Last name: ____________________ First: ____________________

STAT 224 Exam 1, Version 1

Discussion (check one):

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Show your work for partial credit.

1. The four sides of a rectangluar blower enclosure consist of two short pieces chosen randomly from a population with mean length 30 cm and standard deviation .1 cm, and two long pieces chosen randomly from a population with mean length 45 cm and standard deviation .3 cm. Assuming that the four sides are chosen independently, find the standard deviation of the perimeter of the enclosure. (Note that the two short lengths aren't necessarily the same, and the two long lengths aren't necessarily the same.)

   (a) .33

   (b) .38

   (c) .40

   (d) .45

   (e) None of the above

   ANSWER: D
   Let S_1 and S_2 be the two short lengths: $\sigma_{S_i} = .1$. Let L_1 and L_2 be the two long lengths: $\sigma_{L_i} = .3$. The perimeter is $P = S_1+S_2+L_1+L_2$. $\sigma_P^2 = \sigma^2_{S_1+S_2+L_1+L_2} = \sigma^2_{S_1}+\sigma^2_{S_2}+\sigma^2_{L_1}+\sigma^2_{L_2} = .1^2 + .1^2 + .3^2 + .3^2 = .2 \implies \sigma_P = \sqrt{.2} \approx .447$

2. The magnitude of linear momentum p (kg m/s) is calculated as $p = mv$, where m = mass (kg) and v = speed (m/s). Suppose m and v are measured as $m = 10$ and $v = 20$, and it's known that $\sigma_m = .1$ and $\sigma_v = .3$. Estimate σ_p.

   (a) 1.12

   (b) 1.38

   (c) 2.70

   (d) 3.61

   (e) None of the above

   ANSWER: D
   $\sigma_p \approx \sqrt{(\frac{\partial p}{\partial m}\sigma_m)^2 + (\frac{\partial p}{\partial v}\sigma_v)^2} = \sqrt{(v\sigma_m)^2 + (m\sigma_v)^2} = \sqrt{(20 * .1)^2 + (10 * .3)^2} \approx 3.606$

3. Which of these summary statistics are least affected by outliers?

   - IQR = interquartile range
   - M = sample median
   - Q_1 = first quartile
   - s = sample standard deviation
   - $\bar{X}$ = sample mean

   (a) IQR, M, and Q_1

   (b) IQR and s

   (c) M and $\bar{X}$

   (d) s and $\bar{X}$

   (e) None of the above.

   ANSWER: A

4. Suppose asteroids 10 m in diameter collide with Earth according to a Poisson process with rate parameter $\lambda = .2$ per year. What's the probability that at least one will hit Earth in the next ten years?

   (a) .61

   (b) .64

   (c) .87

   (d) .93

   (e) None of the above

   ANSWER: C
   Let X = #colliding in ten years $\sim$ Poisson(λt) = Poisson($2 = (.2/\text{year}) * (10\text{years})$) $\implies$ $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-2} \cdot \frac{2^0}{0!} \approx .865$

5. What's the probability that the time until the next 10 m asteroid hits is less than one year?

   (a) .18

   (b) .24

   (c) .26

   (d) .37

   (e) None of the above

   ANSWER: A
   Let T = waiting time $\sim \text{Exp}(\lambda = .2) \implies P(T < 1) = (1 - e^{-.2(1)}) \approx .181$

6. Suppose X is a continuous random variable with density function $f(x) = \frac{1}{2}x$ if $x \in [0, 2]$ and 0 otherwise. The mean of X is $\mu_X =$

   (a) $\frac{5}{4}$

   (b) $\frac{4}{3}$

   (c) $\frac{3}{2}$

   (d) $\frac{5}{3}$

   (e) None of the above

   ANSWER: B
   $\mu_X = \int_0^2 x \cdot \frac{1}{2}x \, dx = \frac{1}{6}x^3|_0^2 = \frac{4}{3}$

7. Consider tossing a fair coin, which yields H (heads) or T (tails), and then rolling a fair six-sided die, which yields 1, 2, 3, 4, 5, or 6. Let X be the sum of the coin and the die, where we let H count as 1 and T count as 0. Find the expected value (or mean) of X: $\mu_X =$

   (a) 3.8

   (b) 4.0

   (c) 4.2

   (d) 4.4

   (e) None of the above

   ANSWER: B
   The sample space is $S = \{$ T1, T2, T3, T4, T5, T6, H1, H2, H3, H4, H5, H6 $\}$. The corresponding sums are $\{$ 1, 2, 3, 4, 5, 6, 2, 3, 4, 5, 6, 7 $\}$. X's probability mass function is

x	1	2	3	4	5	6	7
$p(x)$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

   $\Longrightarrow \mu_X = \frac{1}{12}(1*1+2*2+3*2+4*2+5*2+6*2+7*1) = 4$
   (Or skip the mass function and use $\mu_X = \frac{1}{12}\sum\{1,2,3,4,5,6,2,3,4,5,6,7\} = 4$)

8. Suppose independent components A, B, and C work with probabilities .5, .4, and .3. Find the probability that the following system works.

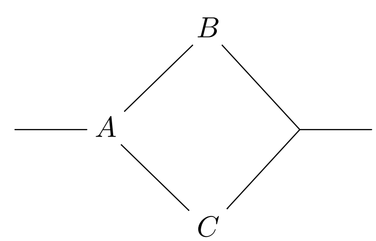
   (a) .29

   (b) .36

   (c) .42

   (d) .58

   (e) None of the above



   ANSWER: A
   $P(\text{works}) = P(A \cap (B \cup C)) = P(A)P(B \cup C)$, by independence $= P(A)[P(B) + P(C) - P(B \cap C)] = .5 * [.4 + .3 - .4 * .3] = .29$
   or
   $P(\text{works}) = P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap A \cap C) = P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$, by independence $= .5 * .4 + .5 * .3 - .5 * .4 * .3 = .29$

9. Suppose drill bit lifetimes (in hours) have a lognormal distribution with $\mu = 4.5$ and $\sigma = .8$. What is the probability that the lifetime of a randomly chosen bit is at least 200?

   (a) .22

   (b) .25

   (c) .28

   (d) .31

   (e) None of the above

   ANSWER: E
   Let X = lifetime $\implies \ln(X) \sim N(4.5, .8^2)$. $P(X > 200) = P(\ln(X) > \ln(200)) = P(\frac{\ln(X)-\mu}{\sigma} > \frac{\ln(200)-4.5}{.8}) \approx P(Z > 1.00) = P(Z < -1) = .1587$.

10. Here are average delays, in minutes, for departures and arrivals of U.S. domestic flights:

Year	Departure	Arrival
2004	50	53
2005	56	61
2006	51	57
2007	54	58
2008	55	59
2009	56	59

   Find the least-squares regression line for predicting arrival delay from departure delay.

   (a) $\hat{y} = 3.9457 + .8597x$

   (b) $\hat{y} = .8597 + 3.9457x$

   (c) $\hat{y} = 6.85 + .95x$

   (d) $\hat{y} = .95 + 6.85x$

   (e) None of the above.

   ANSWER: C

11. A lot contains several thousand components, 10% of which are defective. Nine components are sampled from the lot. Find the probability that exactly two of the nine are defective.

   (a) .07

   (b) .10

   (c) .22

   (d) .28

   (e) None of the above

   ANSWER: E
   Let X = number defective $\sim \text{Bin}(n = 9, p = .1)$. $P(X = 2) = \binom{9}{2} .1^2 (1 - .1)^{9-2} \approx .172$.

12. The length, in days, of a pregnancy from conception to birth is approximately normally distributed with mean $\mu = 272$ and standard deviation $\sigma = 9$. A pregnancy is considered full-term if it lasts between 252 and 298 days. What proportion of pregnancies are full-term?

    (a) .86

    (b) .88

    (c) .92

    (d) .98

    (e) None of the above

    ANSWER: D
    Let X = prenancy length $\sim N(272, 9^2)$. $P(\text{full-term}) = P(252 < X < 298) = P(\frac{252-272}{9} < \frac{X-\mu}{\sigma} < \frac{298-272}{9}) \approx P(-2.22 < Z < 2.89) = P(Z < 2.89) - P(Z < -2.22) = .9981 - .0132 = .9849$

13. A farmer raises chickens with weights, in grams, that are normally distributed with mean 1387 and standard deviation 161. She wants to provide a money-back guarantee that her chickens will weigh at least a certain amount. What minimum weight should she guarantee so that she'll have to give money back only 1% of the time?

    (a) 890.30

    (b) 943.28

    (c) 915.54

    (d) 1063.62

    (e) None of the above

    ANSWER: E
    Let X = weight of a chicken and w = the guaranteed minimum weight. $P(X < w) = .01 \implies P((Z = \frac{X-\mu}{\sigma}) < \frac{w-1387}{161}) = .01 \implies$ (from table) $\frac{w-1387}{161} = -z_{.01} = -2.325 \implies w = -2.325 * 161 + 1387 \approx 1012.7$

14. A roller coaster holds 60 riders. It's maximum safe total rider weight is 12,000 pounds. The weights of adult U.S. men have mean 194 and standard deviation 68 pounds. If a random sample of 60 men ride the coaster, what is the probability the maximum safe weight will be exceeded?

    (a) .16

    (b) .25

    (c) .32

    (d) .36

    (e) None of the above

    ANSWER: B
    Let X_i = weight of i^{th} randomly-chosen man; X_i has $\mu = 194$ and $\sigma = 68$. CLT says $\bar{X} \sim N(194, 68^2/60) = N(194, 8.779^2)(\approx)$.
    $P(\sum_{i=1}^{60} X_i > 12000) = P(\frac{1}{60}\sum X_i > \frac{12000}{60}) = P(\bar{X} > 200) = P(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{200-194}{8.779}) = P(Z > .68) = P(Z <= .68) = .2483$

15. In a cheap automotive parts factory, the probability of producing an oversize piston is .05. In a sample of 300 randomly chosen pistons, what is the probability that fewer than 20 are oversize? (Hint: use the normal approximation to the binomial distribution along with a continuity correction.)

    (a) .64

    (b) .71

    (c) .88

    (d) .93

    (e) None of the above

    ANSWER: C
    Let $X =$ number oversize $\sim \text{Bin}(n = 300, p = .05)$. Let $Y \sim N(np = 15, np(1-p) = 14.25)$.
    Then $P(X < 20) \approx P(Y < 20 - \frac{1}{2}) = P(\frac{Y-\mu}{\sigma} < \frac{19.5-15}{\sqrt{14.25}}) = P(Z < 1.19) = .8830$