Multiple Regression

Model: $y_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_p x_{pi} + \varepsilon_i$
Test $H_0 : \beta_1 = \cdots = \beta_p = 0$ (proceed only if H_0 is rejected):

Source	DF	SS	MS $= \frac{\text{SS}}{\text{DF}}$	F	P
Regression	p	SSR $= \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$	MSR $= \frac{SSR}{p}$	$\frac{\text{MSR}}{\text{MSE}}$	$P(F_{p,n-(p+1)} > F)$
Error	$n - (p+1)$	SSE $= \sum (y_i - \hat{y}_i)^2$	MSE $= \frac{SSE}{n-(p+1)}$		
Total	$n - 1$	SST $= \sum (y_i - \bar{y})^2$ $=$ SSR + SSE			

$\sigma^2 \approx s^2 = \text{MSE}$
$R^2 =$ proportion of y variation explained by regression $= \frac{\text{SSR}}{\text{SST}} = \frac{\text{SST}-\text{SSE}}{\text{SST}} = 1 - \frac{\text{SSE}}{\text{SST}}$

Inference on Coefficients

$\hat{\beta}_j \pm t_{n-(p+1),\alpha/2} s_{\hat{\beta}_j}$ ($s_{\hat{\beta}_j}$ is from software)

$\frac{\hat{\beta}_j - \beta_{j_0}}{s_{\hat{\beta}_j}} \sim t_{n-(p+1)}$ tests $H_0 : \beta_j = \beta_{j_0}$

Factorial Experiments

One Factor: Test $H_0 : \mu_1 = \cdots = \mu_I$

Source	DF	SS	MS $= \frac{\text{SS}}{\text{DF}}$	F	P
Treatment	$I - 1$	SSTr $= \sum_{i=1}^{I} J_i (\bar{X}_{i.} - \bar{X}_{..})^2$	MSTr $= \frac{SSTr}{I-1}$	$\frac{\text{MSTr}}{\text{MSE}}$	$P(F_{I-1,N-I} > F)$
Error	$N - I$	SSE $= \sum_{i=1}^{I} \sum_{j=1}^{J_i} (X_{ij} - \bar{X}_{i.})^2$	MSE $= \frac{SSE}{N-I}$		
Total	$N - 1$	SST $= \sum_{i=1}^{I} \sum_{j=1}^{J_i} (X_{ij} - \bar{X}_{..})^2$ $=$ SSTr + SSE			

Two Factors: $\mu_{ij} = (\mu + \alpha_i + \beta_j) + \gamma_{ij}$

Source	DF	SS	MS $= \frac{\text{SS}}{\text{DF}}$	F
Row (A)	$I - 1$	$JK \sum \hat{\alpha}_i^2 = JK \sum_{i=1}^{I} (\bar{X}_{i..} - \bar{X}_{...})^2$	$\frac{\text{SSA}}{I-1}$	$\frac{\text{MSA}}{\text{MSE}} \sim F_{I-1, IJ(K-1)}$ $(H_0 : \alpha_1 = \cdots = \alpha_I = 0)$
Column (B)	$J - 1$	$IK \sum \hat{\beta}_j^2 = IK \sum_{j=1}^{J} (\bar{X}_{.j.} - \bar{X}_{...})^2$	$\frac{\text{SSB}}{J-1}$	$\frac{\text{MSB}}{\text{MSE}} \sim F_{J-1, IJ(K-1)}$ $(H_0 : \beta_1 = \cdots = \beta_J = 0)$
Int. (AB)	$(I-1)(J-1)$	$K \sum \sum \hat{\gamma}_{ij}^2$ $= K \sum_{i=1}^{I} \sum_{j=1}^{J} (\bar{X}_{ij.} - (\bar{X}_{...} + \hat{\alpha}_i + \hat{\beta}_j))^2$ $= K \sum_{i=1}^{I} \sum_{j=1}^{J} (\bar{X}_{ij.} - \bar{X}_{i..} - \bar{X}_{.j.} + \bar{X}_{...})^2$	$\frac{\text{SSAB}}{(I-1)(J-1)}$	$\frac{\text{MSAB}}{\text{MSE}} \sim F_{(I-1)(J-1), IJ(K-1)}$ $(H_0 : \gamma_{11} = \cdots = \gamma_{IJ} = 0;$ reject $\implies$ quit)
Error	$IJ(K-1)$	$\sum \sum \sum \text{residual}_{ijk}^2$ $= \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (X_{ijk} - \bar{X}_{ij.})^2$	$\frac{\text{SSE}}{IJ(K-1)}$	
Total	$IJK - 1$	$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (X_{ijk} - \bar{X}_{...})^2$		