3 Probability

- 3.1 Basic Ideas
- 3.2 Conditional Probability and Independence
- 3.3 Random Variables
- 3.4 Functions of Random Variables

3.1 Basic Ideas

A *sample space*, S, is the set of ____________________________ of a random process.

An *event*, A, is a ______________ of S.

e.g. Consider tossing two fair coins.

- The sample space is $S =$ ____________________________ (draw tree)
- Events include
  - $A =$ both heads = ____________________________
  - $B =$ at least one head = ____________________________
  - $C =$ three heads = ____________________________

Combining Events

Set notation is convenient for describing compound events:

- $A \cup B =$ "A _______ B" = *union* of A and B = { outcomes in _____________________ }
- $A \cap B =$ "A _______ B" = *intersection* of A and B = { outcomes in ______________ }
- $A^c =$ "_______ A" = *complement* of A = { outcomes that _____________________ to A }

e.g. For tossing two coins,

- $A \cup B =$ ____________________________
- $A \cap B =$ ____________________________
- $A^c =$ ____________________________
- $A \cup A^c =$ ____________________________
- $A \cap A^c =$ ____________________________
- $A^c \cup B =$ ____________________________
- $A^c \cap B =$ ____________________________

Mutually Exclusive Events

Events A and B are *mutually exclusive* if they have __________________________ ($A \cap B = \emptyset$).
Events $A_1, \cdots, A_n$ are mutually exclusive if ____________________ has no outcomes in common.

e.g. For tossing two coins, __________________________ are mutually exclusive.

Axioms of Probability

The *probability* of an outcome of a random process is the ________________ of times the outcome would occur __________________________, if the process were to be repeated ____________________.

e.g. Here are results of computer simulation of n random coin tosses:

n	Data	#Heads	#Tails	$P(\text{Heads}) \approx \frac{\#\text{Heads}}{n}$
1	T	0	1	
10	TTTHHTTHTH	4	6	
100	HTTHTTTTHT $\cdots$	53	47	
1000	HTTHTTTTTT $\cdots$	491	509	
1000000000	THHTTHHHHT $\cdots$	500002628	499997372	

As ____________________ , the proportion of heads is approaching the long-run proportion ______________, which is $P(\text{Heads})$.

The probability of an event A is denoted $P(A)$.

Axioms of probability include

- $P(S) =$ _______
- For any event A, ____________________
- For mutually exclusive events A and B, $P(A \cup B) =$ _____________; for mutually exclusive $A_1, A_2, \cdots$, $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$

Consequences include

- For any event A, $P(A^c) =$ ____________________
- $P(\emptyset) =$ _______

e.g. Here is the distribution of Canadian responses to the question, "What is your mother tongue?"

Language	English	French	Asian/Pacific	Other
Probability	0.59	0.23	0.07	x

- $x =$
- $P(\text{mother tongue isn't English}) =$
- $P(\text{mother tongue is English or French}) =$

e.g. For tossing two coins, each of the four outcomes is equally likely, so

- $P(HH) = P(HT) = P(TH) = P(TT) =$ ____________
- $P(A) =$ ____________
- $P(B) =$ ____________
- $P(B^c) =$ ________________________
- $P(C) =$ ____________

Sample Spaces with Equally Likely Outcomes

If S is a sample space with N equally-likely outcomes, then the probability of each outcome is ____________. If A is an event containing k outcomes, then $P(A) =$ ____________.

e.g. Consider drawing a card randomly from a 52-card deck. $P(\text{Ace of spades}) =$ ____________, $P(\text{Ace}) =$ ____________, and $P(\text{spade}) =$ ____________.

The Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (draw overlapping circles)

e.g. $P(\text{Ace or spade}) =$

Note that the axiom "for mutually exclusive A and B, $P(A \cup B) = P(A) + P(B)$" is the special case where ________________________.

Examples

e.g. (p. 73 #3) Of a certain manufacturer's silicon wafers, 10% have resistances below specification and 5% have resistances above specification.

- What is the probabilty that the resistance of a randomly chosen wafer does not meet the specification?

- If a randomly chosen wafer has a resistance that does not meet the specification, what is the probability that it's too low? (Hint: draw a picture.)

e.g. (#6) Human blood may contain either or both of two antigens, A and B. Type A blood contains only antigen A, type B blood contains only antigen B, type AB blood contains both antigens, and type O blood contains neither. At a certain blood bank, 35% of donors have type A blood, 10% have type B, and 5% have type AB.

- What is the probability that a randomly chosen donor is type O?

- A type A recipient may receive blood from a donor whose blood doesn't contain the B antigen. What is the probability that a randomly chosen donor may donate to a type A recipient?

e.g. (#7) 60% of purchases at a computer store are desktops, 30% are laptops, and 10% are printers. An audit samples one purchase record at random.

- What is the probability that it's a desktop?
- What is the probability that is a desktop or laptop?