## 9.5 Preface

Ronald Fisher (1890-1962), "a _____ who almost single-handedly created the foundations for modern statistical science", said,

> No aphorism is more frequently repeated in connection with field trials, than that we must ask _____ few questions, or, ideally, one question, at a time. The writer is convinced that this view is _____. Nature, he suggests, will best respond to a _____ and _____ questionnaire ...

A factorial experiment allows estimation of the effect of _____ factors, and _____ _____, with the same _____ a simple experiment requires to estimate any one effect with _____ degree of accuracy.

e.g. (Harold Hotelling, 1895-1973) A pan balance used with a set of standard masses indicates the _____ masses in its two pans. Suppose it has independent random errors with mean 0 and standard deviation $\sigma$. How should we find the masses of four diamonds whose (unknown) true masses are $\theta_1, \theta_2, \theta_3$, and $\theta_4$?

- _____: use the balance _____ times, with diamonds arranged in the pans as follows:

| Measurement | Left | Right | Mean |
|---|---|---|---|
| $M_1$ | 1 | empty | |
| $M_2$ | 2 | empty | |
| $M_3$ | 3 | empty | |
| $M_4$ | 4 | empty | |

  For $i = 1$ to 4, $\hat{\theta}_i =$ _____ has standard deviation _____

  (Improving all four $\hat{\theta}_i$'s standard deviations to $\sigma/2$ requires _____ measurements.)

- _____: use the balance _____ times, arranging diamonds in pans as follows:

| Measurement | Left | Right | Mean |
|---|---|---|---|
| $M_1$ | | | |
| $M_2$ | | | |
| $M_3$ | | | |
| $M_4$ | | | |

The estimates are then

$\hat{\theta}_1 =$

$\hat{\theta}_2 =$

$\hat{\theta}_3 = \frac{1}{4}(M_1 - M_2 + M_3 - M_4)$

$\hat{\theta}_4 = \frac{1}{4}(M_1 - M_2 - M_3 + M_4)$

$\hat{\theta}_1$ has variance $\sigma^2_{\hat{\theta}_1} = \sigma^2_{\frac{1}{4}(M_1+M_2+M_3+M_4)} =$ _____

and standard deviation _____

($\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_4$ also have standard deviation _____.)

# 9.5 $2^p$ Factorial Experiments (part 1 of 2)

To study $p$ factors simultaneously, a preliminary experiment can be done in which each factor has only _____ levels, _____ and _____. The experiment then has _____ treatments and is called a $2^p$ *factorial experiment*. It prepares for further experimentation on the important factors at (possibly) _____ levels per factor.

We've already discussed $2^p$ factorial experiments for $p =$ _____ and _____.

e.g. A $2^3$ factorial experiment has _____ factors and _____ treatments.

## $2^3$ Factorial Experiments

Consider a $2^p$ factorial experiment with $p = 3$ factors _____, _____, and _____.

### Treatments

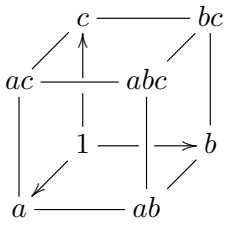
The treatment with all factors at their _____ levels is denoted "_____". Other treatments are denoted by lower-case character strings, where "_____", "_____", or "_____" indicates that the corresponding factor is at its _____ level.

e.g. The treatment

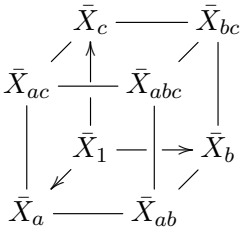
- "1" has all 3 factors _____
- "a" has _____ high and _____ low
- "ac" has _____ high, and _____ low

e.g. The 8 treatments are _____

The 8 treatments may be pictured as:




Corresponding treatment cell means are:




**Main Effects**

A _____ is a linear combination of treatment means whose coefficients add to _____. The contrast of a factor is the _____ of mean responses at the factor's _____ level minus the _____ of mean responses at the factor's _____ level.

e.g. The contrast for factor A is

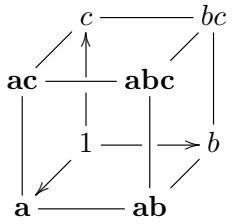
The *main effect* of a factor is the difference in its _____ response with the factor at its _____ level and its _____ response with the factor _____. A main effect estimate is therefore _____ of its contrast.

e.g. The main effect for factor A is denoted $A$ and estimated as

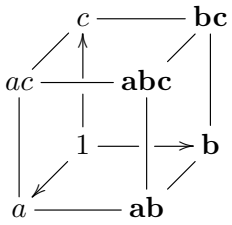
$$\tfrac{1}{4}(\bar{X}_a + \bar{X}_{ab} + \bar{X}_{ac} + \bar{X}_{abc}) - \tfrac{1}{4}(\bar{X}_1 + \bar{X}_b + \bar{X}_c + \bar{X}_{bc})$$

Here are the main effects pictured:

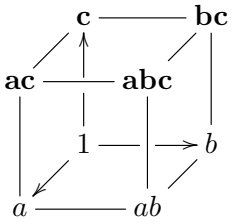
- $A =$ _____ mean minus _____ mean:




- $B =$ _____ mean minus _____ mean:



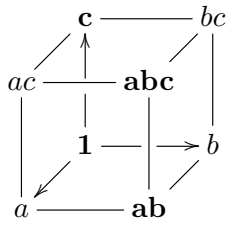
- $C =$ _____ mean minus _____ mean:




**Interactions**

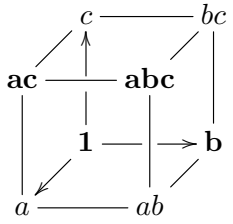
_____, _____, and _____ are the three _____ interactions:

- $AB =$ *half* the difference in the mean ___ effect with _____ and the mean ___ effect with _____ = the diagonal plane $1, ab, c, abc$ mean minus the diagonal plane $a, b, ac, bc$ mean:

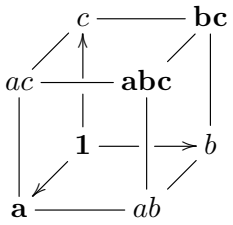



  (The "half" makes the _____ the same for all main effect and interaction estimates.)

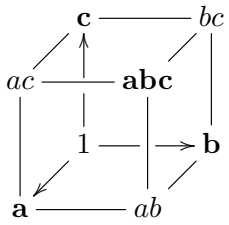
- $AC =$ the diagonal plane $1, b, ac, abc$ mean minus the diagonal plane $a, ab, c, bc$ mean:




- $BC =$ the diagonal plane $1, a, bc, abc$ mean minus the diagonal plane $b, ab, c, ac$ mean:



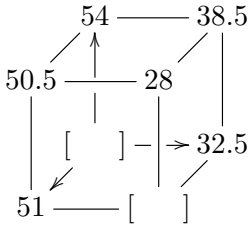
$ABC$, the one three-way interaction, is *half* the difference in the mean AB interaction with C high and the mean AB iteraction with C low = mean of $a, b, c, abc$ minus mean of $1, ab, ac, bc$:

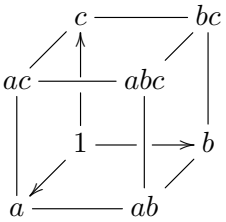



**Example**

e.g. (p. 463 #4) A study on the effects of 3 vitamins, A = nicotinic acid, B = thiamine, and C = biotin, on the yield ($\frac{1}{100}$g/L) of the organic acid pyruvate in a cell culture used two replicates per treatment. In the data, "-1" indicates the low factor level and "1" indicates the high level:

| A | B | C | Treatment | Yields | Mean Yield |
|---|---|---|---|---|---|
| -1 | -1 | -1 | _____ | 55, 49 | _____ |
| 1 | -1 | -1 | $a$ | 60, 42 | 51 |
| -1 | 1 | -1 | $b$ | 37, 28 | 32.5 |
| 1 | 1 | -1 | _____ | 30, 28 | _____ |
| -1 | -1 | 1 | $c$ | 54, 54 | 54 |
| 1 | -1 | 1 | $ac$ | 54, 47 | 50.5 |
| -1 | 1 | 1 | $bc$ | 44, 33 | 38.5 |
| 1 | 1 | 1 | $abc$ | 36, 20 | 28 |




Estimate the main effects and interactions. Which do you think are most important?

- $A =$
- $B =$
- $C = \frac{1}{4}(50.5 + 28 + 54 + 38.5) - \frac{1}{4}(51 + 29 + 52 + 32.5) = 1.625$
- $AB =$
- $AC =$
- $BC = \frac{1}{4}(28 + 38.5 + 51 + 52) - \frac{1}{4}(50.5 + 54 + 29 + 32.5) = .875$
- $ABC =$

Next time we'll see how to test which of these main effects and interactions are _____.