S^2: Efficient Graph Based Active Learning

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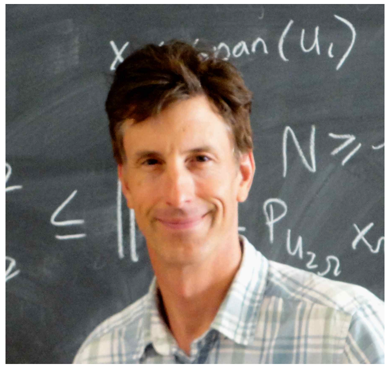
joint work with



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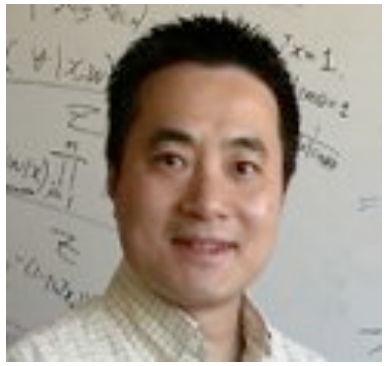

Xiaojin (Jerry) Zhu
Computer Sciences
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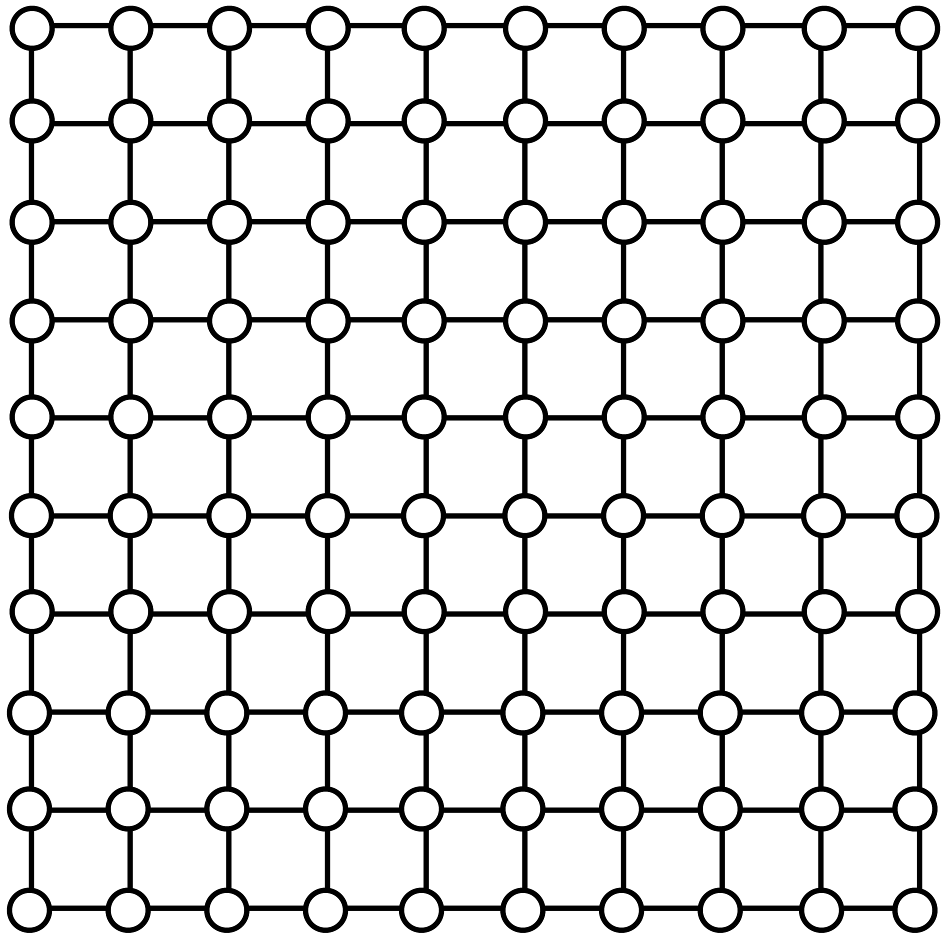
The Problem

$G = ([n], E)$ is a known **undirected graph** on $[n] = \{1, 2, \ldots, n\}$



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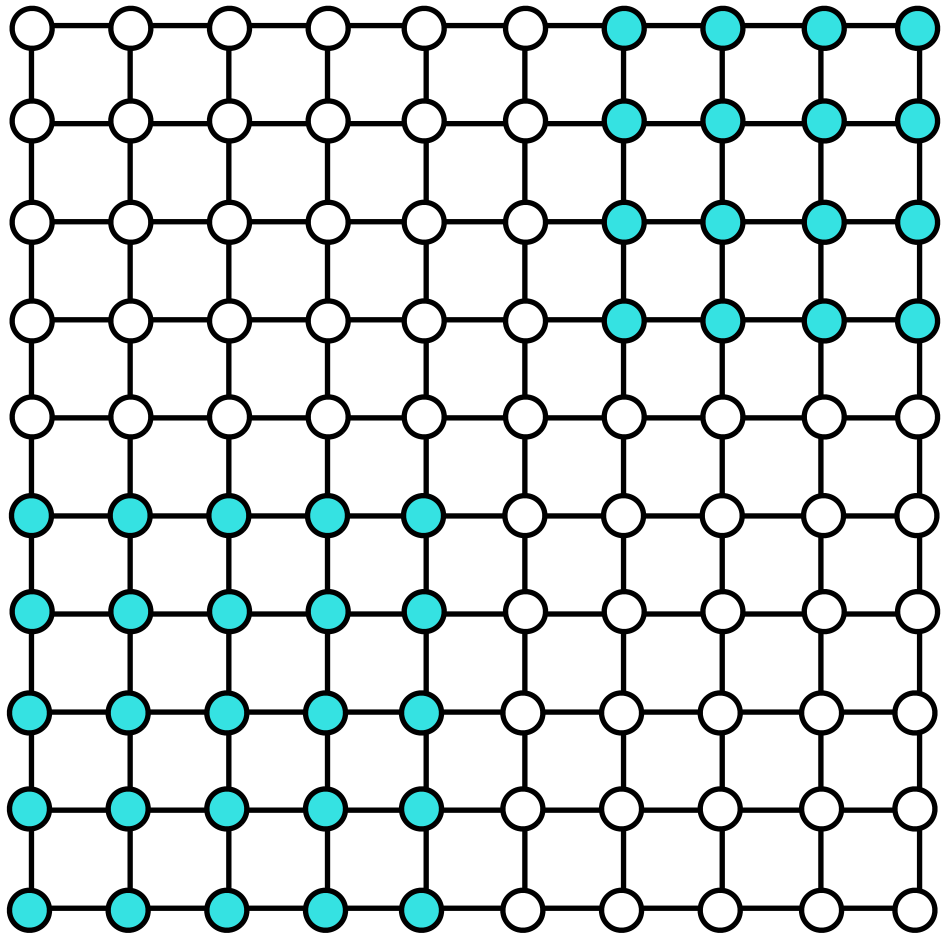

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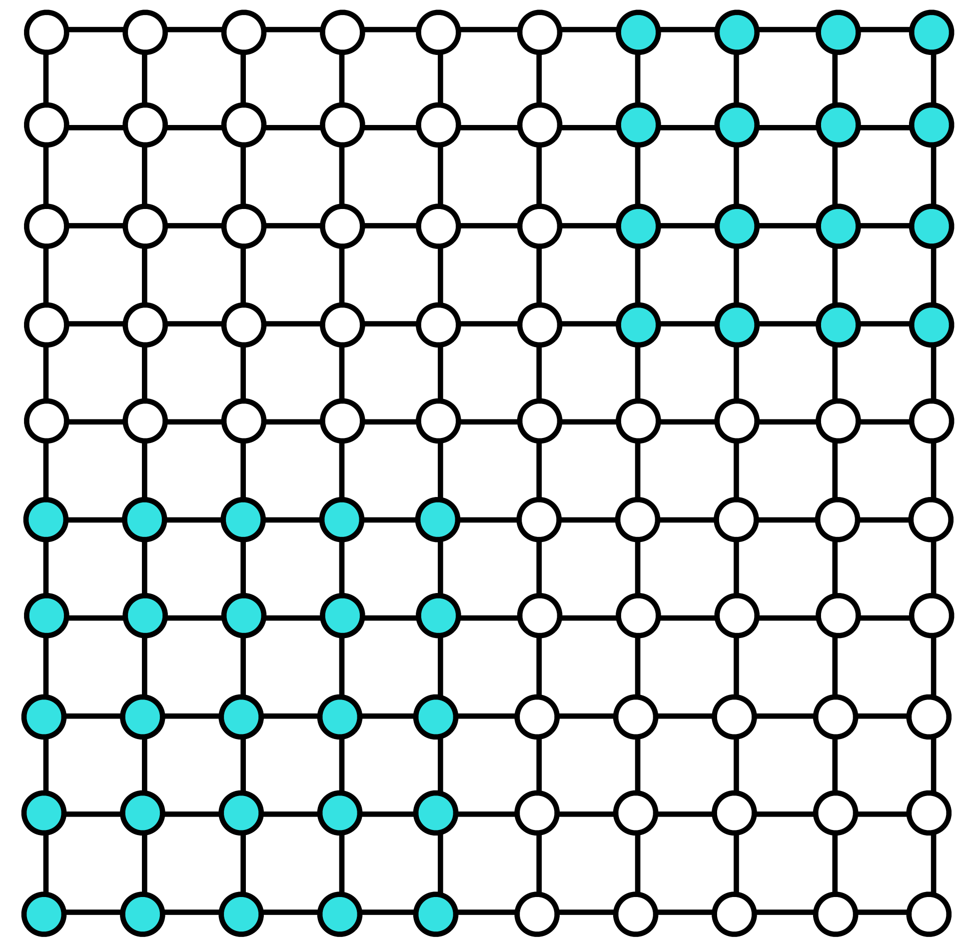
Goal: **Sequentially** and **actively** select a subset $L \subset [n]$ to be labeled.

Predict $\{f(i) : i \notin L\}$.



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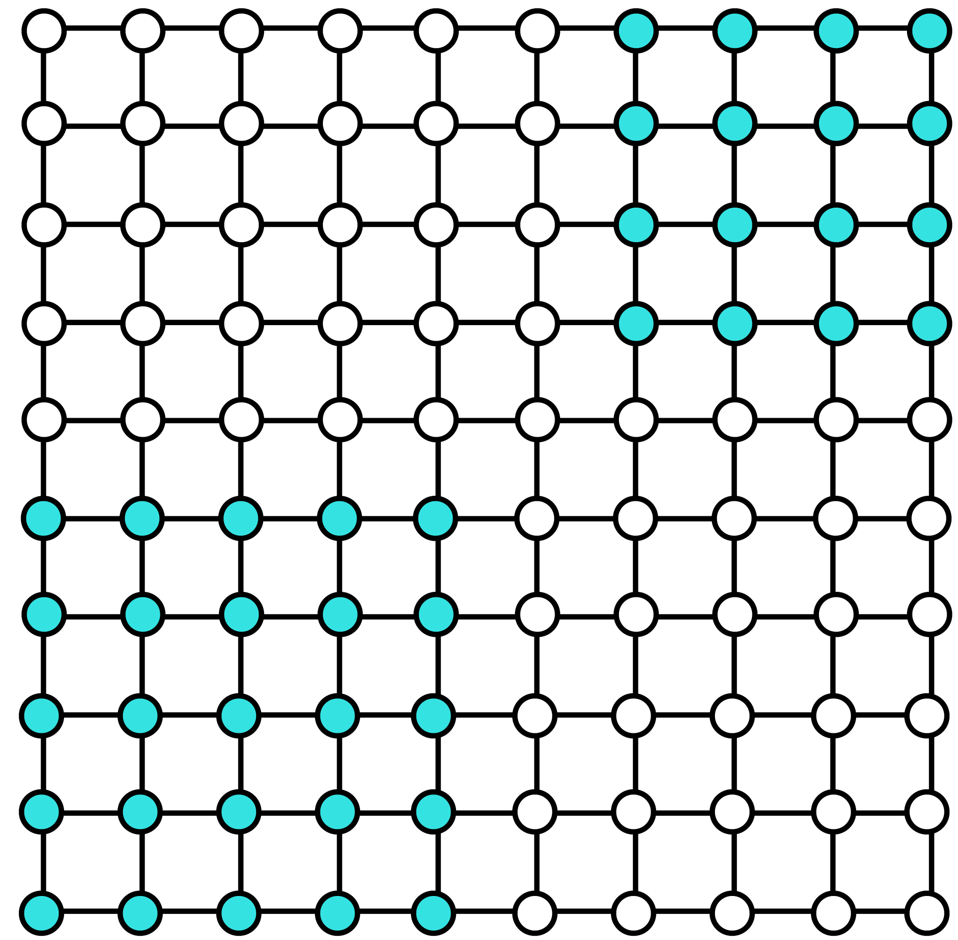
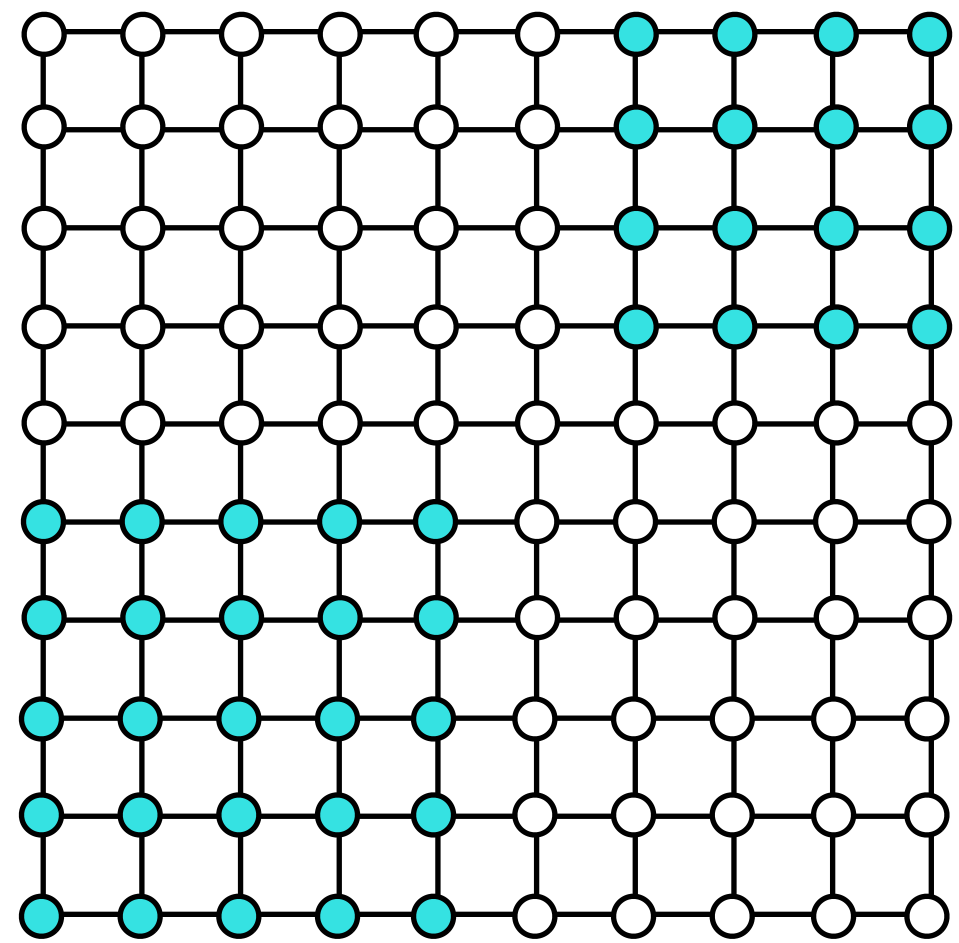


We are naturally interested in questions like:

- How to choose *L* (efficiently) ?
- How big does L have to be ?
- How does the interaction between f and G affect these things?

The S^2 Algorithm

- Randomly query vertices till you find a pair with opposite labels.
- Among all shortest paths connecting oppositely labeled vertices, pick the shortest one and query the vertex at its mid-point.
  - If you find a "cut" edge, remove it and proceed.
- If there are no more oppositely labeled vertices, go back to step 1 (random sampling).



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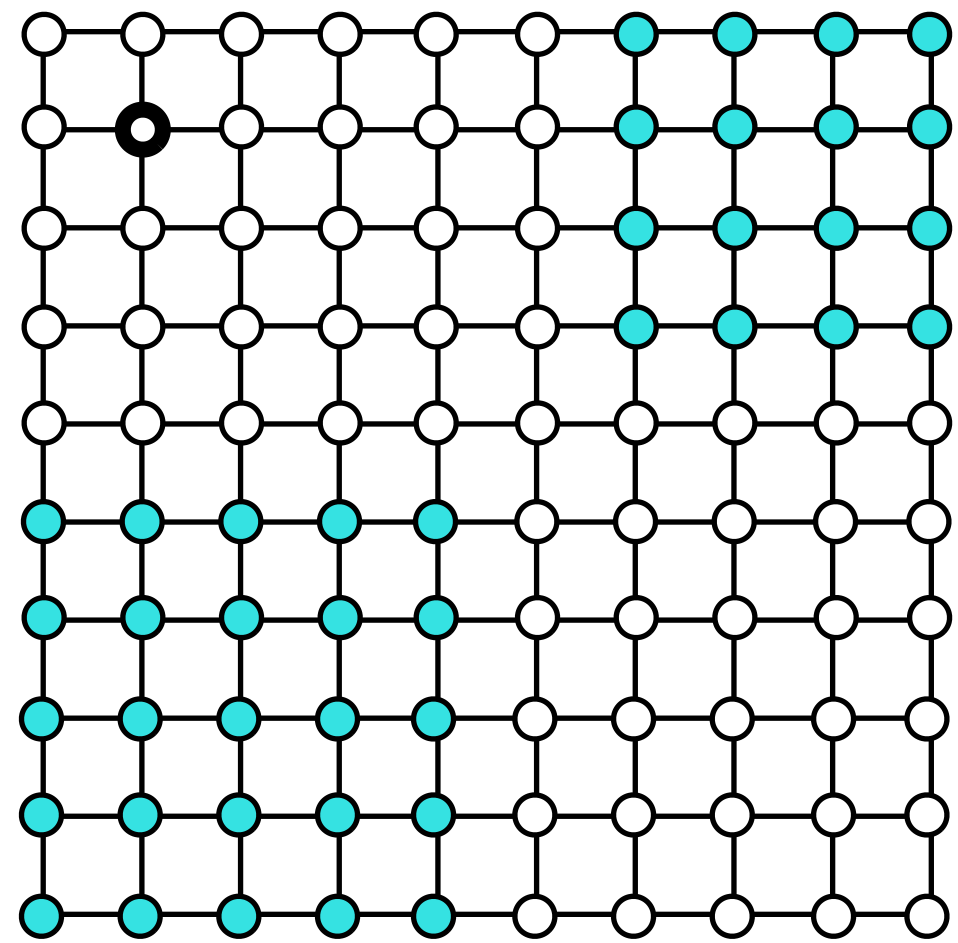

Randomly query

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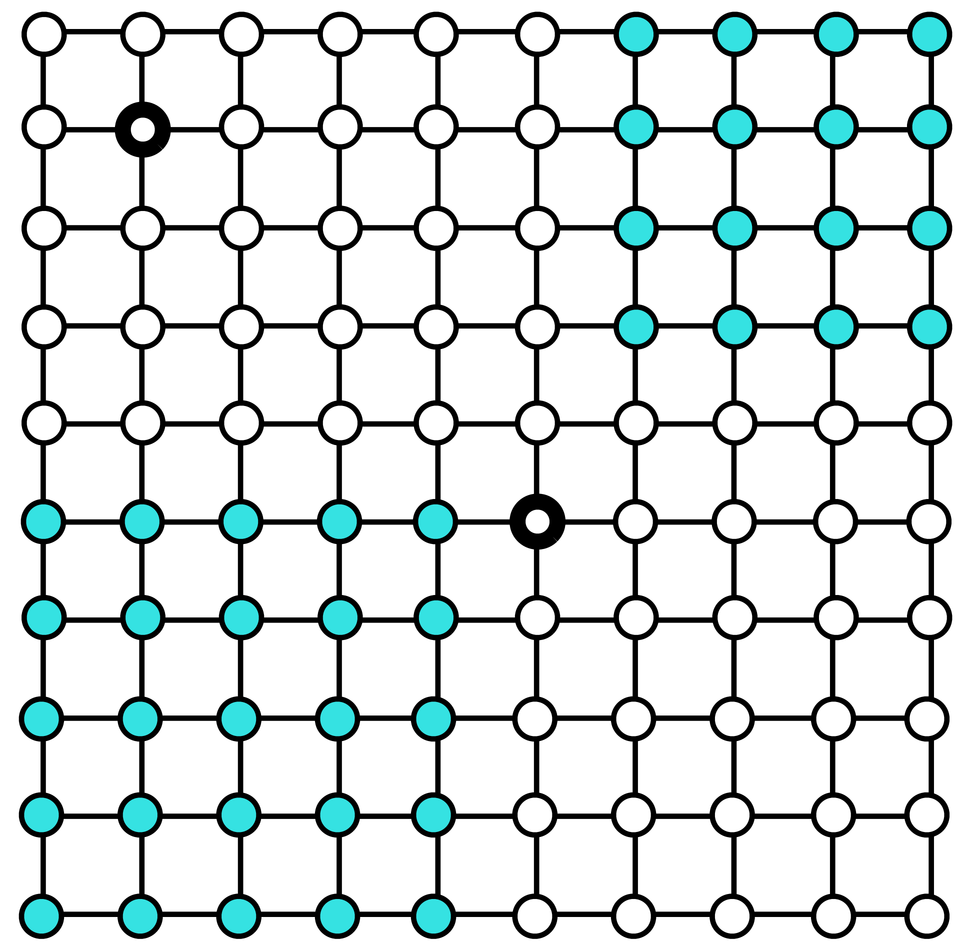

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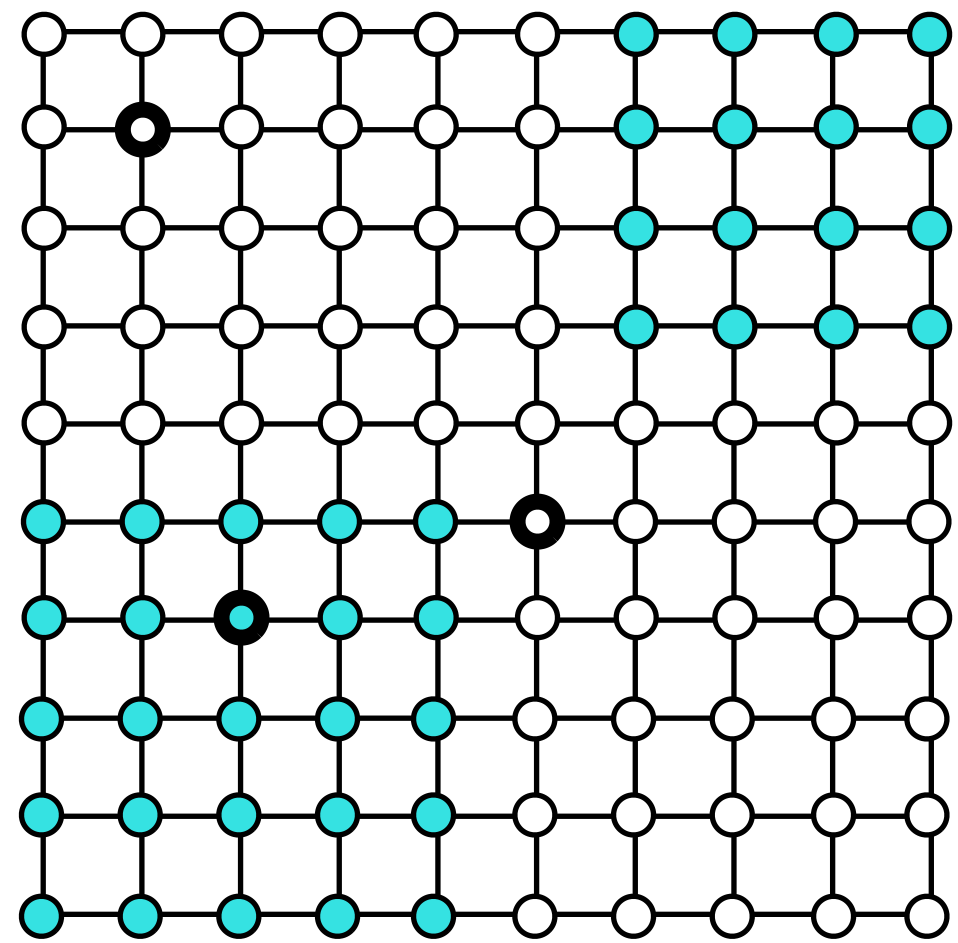

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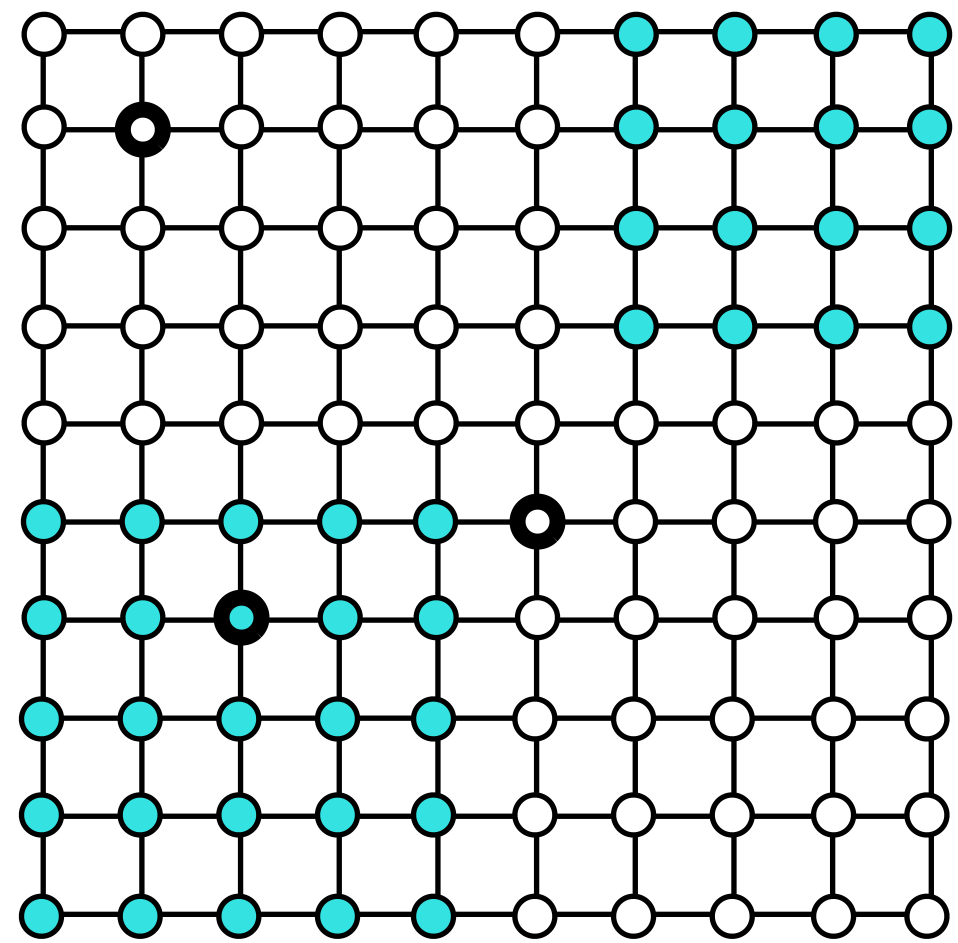

Found oppositely labeled vertices

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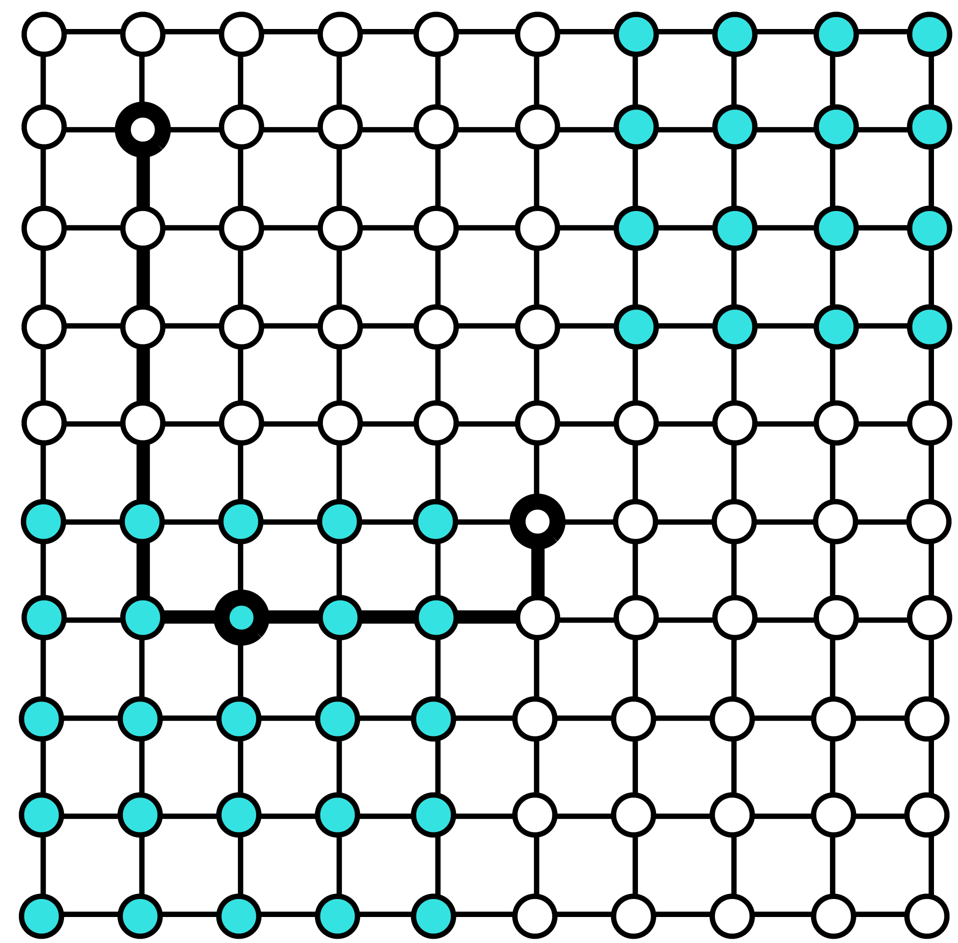

There are 2 shortest paths connecting oppositely labeled vertices.

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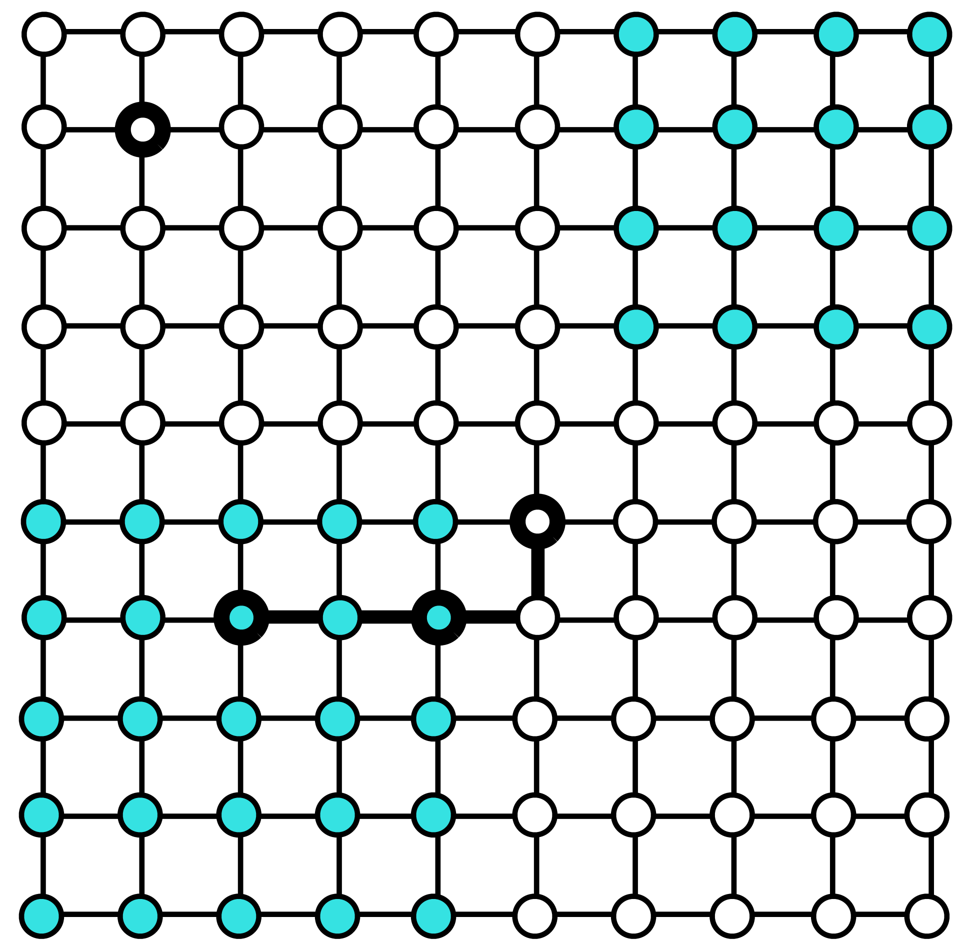

Pick the shortest shortest path and query at midpoint.

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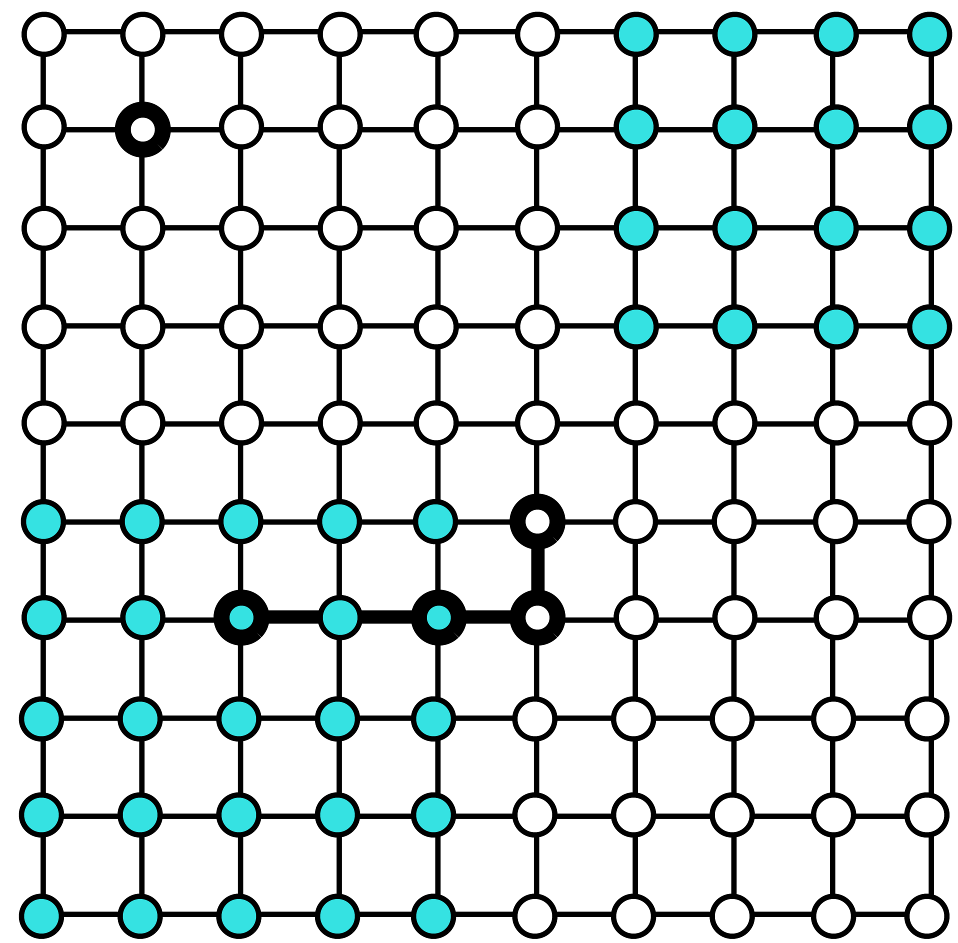

Continue this and remove any cut edges found

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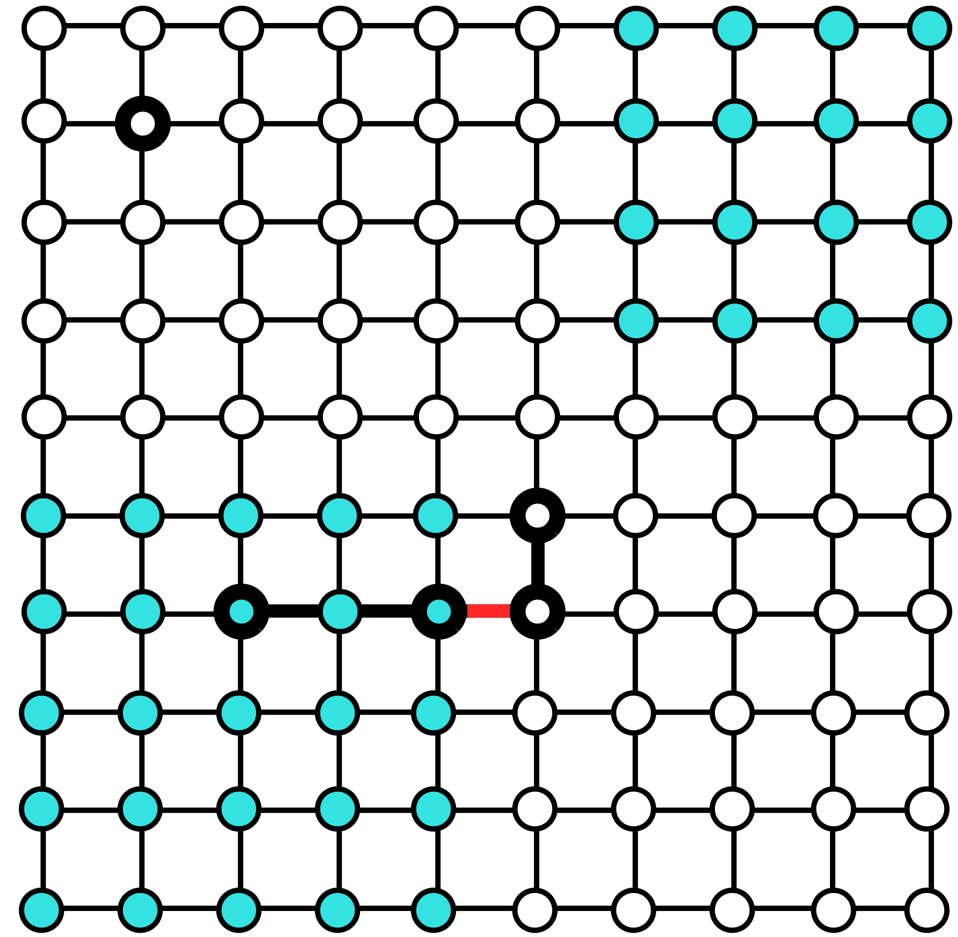

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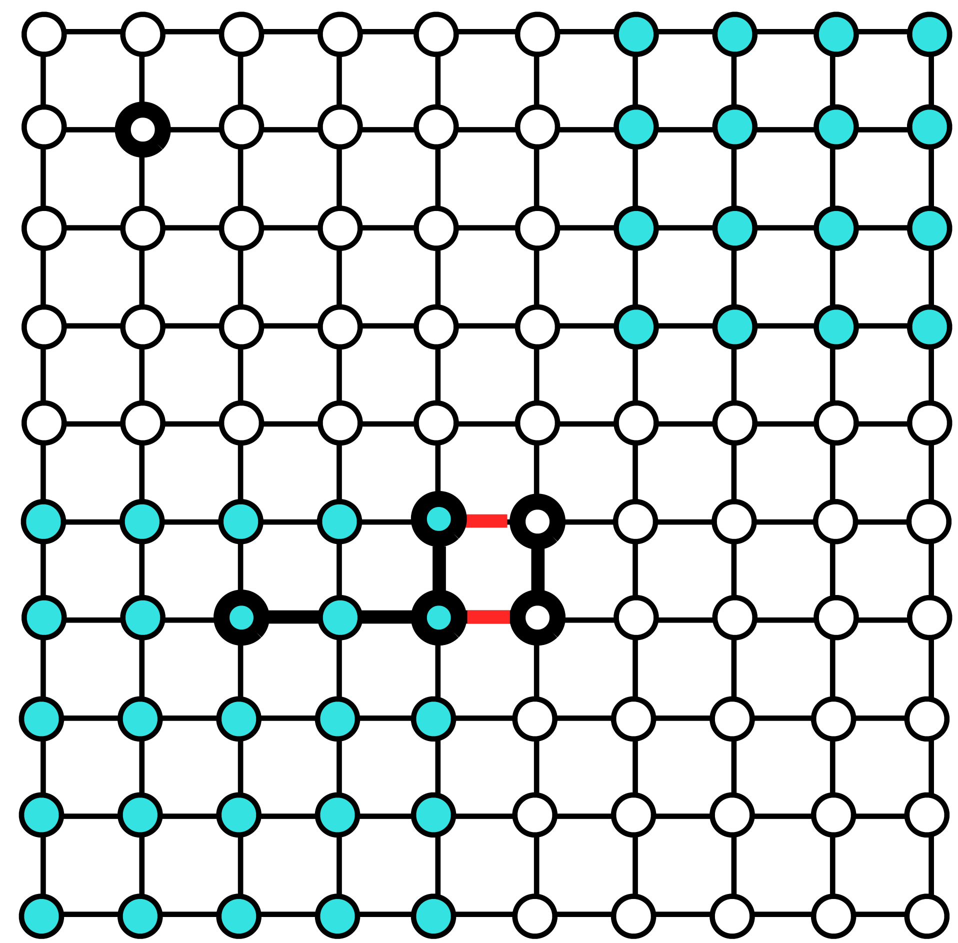

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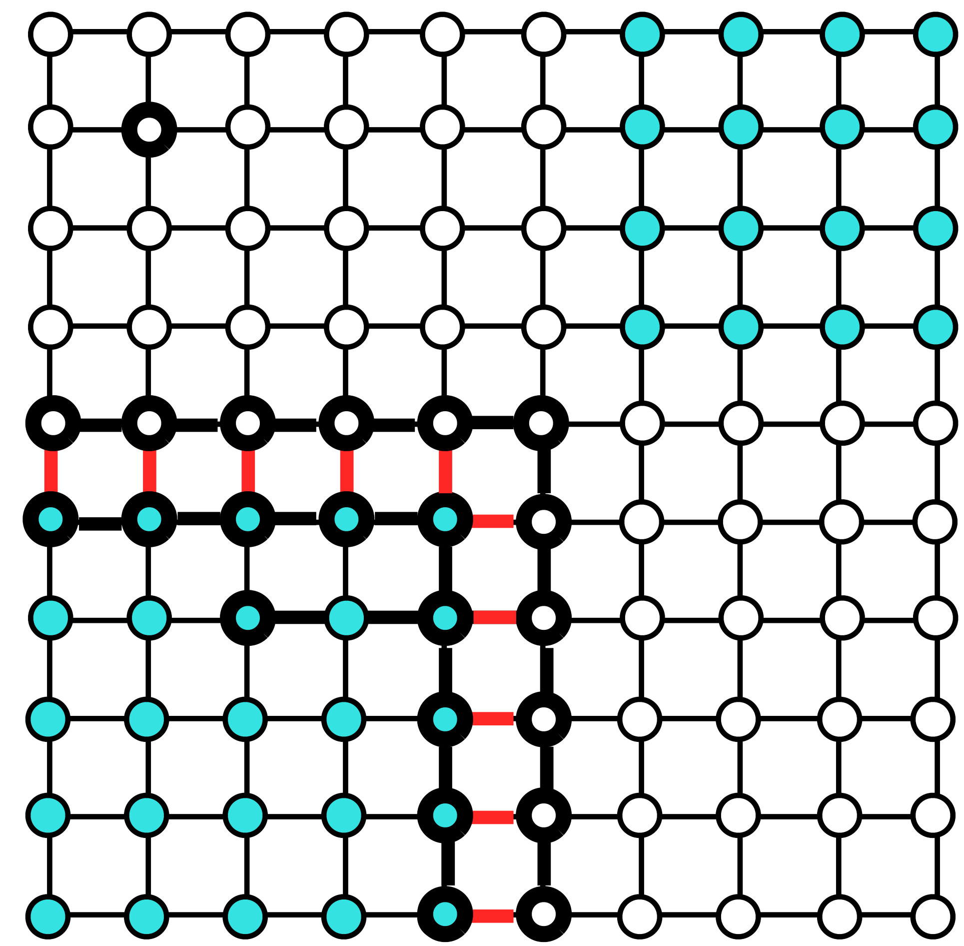

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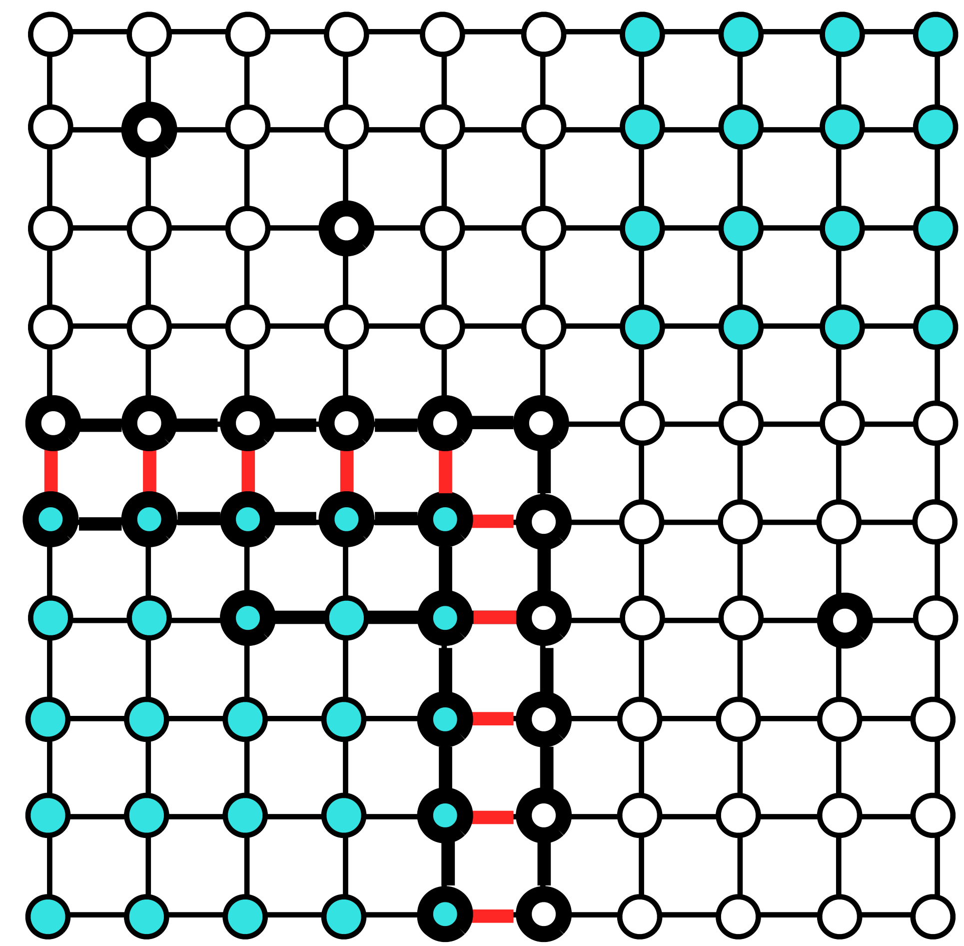

Randomly query again till we find connected pairs of oppositely labeled vertices

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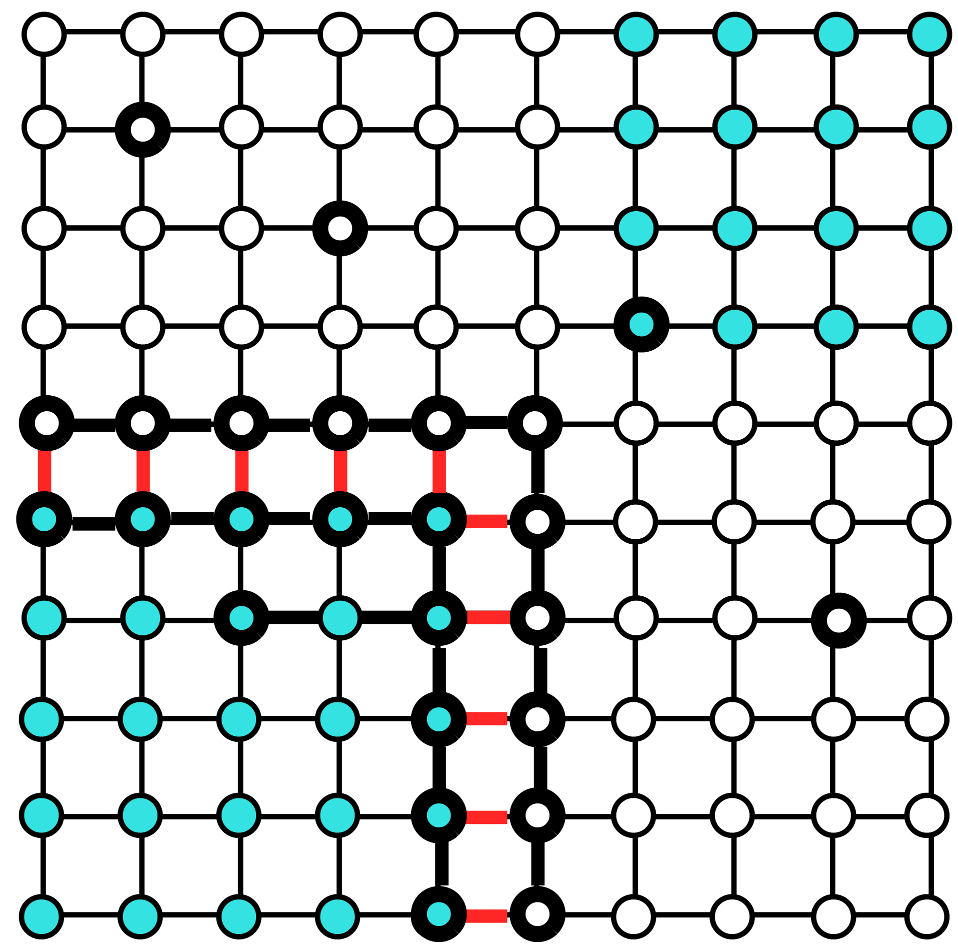

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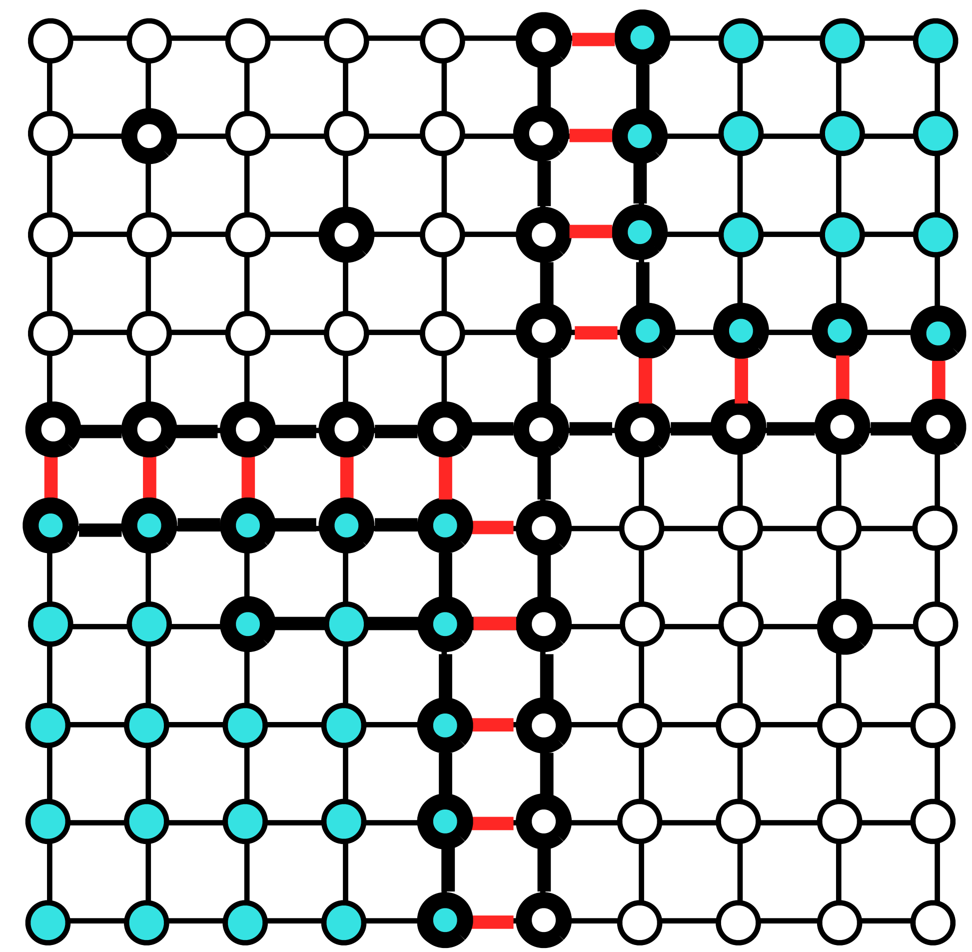

Back to bisecting shortest shortest paths

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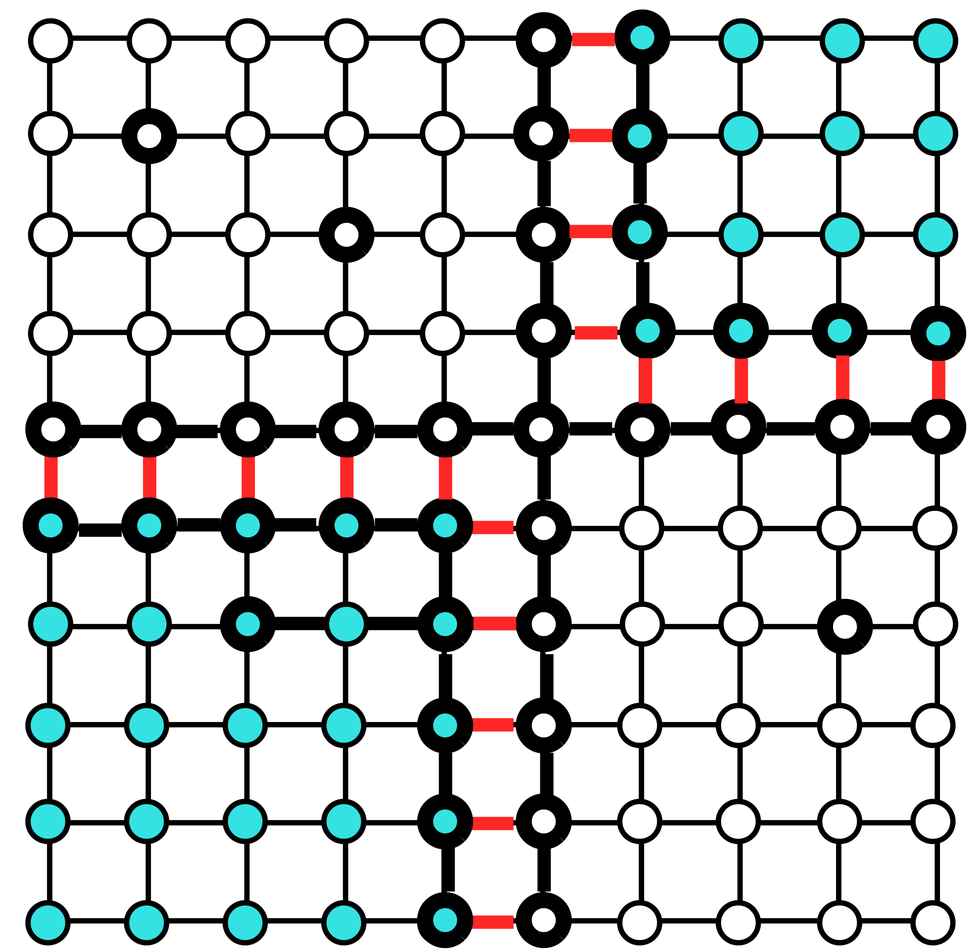


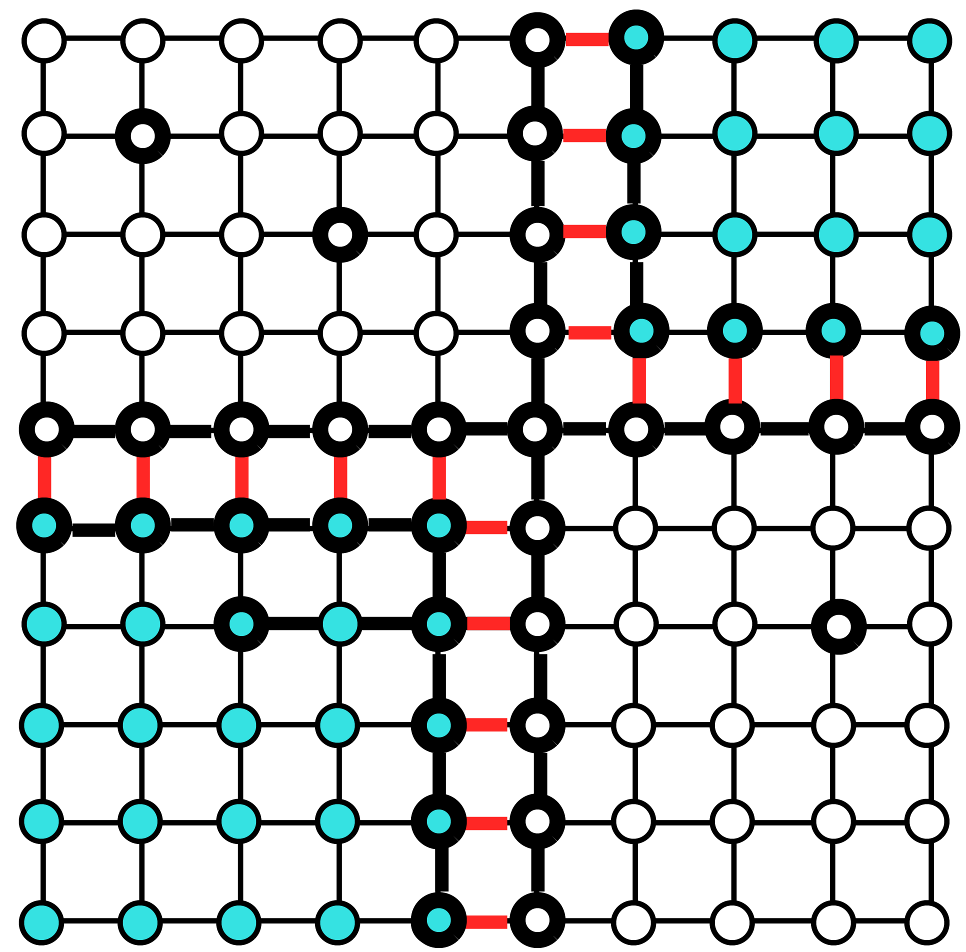
Back to bisecting shortest shortest paths

S^2 thereby very efficiently finds the cut-edges

Visit the poster!

Please visit the poster to:

- Find out about our novel parametrization of the complexity of a binary function with respect to the graph.
  - Size of the cut set (boundary) $|\partial C|$
  - Class balancedness β
  - Cut set composed of m sets of cut edges such that each set is κ clustered.



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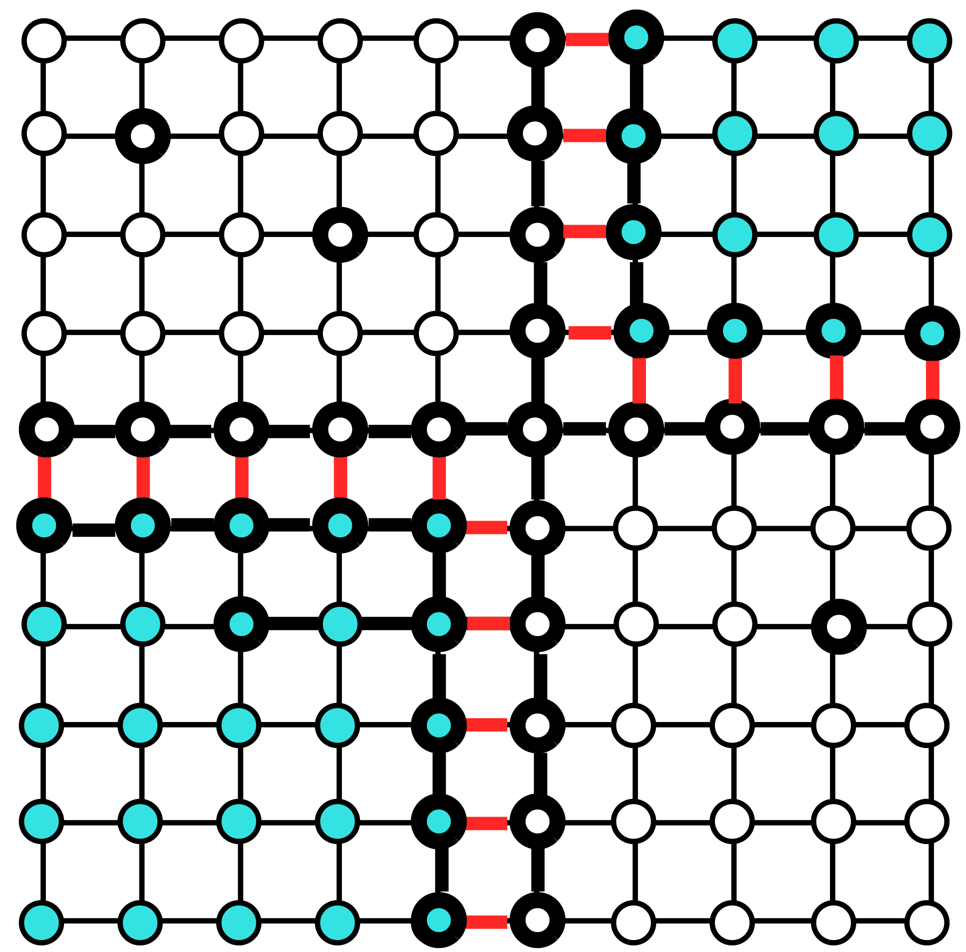
$$\text{Query complexity} \sim \frac{\log(1/\beta\epsilon)}{\log(1/(1-\beta))} + m\log n + |\partial C|\log\kappa$$



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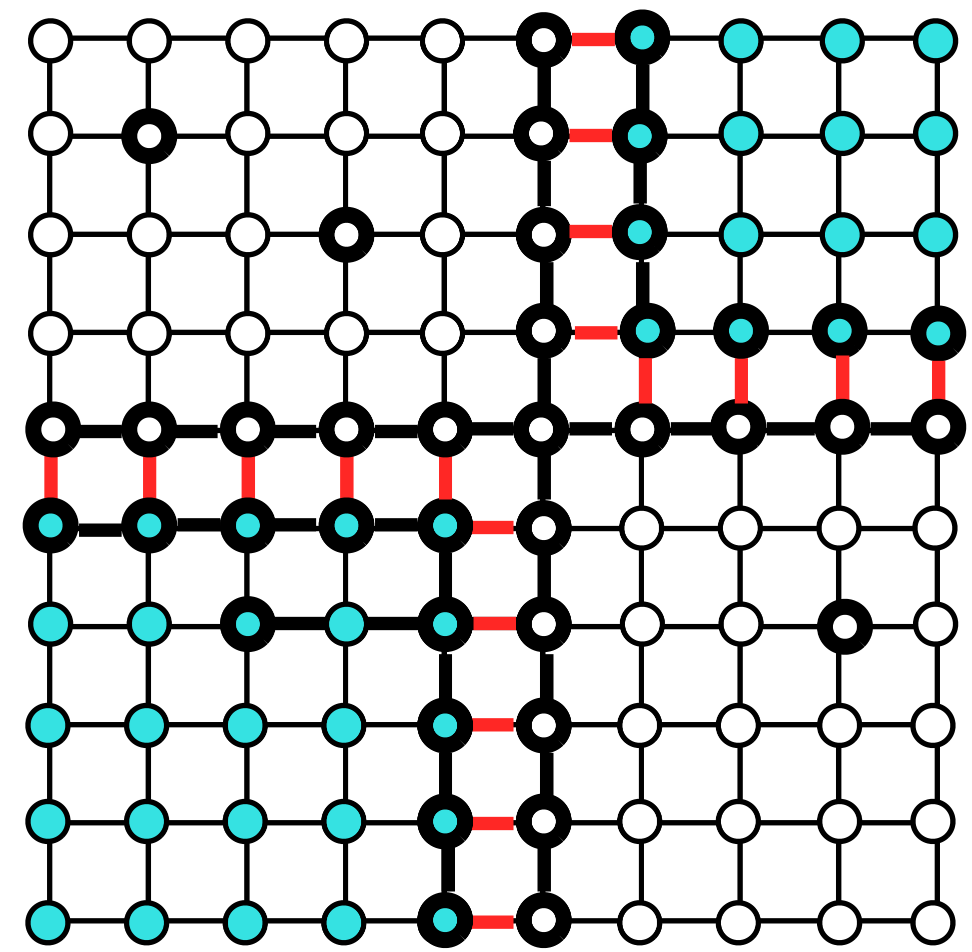

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- Learn how S^2 achieves near minimax optimal sample complexity for nonparametric active classification.
  - Consider the lattice graph and run S^2
  - For a broad class of problems (Bayes decision boundary satisfies some regularity condition), sample complexity is

$$\mathcal{O}\left(\left(\frac{\log n}{n}\right)^{1/d-1}\right)$$

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