6.4 Small-Sample Tests for a Population Mean

Recall from §5.4 that, for a population with mean μ and standard deviation σ,

- CLT says for large n, $\bar{X} \sim N($______, ______$)$ ($\approx$, for almost any population)
- For small n in general, we're out of luck
- If population is ____________, then $\bar{X} \sim N($______, ______$)$ (exactly), even for small n
- Standardizing gives $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$, but we don't know σ and can't use __________ for small n
- $T = \frac{\quad}{\quad} \sim t_{n-1}$, Student's t distribution with $n-1$ degrees of freedom, which
  - looks like ____________
  - is shorter with thicker tails (random ______ causes more variability than fixed ______)
  - approaches $N(0,1)$ as ____________
  - was discovered by William Gosset at ____________

We can use $T = \frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$ to do hypothesis tests for μ from a small sample for which the assumption of a normal population is ____________.

Example: Amber Bubbles

e.g. In §5.4, we considered the N concentration (%) in nine air bubbles trapped in ancient amber:

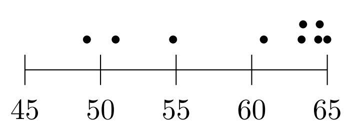
63.4 65.0 64.4 63.3 54.8 64.5 60.8 49.1 51.0

They had mean 59.59 and standard deviation 6.26. Do these data give good reason to think the % N in the air 85 million years ago was different from today's 78.1%?

Test H_0: ____________ vs. H_1: ____________

Check for departure from ____________ (roughly symmetric, single peak, no outliers):

Here's our dotplot from §5.4:



test statistic: $t =$

P-value =

Conclusion:

Summary

Suppose $X_1, \cdots, X_n$ is a (small) random sample from a *normal* population with mean μ and standard deviation σ. To test that μ has a specified value, $H_0 : \mu = \mu_0$,

1. State null and alternative hypotheses, H_0 and H_1
2. Check assumptions
3. Find the test statistic, $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
4. Find the P-value, which is an area under t_{n-1} depending on H_1:

   $H_1 : \mu > \mu_0 \implies P\text{-value} = P(T > t)$, the area right of t (where $T \sim t_{n-1}$)

   $H_1 : \mu < \mu_0 \implies P\text{-value} = P(T < t)$, the area left of t

   $H_1 : \mu \neq \mu_0 \implies P\text{-value} = P(|T| > |t|)$, the sum of areas left of $-|t|$ and right of $|t|$
5. Draw a conclusion.

Beer Bubbles

e.g. Guinness on tap is pressurized by bottled gasses (________________) which force beer through tiny holes in a plate in the tap. Guinness says it takes 119.5 seconds to pour a pint from a tap correctly. Here is a random sample of pouring times (in seconds) by Mike at The Glass Hat.

118 121 113 116 133 117 112 113

Is Mike pouring too quickly?

Test H_0: ______________ vs. H_1: ______________

Check assumptions:

$\bar{x} =$ _______, $s =$ _______, $\nu =$ _______

$t =$

P-value $=$

Conclusion:

Remember

If you like Guinness, you like ____________________.

Soda Bottles

e.g. (p. 238 #7) Two-liter soda bottles are supposed to have average wall thickness 4.0 mils. A quality control engineer samples 7 bottles from a large batch and finds these thicknesses:

4.00 4.04 4.12 4.06 3.97 3.96 4.09

We want to test $H_0 : \mu = 4.0$ vs. $H_1 : \mu \neq 4.0$.

a. Make a dotplot of the data.

b. Should a t-test be used for H_0? If so, do the test. If not, explain why not.

$\bar{x} =$

$s =$

$t =$

$\nu =$

P-value =

Conclusion:

c. Measurements on a different bottle type are 4.07, 3.97, 4.03, 4.01, 4.20, 4.06, and 4.01. Make a dotplot of these values.

d. Should a t-test be used for H_0? If so, do the test. If not, explain why not.

A Pattern to Notice

Many hypothesis tests (including those from §6.1, 6.3, 6.4, 7.1, 7.2, 7.3, and 7.4) use test statistics of the form

$$\frac{\text{(point estimate)} - \text{(parameter value \underline{\hspace{4cm}})}}{\text{(estimated or true) \underline{\hspace{5cm}} of point estimate}}$$

This ________________ point estimate tells how far the estimate is from the parameter, in _________________.