3.2 Conditional Probability and Independence

Conditional Probability

The *conditional probability* of event A, given that event B occurred, is denoted "____________" (and pronounced "probability of A, given B").

e.g. Consider the sample space $S = \{$ outcomes of two fair die rolls $\}$ and these events:

- $A =$ first die is a 3
- $B =$ second die is a 1
- $C =$ the dice sum to 8

Then $|S| =$ ____________ and

- $P(A) =$ ____________
- $P(B) =$ ____________
- $P(C) =$ ____________
- $P(A \cap B) =$ ____________
- $P(A \cap C) =$ ____________
- $P(B \cap C) =$ ____________

Now suppose we know A occurred. Then

- $P(B|A) =$ ____________
- $P(C|A) =$ ____________

If, instead, we know B occurred, then

- $P(A|B) =$ ____________
- $P(C|B) =$ ____________

These quantities are related by the conditional probability formula $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ (draw).

Rearranging gives $P(A \cap B) =$ ____________ (the book's "Multiplication Rule").

(Or, swapping A and B, the original conditional probability formula becomes $P(B|A) =$ ____________ so that $P(A \cap B) =$ __________________.)

Independent Events

Two events A and B are *independent* if and only if $P(A \cap B) =$ ____________. A collection of events is *mutually independent* if and only if, for every finite subset $A_1, \cdots, A_n$, we have $P(A_1 \cap \cdots \cap A_n) = P(A_1) \cdots P(A_n)$.

e.g. Regarding rolling two dice, events ____________ and ____________ are independent.

For independent A and B, the conditional probability formula, $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$, implies $P(A|B) =$ ________________________ and $P(B|A) =$ ________________________.

(Note: The book swaps the definition and consequence.)

e.g. (p. 83 #5) A geneticist is studying two genes. Each gene can be either dominant or recessive. A sample of 100 individuals is categorized as follows:

	Gene 2	
Gene 1	Dominant	Recessive
Dominant	56	24
Recessive	14	6

For a randomly sampled individual, find

a. $P(\text{Gene 1 is dominant}) =$

b. $P(\text{Gene 2 is dominant}) =$

c. $P(\text{Gene 2 is dominant} \mid \text{Gene 1 is dominant}) =$

d. These genes are in "linkage equilibrium" if the events "Gene 1 is dominiant" and "Gene 2 is dominant" are independent. Are they in equilibrium?

A Few Set Algebra Formulas (draw)

- $(A \cup B)^c = A^c \cap B^c$

- $(A \cap B)^c = A^c \cup B^c$

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Application to Reliability Analysis

- If a system relies on *both* of two independent parts A and B, then $P(\text{system success}) =$ __________________________. This corresponds to wiring ____________________. (draw)

- If a system relies on *either* of two independent parts A and B, then $P(\text{system success}) =$ __________________________. This corresponds to wiring ____________________. (draw)

e.g. Suppose you plan to sleep 1500 feet up a stone, and you have two independent carabiners, A and B, each of which has a 90% chance of holding for the night.

- If you hang from them in series, then $P(\text{you live}) =$

- If you hang from them in parallel, then $P(\text{you live}) =$

e.g. Given independent components A, B, C, D, and E, with failure probabilities 1/2, 1/3, 1/4, 1/5, and 1/6, find the probability of success of a system that connects A, B, and C in parallel, D and E in series, and the two subsystems in parallel. (Hint: draw a picture.)

More Examples (if time allows)

e.g. (Part of p. 84 # 10) Show that if A and B are independent, then A^c and B^c are independent.