Last name: ______________________ First: ______________________

STAT 224 Exam 2, Version 2

Discussion (check one):

...

Show your work. Points are indicated in square brackets, "[$\cdots$]".

1. [12] A stainless steel powder is supposed to have a mean particle diameter of $\mu = 15$ μm. A random sample of 87 particles had a mean diameter of 15.2 μm, with standard deviation 1.8 μm. Determine whether this is evidence that the powder doesn't meet its specification by testing $H_0 : \mu = 15$ against $H_1 : \mu \neq 15$ and reporting the P-value.
   ANSWER: P-value $= P(|Z| > z) = P(|Z| > \frac{15.2-15}{.193}) = 2P(Z < -1.04) = 2(.149) = .298 \implies$ retain H_0

2. [12] In a random survey of 500 residents in a town, 274 opposed building a new mall. Is this strong evidence that more than half the town opposes the mall?

   - H_0:
   - H_1:
   - test statistic:
   - P-value:
   - conclusion: yes / no (circle one)

   ANSWER: H_0: $p = .5, H_1$: $p > .5; \hat{p} = \frac{274}{500} = .548; z = \frac{.548-.5}{\sqrt{.5(1-.5)/500}} = 2.1466$; P-value $= P(Z > 2.15) = 0.016 \implies$ reject $H_0 \implies$ yes

3. [12] The stalk-eyed fly has projections from the sides of its head with eyes at the ends. The span, in millimeters, from one eye to the other was measured in a random sample of nine flies:

   8.69 8.15 9.25 9.45 8.96 8.65 8.43 8.79 8.63

   Supposing that the spans have a normal distribution in the population, find a 95% confidence interval for the true mean span.

   ANSWER: $n = 9, \bar{x} = 8.778, s = .398, t_{n-1,\alpha/2} = 2.306 \implies$ interval is $8.778 \pm 2.306 * .398/\sqrt{9} = 8.778 \pm .306$

4. [4] Measurements of the distance (cm) between the canine tooth and last molar for 35 wolf upper jaws were made by a researcher, who found the 95% confidence interval for the mean to be $10.32 \pm .15$, and then the 99% confidence interval to be $10.32 \pm .12$. Without seeing the data, explain why the researcher must have made a mistake.

   ANSWER: The 99% confidence interval must be *larger* than the 95% interval to have a higher probability of capturing the true mean.

5. [18] A portable drill manufacturer says its batteries are strong enough to screw in an average of 125 deck screws, with standard deviation 7. A quality-control engineer samples 100 batteries to test $H_0 : \mu = 125$ against $H_1 : \mu < 125$. Suppose the test is made at the 5% level. What is the power if the true mean is $\mu = 122$?

   ANSWER: level $\alpha = .05 = P(\text{reject } H_0 | H_0 \text{ is true}) = P(\bar{X} < x_{\text{critical}} | \mu = 125) = P(Z < \frac{x-125}{7/\sqrt{100}}) \implies \frac{x-125}{.7} = -z_{.05} = -1.645 \implies x = .7 * (-1.645) + 125 = 123.85$

   power $= P(\text{reject } H_0 | H_0 \text{ is false because } \mu = 122) = P(\bar{X} < 123.85 | \mu = 122) = P(Z < \frac{123.85-122}{.7}) = P(Z < 2.64) = 1 - P(Z < -2.64) = 1 - .004 = .996$

6. [5] Nobel economist Daniel McFaddon worked on the Council of Economic Advisors for President Lyndon Johnson. When McFadden offered a range of forecasts for economic growth, the President replied, "Ranges are for cattle; give me one number." What confidence level is associated with a confidence interval whose error margin is 0?

   ANSWER: margin $= z_{\alpha/2} \frac{s}{\sqrt{n}} = 0 \implies z_{\alpha/2} = 0 \implies \alpha/2 = .5$ => confidence level $= 1 - \alpha = 0$

7. [12] A Department of Energy survey found that 36 out of 100 randomly-selected taxpayers were familiar with the tax incentives for installing energy-saving furnaces. Find a 90% plus-four confidence interval for the population proportion of taxpayers who are familiar with the incentives.
ANSWER: $X = 36, n = 100 \implies \tilde{n} = 100 + 4 = 104, \tilde{p} = \frac{36+2}{104} = \frac{19}{52}; z_{.05} = 1.645$; interval: $\frac{19}{52} \pm 1.645\sqrt{\frac{\frac{19}{52}(1-\frac{19}{52})}{104}} = .365 \pm .078$

8. [15] 16 out of 200 fire trucks produced on a Chicago assembly line required extensive adjustment before they could be shipped, while the same was true for 14 of 400 trucks from an Oshkosh assembly line. (Suppose the 200 trucks may be regarded as a random sample from the Chicago line's long-term work, and similarly for the 400 trucks from Oshkosh.)
Are these data statistically significant evidence at level $\alpha = .01$ to support the claim that the Oshkosh line does superior work? Perform an appropriate test.

- H_0: ANSWER: $p_C - p_O = 0$
- H_1: ANSWER: $p_C - p_O > 0$
- test statistic: ANSWER: difference of two proportions: pooled $\hat{p} = \frac{16+14}{200+400} = .05$, so $Z = \frac{(\frac{16}{200} - \frac{14}{400}) - 0}{\sqrt{.05(1-.05)(\frac{1}{200}+\frac{1}{400})}} = 2.38$
- P-value: ANSWER: P-value $= P(Z > 2.38) = P(Z < -2.38) = .0087$
- conclusion: yes / no (circle one) ANSWER: yes

9. [15] The horned lizard has a fringe of spikes around its head. Researchers wondered whether the spikes help protect the lizard from being eaten. (To collect data, they took advantage of the gruesome behavior of one of the lizard's main predators, the loggerhead shrike, which is a bird. The shrike skewers the lizards it catches on thorns to save for later eating!) The researchers measured the horn lengths on 40 skewered dead lizards, and also on 154 living lizards:

Lizard group	sample mean	sample standard deviation	sample size n
Living	24.28	2.63	154
Dead	21.99	2.71	40

Are these data strong evidence that living lizards' horns are longer than dead lizards' horns?

- H_0: ANSWER: $\mu_L - \mu_D = 0$
- H_1: ANSWER: $\mu_L - \mu_D > 0$
- test statistic: ANSWER: large-samples difference of two means: $Z = \frac{(24.28-21.99)-0}{\sqrt{\frac{2.63^2}{154}+\frac{2.71^2}{40}}} = 4.79$
- P-value: ANSWER: P-value $= P(Z > 4.79) < .0001$
- conclusion: yes / no (circle one) ANSWER: yes

10. [3] (True/False) The conclusion of a test in which the calculated P-value is $= 0.9999$ is "H_0 is almost certainly true."

(a) True

(b) False

ANSWER: False; it's "H_0 is plausible" (other similar H_0s would also be also plausible; the high P-value just means that the point estimate was very close to the H_0 parameter value)

11. [12] Here is a contingency table of observed counts that relates the education level and smoking status of a SRS of 459 French men in order to study the question, "Are education and smoking related?"

	Smoking status				
Education	Nonsmoker	Former	Moderate	Heavy	Total
Primary	56	54	41	36	187
Secondary	37	43	27	32	139
University	53	28	36	16	133
Total	146	125	104	84	459

(a) Under H_0 : "*Education* and *Smoking* are independent", find the expected count of University-educated Former smokers.

ANSWER: $133 * 125/459 = 36.22$

(b) Find the term in the chi-square statistic for University-educated Former smokers.

ANSWER: $(28 - 36.22)^2/36.22 = 1.87$

(c) Find the degrees of freedom for the chi-square test for this two-way table.

ANSWER: $(rows - 1)(columns - 1) = (3 - 1)(4 - 1) = 6$

(d) Software gives a chi-square statistic $\chi^2 = 13.3$ for this table. Find the P-value.

ANSWER: (d) $P(X_6^2 > 13.3) =$ between .025 and .05

12. [12] $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is an unbiased estimator of μ because $\mu_{\bar{X}} = \mu$. What is the bias of $\tilde{X} = \frac{1}{n-1}\sum_{i=1}^{n} x_i$ (a silly statistic that divides the sum by $n - 1$, not n) as an estimator of μ? (Hint: The bias of $\hat{\theta}$ as an estimator of θ is $\mu_{\hat{\theta}} - \theta$.)
ANSWER: $\mu_{\tilde{X}} = \mu_{\frac{n}{n-1}\bar{X}} = \frac{n}{n-1}\mu = \mu + \frac{1}{n-1}\mu \implies$ bias $= \frac{1}{n-1}\mu$

13. [12] In the Northern Hemisphere, dolphins swim predominantly in a counterclockwise direction while sleeping. Researchers watched eight sleeping Southern Hemisphere dolphins and recorded the percentage of time they swam clockwise:
77.7 84.8 79.4 84.0 99.6 93.6 89.4 97.2
Assume that this is a random sample from a normal distribution with mean percentage μ. Are these data strong evidence that $\mu > 50$ (that is, that southern dolphins swim clockwise more than half the time)? Perform a suitable hypothesis test. (Hint: this is a question about an unknown *mean* percentage.)

- H_0: ANSWER: $\mu = 50$
- H_1: ANSWER: $\mu > 50$
- test statistic: ANSWER: $n = 8, \bar{x} = 88.213, s = 8.093, \implies t = \frac{88.213-50}{8.093/\sqrt{8}} = 13.355$
- P-value: ANSWER: P-value $= P(T_{8-1} > 13.335) < .001$
- conclusion: yes / no (circle one) ANSWER: yes

14. [6] A snow-making operation tests thirty water additves to find out whether any increase the amount of artifical snow produced. It uses level $\alpha = .10$. Of the thirty P-values, none are below .05, 3 are between .05 and .10, and 27 are greater than .10. What should the experimenters do?
ANSWER: Re-test the 3 with P-values between .05 and .10. These P-values are low enough to be of interest, but not low enough (less than the Bonferroni $\alpha/N = .10/30 \approx .003$) to be conclusive.