Do not open the exam until I say “go.”

Last name: __________________________ First: __________________________

STAT 224 Exam 1, Version 1

Discussion (check one):

. . .

Good morning, STAT 224 Students.

Before the exam begins, put away everything except a pencil, a calculator, and your one-page (two sides) formula sheet.

Your exam version is in the line below the blanks for your name. Write it in Special Code ”B” on your Scantron form. Also write your name and student ID, and fill in the corresponding bubbles.

Show your work. The Scantron office will score your exam, and then TAs will assign partial credit.

Round your answers to 2 places after the decimal point. If you find a question ambiguous, resolve the ambiguity in writing, and we’ll consider grading accordingly.

“None of the above” is a legitimate answer.

The exam will end when I call time (at the bell, if it rings). If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)

You’re welcome to turn your exam and Scantron form in to me at least five minutes before the period ends. But, if you’re still here in the last five minutes, please remain seated until I’ve picked up all the exams and Scantron forms.

Good luck!

1. A high school student applies to seven colleges, each of which makes an admission decision by rolling a fair six-sided die and admitting a student if a 1 is rolled. What is the probability the student will be admitted to exactly two colleges?

   (a) .04

   (b) .17

   (c) .23

   (d) .31

   (e) None of the above

   ANSWER: C
   Let X = #colleges to which student is admitted $\sim$ Bin$(n = 7, p = \frac{1}{6})$. $P(X = 2) = \binom{7}{2}(\frac{1}{6})^2(1 - \frac{1}{6})^{7-2} \approx .2344$

2. A trapezoid has area $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the two parallel sides, and h is the height (or distance between the parallel sides). Suppose these dimensions are measured (in centimeters) as $a = 1$, $b = 3$, and $h = 4$, and it's known that $\sigma_a = \sigma_b = .1$ and $\sigma_h = .2$. Estimate σ_A, the standard deviation of the calculated area.

   (a) .04

   (b) .13

   (c) .37

   (d) .49

   (e) None of the above

   ANSWER: D
   $\sigma_A \approx \sqrt{(\frac{\partial A}{\partial a}\sigma_a)^2 + (\frac{\partial A}{\partial b}\sigma_b)^2 + (\frac{\partial A}{\partial h}\sigma_h)^2} = \sqrt{(\frac{h}{2}\sigma_a)^2 + (\frac{h}{2}\sigma_b)^2 + (\frac{a+b}{2}\sigma_h)^2} = \sqrt{((4/2) * .1)^2 + ((4/2) * .1)^2 + (((1 + 3)/2) * .2)^2} \approx .4899$

3. Suppose X is a continuous random variable with density function $f(x) = 3x^2$ if $x \in [0, 1]$ and 0 otherwise. Find $P(X < \frac{1}{2})$.

   (a) $\frac{1}{8}$

   (b) $\frac{3}{8}$

   (c) $\frac{1}{2}$

   (d) $\frac{5}{8}$

   (e) None of the above

   ANSWER: A
   $P(X < \frac{1}{2}) = \int_0^{\frac{1}{2}} 3x^2\, dx = x^3|_0^{\frac{1}{2}} = \frac{1}{8}$.

4. Suppose telephone calls come into a customer service center according to a Poisson process with rate parameter $\lambda = 6$ per hour. What is the probability of exactly two calls in the next five minutes?

   (a) .02

   (b) .08

   (c) .16

   (d) .22

   (e) None of the above

   ANSWER: B
   Let X = #calls in five minutes $\sim$ Poisson(λt) = Poisson(.5 = $(\frac{6}{\text{hour}})(5\text{ minutes})(\frac{1\text{ hour}}{60\text{ minutes}})$) $\implies P(X = 2) = e^{-.5}\frac{.5^2}{2!} \approx .0758$

5. What's the probability that the time until the next call is less than five minutes?

   (a) .52

   (b) .62

   (c) .72

   (d) .82

   (e) None of the above

   ANSWER: E
   Let T = waiting time $\sim$ Exp(.1 = $\frac{6}{\text{hour}}\frac{1\text{ hour}}{60\text{ minutes}}$) $\implies P(T < 5) = 1 - e^{-.1(5)} \approx .3935$

6. The lifetime of a lightbulb used in refrigerators is normally distributed with mean $\mu = 1000$ hours and standard deviation $\sigma = 150$ hours. What minimum lifetime should the producer promise if it wants the bulbs to meet (or exceed) the promised lifetime 99.7% of the time?

   (a) 587.50

   (b) 601.30

   (c) 640.20

   (d) 692.40

   (e) None of the above

   ANSWER: A
   Let X = lifetime of a bulb and t = the unknown promised lifetime. $P(X > t) = .997 \implies P((Z = \frac{X-\mu}{\sigma}) > \frac{t-1000}{150}) = .997 \implies$ (from table)$\frac{t-1000}{150} = z$, where $P(Z > z) = .997$, so that $z = -2.75 \implies t = -2.75 * 150 + 1000 \approx 587.5$

7. These data relate radiative heat flux $\left(\frac{\text{kW}}{\text{m}^2}\right)$ and electromotive force reading (V) of a radiometer:

Heat flux (y)	15	31	51	55	67	89
Signal output (x)	1.1	2.4	4.2	4.5	5.2	6.9

   The regression line for predicting heat flux (y) from signal output (x) for these data is

   (a) $\hat{y} = -0.06 + 12.69x$

   (b) $\hat{y} = 12.69 + -0.06x$

   (c) $\hat{y} = 0.01 + 7.90x$

   (d) $\hat{y} = 7.90 + 0.01x$

   (e) None of the above.

   ANSWER: A

8. Here are sorted average commute times (in minutes) for each of 51 states (or, rather, 50 states and the District of Columbia):

15.2	15.4	**16.5**	16.9	17.5	17.5	18.1	18.9	19.1	19.4
19.5	19.7	19.9	20.3	20.4	21	21.2	21.6	21.7	21.8
21.8	22.1	22.1	22.5	22.6	22.7	22.7	22.9	23	23.2
23.3	23.3	23.4	23.4	23.6	23.7	23.8	24.5	24.6	24.7
24.8	24.8	25.8	26	26.1	26.5	27	28.4	28.5	30.2
30.4									

Summary statistics include $\bar{x} = 22.4$ and $s = 3.5$. A probability plot for checking whether these data plausibly came from $N(22.4, 3.5^2)$ has 51 points of the form (x_i, y_i). The coordinates of the third point are $(x_3, y_3) = (16.5, y_3)$. What is y_3? (Hint: you do not need to use the raw data.)

(a) 16.5

(b) 16.6

(c) 16.7

(d) 16.8

(e) None of the above

ANSWER: B
$c_3 = \frac{(3-\frac{1}{2})}{51} \approx .0490$. y_3 satisfies $P(X < y_3) = c_3 = .0490 \implies P\left(\frac{X-\mu}{\sigma} < \frac{y_3-22.4}{3.5}\right) = .0490 \implies P\left(Z < (z = \frac{y_3-22.4}{3.5})\right) = .0490 \implies$ (from table) $z \approx -1.655 \implies y_3 = -1.655 * 3.5 + 22.4 \approx 16.6075$

9. A gondola (cable car) at a ski area holds 50 people. It's maximum safe load is 10000 pounds. The weights of skiers (with their equipment) have mean 190 and standard deviation 40 pounds. If 50 (randomly chosen) skiers take the gondola, what is the probability its maximum safe load will be exceeded?

   (a) .02

   (b) .04

   (c) .06

   (d) .08

   (e) None of the above

   ANSWER: B
   Let X_i = weight of i^{th} randomly-chosen skier. X_i has $\mu = 190$ and $\sigma = 40$. CLT says $\bar{X} \sim N(190, 40^2/50)(\approx)$.
   $P(\sum_{i=1}^{50} X_i > 10000) = P(\frac{1}{50}\sum X_i > \frac{10000}{50}) = P(\bar{X} > 200) = P(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{200-190}{40/\sqrt{50}}) = P(Z > 1.7678) \approx P(Z < -1.77) = .0384.$

10. The number of deck screws a cordless drill can drive (until recharging its battery is necessary) has a lognormal distribution with $\mu = 5$ and $\sigma = 0.4$. What is the probability that the number of screws the drill can drive is less than 100?

   (a) .04

   (b) .10

   (c) .16

   (d) .21

   (e) None of the above

   ANSWER: C
   Let X = #screws $\sim$ lognormal$(\mu = 5, \sigma = 0.4) \implies \ln(X) \sim N(5, 0.4^2)$. $P(X < 100) = P(\ln(X) < \ln(100)) = P\left(\frac{\ln(X)-\mu}{\sigma} < \frac{\ln(100)-5}{0.4}\right) = P(Z < -0.987) \approx P(Z < -0.99) \approx .1611$

11. Suppose independent components A, B, C, and D work with probabilities .1, .2, .3, and .4. Find the probability that the following system works.

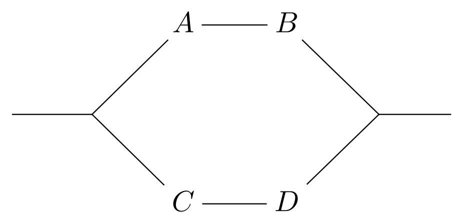
   (a) .03

   (b) .07

   (c) .10

   (d) .14

   (e) None of the above



ANSWER: D

$$
\begin{aligned}
P(\text{works}) &= P(A \cap B) \cup (C \cap D)) \\
&= P(A \cap B) + P(C \cap D) - P((A \cap B) \cap (C \cap D)) \\
&= P(A)P(B) + P(C)P(D) - P(A)P(B)P(C)P(D) \\
&= .1 * .2 + .3 * .4 - .1 * .2 * .3 * .4 \\
&\approx .1376 \\
&\text{or} \\
P(\text{works}) &= 1 - P(\text{fails}) \\
&= 1 - P((A^c \cup B^c) \cap (C^c \cup D^c)) \\
&= 1 - P(A^c \cup B^c)P(C^c \cup D^c) \\
&= 1 - [P(A^c) + P(B^c) - P(A^c)P(B^c)][P(C^c) + P(D^c) - P(C^c)P(D^c)] \\
&= 1 - ((1 - .1) + (1 - .2) - (1 - .1) * (1 - .2)) * ((1 - .3) + (1 - .4) - (1 - .3) * (1 - .4)) \\
&\approx .1376
\end{aligned}
$$

12. R was used to get summary statistics on data on the average commute time (in minutes) for each of 51 states (or, rather, 50 states and the District of Columbia):

```
commute = c(15.2, 15.4, 16.5, 16.9, 17.5, 17.5, 18.1, 18.9, 19.1, 19.4, 19.5, 19.7,
  19.9, 20.3, 20.4, 21, 21.2, 21.6, 21.7, 21.8, 21.8, 22.1, 22.1, 22.5, 22.6,
  22.7, 22.7, 22.9, 23, 23.2, 23.3, 23.3, 23.4, 23.4, 23.6, 23.7, 23.8, 24.5,
  24.6, 24.7, 24.8, 24.8, 25.8, 26, 26.1, 26.5, 27, 28.4, 28.5, 30.2, 30.4)
> summary(commute)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  15.20   20.10   22.70   22.43   24.55   30.40
>
```

What is the interquartile range (IQR) of the commute times? (Hint: you do not need to use the raw data.)

(a) 1.85

(b) 2.60

(c) 7.80

(d) 15.20

(e) None of the above.

ANSWER: E
IQR = $Q_3 - Q_1$ = 24.55 - 20.10 = 4.45