___ Optional Q&A review at 4:00 on Friday, November 22, in Psychology 107

7.5 The F Test for Equality of Variance

Let $X_1, \cdots, X_{n_X}$ and $Y_1, \cdots, Y_{n_Y}$ be independent random samples from *normal* populations with (unknown) means μ_X and μ_Y (which we _____) and (unknown) variances σ_X^2 and σ_Y^2. (Non-normal $\implies$ _____.)

Recall that the sample variances are

$$s_X^2 = \frac{1}{n_X - 1}\sum_{i=1}^{n_X}(X_i - \bar{X})^2 \quad \text{and} \quad s_Y^2 = \frac{1}{n_Y - 1}\sum_{i=1}^{n_Y}(Y_i - \bar{Y})^2$$

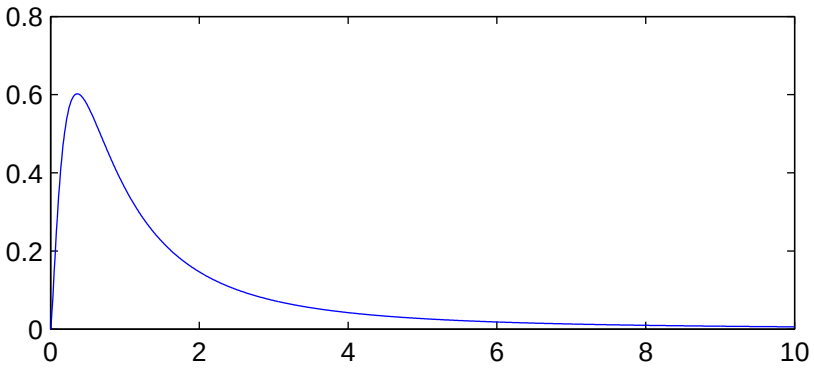
To test $H_0 : \sigma_X^2 = \sigma_Y^2$, use a test statistic $F = \dfrac{s_X^2}{s_Y^2}$. Under H_0, this statistic has the F *distribution* with _____ and _____ degrees of freedom.

The F Distributions

(More generally, if $X \sim \chi^2_{\nu_1}$ and $Y \sim \chi^2_{\nu_2}$ are independent, then $\dfrac{X/\nu_1}{Y/\nu_2} \sim F(\nu_1, \nu_2)$.)

An F distribution is specified by two values for degrees of freedom, ν_1 and ν_2. Its properties include:

- ν_1 corresponds to F's _____: ν_1 and ν_2 are not interchangeable
- $F \geq 0$ (it's a ratio of _____ numbers)
- Each F_{ν_1,ν_2} density function is skewed right
- e.g. Here's $F_{5,3}$:



- Table A.6 (pp. 526-533) gives, in column ν_1, row ν_2, and subrow α, the point $F_{\nu_1,\nu_2,\alpha}$ with area _____ to its right.
  e.g. $F_{5,3,.05}$ = _____ (draw)

- If $\sigma_X^2 = \sigma_Y^2$, we expect $s_X^2 \approx s_Y^2$, so F should be near _____; large $F \implies$ _____, and small $F \implies$ _____ (this is the key to the test)

The F Test

Let $X_1, \cdots, X_{n_X}$ and $Y_1, \cdots, Y_{n_Y}$ be independent random samples from normal populations with variances σ_X^2 and σ_Y^2. To test $H_0 : \sigma_X^2 = \sigma_Y^2$,

1. State null and alternative hypotheses, H_0 and H_1
2. Check assumptions
3. Find the test statistic $f = \dfrac{s_X^2}{s_Y^2}$, _____ *X and Y if necessary so* _____
4. Find the P-value, which is an area under the F_{n_X-1,n_Y-1} curve depending on H_1:
   - $H_1 : \sigma_X^2 > \sigma_Y^2 \implies P$-value $= P(F_{n_X-1,n_Y-1} > f)$, the area _____
   - $H_1 : \sigma_X^2 \neq \sigma_Y^2 \implies P$-value = _____ the P-value from a one-sided test

   (• $H_1 : \sigma_X^2 < \sigma_Y^2$ is handled by the _____ in step 3)
5. Draw a conclusion

Caution

The F test is _____ for populations that aren't very close to _____.

Examples

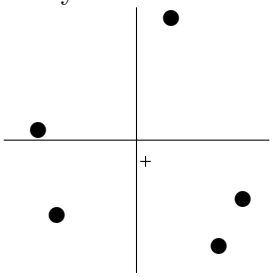
- e.g. $F = \frac{s_X^2}{s_Y^2}$ is calculated from samples of sizes $n_X = 10$ and $n_Y = 8$.

  a. What is the upper 5% critical value for this F?

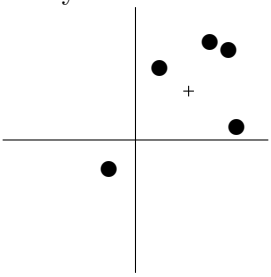
  b. In a test of equality of variances against the two-sided alternative, this statistic has the value $f = 3.45$. Is this value significant at the 10% level? At the 5% level?

- e.g. Molly and Jenny test themselves by hustling up Green Mountain and immediately firing their .50 BMG rifles at ($100'' \times 100''$) targets a mile away on Bear Peak, with these results:

Molly:



Jenny:



The targets suggest that Molly's shots are centered better (+) than Jenny's. (§5.1 might say Molly is more _____.) Jenny can adjust her scope _____.

To simplify things, project to the x-axis, reducing to 1 dimension:

Molly:

Jenny:

The x-coordinates of the shots (in inches) are:

						$\bar{x}$	s
Molly	31	40	−37	−30	13	3.40	35.1
Jenny	−10	38	35	9	28	20.00	20.2

The targets suggest that Jenny's shots are closer together. (§5.1 might say Jenny is more _____.) Test $H_0 : \sigma_M^2 = \sigma_J^2$ against $H_1 : \sigma_M^2 > \sigma_J^2$.

- e.g. Grace runs a doll umbrella production line that includes an automated caliper to check lengths of titanium stretchers to the nearest .001 mm. She sends a single stretcher through the caliper 8 times. Then she increases the caliper's jaw speed (which would increase the line's capacity), and sends the same stretcher through 7 more times, with these results (mm):

									$\bar{x}$	s
Before	21.529	21.528	21.529	21.529	21.528	21.530	21.529	21.529	21.529	.0006
After	21.531	21.525	21.528	21.531	21.520	21.535	21.542		21.530	.0071

Test whether the increased speed changes the variability of the measurements.