CS/Math 240: Introduction to Discrete Mathematics		7/12/2007
	Homework 4	
Instructor: Jeff Kinne		TA: Mike Kowalczyk

This homework is due at the beginning of class on Thursday July 19, 2007. Mike will hold a review session at 12:45-1:45 on July 19 to discuss the solutions to these problems (that is after you have handed in the homework).

Note: All logarithms are base 2 unless otherwise specified.

Problem 1

In class, we mentioned that both breadth first search and depth first search can be used to find a spanning tree (not necessarily minimal) in an undirected connected graph.

An alternative approach was also suggested - remove edges from cycles until there are no more cycles. You will develop this approach in this problem.

Part a

Give pseudo-code for an algorithm that takes as input an undirected connected graph G, and outputs a spanning tree for G. Your algorithm should follow the strategy mentioned above - remove edges from cycles until there are no more cycles. You can assume that you have a procedure that: finds an edge that is part of a cycle if there is a cycle in the graph, and determines that there are no cycles if given an acyclic graph.

Part b

Prove that the algorithm you gave in Part a) is correct - that it outputs a spanning tree.
Hint: try using induction on the number of edges in the graph.

Problem 2

In class we discussed the Monty Hall 3-door problem. We repeat it here as well. You play a game of chance with the following sequence of events:

- The game-show host picks at random one of 3 doors to hide a prize behind.
- You pick a door at random as your initial guess.
- The game-show host opens one of the doors that is not your guess and is not the "prize door". So two doors remain - your initial guess and another door.
- You are given the chance to change your guess to the other door.
- The game-show host opens the door you have chosen, and you get the prize if it is there.

We showed that by following the strategy of changing your guess, you have 2/3 probability of winning - better than the 1/3 probability of winning if you do not change your guess.

Part a

Suppose we use 4 doors rather than 3, and the game proceeds just as before but now with 4 doors instead of 3. Compute the probability of winning for both cases for the 4-door game: for the case where you don't change your guess, and the case where you do.

Part b

Let k be any integer that is at least 3. Compute the probabilities for winning for both cases if we use k doors.

Assuming that the player chooses the strategy that maximizes his/her probability of winning, what is the minimum number of doors required so that the player loses with probability at least 1/2?

Problem 3

In class, we solved the Birthday Problem - the probability that out of a set of n people, there are at least 2 people that have the same birthday. In this problem, we consider a modification of the problem.

Part a

Consider a set of 10 people, with each person's birthday chosen at random from 366 possible days. Compute the probability that out of the 10 people, there are at least 3 that have the same birthday.

Part b

Extend your result from Part a) to give a formula for the probability that out of n people, there are at least 3 that have the same birthday.

Problem 4

For this problem, consider the experiment of rolling 5 six-sided die in a row.

Part a

Compute the probability that there are no pairs - that no number comes up twice.

Part b

Compute the probability that there is a number that comes up exactly 3 times out of the five rolls.

Part c

Compute the probability that there is a number that comes up exactly 5 times out of the five rolls.