9.5 2^p Factorial Experiments (part 2 of 2)

An Alternate Presentation: The Sign Table

A 2^p ______ *table* provides an another way of presenting the main effects ____________ and interactions __________________ and ____________. It indicates

- for each of the ____ main effects whether that factor is at its high (____) or low (____) level
- for each interaction, the ________________________ of its main effects

Treatment	Cell Mean	Main Effects: A	B	C	Interactions: AB	AC	BC	ABC
1	$\bar{X}_1$	$-$	$-$		$+$	$+$		
a		$+$	$-$	$-$	$-$	$-$	$+$	$+$
b	$\bar{X}_b$	$-$	$+$	$-$	$-$	$+$	$-$	$+$
	$\bar{X}_{ab}$							
c	$\bar{X}_c$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
ac	$\bar{X}_{ac}$	$+$	$-$	$+$	$-$	$+$	$-$	$-$
bc	$\bar{X}_{bc}$	$-$	$+$	$+$	$-$	$-$	$+$	$-$
abc	$\bar{X}_{abc}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$

The estimated contrast for a main effect or interaction is the __________________ of its column with the __________________ column.

e.g. Factor A's estimated contrast is still

A's estimated main effect is still ______ of its contrast, which can be rearranged to estimated $A = \frac{1}{4}(\bar{X}_a + \bar{X}_{ab} + \bar{X}_{ac} + \bar{X}_{abc}) - \frac{1}{4}(\bar{X}_1 + \bar{X}_b + \bar{X}_c + \bar{X}_{bc})$

e.g. Estimate the three-way interaction ABC:

Assumptions (same as in §9.3)

1. The design is ____________
2. The design is ____________
3. The #replicates $K \geq$ ______ for each treatment
4. Observations in each treatment are a simple random sample from a ________ population
5. All treatment populations have ________________________

An ANOVA table (as in §9.3) helps test, for each main effect and interaction, the H_0 that it is ______. The test statistics require ______ squares from ____________ squares from ______________.

Sums of Squares, Mean Squares, and Tests

- The sum of squares for residuals is

  $$\text{SSE} = \sum_{\text{all } 2^p \text{ treatments } t} \; \sum_{t\text{'s } K \text{ observations}} (\text{observation} - t \text{ sample mean})^2 = (K-1) \sum_{\text{all } 2^p \text{ treatments } t} s_t^2$$

  SSE has ____________________ degrees of freedom, so MSE $= \frac{\text{SSE}}{\quad}$

  As before, MSE is an estimate of _______.

- The sum of squares for a main effect or interaction $X \in \{A, B, C, AB, AC, BC, ABC\}$ is

  $$SS_X = 2^{p-1}K[(\text{X high mean} - \text{overall mean})^2 + (\text{X low mean} - \text{overall mean})^2]$$

  (This is equivalent to the book's $SS_X = \frac{K(\text{contrast for X})^2}{2^p}$.)

  Each of these $2^p - 1$ sums of squares has _______ degree of freedom, so $MS_X = \frac{SS_X}{\quad}$

- As in §9.3, test $H_0 : X =$ _______ by calculating the test statistic $F = \frac{\quad}{\quad}$ and finding the P-value as a right-tail probability, P-value $= P(F_{\quad,\quad} > F)$

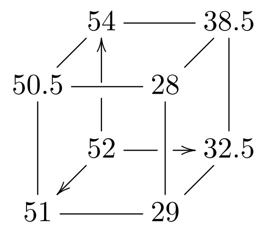
Summary: ANOVA Table

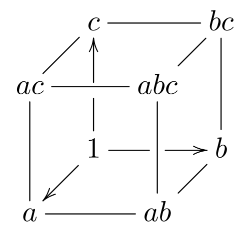
Source	Effect	DF	SS	MS	F	P
⋮	⋮	⋮	⋮	⋮	⋮	⋮
X	X	1	SS_X	$\frac{\text{SS}_\text{X}}{1}$	$\frac{\text{MS}_\text{X}}{\text{MSE}}$	$P(F_{1,2^p(K-1)} > F)$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Error		$2^p(K-1)$	SSE	$\frac{\text{SSE}}{2^p(K-1)}$		
Total		$2^pK - 1$	SST			

- There's one row for each of the $2^p - 1$ sources X ∈ { A, B, ..., AB, ...}
- Effect X = (mean response with X high) − (mean response with X low)
- $\text{SS}_X = 2^{p-1}K[(\text{X high mean} - \text{overall mean})^2 + (\text{X low mean} - \text{overall mean})^2]$
- $$\text{SSE} = \sum_{\text{all } 2^p \text{ treatments } t} \; \sum_{t\text{'s } K \text{ observations}} (\text{observation} - t \text{ sample mean})^2$$
  $$= (K-1) \sum_{\text{all } 2^p \text{ treatments } t} s_t^2$$
- $$\text{SST} = \sum_{\text{all } 2^pK \text{ observations}} (\text{observation} - \text{overall mean})^2$$

e.g. (p. 463 #4, continued) The study on vitamins and pyruvate has these data:

Treatment	Yields	Mean Yield
1	55, 49	52
a	60, 42	51
b	37, 28	32.5
ab	30, 28	29
c	54, 54	54
ac	54, 47	50.5
bc	44, 33	38.5
abc	36, 20	28
	Overall Mean	41.9





a. Estimate the main effects and interactions (done last time). Find the sum of squares and P-value for each.

Source	Effect	DF	SS	MS	F	P
A	−4.62	___	___	___	___	___
B	−19.87	1	1580.1	1580.1	29.03	.0007
C	1.63	1	10.6	10.6	.19	.67
AB	−2.37	1	22.6	22.6	.41	.54
AC	−2.38	1	22.6	22.6	.41	.54
BC	0.88	1	3.1	3.1	.06	.82
ABC	−1.13	1	5.1	5.1	.09	.77
Error		___	435.5	___		
Total		15	2164.9			

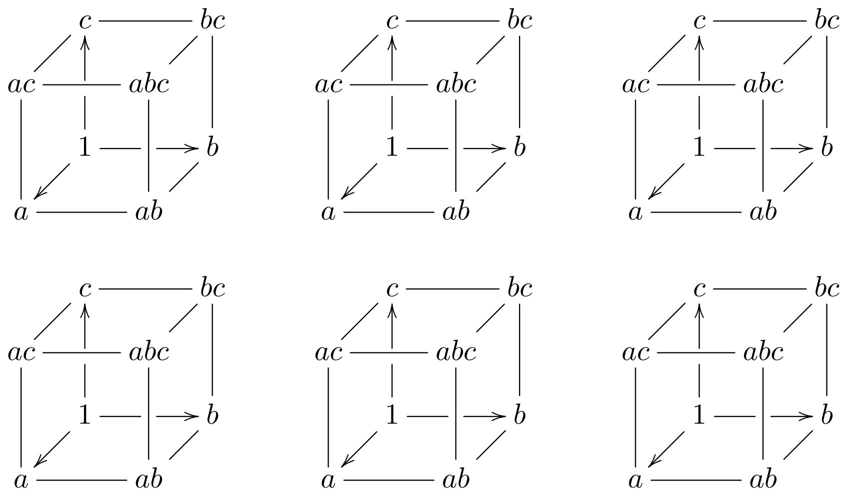
b. Is the additive model appropriate? (That is, is it plausible that all the interactions are ____?)

c. What conclusions about the factors can be drawn from these results?

Fractional Factorial Experiments (Optional)

Running an experiment with 2^p treatments becomes ____________________ as p increases. A key insight is that we can _______ some treatments and still get ______________ of information. If you could only afford to run 4 treatments, but you wanted information on all the main effects and two-way interactions (sacrificing the ___________ interaction), which 4 treatments would you use?

Here are a few copies of the 8 treatments for experimenting:



This idea generalizes to _______.