Last name: ___________________________ First: ____________________ (**legible $\implies$ [1] point.**)

STAT 224 Exam 2, Version 1

Discussion (**check one, for [1] point**):

...

Show your work. Points are indicated in square brackets, "[$\cdots$]".

1. [15] A random survey of teachers found that 224 of 395 elementary school teachers, and 126 of 266 high school teachers, were very satisfied with their work. Give a 95% (plus-four) confidence interval for the difference (elementary minus high school) in proportions of teachers who are very satisfied with their work.
   ANSWER: Let X = #elementary satisifed and Y = #high school satisfied. $n_X = 395, X = 224, n_Y = 266, Y = 126 \implies \tilde{n}_X = 397, \tilde{p}_X = \frac{224+1}{397}, \tilde{n}_Y = 268, \tilde{p}_Y = \frac{126+1}{268}, z_{\alpha/2} = z_{.025} = 1.96 \implies$ interval $= (\frac{225}{397} - \frac{127}{268}) \pm 1.96\sqrt{\frac{225}{397}(1-\frac{225}{397})/397 + \frac{127}{268}(1-\frac{127}{268})/268} \approx .093 \pm .077 = (.016, .170)$

2. [10] A scientist computes a confidence interval for an unknown mean μ from a random sample for each of several confidence levels:

Confidence level	Interval
90%	(4.38, 6.02)
95%	(4.22, 6.18)
99%	(3.91, 6.49)

   Now she wants to test $H_0 : \mu = 4$ versus $H_1 : \mu \neq 4$. What can you say about the P-value of her test?
   ANSWER: 4 is outside the 95% interval (and the 90% interval), so she would reject H_0 at level $\alpha = .05$ (and at level $\alpha = .10$): $P < .05$ (and $P < .10$). 4 is inside the 99% interval, so she would retain H_0 at level $\alpha = .01$: $P > .01$. Therefore $.01 < P < .05$.

3. [15] A random sample of 5 Phillips Hall students weighed 132, 145, 162, 166, and 175 pounds. A random sample of 4 Tripp Hall students weighed 137, 147, 158, and 170 pounds. Supposing each hall's population of weights is normally distributed, find a 98% confidence interval for $\mu_P - \mu_T$, where μ_P = the mean Phillips weight and μ_T = the mean Tripp weight. (Hint: you may use $\nu = 6$.)

   ANSWER: Let X = Phillips weight and Y = Tripp weight. $\bar{X} = 156, \bar{Y} = 153, s_X = 17.277, s_Y = 14.213, \text{SE} = \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}} = 10.498, t_{\nu,\alpha/2} = t_{6,.01} = 3.143 \implies$ interval $= (\bar{X} - \bar{Y}) \pm t_{\nu,\alpha/2}\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}} \approx 3 \pm 33.00 = (-30, 36)$

4. [20] Consider whether the month in which you were born predicts attributes of your life. For example, do professional athletes tend to be born in the months just after the age cutoff for separating levels for young players? These players are the oldest in their groups, so they should be bigger and better, and then get more coaching attention as they develop, and so enter into a positive feedback loop. Here are data on the number of 2006 National Hockey League (NHL) players born in each month, along with the proportion of the Canadian population born in each month (in 2000-2005).

Month	#NHL players
January	141
February	128
March	122
April	120
May	117
June	122
July	95
August	91
September	80
October	89
November	74
December	88
Total	1267

Month	Proportion of Canadian births
January	0.0804
February	0.0757
March	0.0860
April	0.0847
May	0.0880
June	0.0854
July	0.0881
August	0.0865
September	0.0863
October	0.0833
November	0.0780
December	0.0776
Total	1.0000

Consider performing a χ^2 test to determine whether NHL births are plausibly from the broader distribution of births in the Canadian population.

(a) Under H_0 : "The NHL birth month data came from the Canadian population distribution," find the expected number of NHL players born in January.

ANSWER: $1267 * 0.0804 = 101.8668 \approx 101.9$

(b) Find the term in the chi-square statistic for January.

ANSWER: $(141 - 101.9)^2/101.9 = 15.0$

(c) Find the degrees of freedom for the chi-square test for these data.

ANSWER: $(\#\text{months} - 1) = 12 - 1 = 11$

(d) Software gives a chi-square statistic $X^2 = 54.3$ for these data. Find the P-value.

ANSWER: $P(\chi^2_{11} > 54.3) < .001$

5. [8] A preliminary study found that, in a sample of 60 coal pieces from a coal seam, the average helium porosity was .055, with standard deviation .0075. What sample size is necessary to estimate the true coal seam average porosity to within .002 with 96.6% confidence?

   ANSWER: $1 - \alpha = .966 \implies \alpha = .034 \implies z_{\alpha/2} = z_{.017} = 2.12; m = .002; n = \left(\frac{z_{\alpha/2}\sigma}{m}\right)^2 = \left(\frac{2.12 \times .0075}{.002}\right)^2 \approx 63.20 \implies$ use $n = 64$

6. [20] A study on pavement deflection in an airport runway ran a Boeing 777 aircraft and a Boeing 747 aircraft across four sections of pavement, with these deflections (in mm):

	Section			
	1	2	3	4
Boeing 777	4.57	4.48	4.36	4.43
Boeing 747	4.01	3.87	3.72	3.76

   Can you conclude that the (population) mean deflection is greater for the Boeing 777?

   - Hypotheses:
   - Assumptions you're making:
   - Test statistic:
   - P-value:
   - Conclusion:

   ANSWER: The samples are not independent–they are paired by pavement section. Let $X =$ Boeing 777 deflection, $Y =$ Boeing 747 deflection, and $D = X - Y$.

   - Hypotheses: $H_0 : \mu_D = 0, H_1 : \mu_D > 0$
   - Assumptions: differences are a random sample from a normal population
   - Test statistic: $\{D_i\} = .56, .61, .64, .67, n = 4, \bar{D} = .62$, and $s_D = .0469 \implies t = \frac{\bar{D} - \mu_0}{s_D/\sqrt{n}} = \frac{.62 - 0}{.0469/\sqrt{4}} = 26.44$
   - P-value: $P(t_{4-1} > 26.44) \approx 0$ (The table says it's $< .0005$.)
   - Conclusion: Yes. (The data are strong evidence that Boeing 777 mean deflection is greater.)

7. [10] The following Minitab output presents a confidence interval for a population mean.

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One-sample T: X

Variable   N   Mean       StDev    SE Mean        95% CI
X         10   6.59635   0.11213   0.03546   (6.51613, 6.67656)
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(a) How many degrees of freedom does the Student's t distribution have?

ANSWER: 10-1 = 9

(b) Use the information in the output, along with a table, to find a 99% confidence interval for the mean.

ANSWER: $n = 10, \bar{x} = 6.59635, s = 0.11213, t_{n-1,\alpha/2} = t_{9,.005} = 3.25 \implies \bar{x} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} = 6.59635 \pm 3.25 * 0.11213/\sqrt{10} = 6.59635 \pm .11524 \approx 6.60 \pm .12 = (6.48, 6.72)$

8. [20] Here are data on the compressive modulus (in psi $\times 10^6$) for two concrete-repair polymers:

					$\bar{x}$	s
Epoxy	1.75	2.12	2.05	1.97	1.97	.160
MMA	1.77	1.59	1.70	1.69	1.69	.074

Can you conclude that the (population) variance of Epoxy is greater than the population variance of MMA? Your answer should include

- Hypotheses:
- Assumptions you're making:
- Test statistic:
- P-value:
- Conclusion:

ANSWER:
Let X = Epoxy modulus, Y = MMA modulus.

- $H_0 : \sigma_X^2 = \sigma_Y^2, H_1 : \sigma_X^2 > \sigma_Y^2$
- Assumptions: each set of 4 measurements is a random sample from a normal population
- $f = \frac{.160^2}{.074^2} \approx 4.67$ (software from raw data: 4.69)
- P-value = $P(F_{3,3} > 4.67) > .10$ (software: .12)
- No. (The data are compatible with H_0.)

9. [20] A scooter maker says its wheels last an average of 50 miles, with standard deviation 5. A quality-control engineer samples 100 wheels to test $H_0 : \mu = 50$ against $H_1 : \mu < 50$. Suppose the test is made at the 5% level. What is the power if the true mean is $\mu = 49$?
ANSWER: level $\alpha = .05 = P(\text{reject } H_0 | H_0 \text{ is true}) = P(\bar{X} < x_{\text{critical}} | \mu = 50) = P(Z < \frac{x-50}{5/\sqrt{100}})$; $\implies \frac{x-50}{5/\sqrt{100}} = -z_{.05} = -1.645 \implies x = .5(-1.645) + 50 = 49.1775$
power $= P(\text{reject } H_0 | H_0 \text{ is false because } \mu = 49) = P(\bar{X} < 49.1775 | \mu = 49) = P(Z < \frac{49.1775-49}{.5}) = P(Z < .355) = 1 - P(Z < -.355) \approx 1 - P(Z < -.36) = 1 - .359 = .641$

10. [10] Six chemicals are tested on cherry trees to see if any reduce damage to leaves by the Japanese beetle. The six P-values (for appropriate H_0) are $.54, .48, .15, .13, .13$, and $.02$. Can we conclude, using level $\alpha = .05$, that the chemical whose P-value is .02 reduces damage? Explain.
ANSWER: No; Bonferroni says to adjust P-value to $NP = 6 * .02 = .12 > (\alpha = .05)$. We should re-test that chemical.