

Homework 4

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Question 1 (20 points):

Part a: Express the relations given in Figure 1 using the relations given in class and various operators (of course!). Please give a short justification for your answer.

Part b: Let $\text{CousinsFatherSide}(n)$, $\text{CousinsMotherSide}(n)$, and $\text{Cousins}(n)$ be the set of cousins that are separated by n -steps on the father's side, mother's side, and father or mother's side. Write a recursive expression for these relations, i.e., express the relations with parameter n in terms of relations with parameter $n - 1$ and the the basic relations introduced in the

FatherInLaw : People \leftrightarrow Males
SisterInLaw : People \leftrightarrow Females
FirstCousinsFatherSide : People \leftrightarrow People
FirstCousinsMotherSide : People \leftrightarrow People
FirstCousins : People \leftrightarrow People
SecondCousinsFatherSide : People \leftrightarrow People
SecondCousinsMotherSide : People \leftrightarrow People
SecondCousins : People \leftrightarrow People

Figure 1: Some relations

class.

Question 2 (20 points): Establish whether the following equations between the relations are true or not. Please justify your answer. You should enter these equations in *ladybug* and play around with them. Assume that the relations have appropriate types.

$$\begin{aligned}(P; Q)^\top &= Q^\top; P^\top \\ (P \subseteq Q) &\Rightarrow (P; P \subseteq Q) \\ (Q \neq \emptyset) &\Rightarrow (\text{Un}; Q; \text{Un} = \text{Un})\end{aligned}$$

Note: The symbol \emptyset denotes the empty relation.

Question 3 (60 points): In this question you will write a specification for the problem described. Our aim is to model a library. There are two types `PERSONS` and `BOOKS`. There is a function `issued : BOOKS \rightarrow PERSONS` and a set `Library : Set BOOKS`. If book b is issued to a person p , then `issued(b) = p` . `Library` represents the set of books in the library. There is also a set of books on reserve given by the set `Reserve : Set BOOKS`. The operations are:

- *Issue a book*
This operation issues a book to a person. A book on reserve or already issued cannot be issued.
- *Return a book*
This operation models the act of a person returning a book.
- *Adding a book to the library*
This operations models a new book being added to the library.
- *Putting a book on reserve*
This operations models a book being put on reserve. A book which is currently issued cannot be put on reserve.
- *Taking a book off reserve*
This operation models a book being taken off reserve.

The claims (or assertions in Alloy) are:

- Issuing a book and then returning it results in the same state.
- If a book is on reserve, it should never be issued. This claim should be true before and after every operation.

Part a: Write a mathematical description for all the operations and claims and explain your answer.

Part b: Now write the specification in *ladybug*.

Part c: Now express your design in *Alloy*.

Part d: Based on your experience for part b and c, compare ladybug and Alloy.