6.6 Fixed-Level Testing
6.7 Power (_____________)

Example

Suppose you are writing a contract between the producer of spliced ropes and the consumer, a parachute maker needing lines to attach a canopy to a harness.

- The producer promises that the mean breaking strength of the lines is $\mu = 100$ pounds, with $\sigma = 16$.
- An independent lab will find $\bar{X}$ from a SRS of $n = 10$ lines to test $H_0 : \mu = (\mu_0 = 100)$ vs. $H_1 : \mu < \mu_0$.
- A draft contract specifies $\bar{x}_{\text{critical}} = 97$, above which H_0 is retained, and below which H_0 is rejected.
- Suppose strengths $\sim N(\dots)$, and the promised $\sigma = 16$ is correct. Then $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(\dots)$ (even with small n), so we can use a Z distribution instead of T.

Suppose you work for the producer (the splicing shop). If H_0 is true, what is the probability that the test will reject H_0? (In this case, the shipment of lines will be discarded, and you will not be paid.) Draw a picture of the distribution of $\bar{X}$, indicating μ_0 and $\bar{x}_{\text{critical}}$, and shading the required probability. Which way would you like to move $\bar{x}_{\text{critical}}$ to decrease your risk?

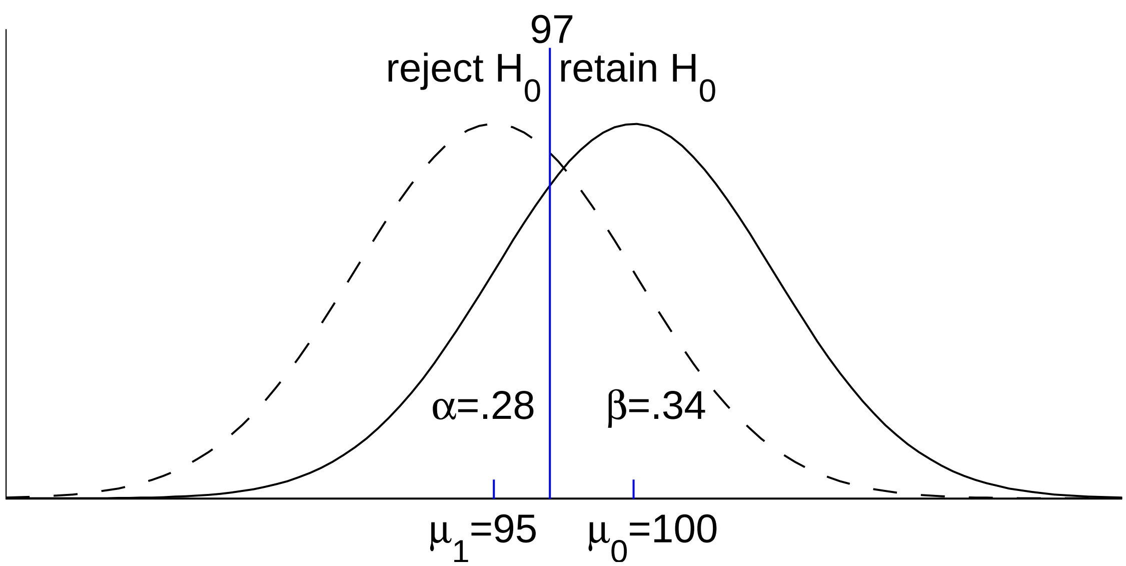
Suppose you work for the consumer (the parachute maker). You can't use the lines if $\mu = 95$ (unless you redesign your parachute to use more of the weaker lines). If H_0 is false because $\mu = (\mu_1 = 95)$, what is the probability that the test will retain H_0? (In this case, you'll use a defective shipment of lines, and then sell defective parachutes.) Draw a picture of the distribution of $\bar{X}$, indicating μ_1 and $\bar{x}_{\text{critical}}$, and shading the required probability. Which way would you like to move $\bar{x}_{\text{critical}}$ to decrease your risk?

Terminology

- A *type I error* occurs when we reject H_0 when it's true
- $\alpha = P(\text{type I error})$ = producer's risk of a shipment meeting the specification being rejected
- A *type II error* occurs when we retain H_0 when it's false because $\mu = \mu_1$ (where $\mu_1 \neq \mu_0$)
- $\beta = P(\text{type II error})$ = consumer's risk of using a defective shipment

Superimpose the pictures

- Producer, use the solid curve for your $H_0 : \mu = 100$ distribution of $\bar{X}$. Mark your risk as α.
- Consumer, use the dashed curve for your $H_1 : \mu = 95$ distribution. Mark your risk as β.



Why does it seem that the producer and consumer may not agree to a contract?

We have a problem! Identify at least one thing each of the following parties can do to resolve it.

Hint: for each of the quantities mentioned below, figure out the role it plays in the preceding figure.

- Business people from both producer and consumer control $\bar{x}_{\text{critical}}$ and the price.

- Producer engineers control the quality of the spliced lines and can change μ_0 and σ.

- Consumer engineers control the use of the lines and can change μ_1.

- Quality control engineers collect and analyze the sample and can change the sample size, n.

Which of these options is likely to be easiest?

Loose ends

6.6 Fixed-Level Testing

Critical Point and Rejection Region

§6.2 called a test result *statistically significant* (and rejected H_0) at the (100%)α level if ________ .

In a *fixed-level test*, we decide α, compute a P-value, and then reject $H_0 \iff$ ______________ .

A decision threshold is expressed in terms of a ____________________ (instead of the P-value) by finding the *critical point* of the statistic that produces a P-value equal to α. The *rejection region* of the statistic consists of values ___________________________ as the critical point (draw for example, below).

e.g. The producer found that our test of parachute lines could be done at level $\alpha =$ ______ with critical $z =$ ______ and rejection region $z <$ ______ . But suppose the producer wants $\alpha = .05$:

- Find the critical point and rejection in terms of $\bar{X}$ and in terms of z.

- What will the consumer think of this?

6.7 Power

The *power* of a test $H_0 : \mu = \mu_0$ is its probability of ___________ H_0 when it is false because $\mu = \mu_1$. Power = _______ .

We'd like low α and low β (high power), but these goals are ______________ . ____________________ can allow both goals to be met.

e.g. For the parachute lines contract and test of $H_0 : \mu = (\mu_0 = 100)$ vs. $H_1 : \mu < \mu_0$, suppose

- the producer won't change $\mu_0 = 100$ or $\sigma = 16$
- the consumer won't change $\mu_1 = 95$
- the producer wants level $\alpha = .05$
- the consumer wants power $= .8$ when $\mu = (\mu_1 = 95)$

Two variables are still in play: find the required $\bar{x}_{\text{critical}}$ (call it x to save writing) and n. n will be larger than the original 10, which will tighten up both $\bar{X}$ distributions. Hint:

- producer: draw $\alpha = .05$ under the $\mu_0 = 100$ solid curve, and solve for x in terms of n
- consumer: draw power $= .8$ for $\mu_1 = 95$ under the dashed curve, and solve for x in terms of n
- both: set your x's equal to solve for n
- use n in either of the first two equations to find x