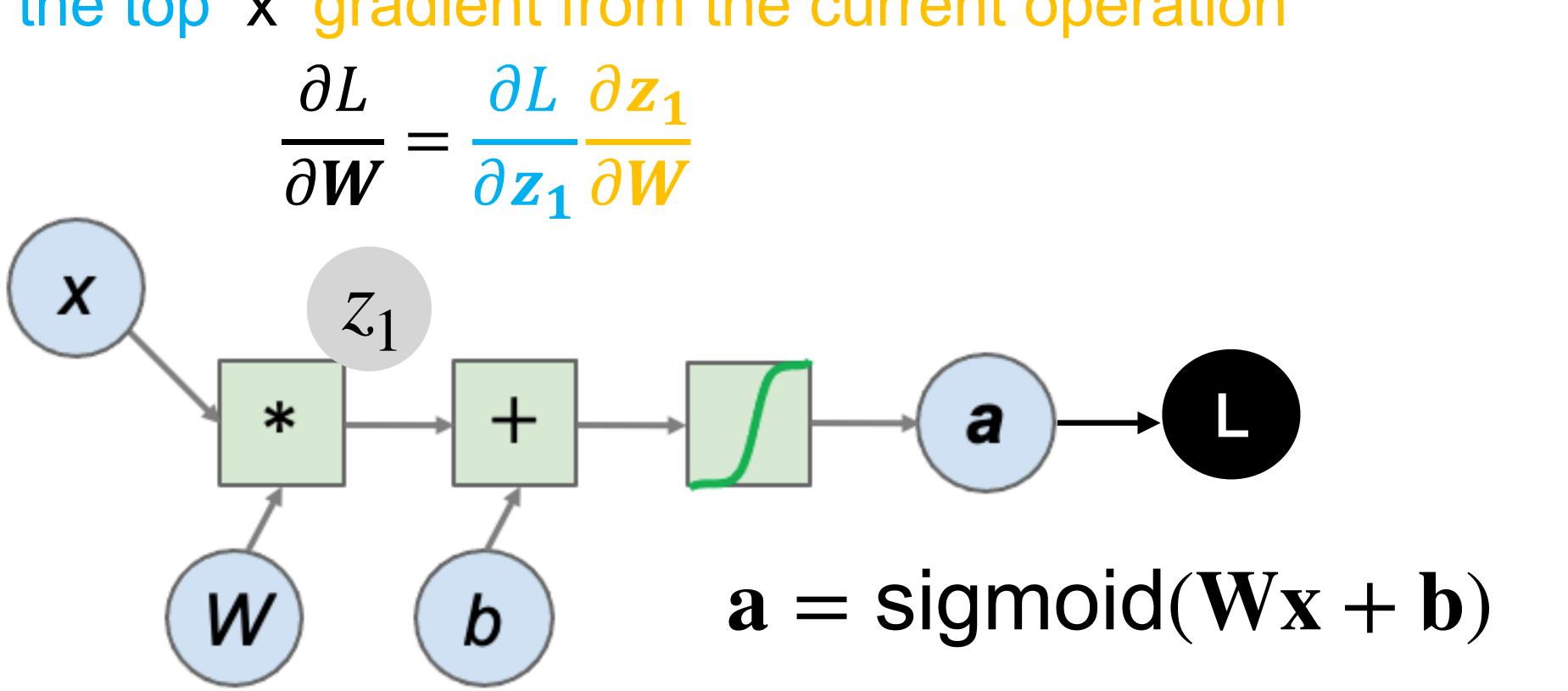
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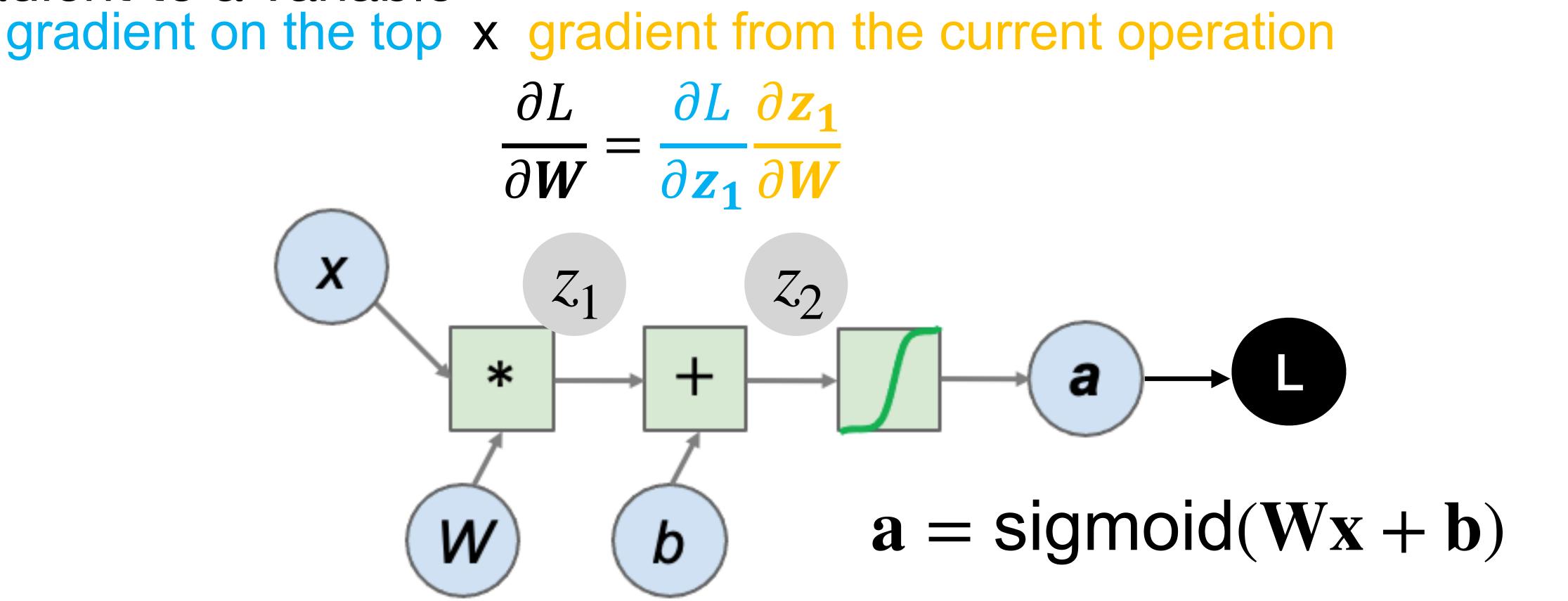
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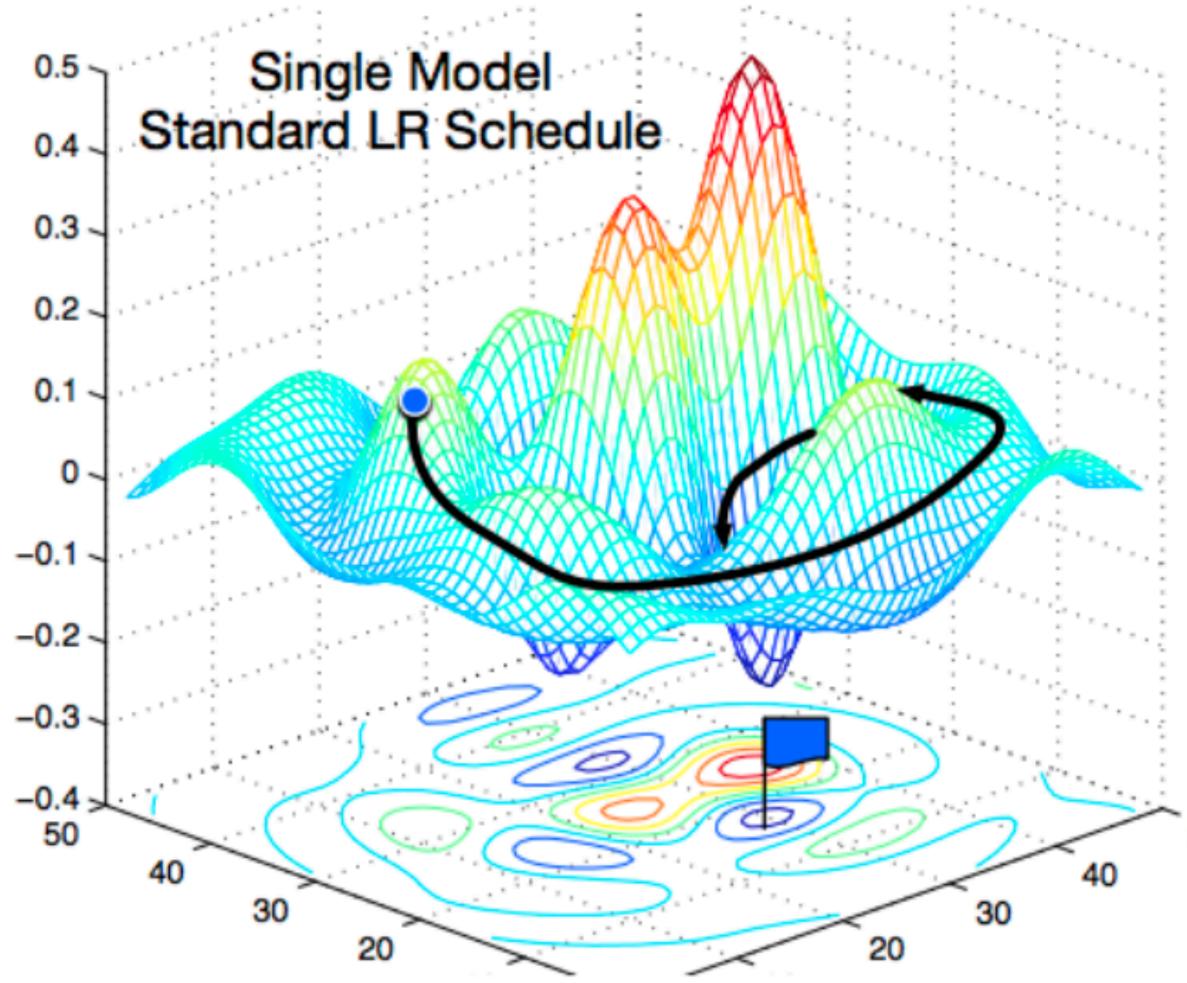


gradient on the top x gradient from the current operation

- weights and biases.
- Gradient to a variable =



Non-convex Optimization



[Gao and Li et al., 2018]



How to classify Cats vs. dogs?







How to classify Cats vs. dogs?









Dual 1210P wide-angle and

telephoto cameras

36M floats in a RGB image!

Fully Connected Networks

Cats vs. dogs?







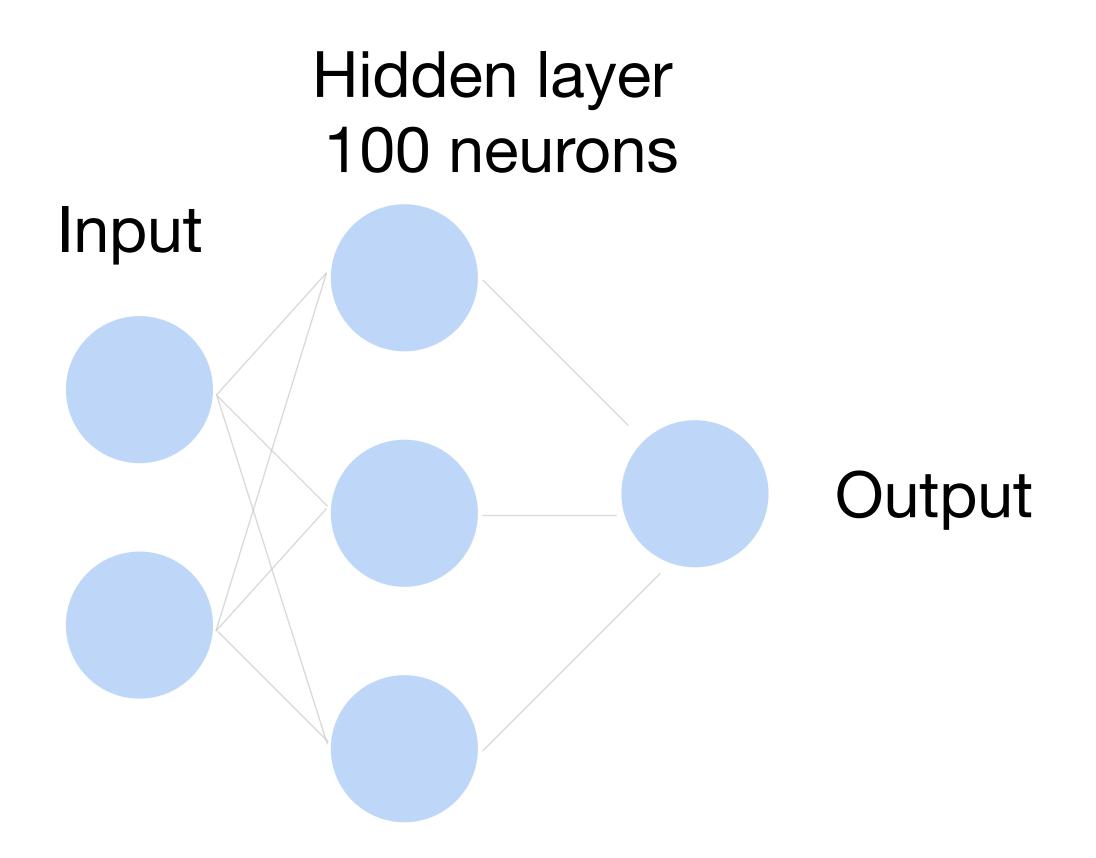
Fully Connected Networks

Cats vs. dogs?









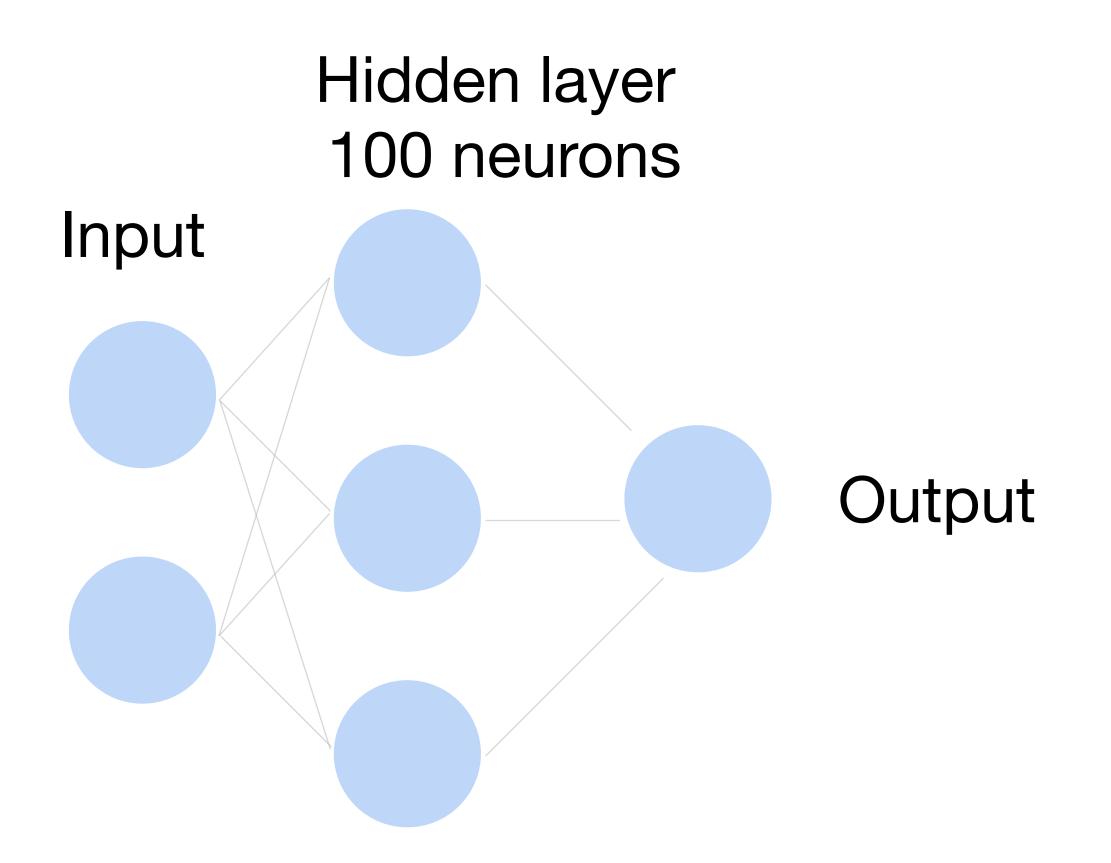
Fully Connected Networks

Cats vs. dogs?









~ 36M elements x 100 = ~3.6B parameters!

Convolutions come to rescue!

Where is Waldo?





Why Convolution?

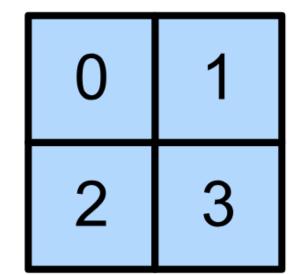
- Translation
 Invariance
- Locality



Input

Kernel

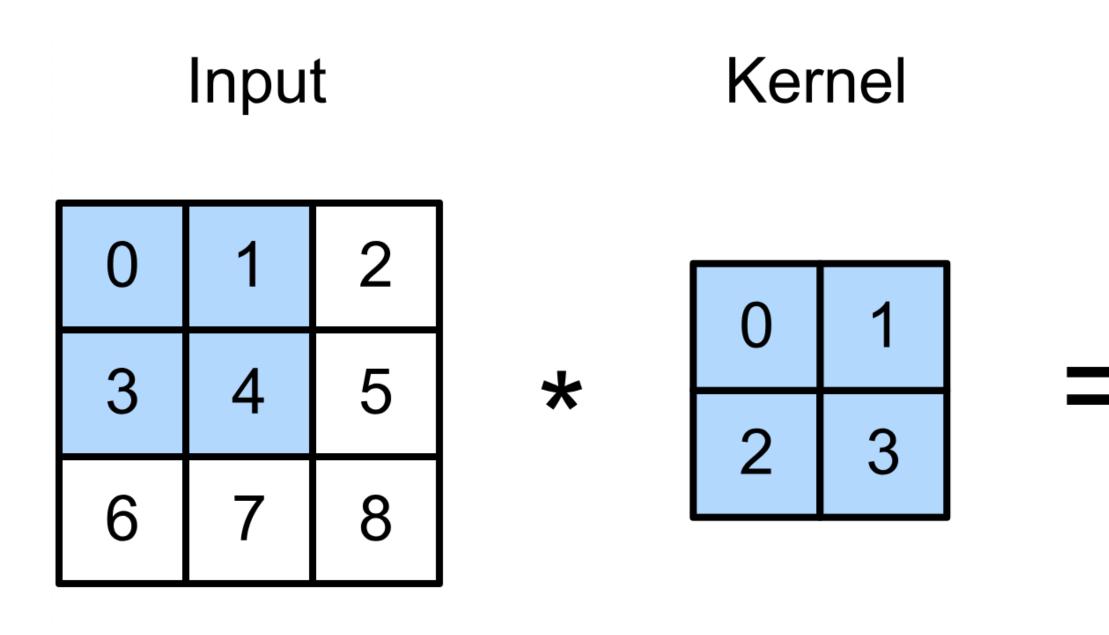
0	1	2
3	4	5
6	7	8



*

Output

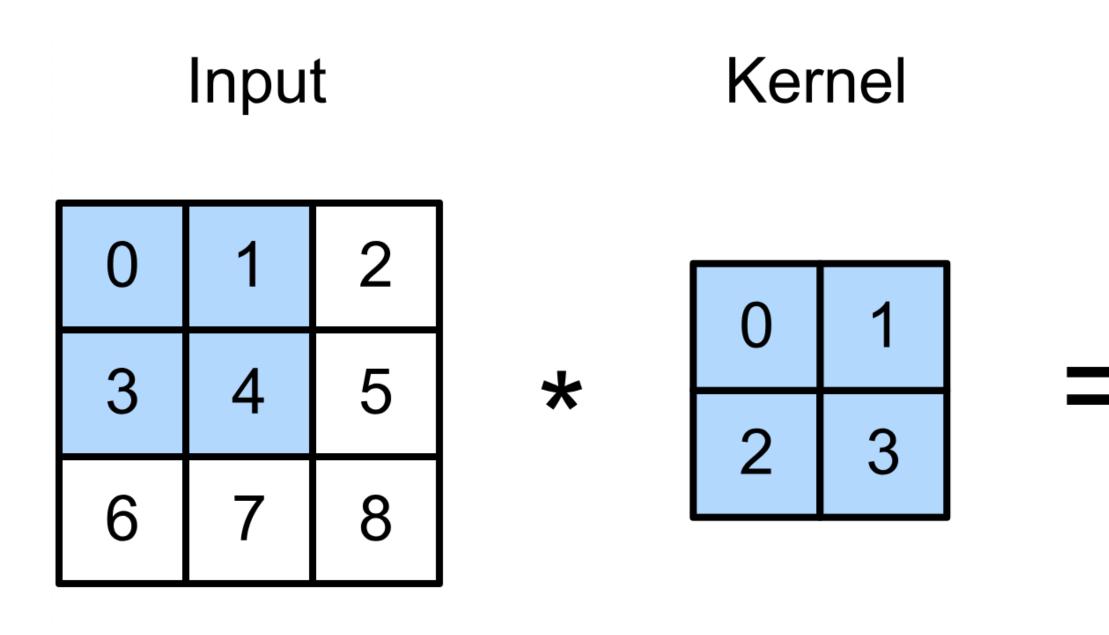
19	25
37	43



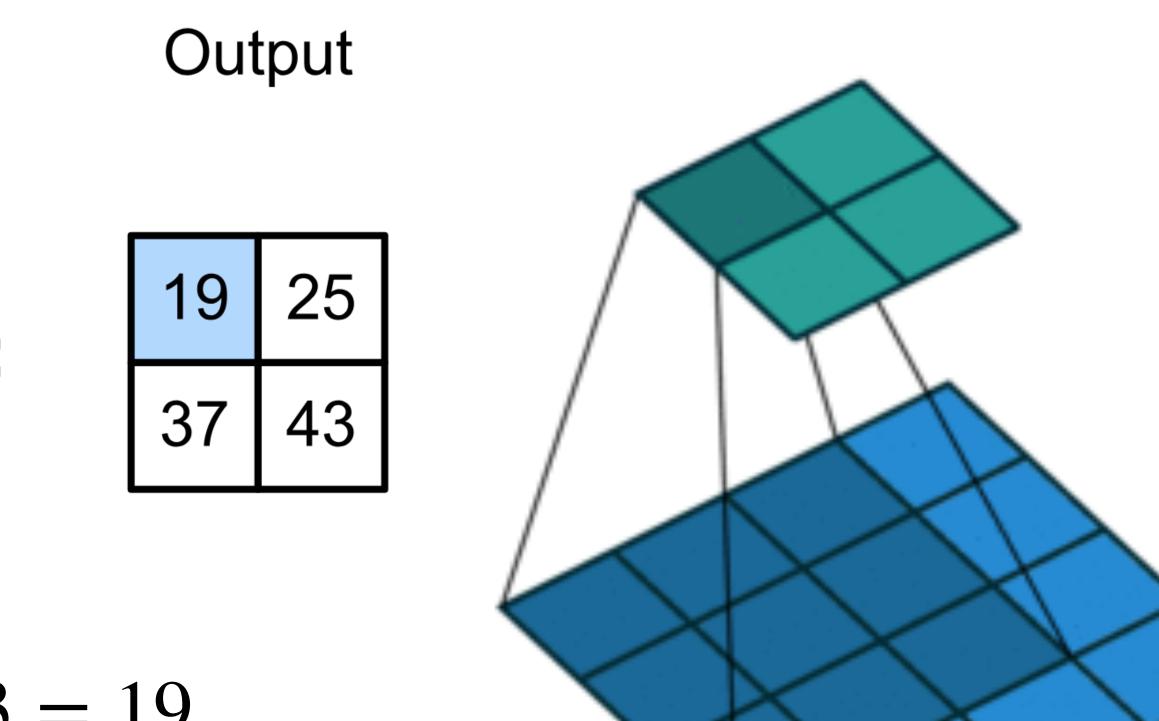
 $0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19$, $1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25$, $3 \times 0 + 4 \times 1 + 6 \times 2 + 7 \times 3 = 37$, $4 \times 0 + 5 \times 1 + 7 \times 2 + 8 \times 3 = 43.$

Output

19	25
37	43



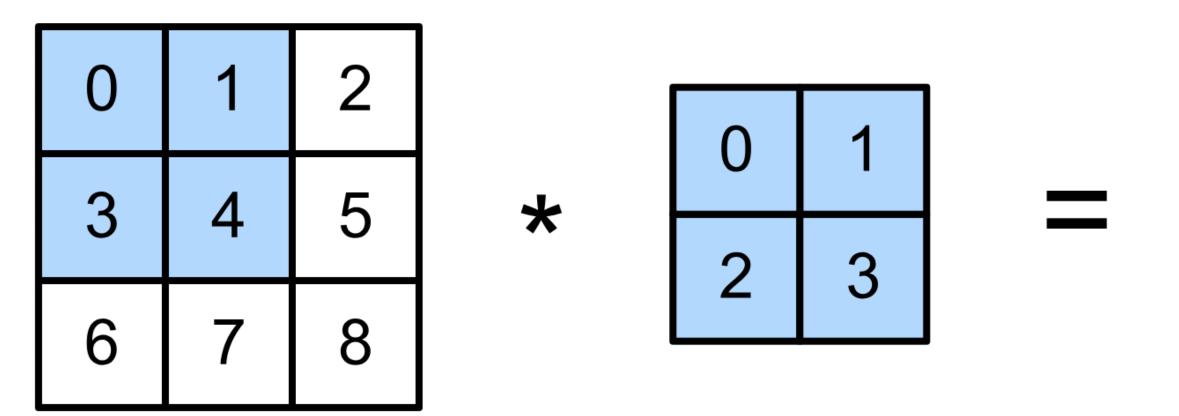
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(vdumoulin@ Github)



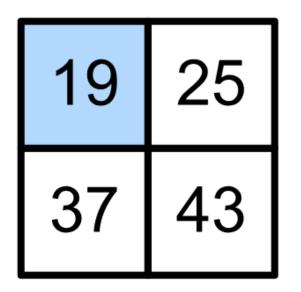
2-D Convolution Layer



• $\mathbf{X}: n_h \times n_w$ input matrix

- W: $k_h \times k_w$ kernel matrix
- b: scalar bias
- **Y** : $(n_h k_h + 1) \times (n_w k_w + 1)$ output matrix

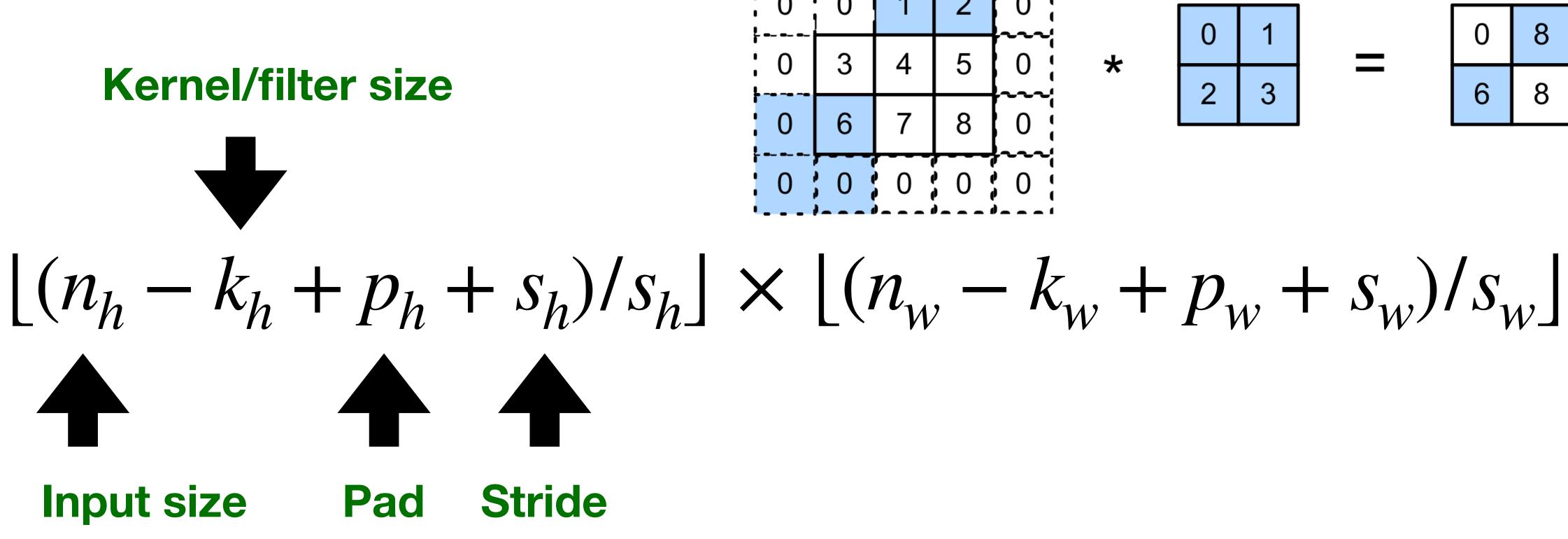
• W and b are learnable parameters

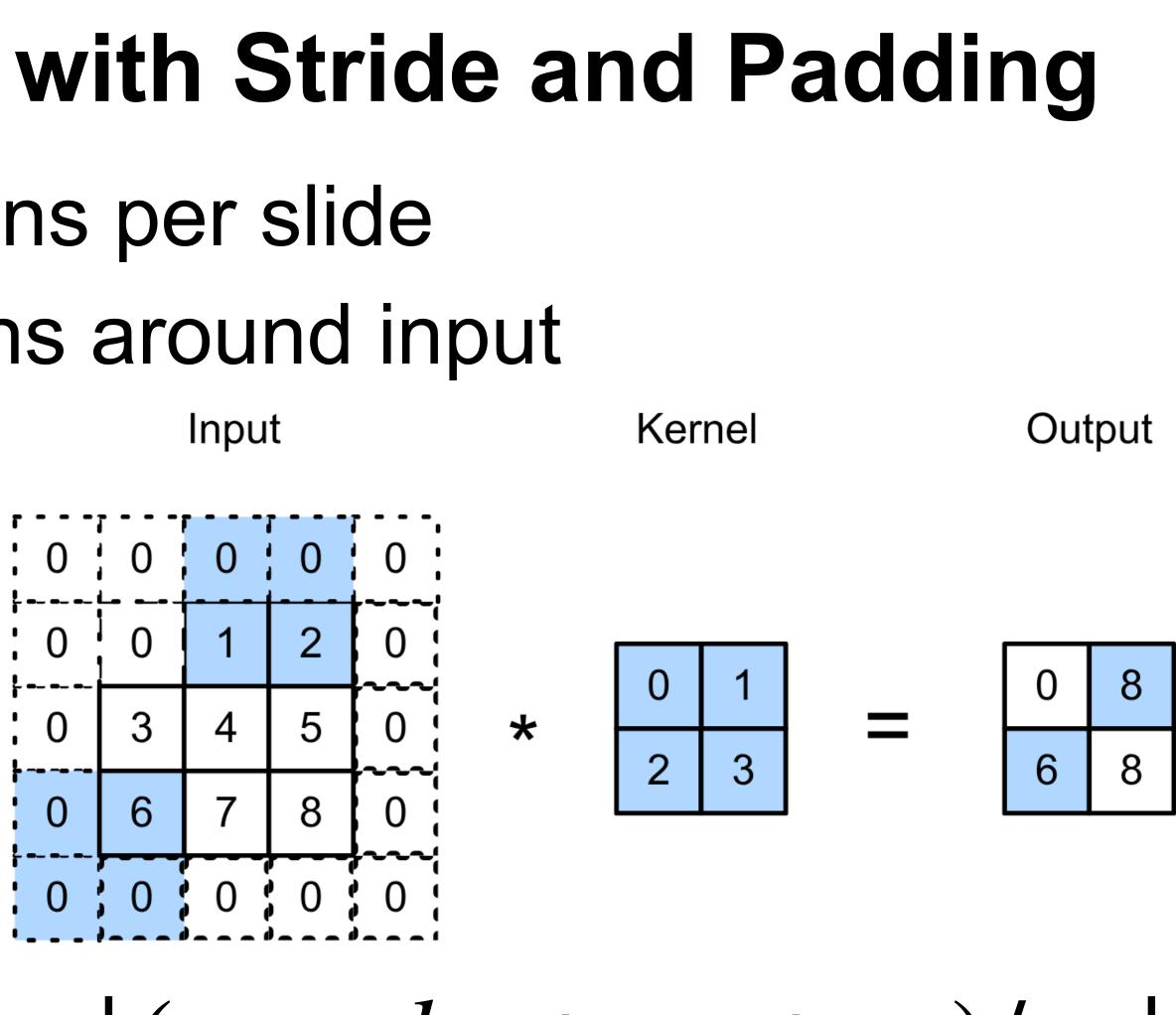


$\mathbf{Y} = \mathbf{X} \star \mathbf{W} + b$

2-D Convolution Layer with Stride and Padding

- Stride is the #rows/#columns per slide
- Padding adds rows/columns around input
- Output shape

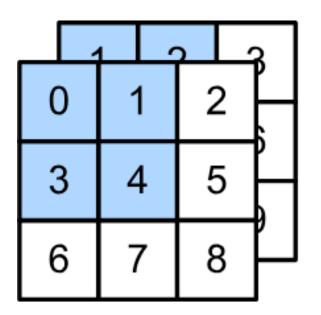




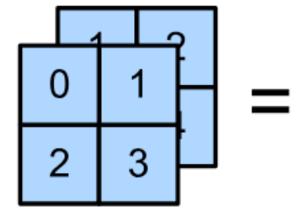
- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- channels

Input

Kernel



*



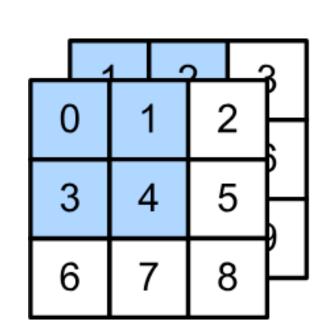
Have a kernel for each channel, and then sum results over

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- channels

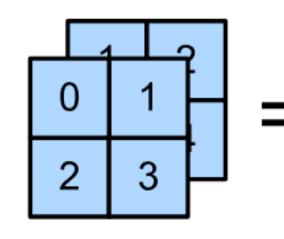
Input

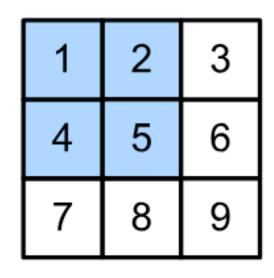
Kernel

Input



*

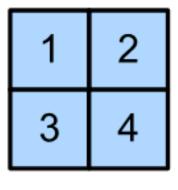




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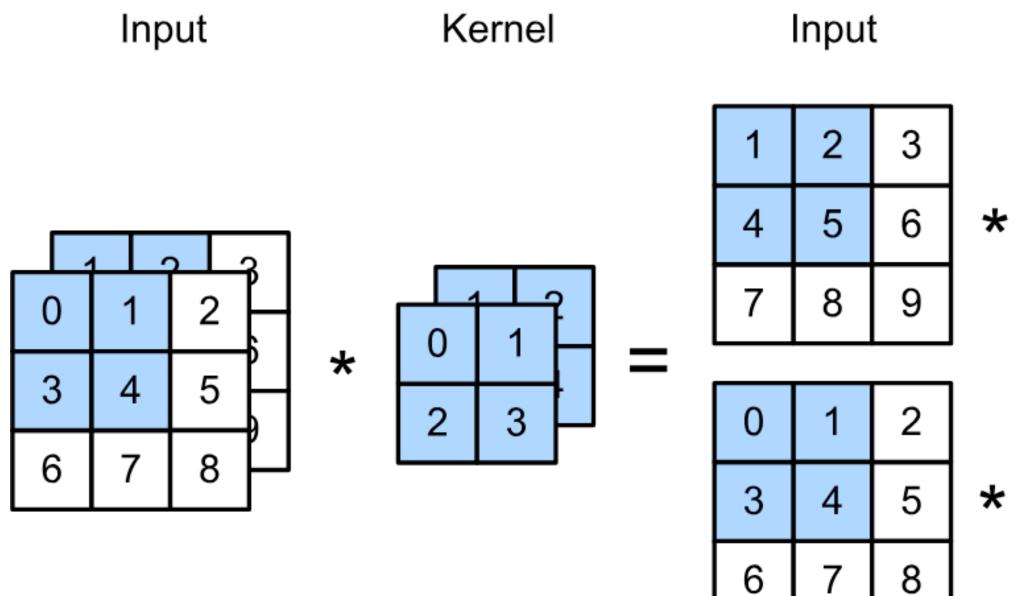
Have a kernel for each channel, and then sum results over

Kernel



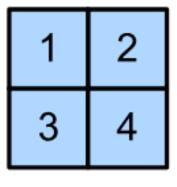
+

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- channels

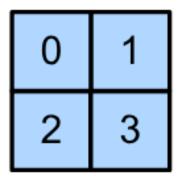


Have a kernel for each channel, and then sum results over

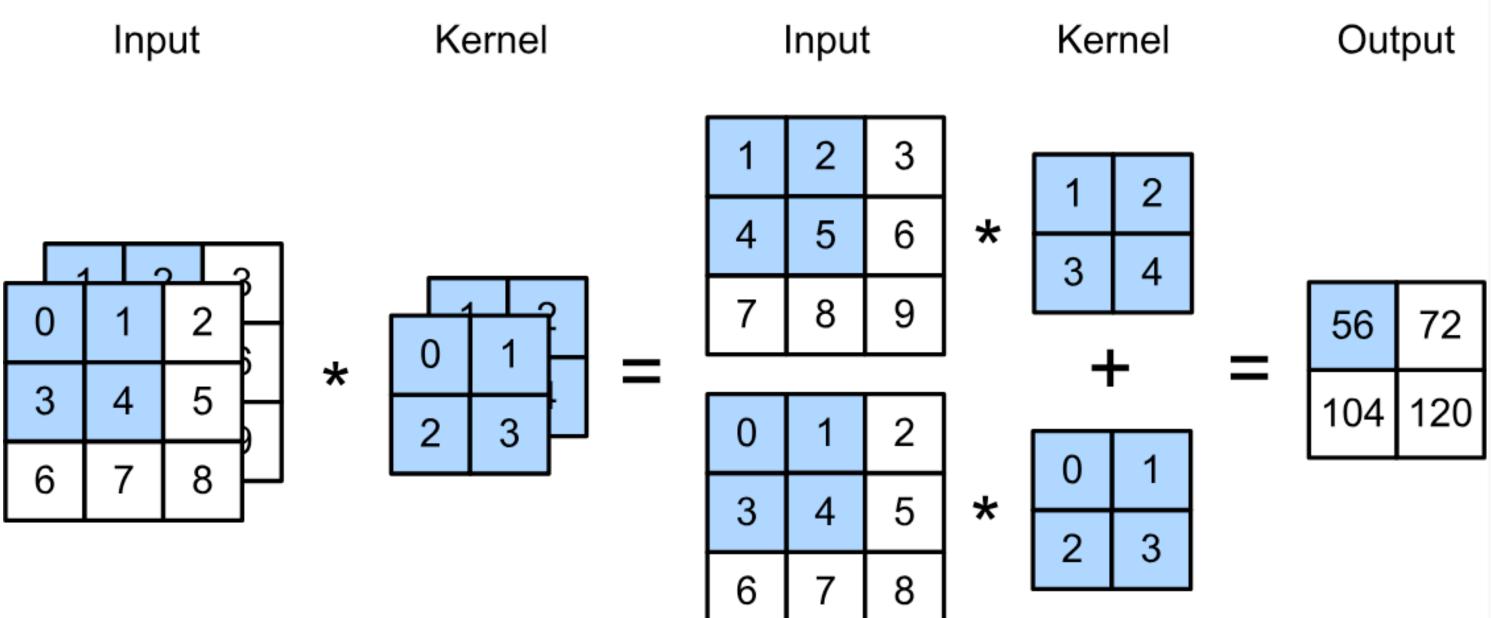
Kernel



╋

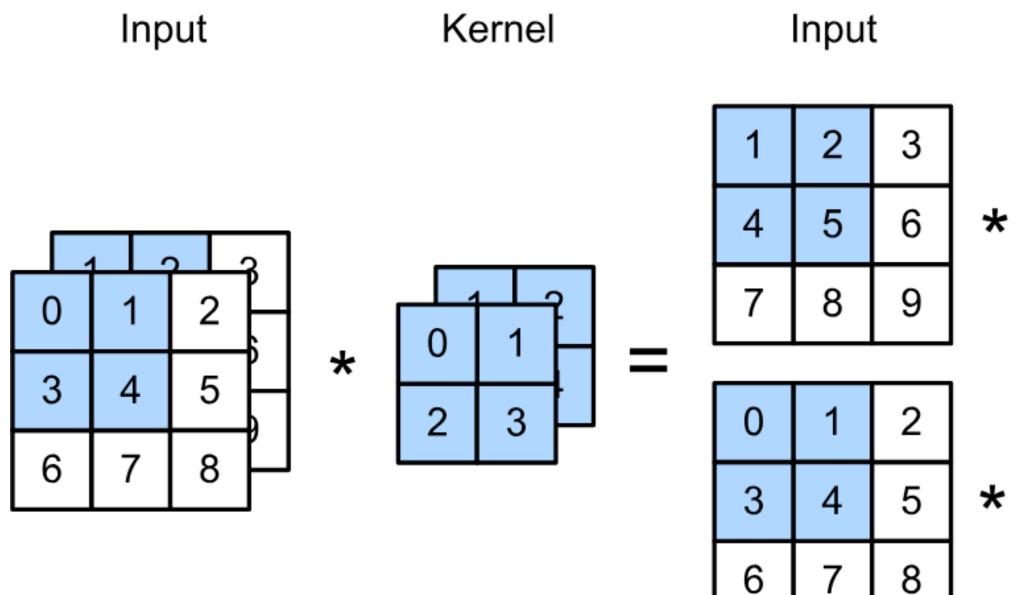


- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- channels

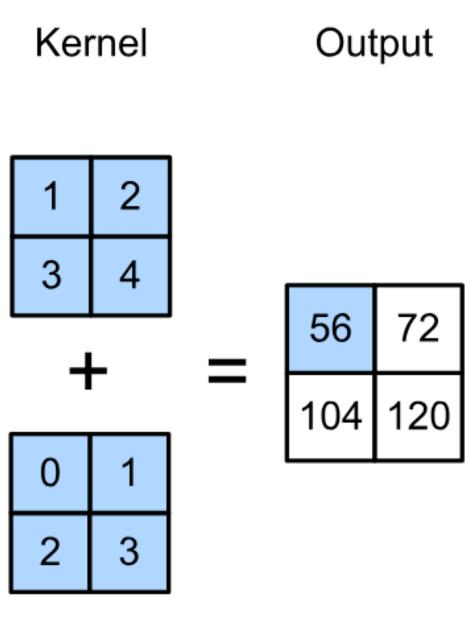


Have a kernel for each channel, and then sum results over

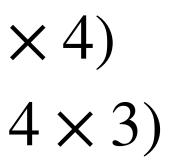
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- channels



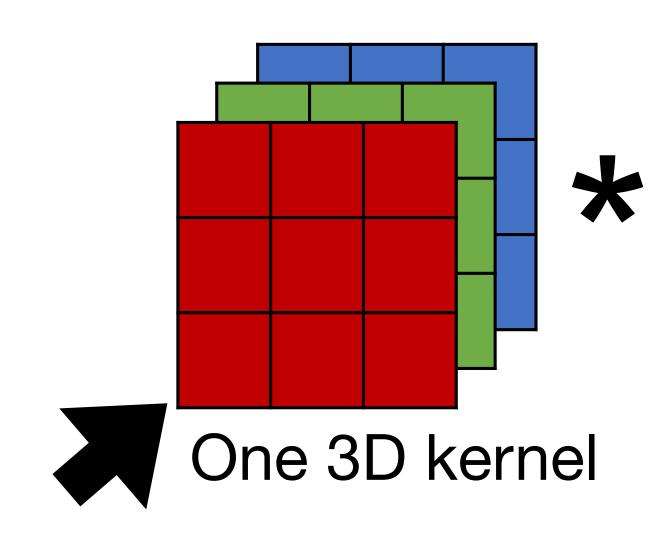
Have a kernel for each channel, and then sum results over



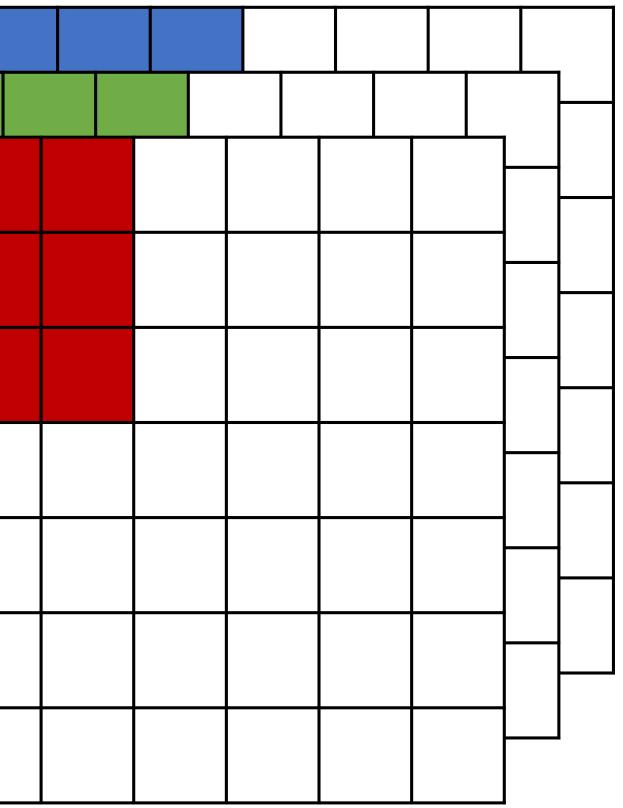
$(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4)$ $+(0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3)$ = 56



- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- channels

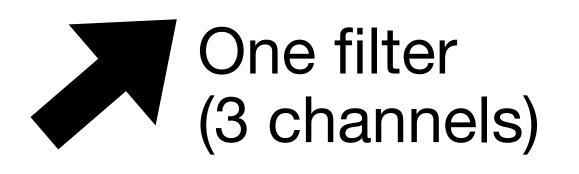


Have a 2D kernel for each channel, and then sum results over





- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- output channel (due to summation over channels)

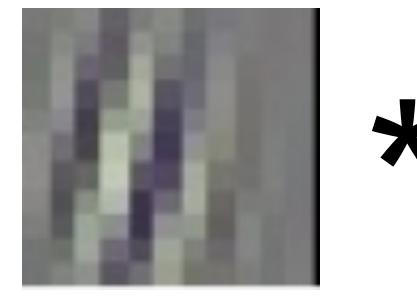


Also call each 3D kernel a "filter", which produce only one





Multiple filters (in one layer) • Apply multiple filters on the input Each filter may learn different features about the input Each filter (3D kernel) produces one output channel





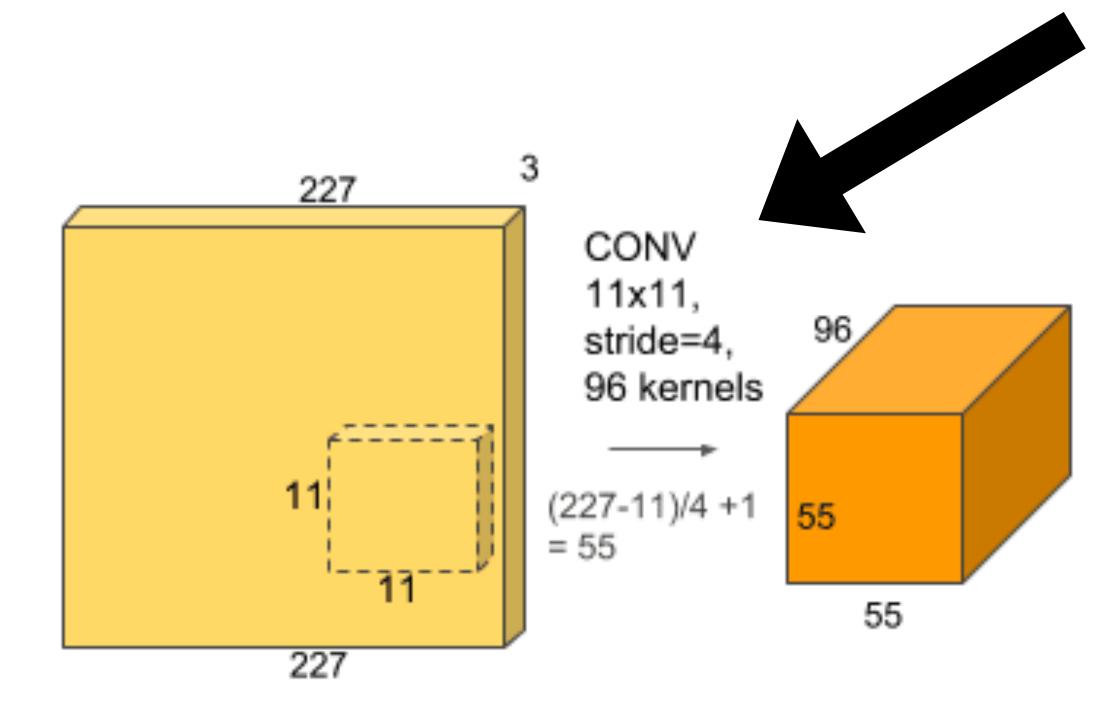


RGB (3 input channels)

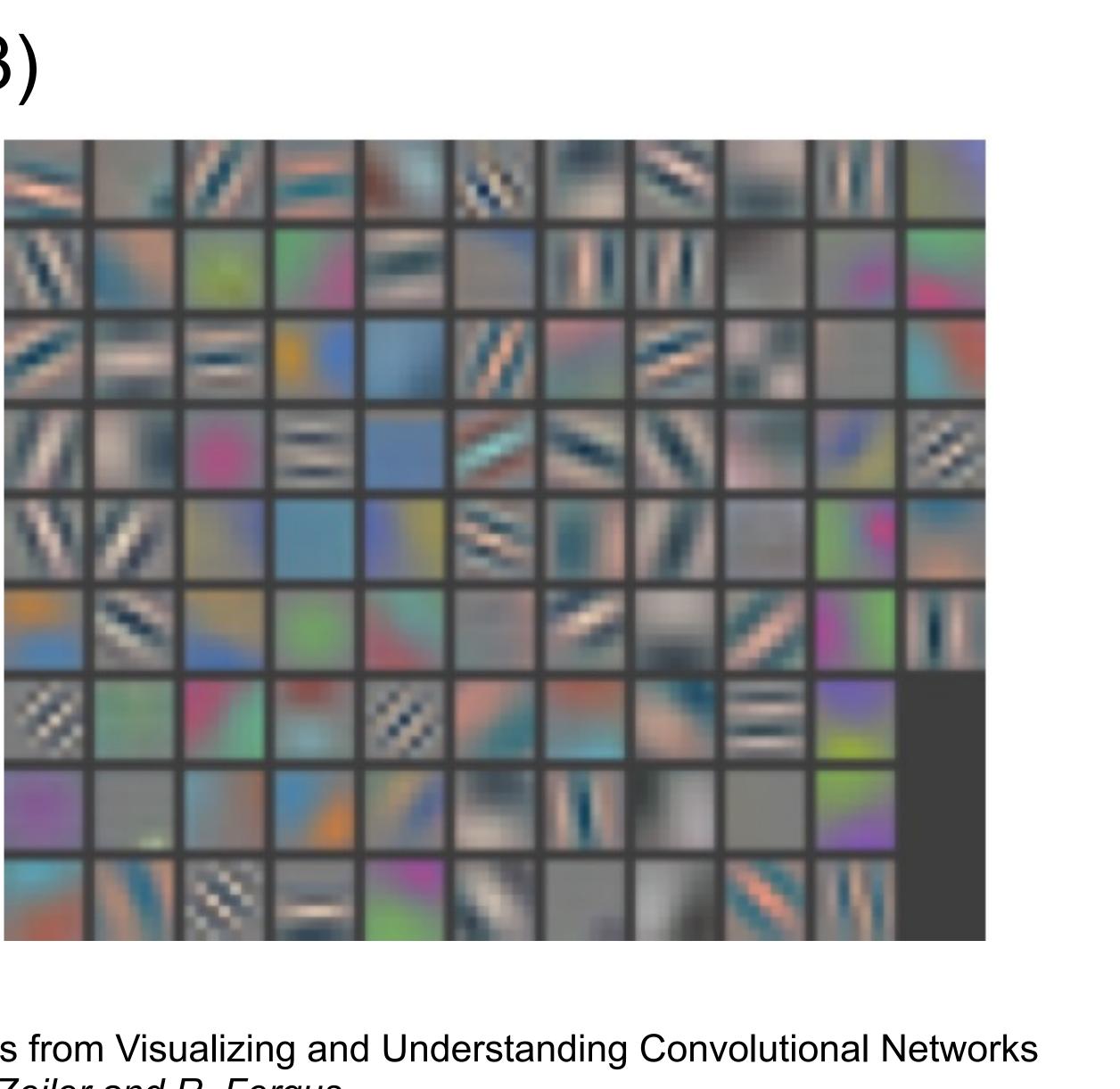


Conv1 Filters in AlexNet

- 96 filters (each of size 11x11x3)
- Gabor filters







Figures from Visualizing and Understanding Convolutional Networks by M. Zeiler and R. Fergus

Multiple Output Channels

- The # of output channels = # of filters
- Input $\mathbf{X}: c_i \times n_h \times n_w$
- Kernel W : $c_o \times c_i \times k_h \times k_w$
- Output $\mathbf{Y} : c_o \times m_h \times m_w$

IS # of filter

> $Y_{i,:,:} = X \star W_{i,:,:,:}$ for $i = 1, ..., c_o$

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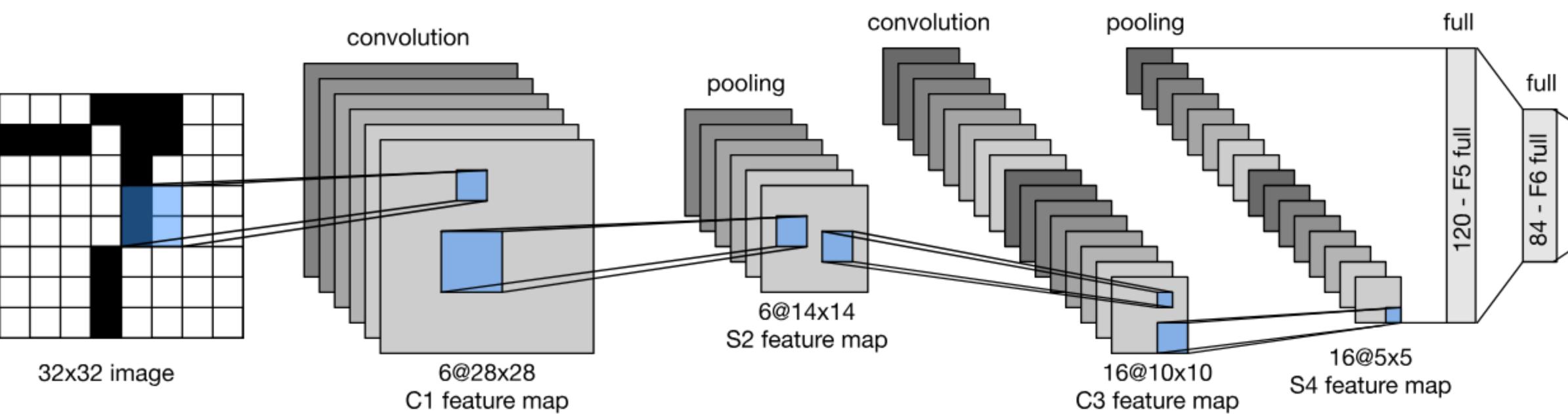
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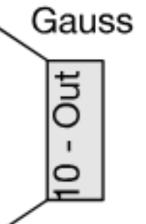
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Convolutional Neural Networks

LeNet Architecture

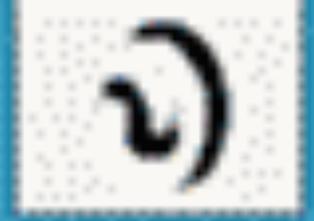


gluon-cv.mxnet.io





















































LeNet 5

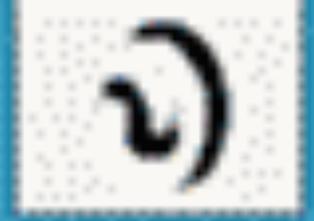


Y. LeCun, L. Bottou, Y. Bengio, P. Haffner, 1998 Gradient-based learning applied to document recognition





















































LeNet 5



Y. LeCun, L. Bottou, Y. Bengio, P. Haffner, 1998 Gradient-based learning applied to document recognition



Quiz break

Which one of the following is NOT true?

- A. LeNet has two convolutional layers
- B. The first convolutional layer in LeNet has 5x5x6x3 parameters, in case of RGB input C. Pooling is performed right after convolution
- D. Pooling layer does not have learnable parameters



Quiz break

Which one of the following is NOT true?

A. LeNet has two convolutional layers

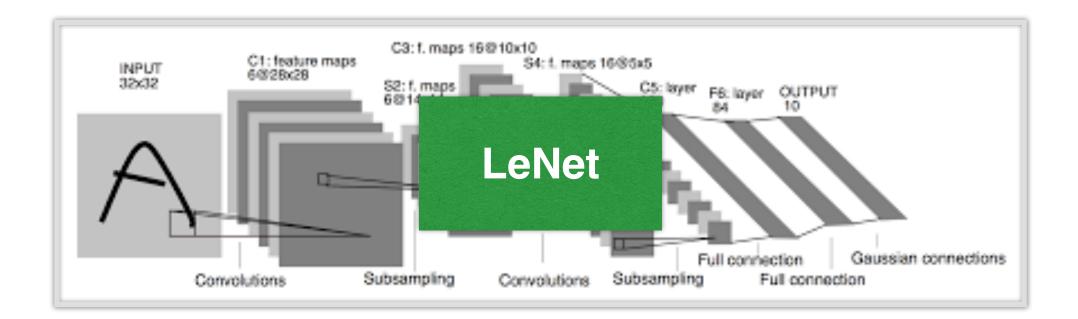
- B. The first convolutional layer in LeNet has 5x5x6x3 parameters, in case of RGB input C. Pooling is performed right after convolution
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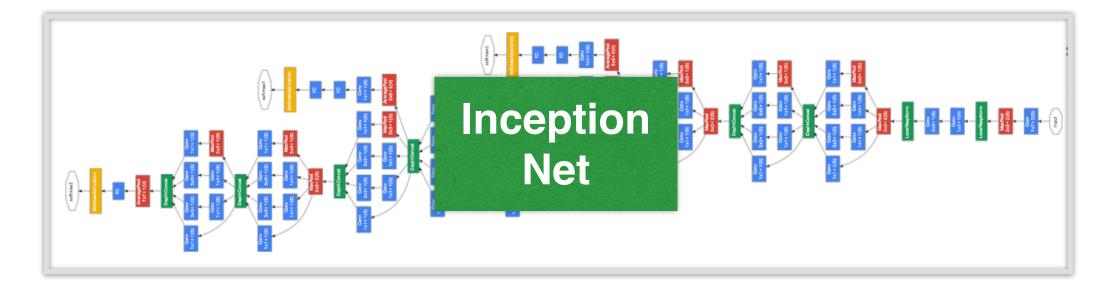
Pooling is performed after ReLU: conv->relu->pooling

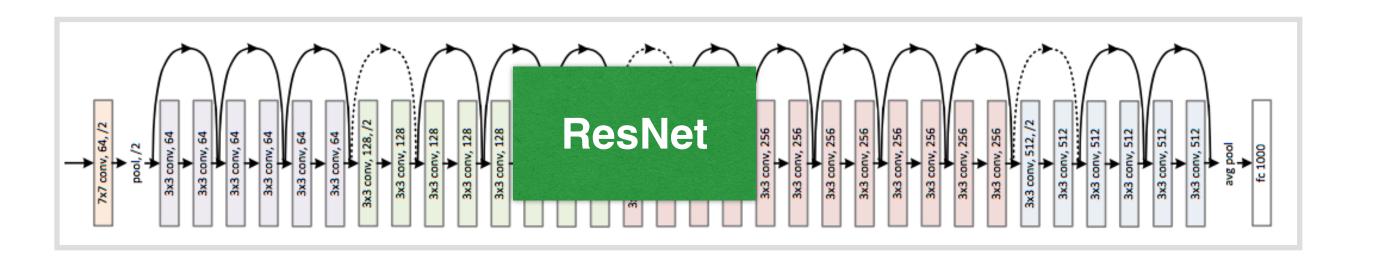


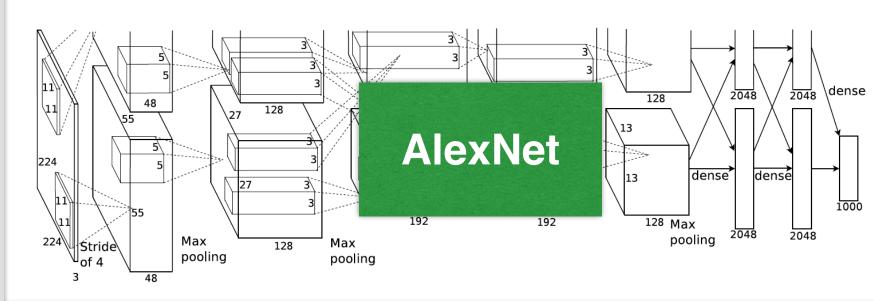
Evolution of neural net architectures

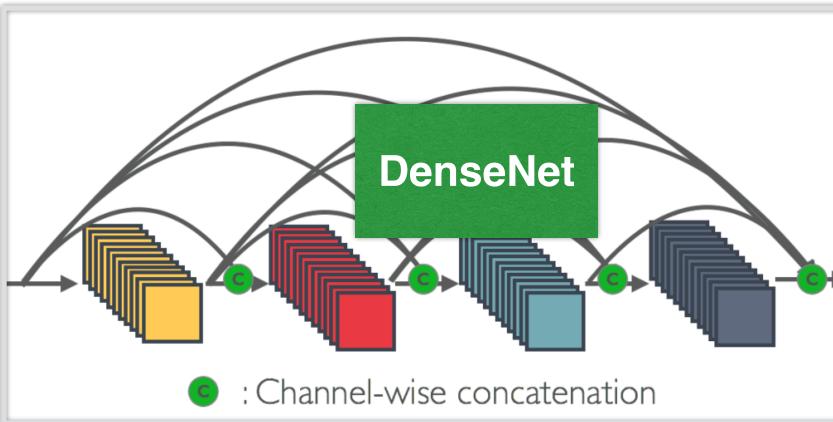
Evolution of neural net architectures











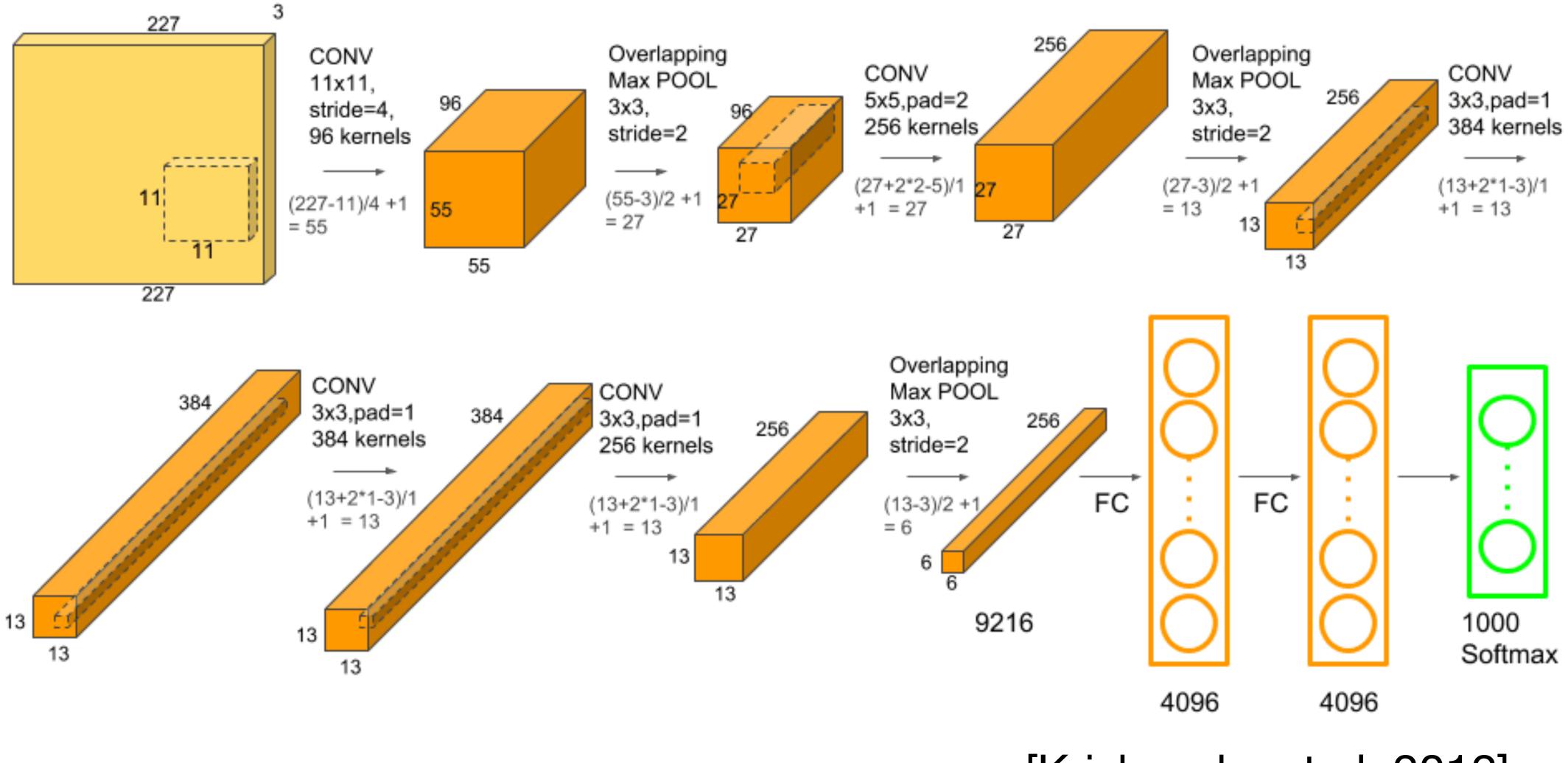




Deng et al. 2009

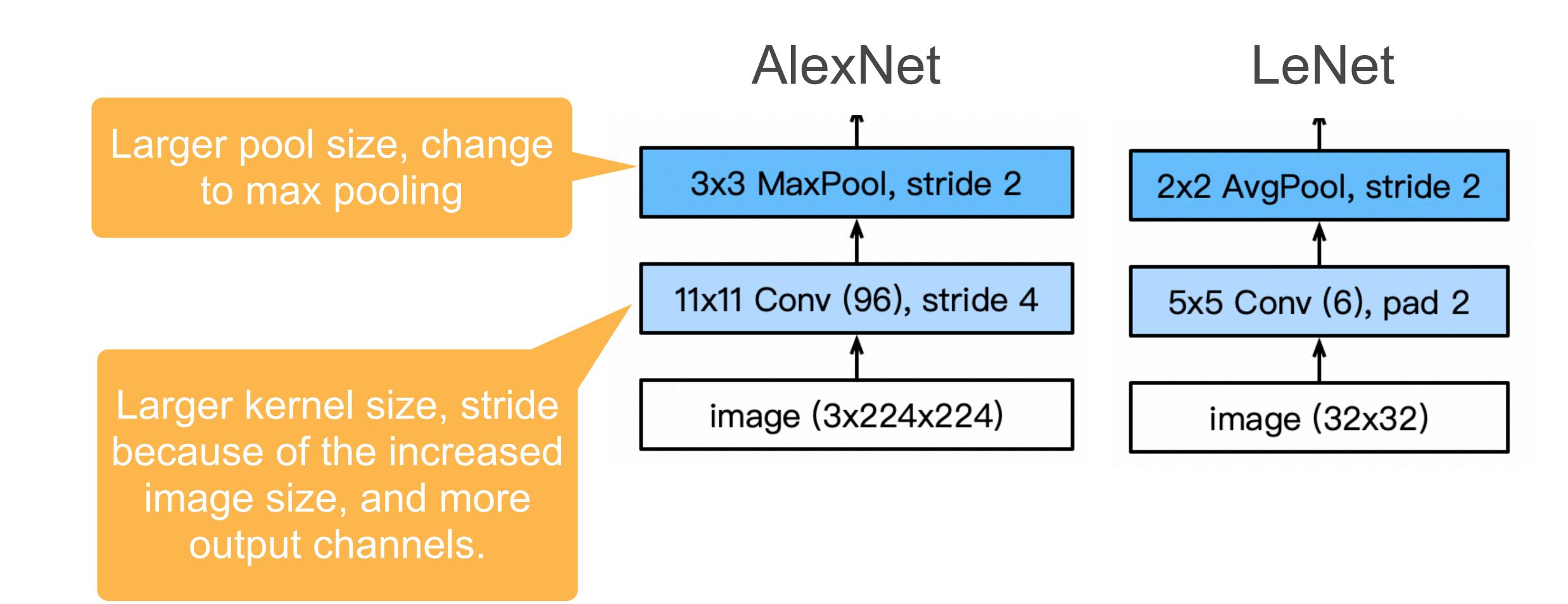




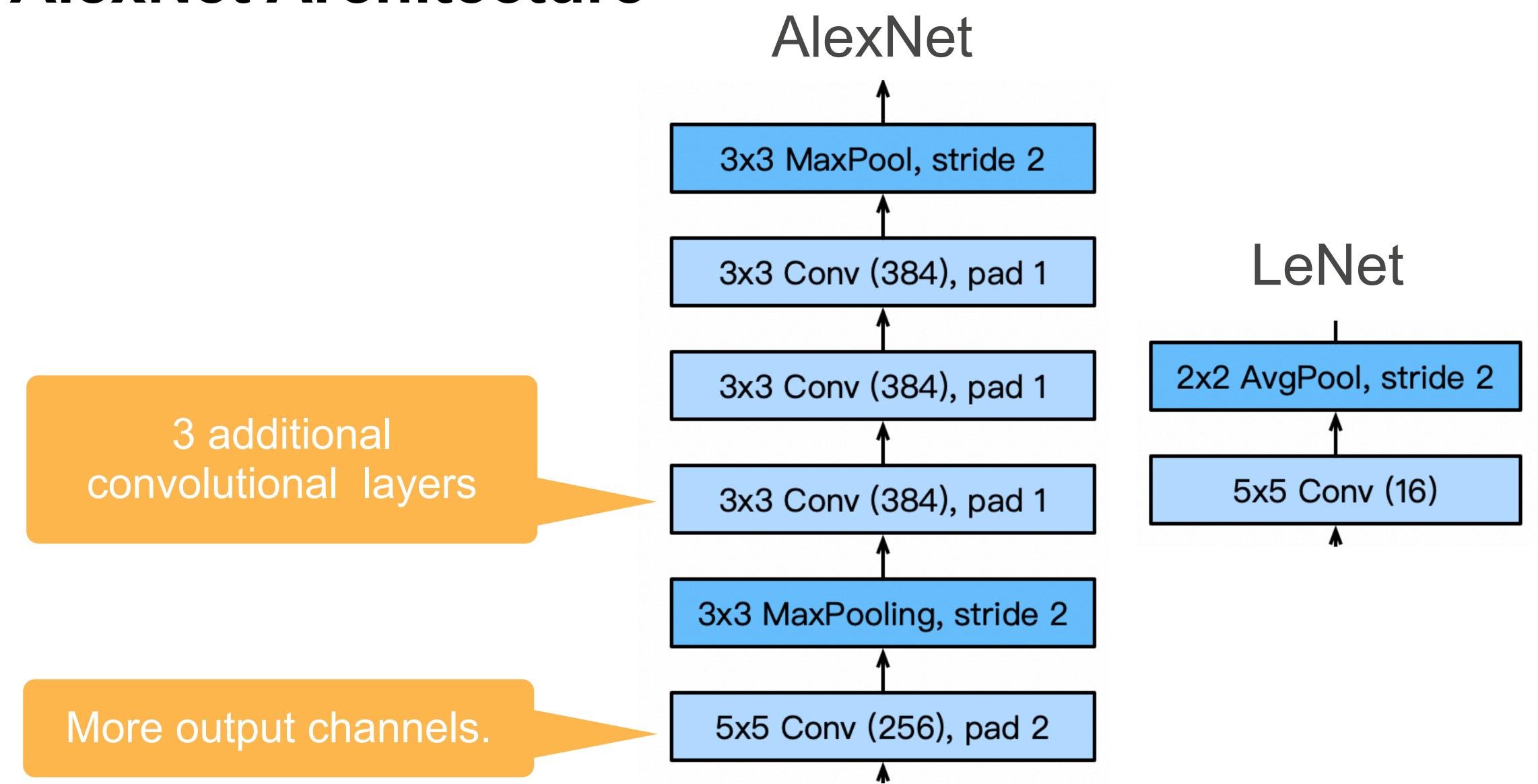


[Krizhevsky et al. 2012]

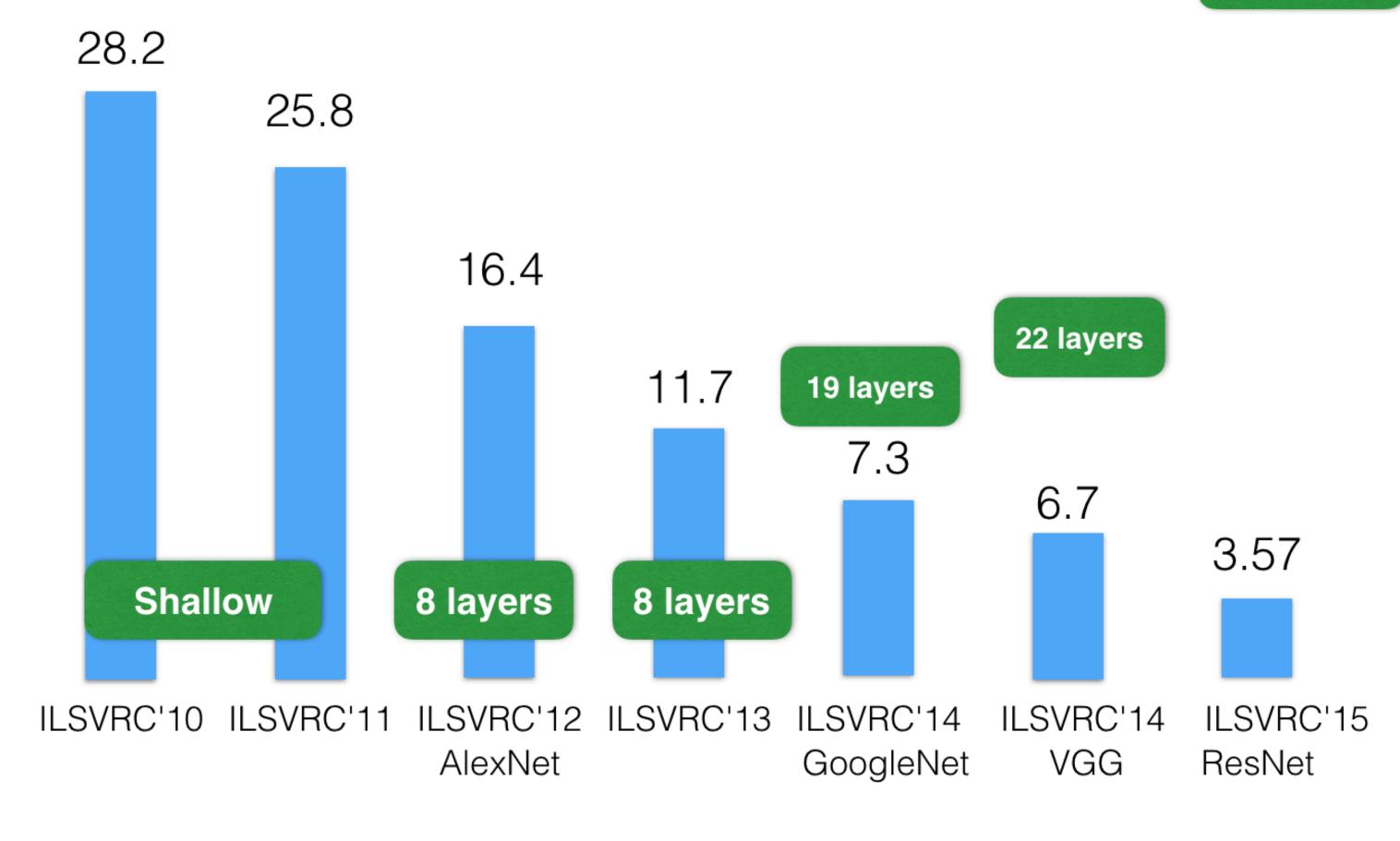
AlexNet vs LeNet Architecture



AlexNet Architecture



ResNet: Going deeper in depth



ImageNet Top-5 error%

152 layers

[He et al. 2015]



Convolutional neural networks are one of many special types of layers.

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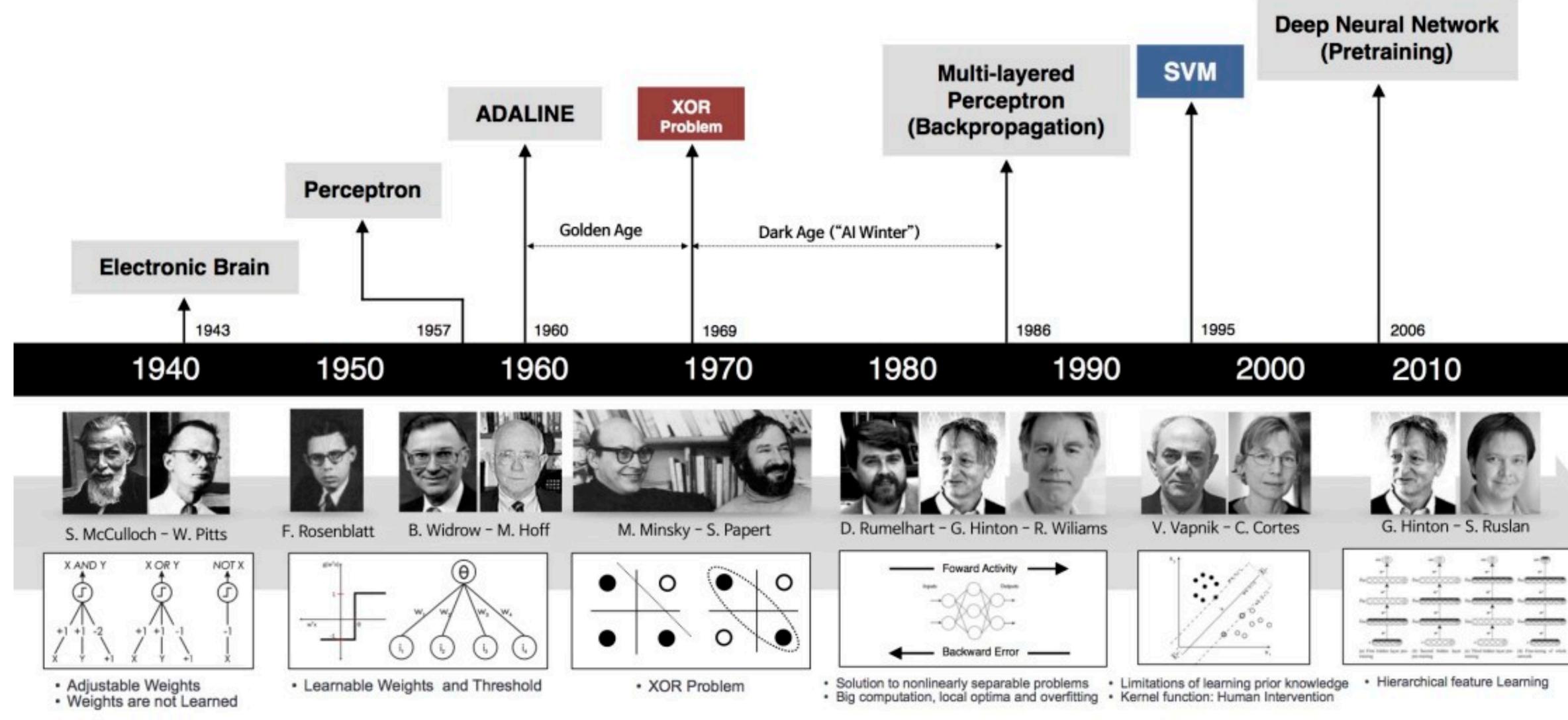
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 - attention to.

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Transformers: take sequences as input and learn what parts of input to pay

Brief history of neural networks





• Modeling a single neuron

- Modeling a single neuron
 - Linear perceptron

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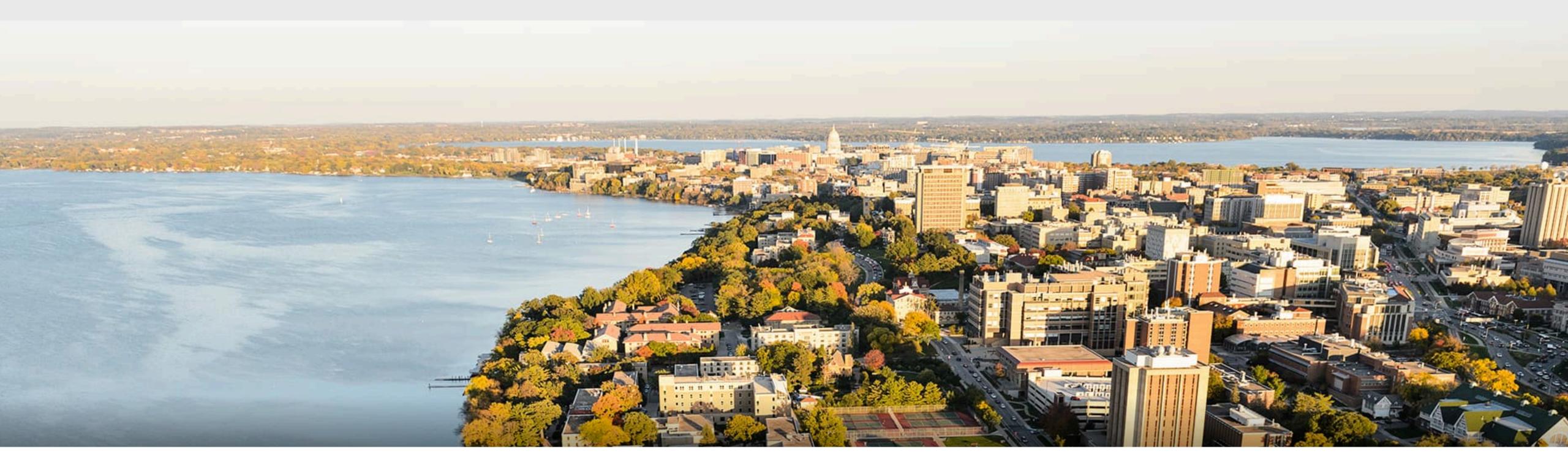
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 - More advanced architectures (AlexNet, ResNet etc)



Thank you!

Some of the slides in these lectures have been adapted from materials developed by Alex Smola and Mu Li: <u>https://courses.d2l.ai/berkeley-stat-157/index.html</u>