CS/Math 240: Introduction to Discrete Mathematics 7/19/2007

Homework 5

Instructor: Jeff Kinne TA: Mike Kowalczyk

This homework is due at the beginning of class on Thursday July 26, 2007. Mike will hold a review session at 11:45-12:45 on July 26 to discuss the solutions to these problems (that is after you have handed in the homework).

Note: All logarithms are base 2 unless otherwise specified.

Problem 1

The goal of this problem is to compute the probability that if we flip $2n$ independent fair coins in a row, that at some point there are at least n heads in a row. To do this, we first count the number of possible ways that this can happen (and then divide by 2^{2n}).

Part a

Determine the number of possible ways that the first "n heads in a row" can occur as the first coin flips. For $n = 3$, one example is (H,H,H,T,H,T) and another is (H,H,H,H,T,T).

Part b

Determine the number of possible ways that the first "n heads in a row" can occur as the 2nd through n+1st coin flips. For $n = 3$, one example is (T,H,H,H,T,H) and another is (T,H,H,H,H,H).

Part c

Now determine the number of possible ways that there can be at least n heads in a row in a sequence of $2n$ coin flips.

Hint: Break this event up into $n + 1$ disjoint events, compute the number of ways for each of these, then add them up. You have already computed 2 of these for Parts a) and b).

Is the probability of getting at least n heads in a row at some point greater than, less than, or equal to 1/2?

Problem 2

Show that if E and F are independent events, then $\overline{E}$ and $\overline{F}$ are independent events.

Problem 3

In this problem, we examine an *incorrect* way to try to solve the birthday problem. You have a friend who is trying to solve the birthday problem for a group of n people. His reasoning is as follows. Each pair of people have probability 1/366 of having the same birthday. So there are $n - 1$

independent Bernoulli trials, and if one of them is successful, then there are at least two people that have the same birthday.

Part a

Determine the probability that your friend claims for the probability that out of a group of n people there are at least 2 with the same birthday. That is, determine the probability that there is at least 1 success in $n-1$ independent Bernoulli trials that each have probability of success equal to $1/366$.

Part b

How large does n have to be for your friends claimed probability to be greater than $1/2$?

Part c

Is your friend's reasoning correct? If so, explain why; if not, explain why not. If your friend's reasoning is not correct, what event have they computed the probability of?

Problem 4

This problem considers the use of DNA evidence in trials. Suppose you are on trial, and your DNA matches DNA taken from the crime scene that was left by the perpetrator. The DNA test has the property that there is a false positive (a match between people with different DNA) with probability $\frac{1}{10^6}$. The test has the additional property that false negatives (a non-match between two samples of the same DNA) never occur.

The prosecutor claims that we can therefore conclude that the probability you are innocent is $\frac{1}{10^6}$. Assuming there is no other evidence in the case, is this true? That is what we aim to answer in this problem.

Part a

Let D be the event that your DNA matches the DNA from the crime scene, and let I be the event that you are innocent of the crime.

Suppose the city you live in has a population of 10^8, and we know with certainty that the perpetrator is one of these people. Assuming no evidence at all (not even the DNA evidence above), what is a reasonable value for $\Pr[I]$? What is a correct value for $\Pr[D]$?
Hint: For $\Pr[D]$, *use the law of total probability.*

Part b

Use Bayes' rule and your results from Part a) to compute a value for $\Pr[I|D]$.

Is the prosecutor correct in his/her claim?