Announcements

Homeworks:
- Homework 9 due Thursday April 27
- Homework 10 due Thursday May 4

Course Evaluation:
- Complete by Friday May 5

Class roadmap:

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<td>Thursday, May 4</td>
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Final Exam: May 12 5:05 - 7:05 pm
Outline
Outline

• Introduction to reinforcement learning
  – Basic concepts, mathematical formulation, MDPs, policies.
Outline

• Introduction to reinforcement learning
  – Basic concepts, mathematical formulation, MDPs, policies.

• Learning policies
  – Q-learning, action-values, exploration vs exploitation.
Back to Our General Model

We have an agent interacting with the world
Back to Our General Model

We have an agent interacting with the world

Agent
Back to Our General Model

We have an **agent** interacting with the **world**
Back to Our General Model

We have an **agent** interacting with the **world**

[Diagram showing an agent interacting with the world through actions]
Back to Our General Model

We have an **agent** interacting with the **world**

![Diagram showing a stick figure (Agent) interacting with the world through actions and observations.](image-url)
Back to Our General Model

We have an **agent interacting** with the **world**

- Agent receives a reward based on state of the world
Back to Our General Model

We have an **agent interacting** with the **world**

- **Agent** receives a reward based on state of the world
  - **Goal**: maximize reward / utility
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  - Note: **data** consists of actions & observations
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We have an **agent interacting** with the **world**

- Agent receives a reward based on state of the world
  - **Goal**: maximize reward / utility  ($$$)
  - Note: **data** consists of actions & observations
    - Compare to unsupervised learning and supervised learning
Examples: Gameplay Agents

AlphaZero:
Examples: Gameplay Agents

AlphaZero:
Examples: Gameplay Agents

**AlphaZero:**
Examples: Gameplay Agents

AlphaZero:

https://deepmind.com/research/alphago/
Examples: Video Game Agents

Pong, Atari

Mnih et al, “Human-level control through deep reinforcement learning”

A. Nielsen
Examples: Video Game Agents

Pong, Atari

Mnih et al, “Human-level control through deep reinforcement learning”
Examples: Video Game Agents

Minecraft, Quake, StarCraft, and more!
Examples: Video Game Agents

Minecraft, Quake, StarCraft, and more!

Shao et al, "A Survey of Deep Reinforcement Learning in Video Games"
Examples: Robotics

Training robots to perform tasks (e.g., grasp objects!)
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Ibarz et al, "How to Train Your Robot with Deep Reinforcement Learning – Lessons We’ve Learned"
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Building The Theoretical Model

Basic setup:
Building The Theoretical Model

Basic setup:

Agent

World

Actions

Observations
Building The Theoretical Model

Basic setup:

- Set of states, $S$
Building The Theoretical Model

Basic setup:
- Set of states, $S$
- Set of actions $A$
Building The Theoretical Model

Basic setup:

- Set of states, $S$
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- Information: at time $t$, observe state $s_t \in S$. Get reward $r_t$
Building The Theoretical Model

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Goal: find a map from states to actions that maximize rewards.
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A “policy”
Markov Decision Process (MDP)
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The formal mathematical model:
Markov Decision Process (MDP)

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Markov Decision Process (MDP)

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- More generally: $r(s_t, a_t)$, $P(r_t, s_{t+1} \mid s_t, a_t)$
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- **Policy**: $\pi(s) : S \rightarrow A$ action to take at a particular state
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$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots$
Example of MDP: Grid World

Robot on a grid; goal: find the best policy
Example of MDP: Grid World

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Robot on a grid; goal: find the best policy

Source: P. Abbeel and D. Klein
Example of MDP: Grid World

Note: (i) Robot is unreliable    (ii) Reach target fast
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![Grid World Diagram]
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Grid World Optimal Policy

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Reinforcement Learning Challenges
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Credit-assignment:
Reinforcement Learning Challenges

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- Example: you study 15 minutes a day all semester. The morning of the final exam, you eat a bowl of yogurt. You receive an A on the final. Was it the studying or the yogurt that led to the A?
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Exploration vs. Exploitation:
Reinforcement Learning Challenges

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Exploration vs. Exploitation:

- Transition probabilities and reward may be unknown to the learner.
Reinforcement Learning Challenges

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Exploration vs. Exploitation:

- Transition probabilities and reward may be unknown to the learner.

- Should you keep trying actions that led to reward in the past or try new actions that might lead to even more reward?
Q 1.1 Which of the following statement about MDP is not true?

- A. The reward function must output a scalar value
- B. The policy maps states to actions
- C. The probability of next state can depend on current and previous states
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards
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Q 1.1 Which of the following statement about MDP is not true?

- A. The reward function must output a scalar value (True: need to be able to compare)
- B. The policy maps states to actions (True: a policy tells you what action to take for each state).
- C. The probability of next state can depend on current and previous states (False: Markov assumption).
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards (True: want to maximize rewards overall).
Defining the Optimal Policy
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For policy $\pi$, \textbf{expected utility} over all possible state sequences from $s_0$ produced by following that policy:

Called the \textbf{value function} (for $\pi$, $s_0$)
Defining the Optimal Policy

For policy \( \pi \), **expected utility** over all possible state sequences from \( s_0 \) produced by following that policy:

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V^\pi(s_0) = \sum_{\text{sequences starting from } s_0} P(\text{sequence}) U(\text{sequence})
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Discounting Rewards

One issue: these are possibly infinite series. 
Convergence?
Discounting Rewards

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- Solution: discount future rewards.
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U(s_0, s_1 \ldots) = r(s_0) + \gamma r(s_1) + \gamma^2 r(s_2) + \ldots = \sum_{t \geq 0} \gamma^t r(s_t)
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  - Set according to how important **present** is VS **future**
  - Note: has to be less than 1 for convergence
From Value to Policy
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Now that $V^\pi(s_0)$ is defined, what $a$ should we take?
From Value to Policy

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- First, let $\pi^*$ be the **optimal** policy for $V^\pi(s_0)$, and $V^*(s_0)$ its expected utility.
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- What’s the expected utility following an action?
  - Specifically, action $a$ in state $s$?
    $$\sum_{s'} P(s'|s,a)V^*(s')$$
All the states we could go to
Transition probability
Expected rewards
Slight Problem...

Now we can get the optimal policy by doing
Slight Problem...

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$$\pi^*(s) = \arg\max_a \sum_{s'} P(s'|s, a)V^*(s')$$
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  - So we need some other approach to get $V^*(s)$.
  - Instead, learn about the utility of actions directly.
The Q*(s,a) function
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- Starting from state $s$, perform (perhaps suboptimal) action $a$. THEN follow the optimal policy
The $Q^*(s,a)$ function

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$$Q^*(s, a) = r(s) + \gamma \sum_{s'} P(s' \mid s, a) V^*(s')$$
The $Q^*(s,a)$ function

- Starting from state $s$, perform (perhaps suboptimal) action $a$. THEN follow the optimal policy

\[
Q^*(s, a) = r(s) + \gamma \sum_{s'} P(s'|s, a)V^*(s')
\]

- Equivalent to

\[
Q^*(s, a) = r(s) + \gamma \sum_{s'} P(s'|s, a) \max_{a'} Q^*(s', a')
\]
Q-Learning
Q-Learning
Q-Learning

- Our first reinforcement learning algorithm.
Q-Learning

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- Does not require knowing r or P. Learn from data of the form: \( \{(s_t, a_t, r_t, s_{t+1})\} \).
Q-Learning

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• Does not require knowing r or P. Learn from data of the form: \(\{(s_t, a_t, r_t, s_{t+1})\}\).
• Learns an action-value function \(Q^*(s,a)\) that tells us the expected value of taking a in state s.
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  - Note: \( V^*(s) = \max_a Q^*(s, a) \).
Q-Learning

• Our first reinforcement learning algorithm.
• Does not require knowing $r$ or $P$. Learn from data of the form: $\{(s_t, a_t, r_t, s_{t+1})\}$.
• Learns an action-value function $Q^*(s,a)$ that tells us the expected value of taking $a$ in state $s$.
  
  Note: $V^*(s) = \max_a Q^*(s,a)$.

• Optimal policy is formed as $\pi^*(s) = \arg \max_a Q^*(s,a)$. 
Q-Learning

Estimate $Q^*(s,a)$ from data $\{(s_t, a_t, r_t, s_{t+1})\}$:

1. Initialize $Q(.,.)$ arbitrarily (eg all zeros)
   1. Except terminal states $Q(s_{\text{terminal}},.)=0$

2. Iterate over data until $Q(.,.)$ converges:

   $$Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha(r_t + \gamma \max_b Q(s_{t+1}, b))$$
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   Learning rate
Q-learning Algorithm

Input: step size $\alpha$, exploration probability $\epsilon$
1. set $Q(s,a) = 0$ for all $s$, $a$.
2. For each episode:
3. Get initial state $s$.
4. While (s not a terminal state):
5. Perform $a = \epsilon$-greedy($Q$, $s$), receive $r$, $s'$
6. $Q(s, a) = (1 - \alpha)Q(s, a) + \alpha(r + \gamma \max_{a'} Q(s', a'))$
7. $s \leftarrow s'$
8. End While
9. End For
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Explore: take action to see what happens.
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Explore: take action to see what happens.
Update action-value based on result.
Converges to $Q^*(s,a)$ in limit if all states and actions visited infinitely often.
Exploration Vs. Exploitation

General question!
Exploration Vs. Exploitation

General question!
- **Exploration**: take an action with unknown consequences
Exploration Vs. Exploitation

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• **Exploration**: take an action with unknown consequences
  – **Pros**:
    • Get a more accurate model of the environment
    • Discover higher-reward states than the ones found so far
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Exploration vs. Exploitation

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Q-Learning: $\varepsilon$-Greedy Behavior Policy
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Getting data with both **exploration and exploitation**

- With probability $\varepsilon$, take a random action; else the action with the highest (current) $Q(s,a)$ value.
Q-Learning: $\epsilon$-Greedy Behavior Policy

Getting data with both **exploration and exploitation**

- With probability $\epsilon$, take a random action; else the action with the highest (current) $Q(s, a)$ value.

$$a = \begin{cases} \arg\max_{a \in A} Q(s, a) & \text{uniform}(0, 1) > \epsilon \\ \text{random } a \in A & \text{otherwise} \end{cases}$$
Q-Learning Iteration

How do we get $Q(s,a)$?

- Iterative procedure

Idea: combine old value and new estimate of future value.

Note: We are using a policy to take actions; based on the estimated $Q$!
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$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r(s_t) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

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Break & Quiz

Q 2.1 For Q learning to converge to the true Q function, we must

- A. Visit every state and try every action
- B. Perform at least 20,000 iterations.
- C. Re-start with different random initial table values.
- D. Prioritize exploitation over exploration.
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Q 2.1 For Q learning to converge to the true Q function, we must

- **A. Visit every state and try every action**
- B. Perform at least 20,000 iterations. *(No: this is dependent on the particular problem, not a general constant).*
- C. Re-start with different random initial table values. *(No: this is not necessary in general).*
- D. Prioritize exploitation over exploration. *(No: insufficient exploration means potentially unupdated state action pairs).*
Summary

• Reinforcement learning setup
• Mathematical formulation: MDP
• The Q-learning Algorithm
Obtaining the Optimal Policy
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We know the expected utility of an action

- So, to get the optimal policy, compute
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All the states we could go to  Transition probability  Expected rewards
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Credit L. Lazbenik
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  - Need some other property of the value function!
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Let’s walk over one step for the value function:

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Value Iteration: Demo

Source: POMDPBGallery Julia Package
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Q 2.1 Consider an MDP with 2 states \{A, B\} and 2 actions: “stay” at current state and “move” to other state. Let \( r \) be the reward function such that \( r(A) = 1, r(B) = 0 \). Let \( \gamma \) be the discounting factor. Let \( \pi: \pi(A) = \pi(B) = \text{move} \) (i.e., an “always move” policy). What is the value function \( V_{\pi}(A) \)?

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- C. \( 1/(1-\gamma^2) \) (States: A,B,A,B,... rewards 1,0, \( \gamma^2 \), 0, \( \gamma^4 \), 0, ...)
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Summary

• Reinforcement learning setup
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• Value functions & the Bellman equation
• Value iteration