CS540 Introduction to Artificial Intelligence

Convolutional Neural Networks (I)

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University of Wisconsin-Madison

November 2, 2021

Slides created by Sharon Li [modified by Josiah Hanna]

Congrats on getting midterm done!

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Reminder: HW6 deadline has been extended to Thursday 11:00am

Outline

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- Intro of convolutional computations

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    - 2D convolution

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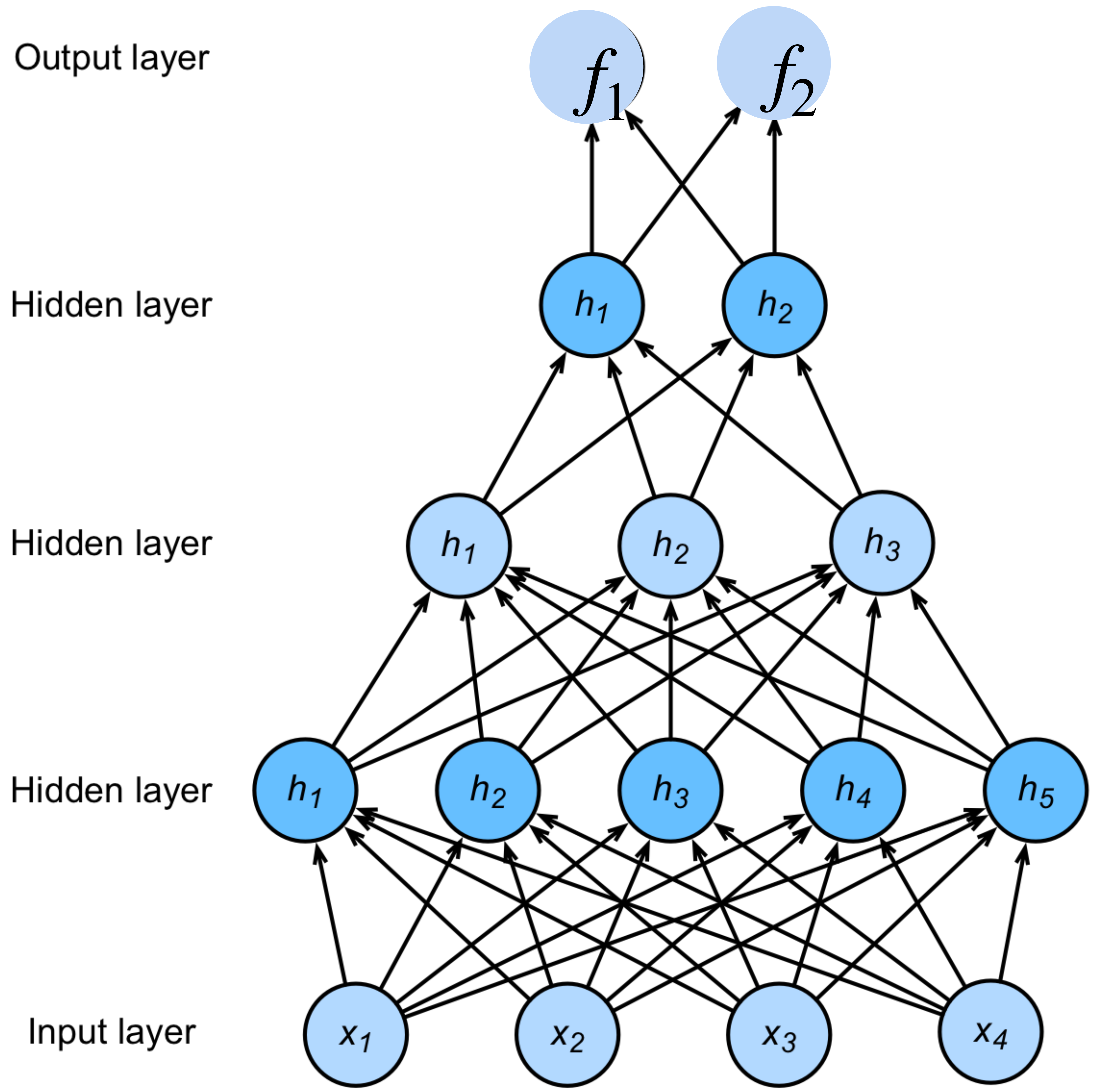
    - Multiple input and output channels
    - Pooling
- Basic Convolutional Neural Networks
    - LeNet

Review: Deep neural networks (DNNs)



$$\mathbf{h}_1 = \sigma(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$

$$\mathbf{h}_2 = \sigma(\mathbf{W}_2\mathbf{h}_1 + \mathbf{b}_2)$$

$$\mathbf{h}_3 = \sigma(\mathbf{W}_3\mathbf{h}_2 + \mathbf{b}_3)$$

$$\mathbf{f} = \mathbf{W}_4\mathbf{h}_3 + \mathbf{b}_4$$

$$\mathbf{y} = \text{softmax}(\mathbf{f})$$

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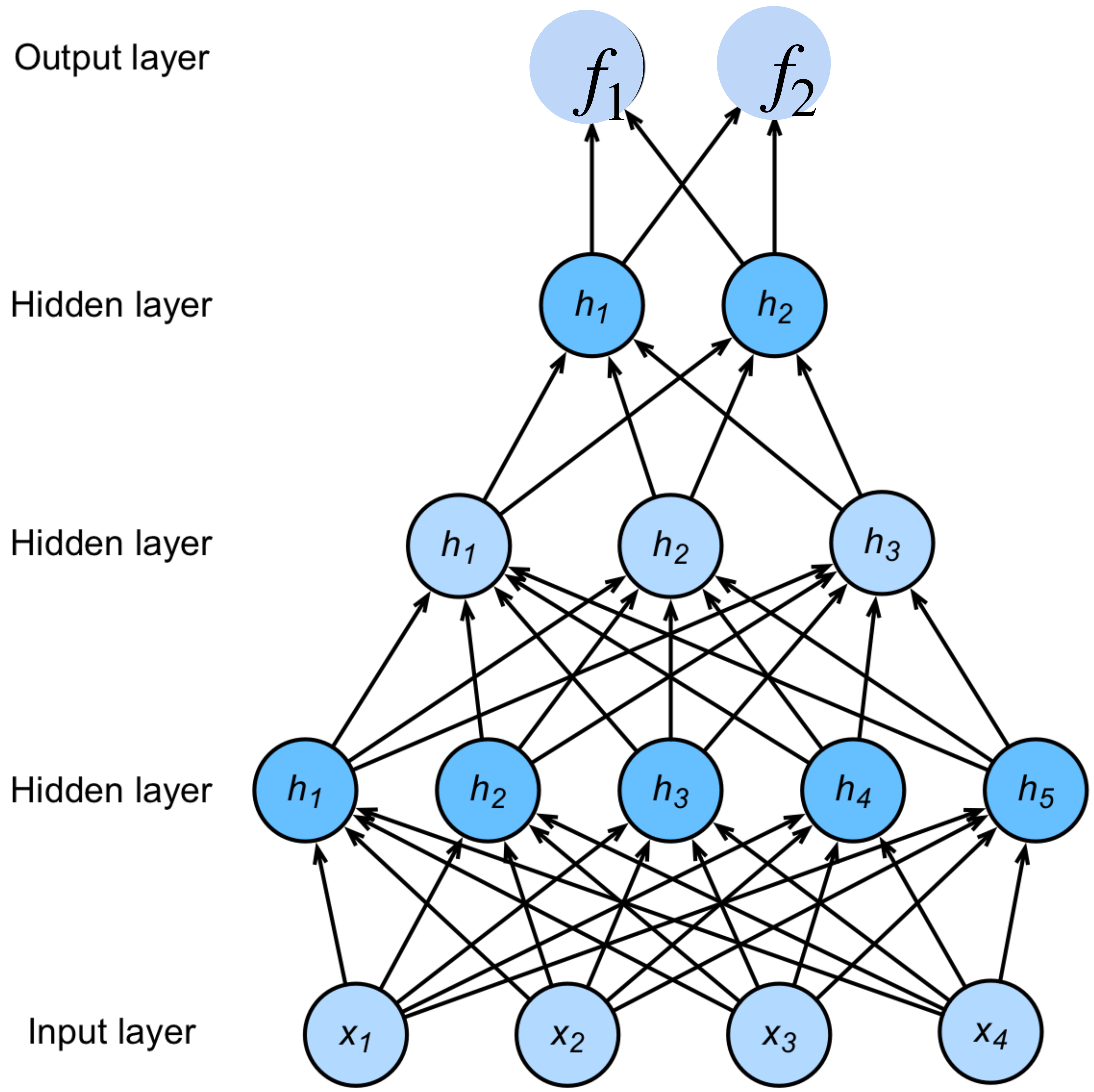

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NNs are composition of nonlinear functions

How to classify

Cats vs. dogs?

How to classify

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How to classify

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Cats vs. dogs?

Dual

12MP

wide-angle and
telephoto cameras

36M floats in a RGB image!

Fully Connected Networks

Cats vs. dogs?

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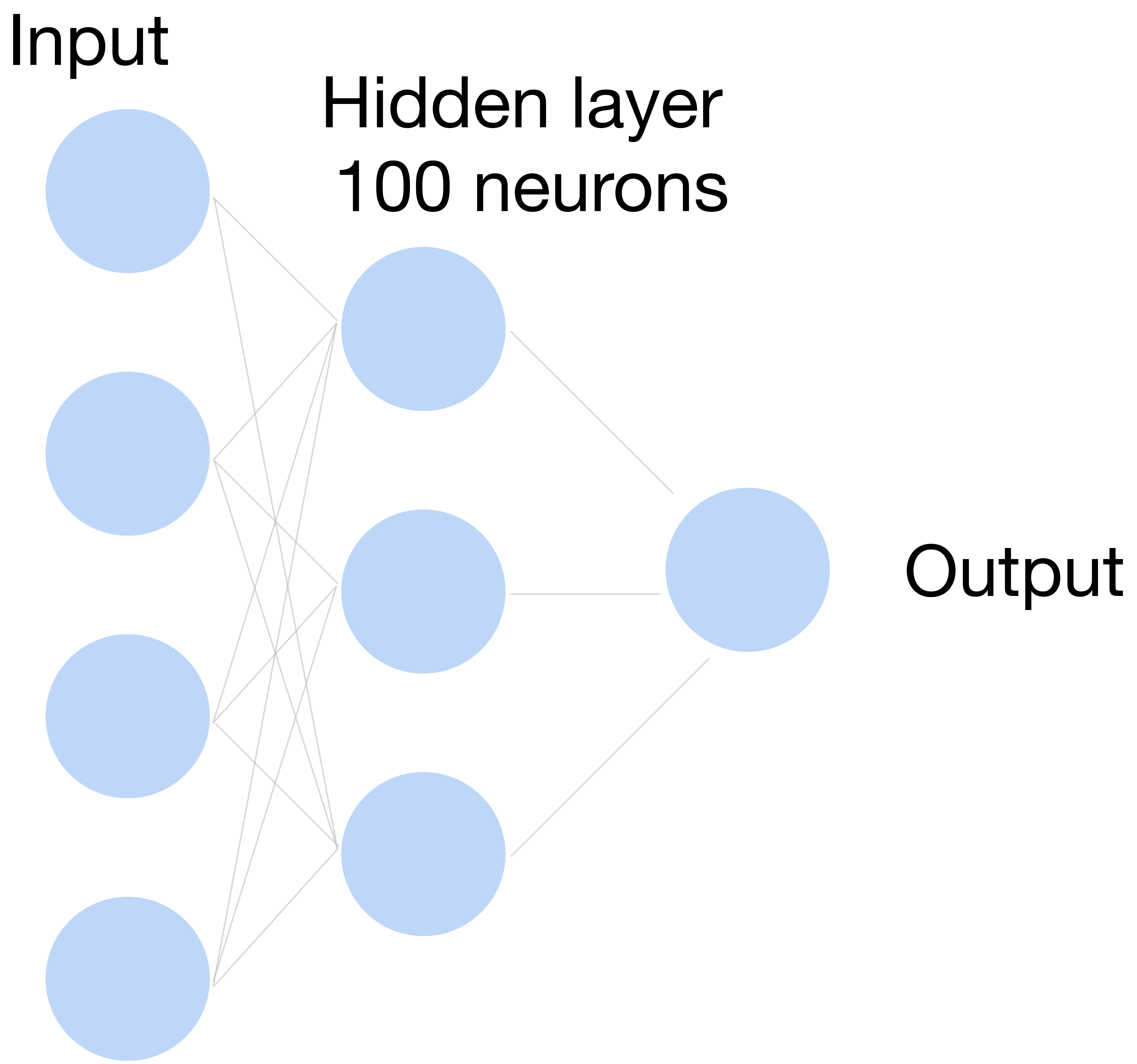
Cats vs. dogs?



Fully Connected Networks

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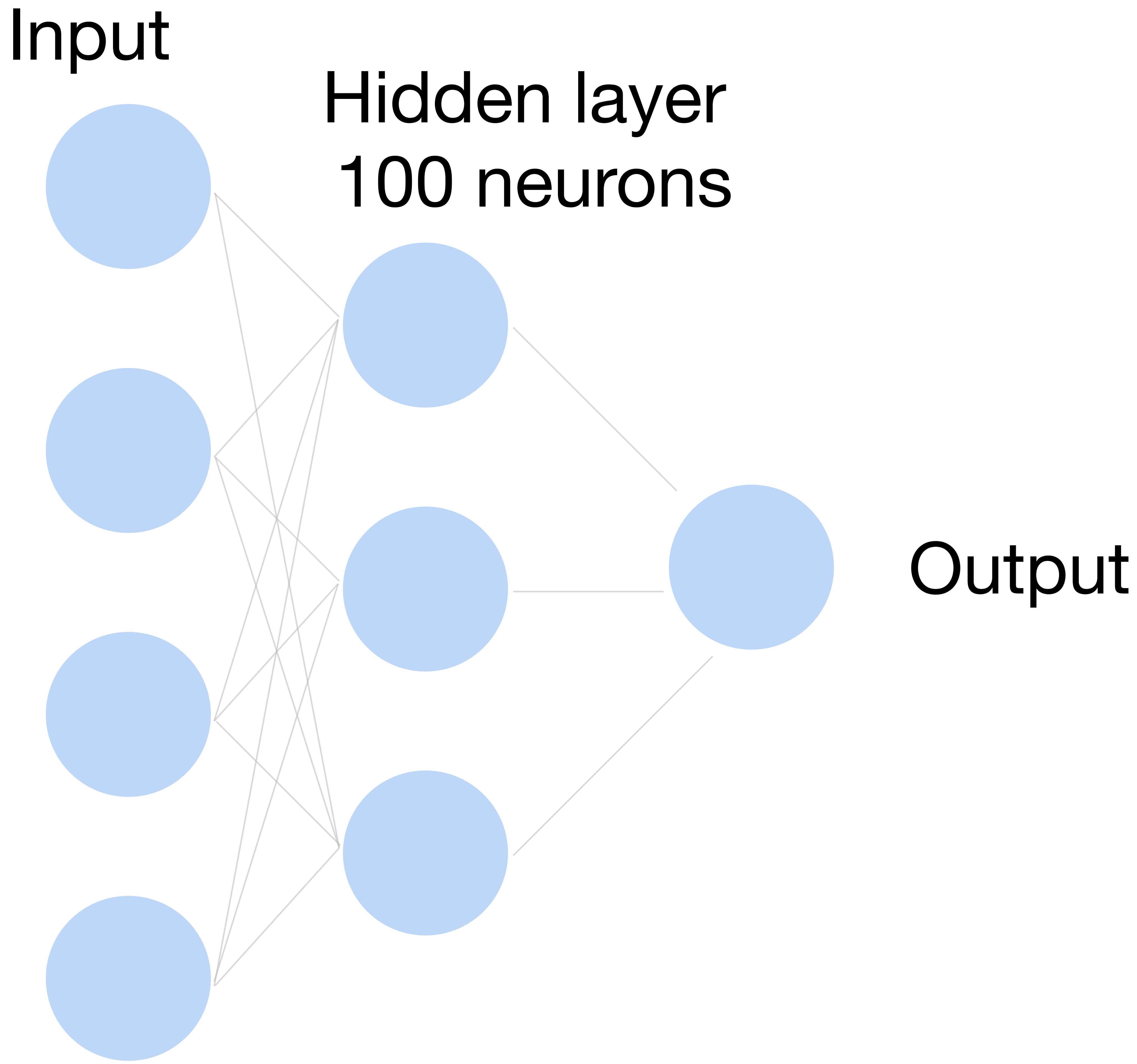


~ 36M elements x 100 = ~**3.6B** parameters!

Convolutions come to rescue!

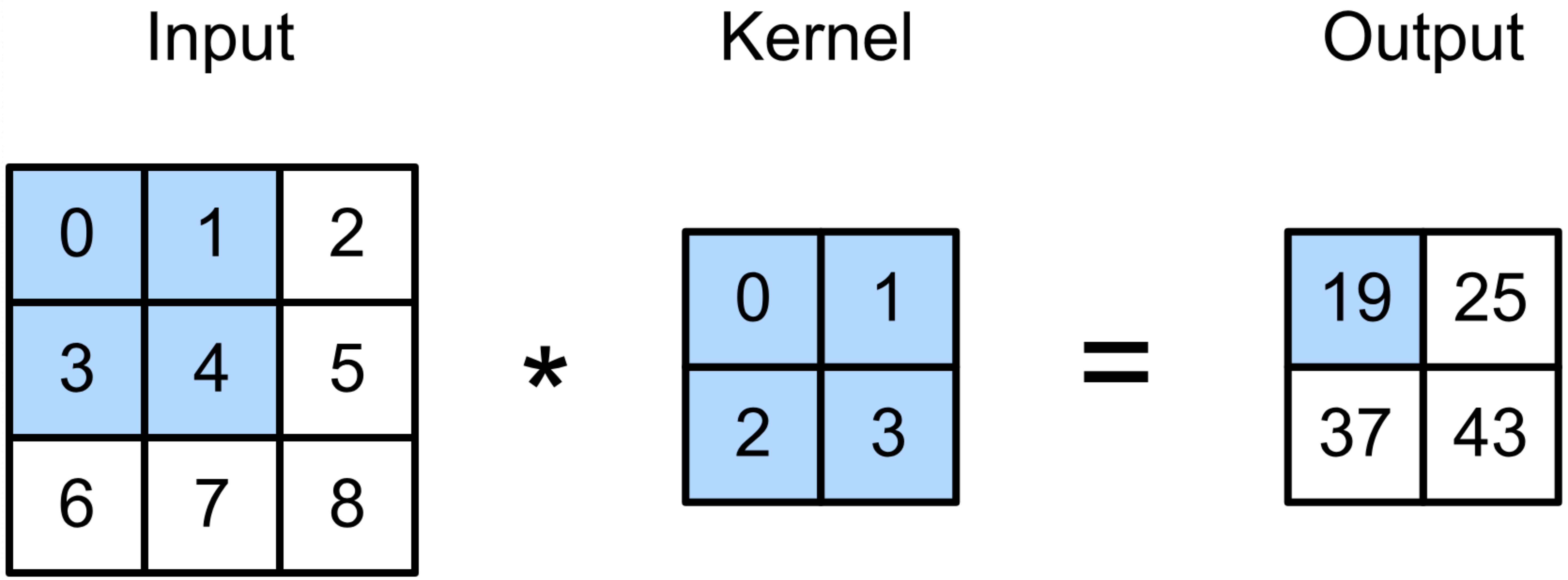
Where is Waldo?

Why Convolution?

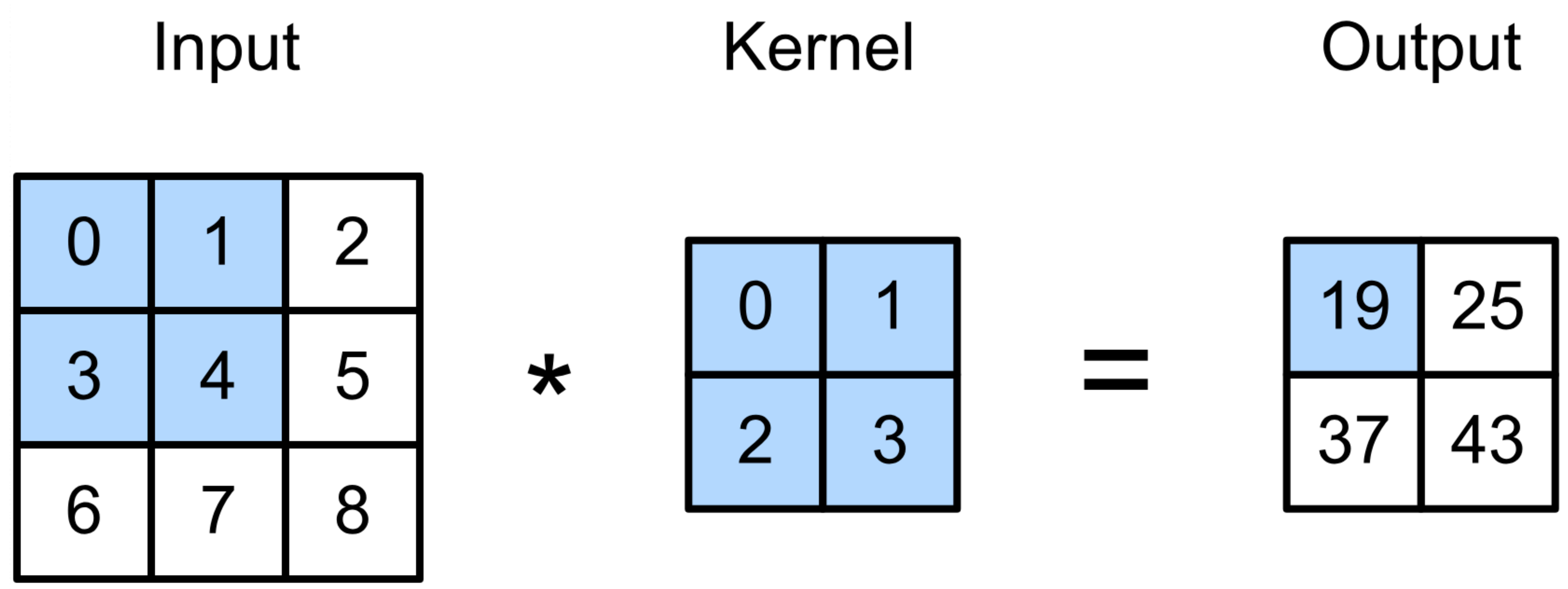
- Translation Invariance
- Locality

2-D Convolution

2-D Convolution

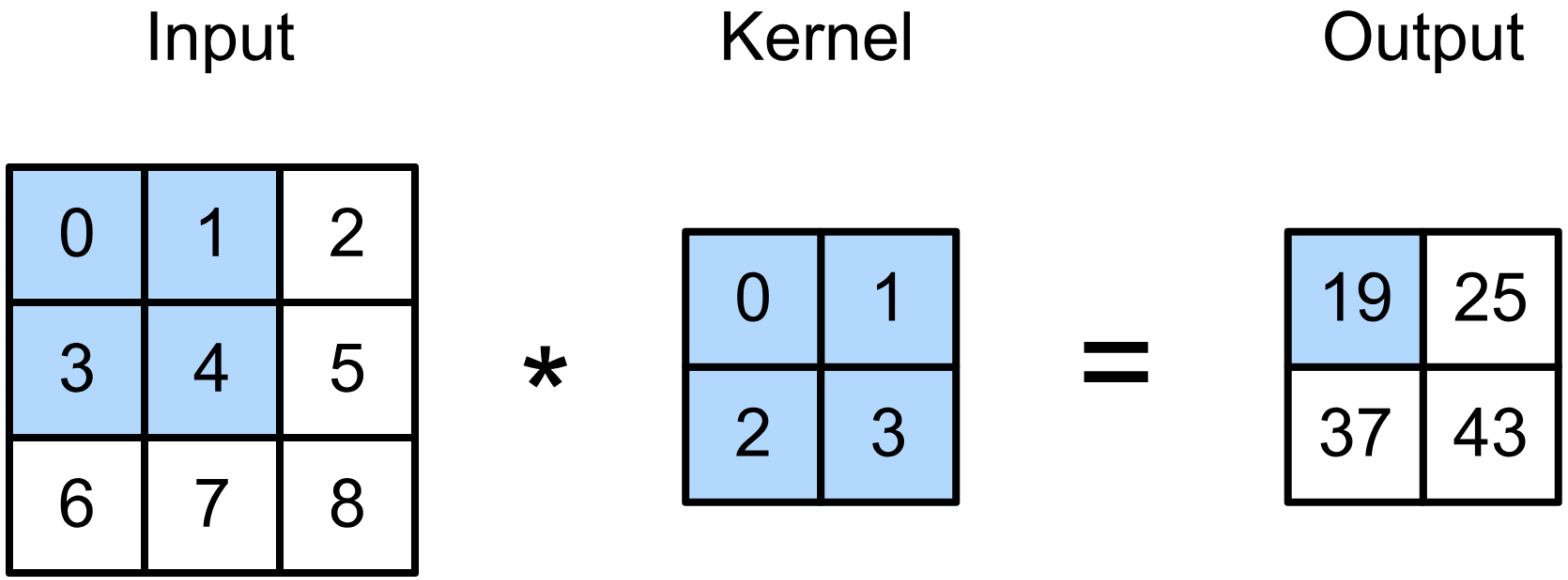


2-D Convolution



$$0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19,$$
$$1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25,$$
$$3 \times 0 + 4 \times 1 + 6 \times 2 + 7 \times 3 = 37,$$
$$4 \times 0 + 5 \times 1 + 7 \times 2 + 8 \times 3 = 43.$$

2-D Convolution



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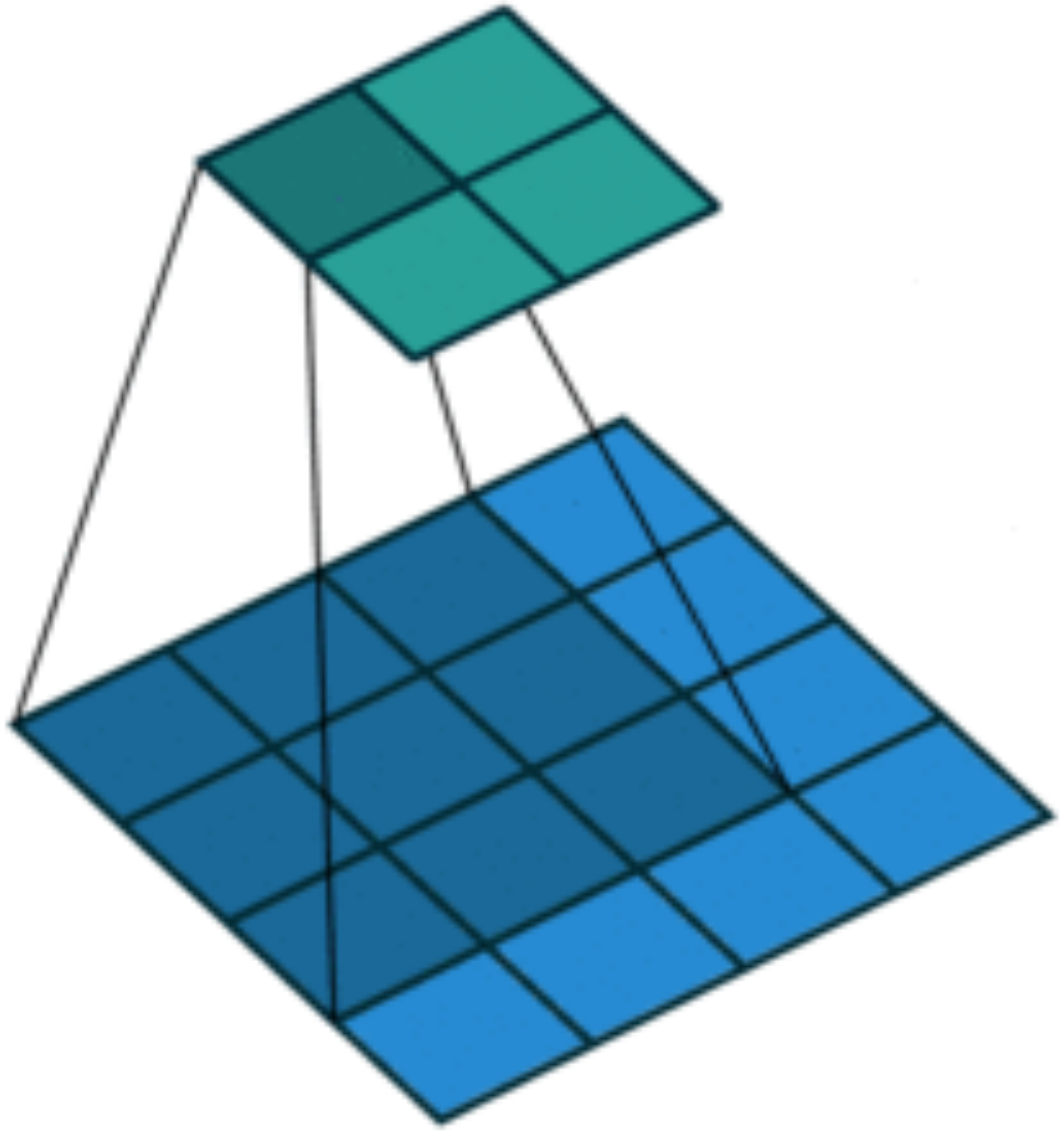
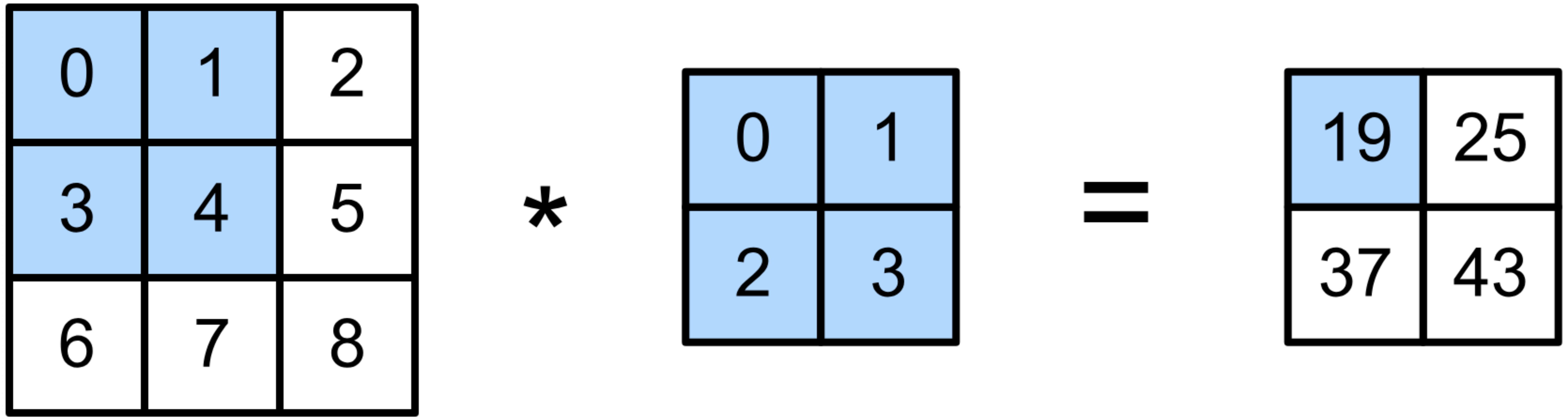


(vdumoulin@ Github)

2-D Convolution Layer

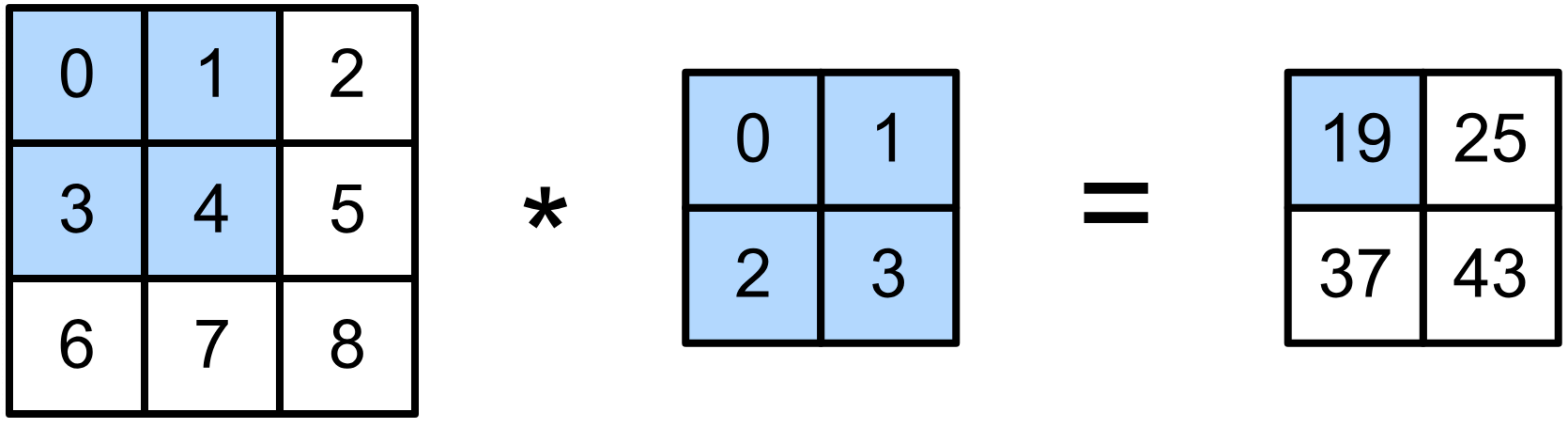


- $\mathbf{X} : n_h \times n_w$ input matrix
- $\mathbf{W} : k_h \times k_w$ kernel matrix
- b: scalar bias
- $\mathbf{Y} : (n_h - k_h + 1) \times (n_w - k_w + 1)$ output matrix

$$\mathbf{Y} = \mathbf{X} \star \mathbf{W} + b$$

- **W** and *b* are learnable parameters

2-D Convolution Layer



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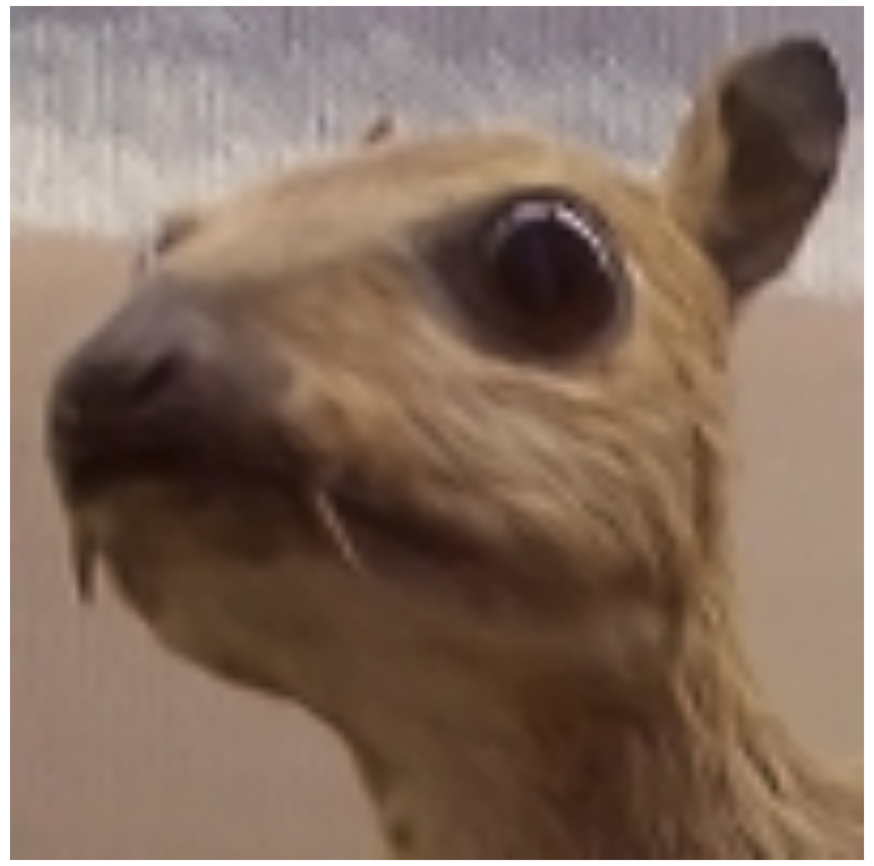
Convolution Operator (not matrix multiply)

- **W** and *b* are learnable parameters

Examples

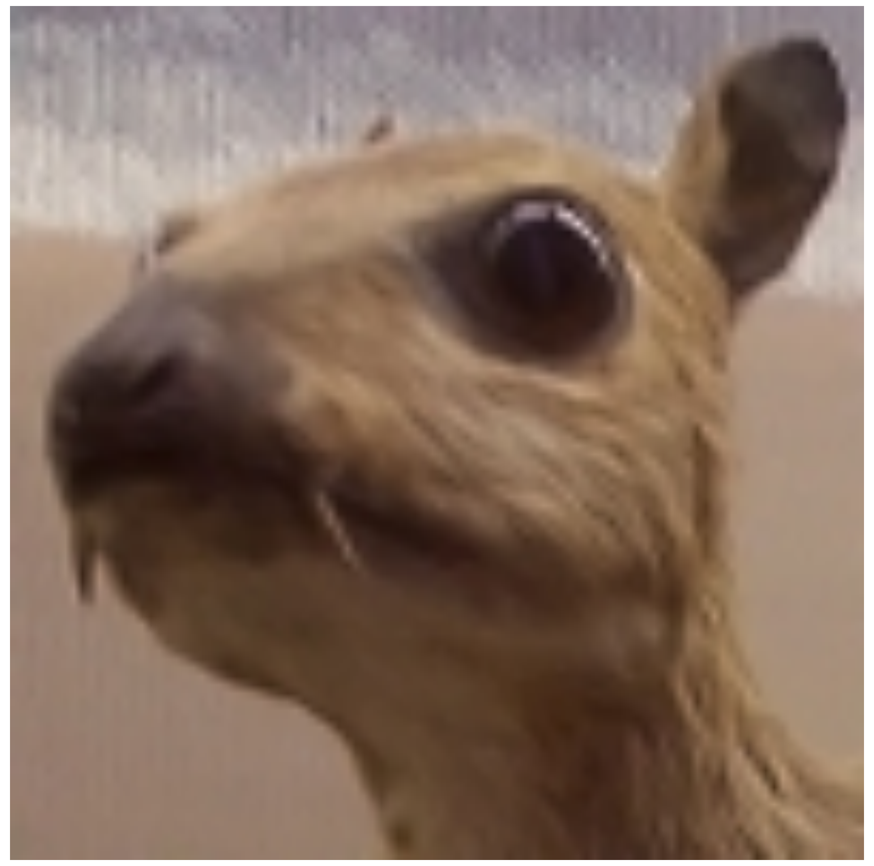


(wikipedia)

Examples

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Edge Detection



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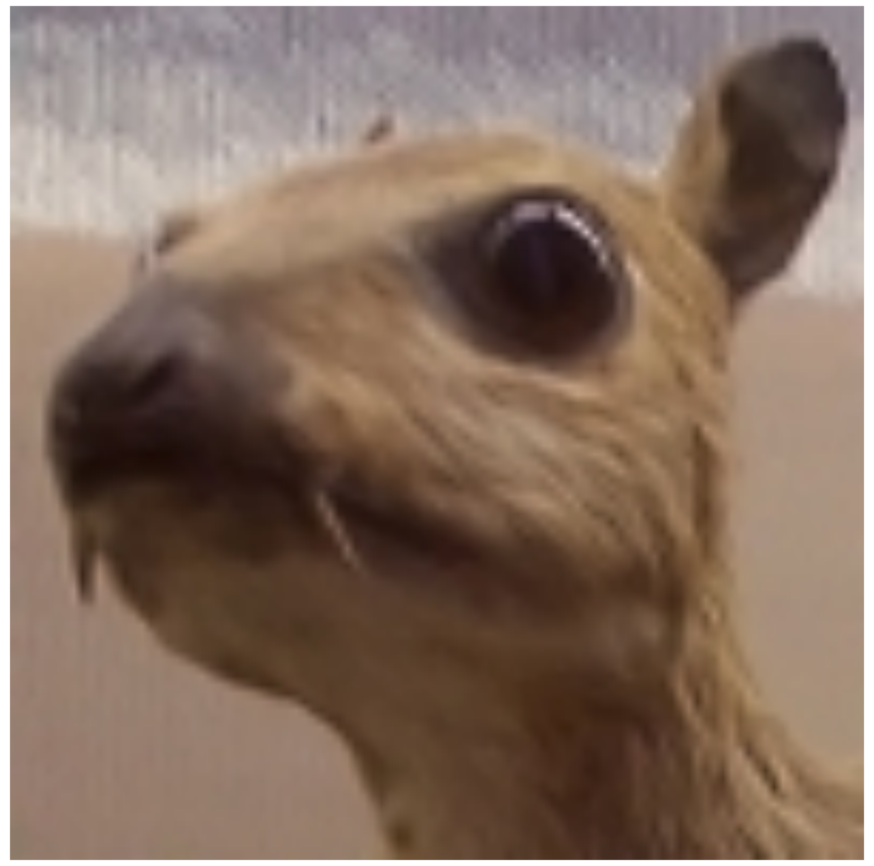

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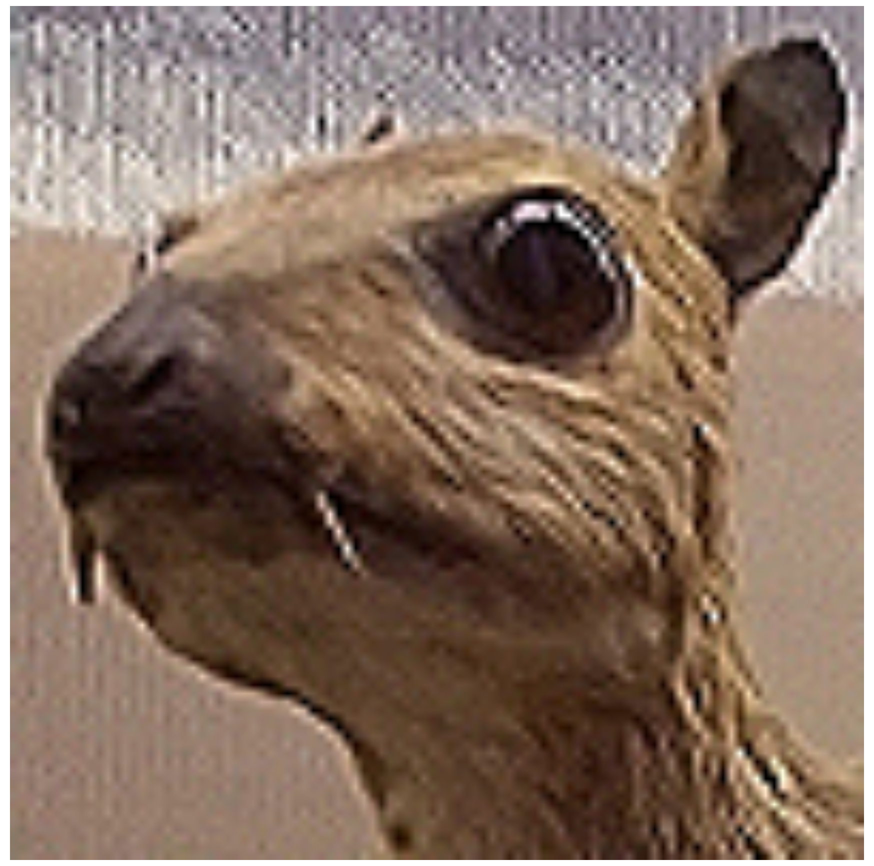
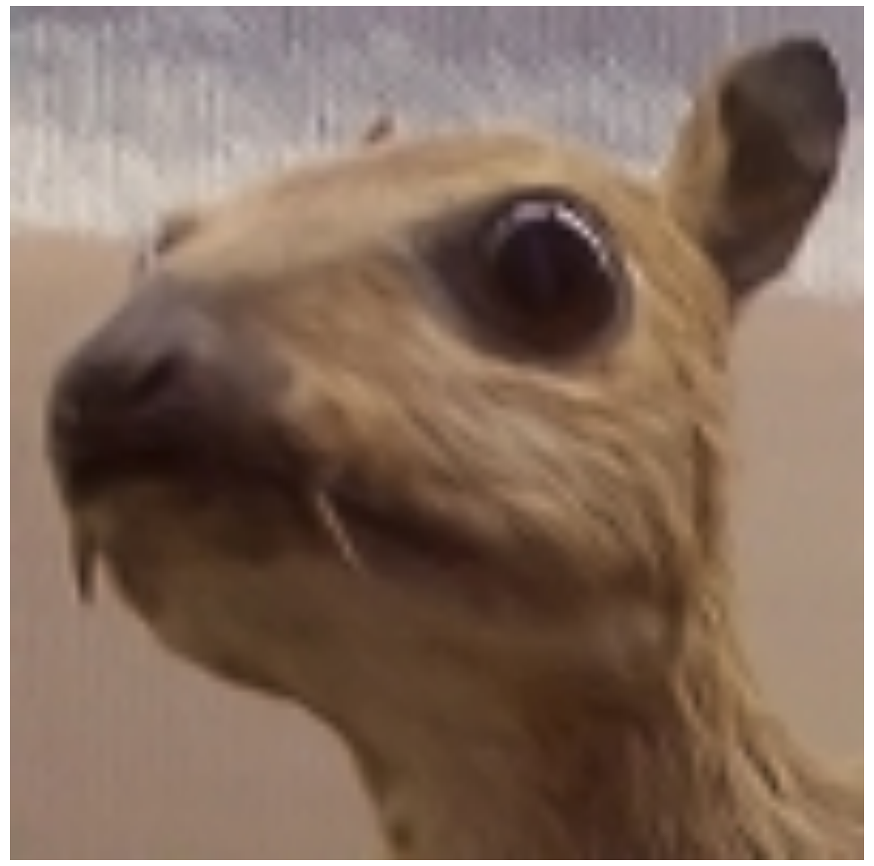


Sharpen

Examples



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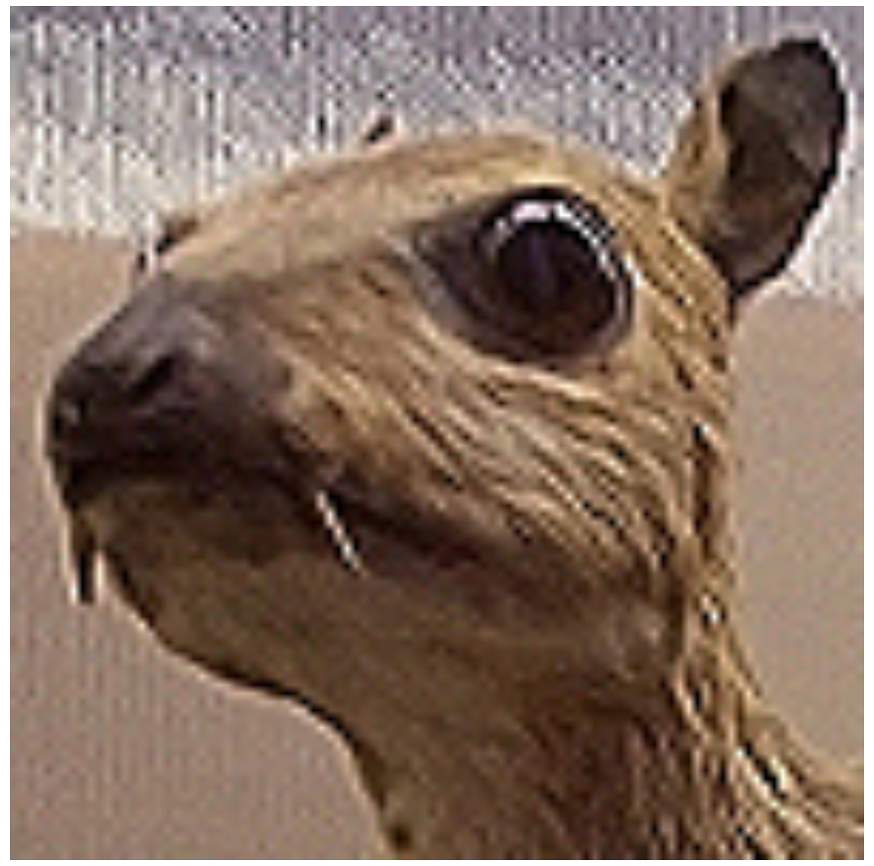


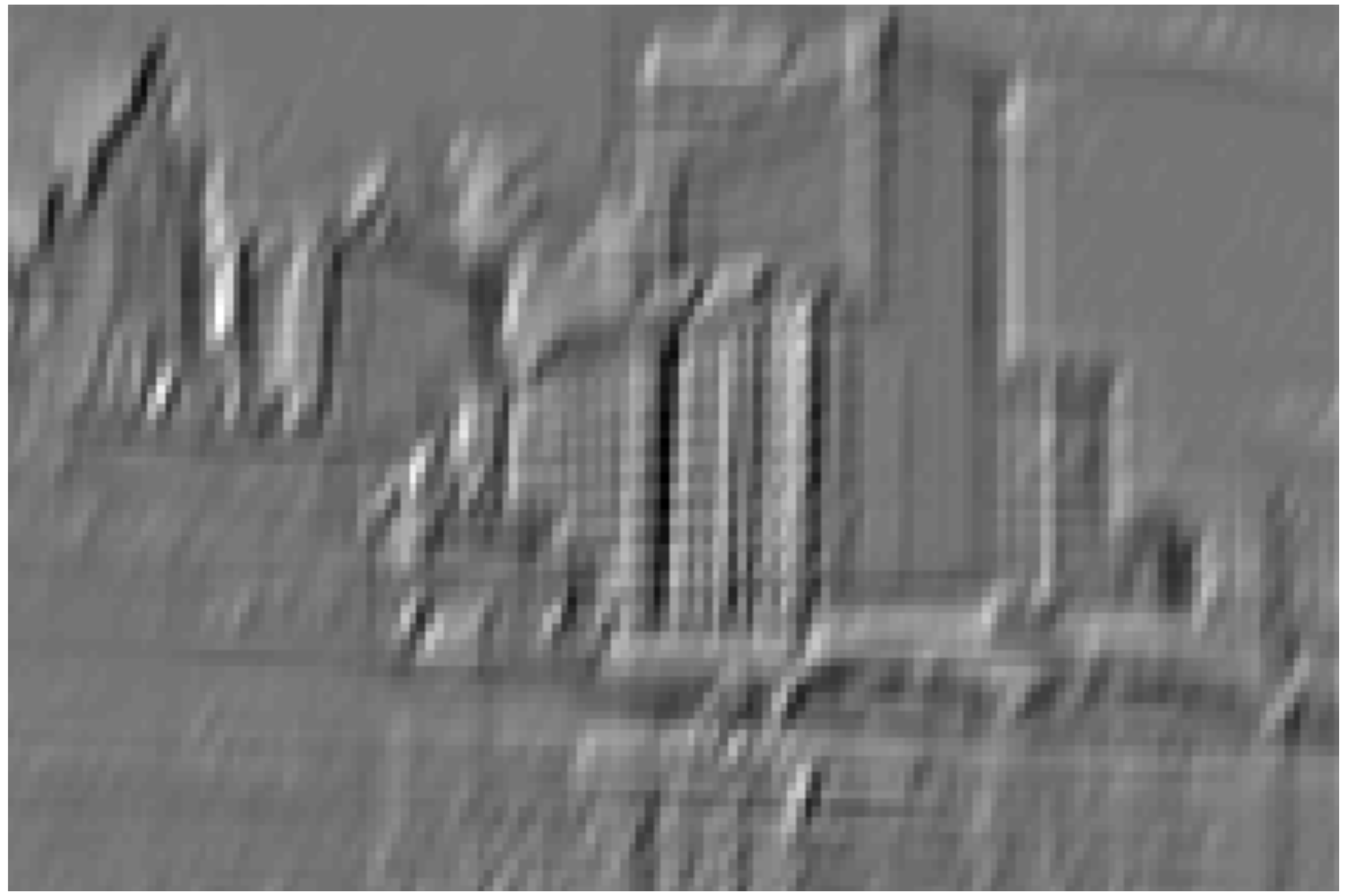
Sharpen

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Gaussian Blur

Examples

(Rob Fergus)



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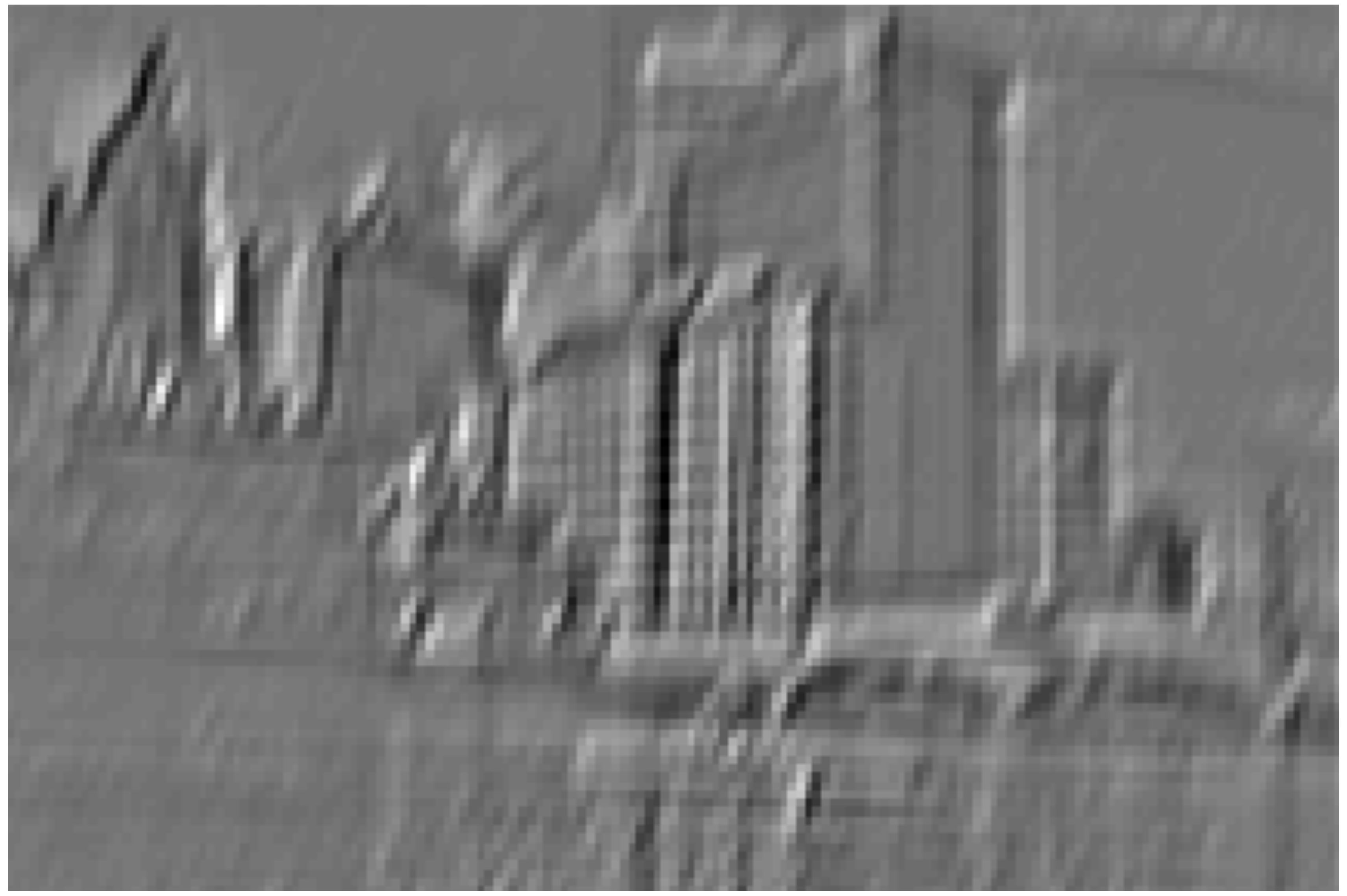


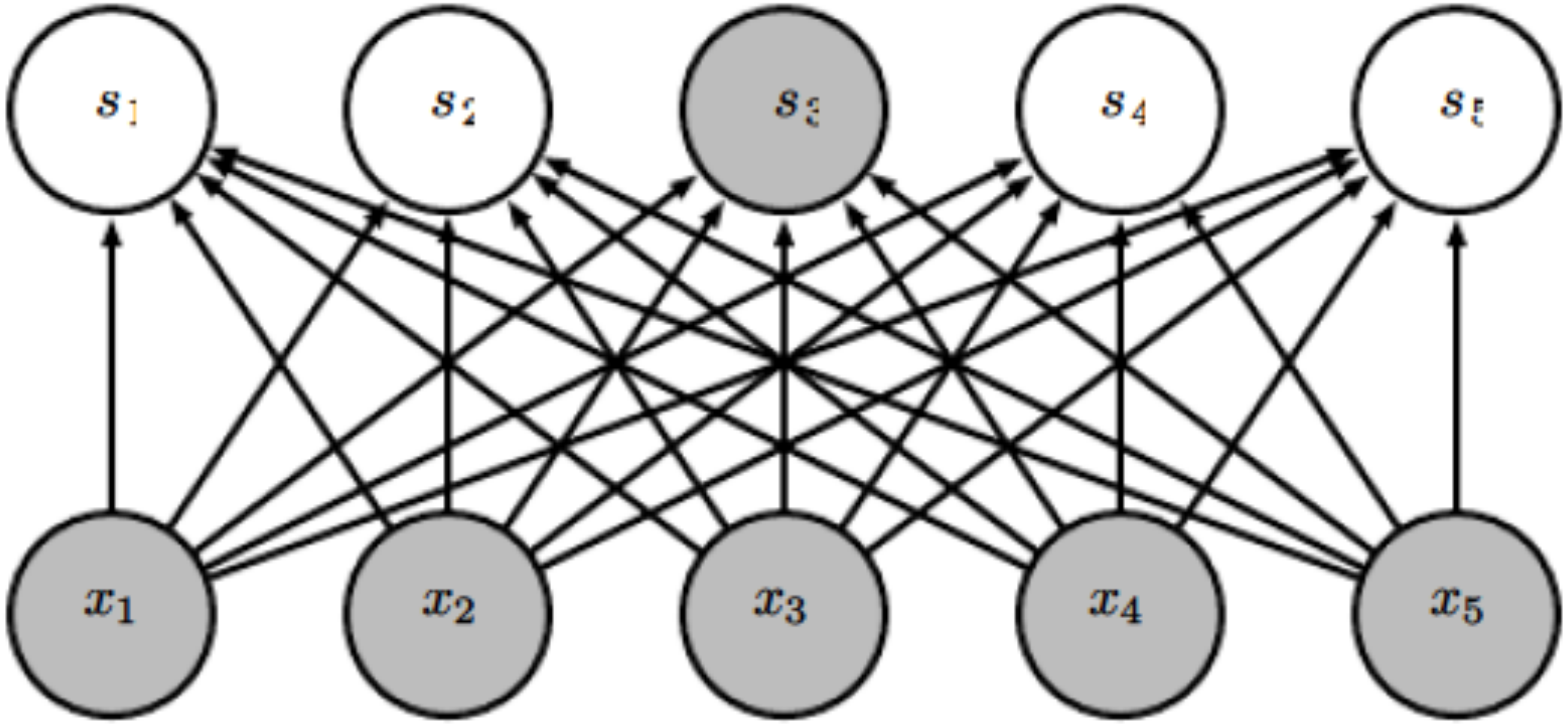
Convolutional Neural Networks

- Convolutional networks: neural networks that use convolution in place of general matrix multiplication in at least one of their layers

- Strong empirical application performance
  - Particularly on computer vision tasks.

Advantage: sparse interaction

Fully connected layer, $m \times n$ edges



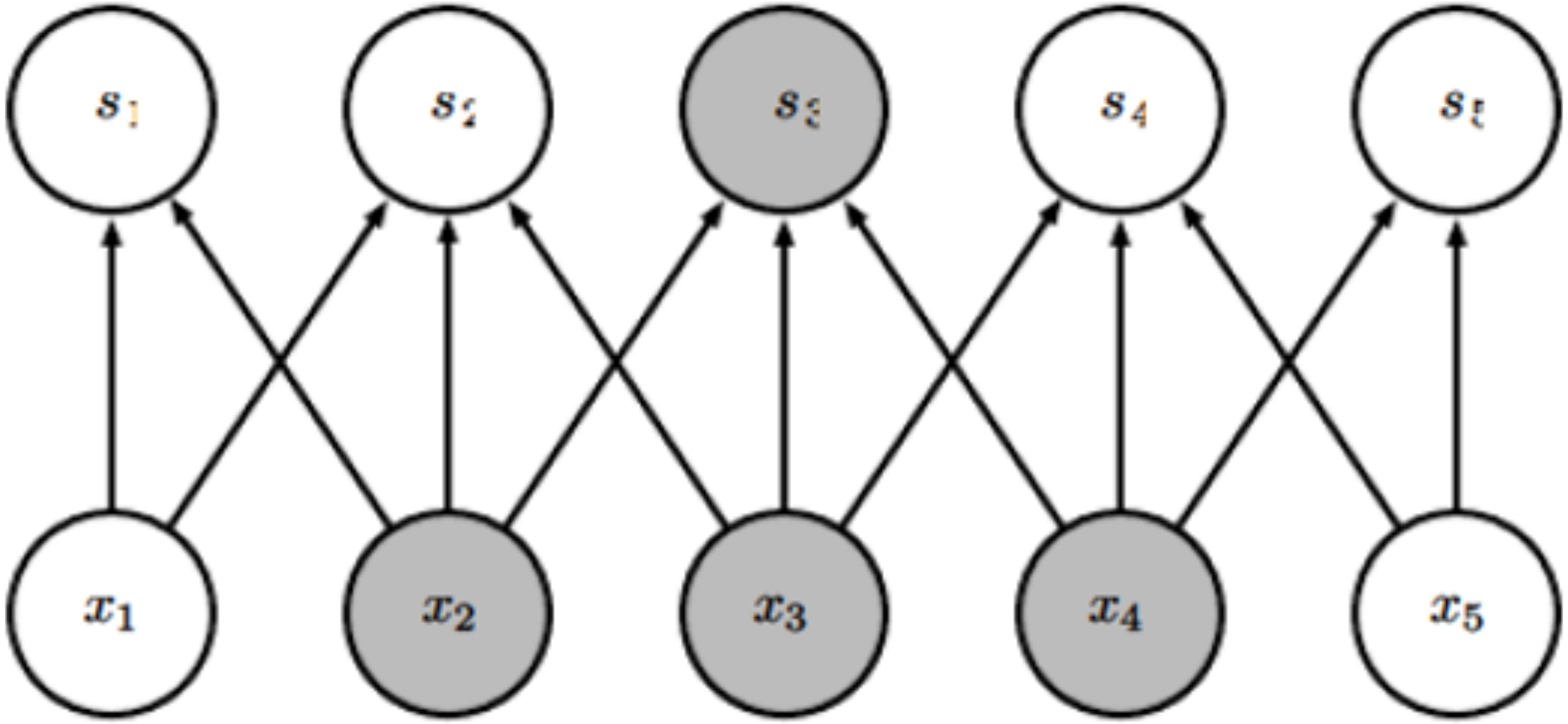
m output nodes

n input nodes

Figure from *Deep Learning,* by Goodfellow, Bengio, and Courville

Advantage: sparse interaction

Convolutional layer, $\leq m \times k$ edges



m output nodes

k kernel size

n input nodes

Figure from *Deep Learning,* by Goodfellow, Bengio, and Courville

Efficiency of Convolution

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- Input size: 320 x 280
- Kernel Size: 2 x 1
- Output size: 319 x 280

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Stored floats		
Float muls or adds		

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	Convolution	Dense matrix
Stored floats	2	
Float muls or adds		

Efficiency of Convolution

- Input size: 320 x 280
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	Convolution	Dense matrix
Stored floats	2	319*280*320*280 $> 8e9$
Float muls or adds		

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2 multiplies plus 1 addition = 3 ops / output

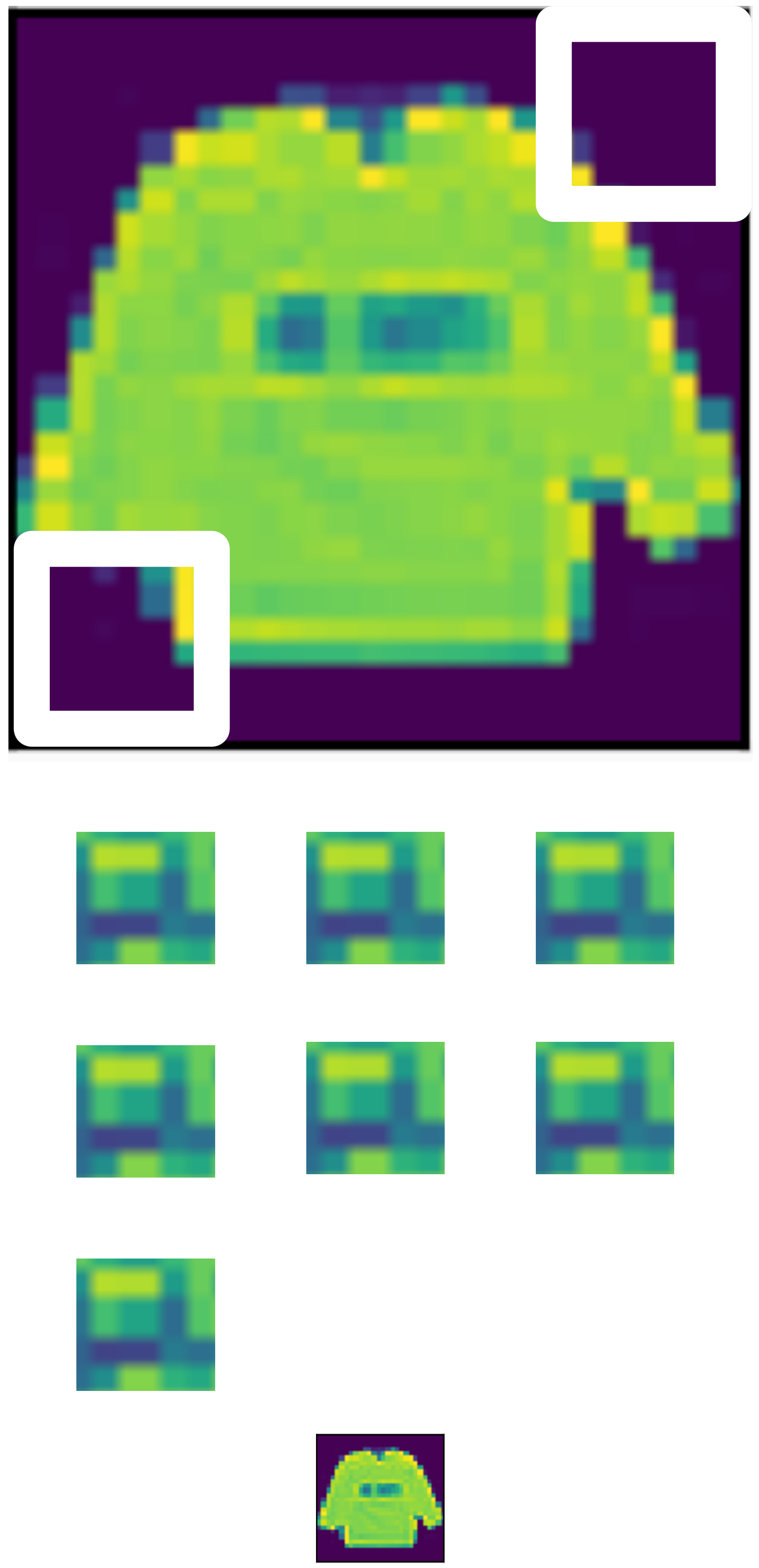
1 multiplication per input and add together per output.

Padding and Stride

Padding

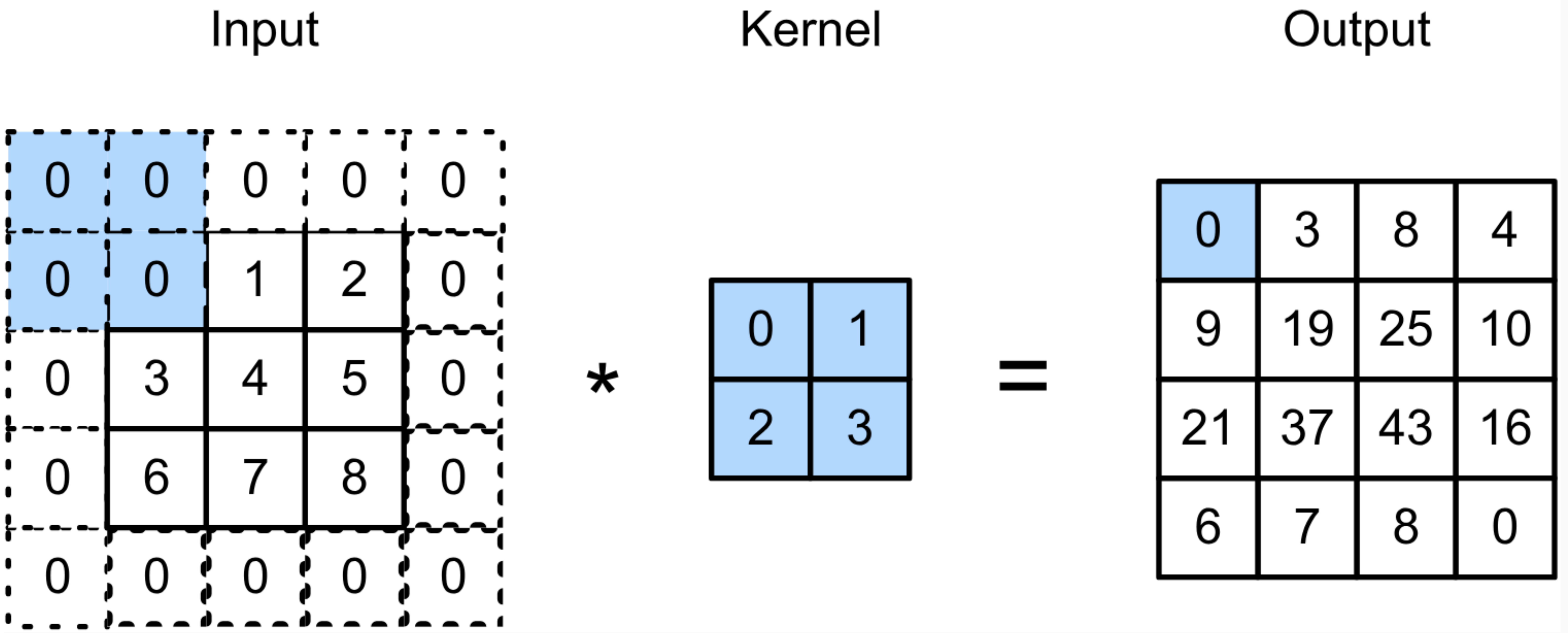
- Given a 32 x 32 input image
- Apply convolution with 5 x 5 kernel
  - 28 x 28 output with 1 layer
  - 4 x 4 output with 7 layers
- Shape decreases faster with larger kernels
  - Shape reduces from $n_h \times n_w$ to

$$(n_h - k_h + 1) \times (n_w - k_w + 1)$$



Padding

Padding adds rows/columns around input



$$0 \times 0 + 0 \times 1 + 0 \times 2 + 0 \times 3 = 0$$

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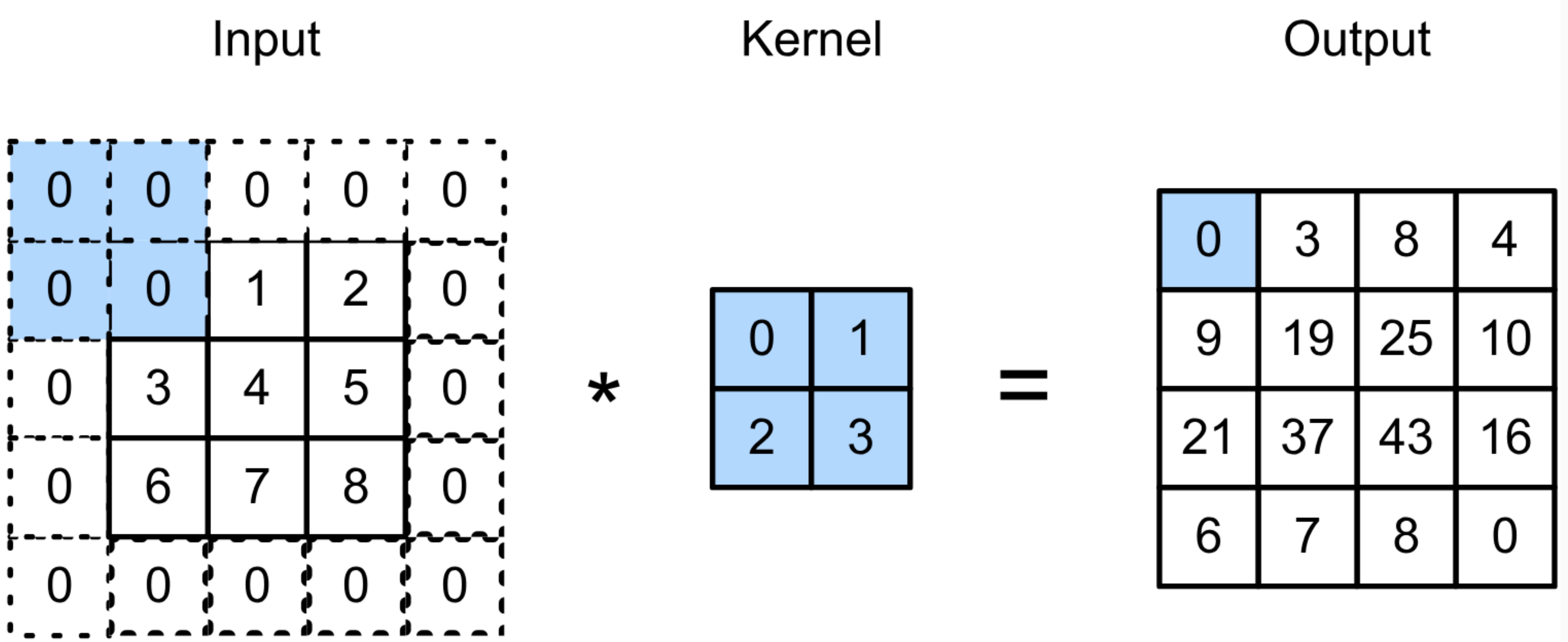

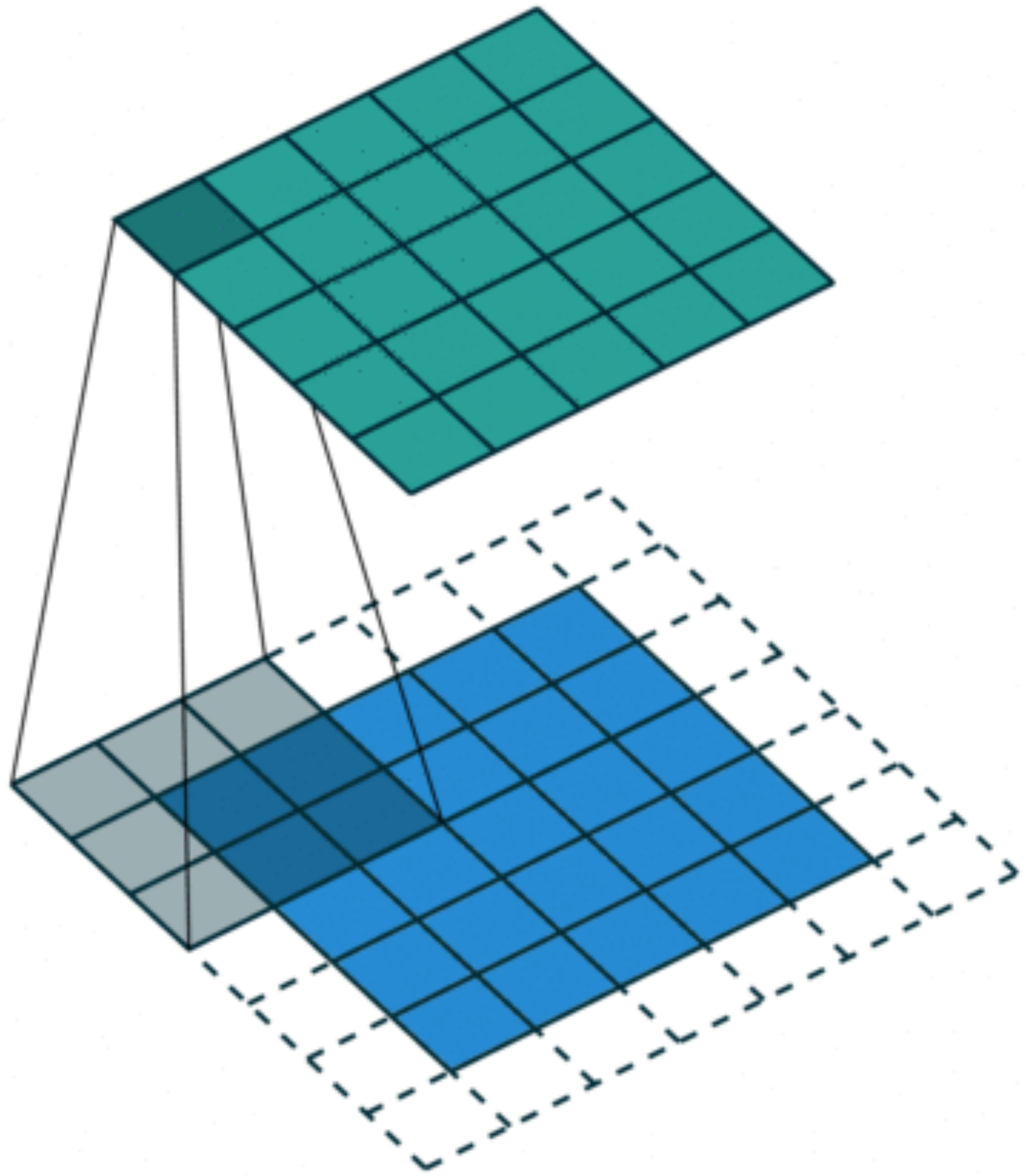

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Padding

- Padding p_h rows and p_w columns, output shape will be

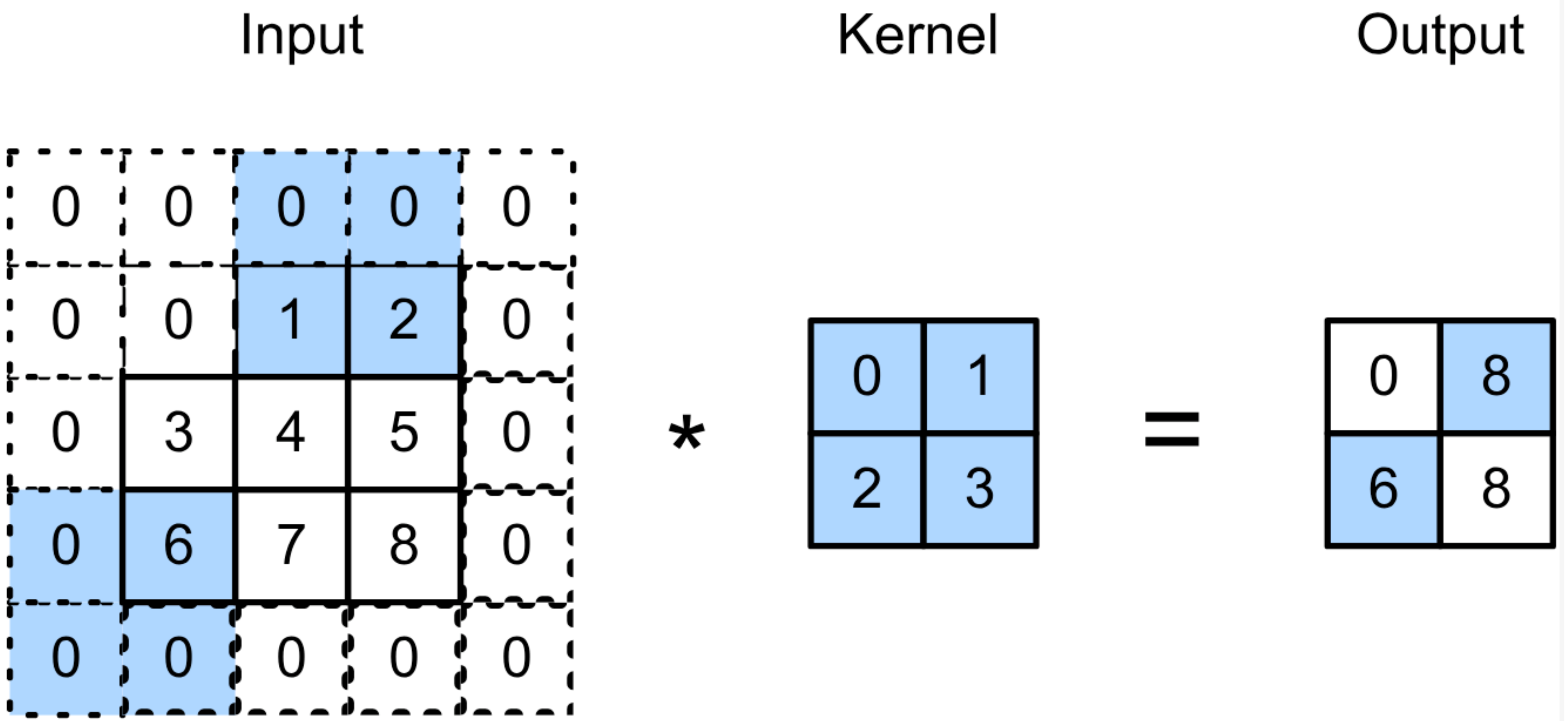
$$(n_h - k_h + p_h + 1) \times (n_w - k_w + p_w + 1)$$

- A common choice is $p_h = k_h - 1$ and $p_w = k_w - 1$
  - Odd k_h: pad $\frac{p_h}{2}$ on both sides
  - Even k_h: pad $\lceil \frac{p_h}{2} \rceil$ on top, $\lfloor \frac{p_h}{2} \rfloor$ on bottom

Stride

- Stride is the #rows/#columns per slide

Strides of 3 and 2 for height and width



$$0 \times 0 + 0 \times 1 + 1 \times 2 + 2 \times 3 = 8$$
$$0 \times 0 + 6 \times 1 + 0 \times 2 + 0 \times 3 = 6$$

Stride

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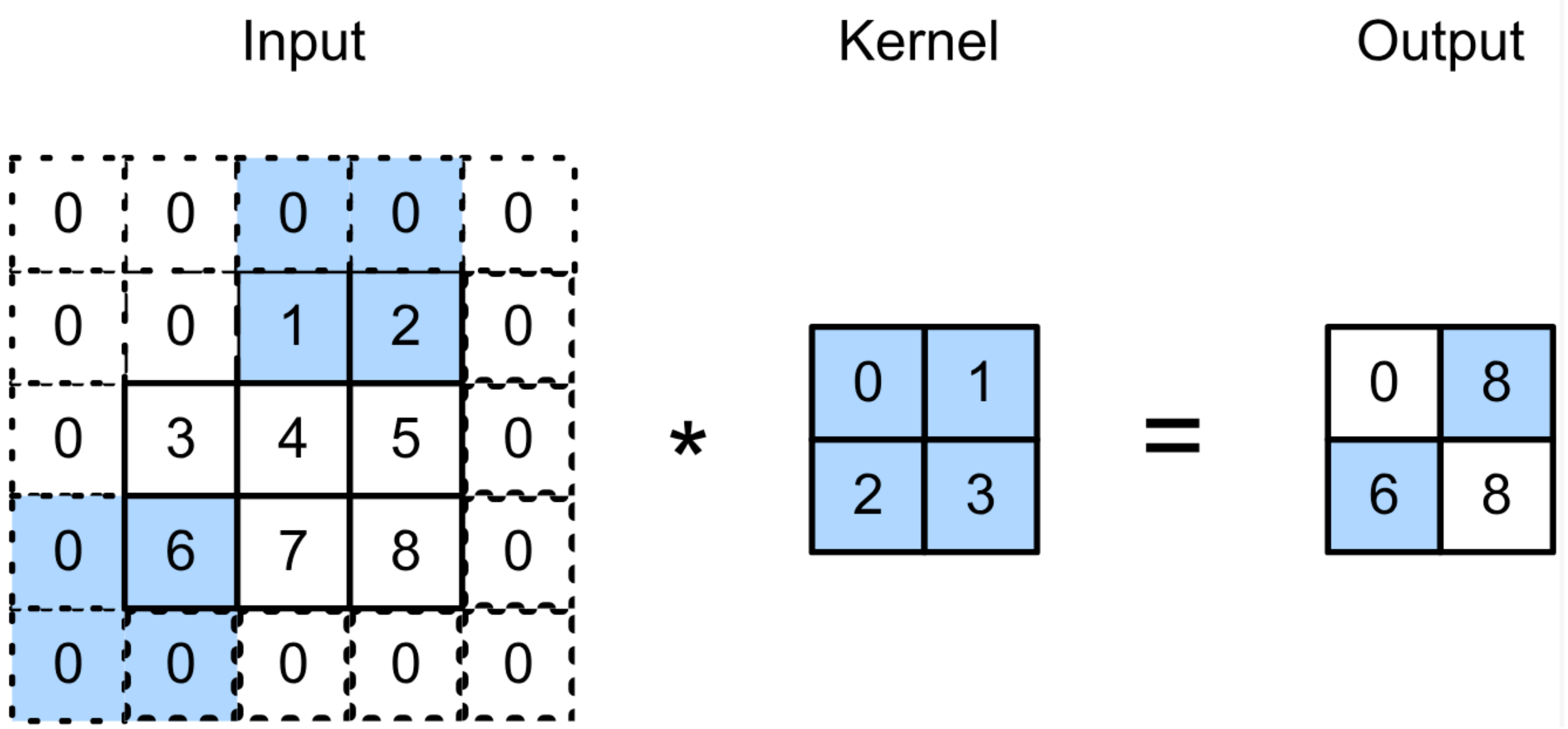
Strides of 3 and 2 for height and width



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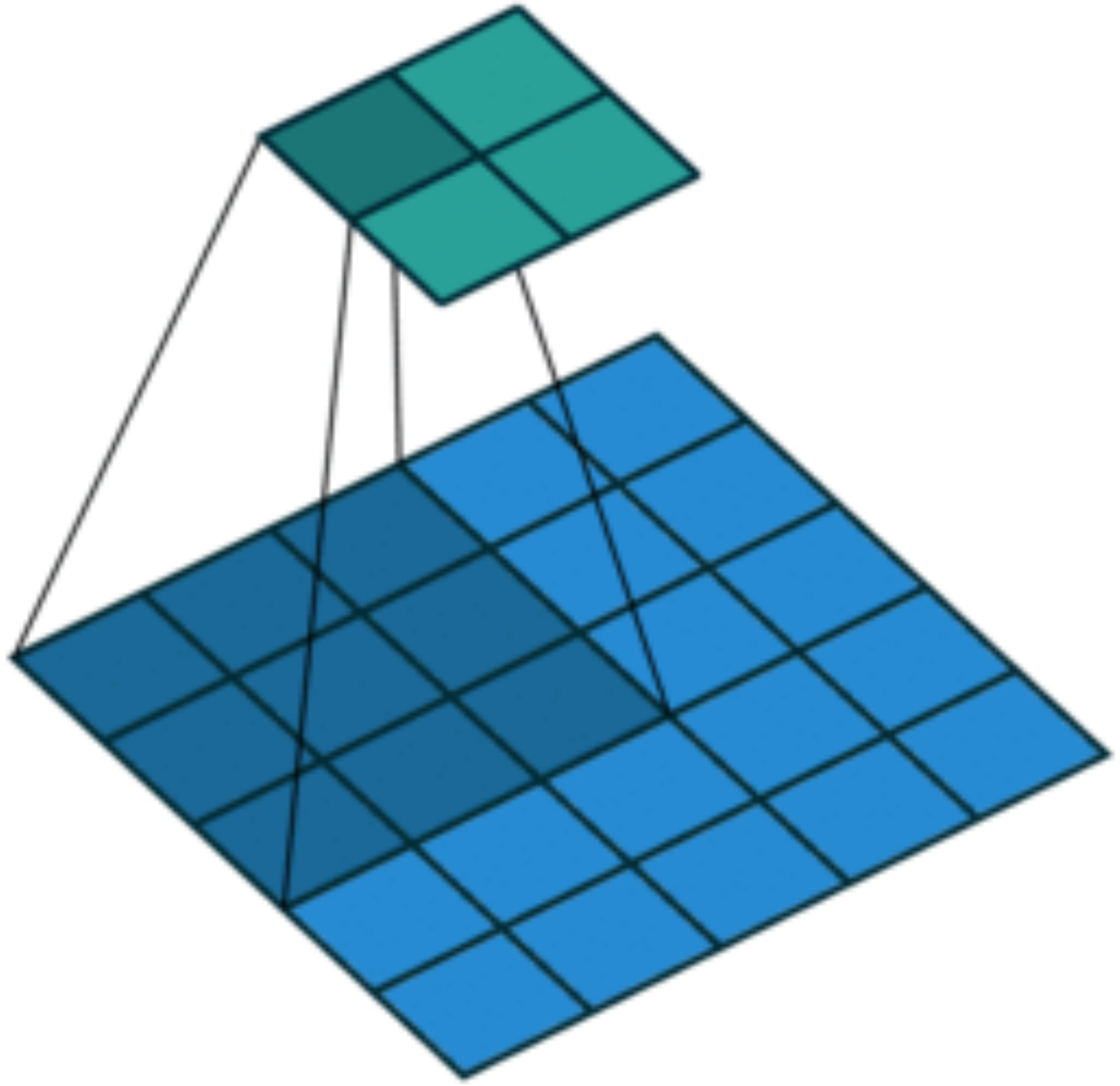


Stride

- Given stride s_h for the height and stride s_w for the width, the output shape is

$$\lfloor (n_h - k_h + p_h + s_h)/s_h \rfloor \times \lfloor (n_w - k_w + p_w + s_w)/s_w \rfloor$$

- With $p_h = k_h - 1$ and $p_w = k_w - 1$

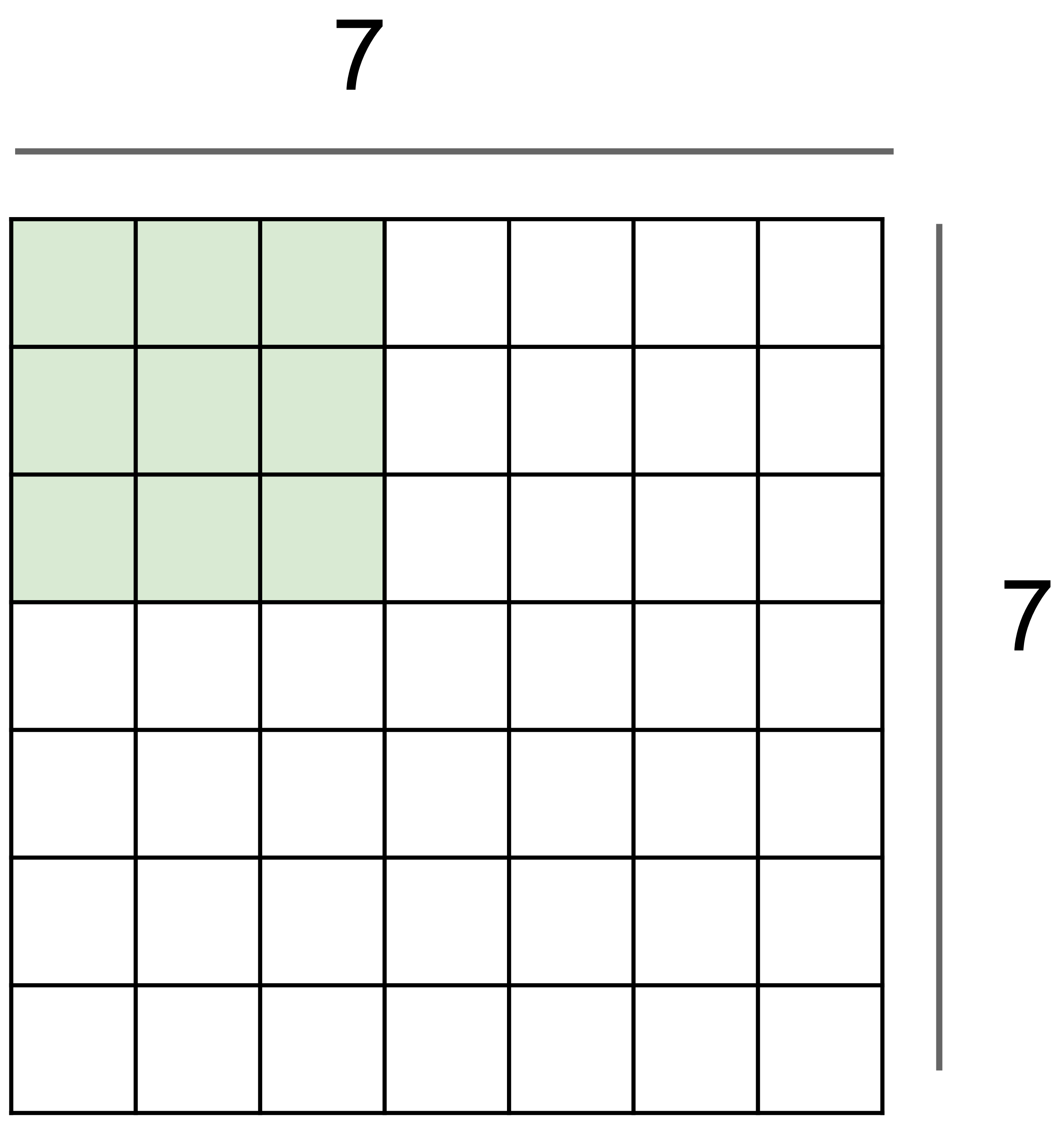
$$\lfloor (n_h + s_h - 1)/s_h \rfloor \times \lfloor (n_w + s_w - 1)/s_w \rfloor$$

- If input height/width are divisible by strides

$$(n_h/s_h) \times (n_w/s_w)$$

Q1. Suppose we want to perform convolution on a single channel image of size 7x7 (no padding) with a kernel of size 3x3, and stride = 2. What is the dimension of the output?

A. 3x3
B. 7x7
C. 5x5
D. 2x2



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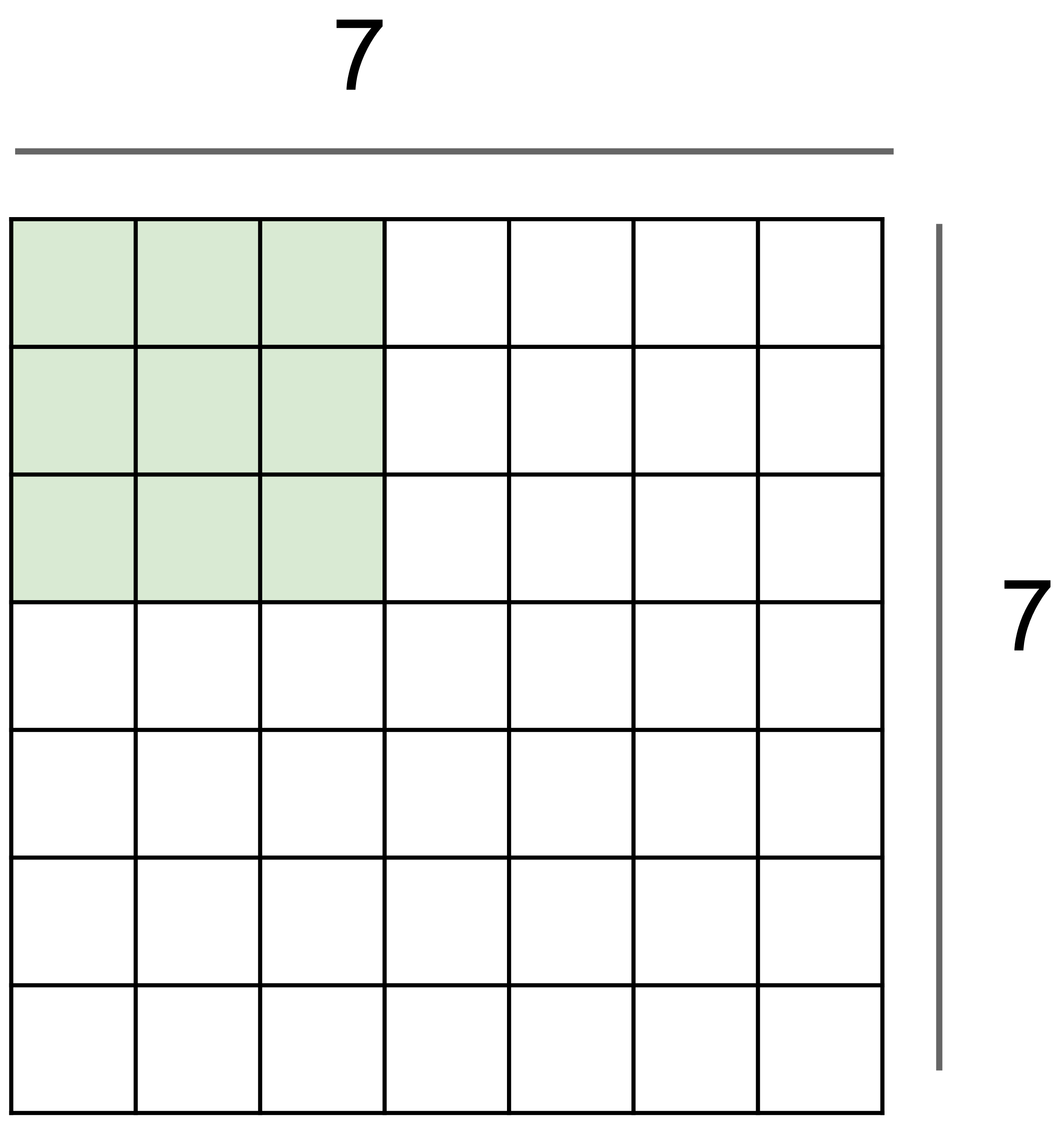

$$\lfloor (n_h - k_h + p_h + s_h)/s_h \rfloor \times \lfloor (n_w - k_w + p_w + s_w)/s_w \rfloor$$

Multiple Input and Output Channels

Linear Algebra Terminology Review

- Vector: 1 x n list of real values.

- Matrix: m x n block of real values.
  - m vectors concatenated together.

- Tensor: k x … x m x n block of real values.
  - Generalization of matrix to higher dimensions.
  - k x m x n is k of m x n matrices stacked together.

Multiple Input Channels

- Color image may have three RGB channels
- Converting to grayscale loses information

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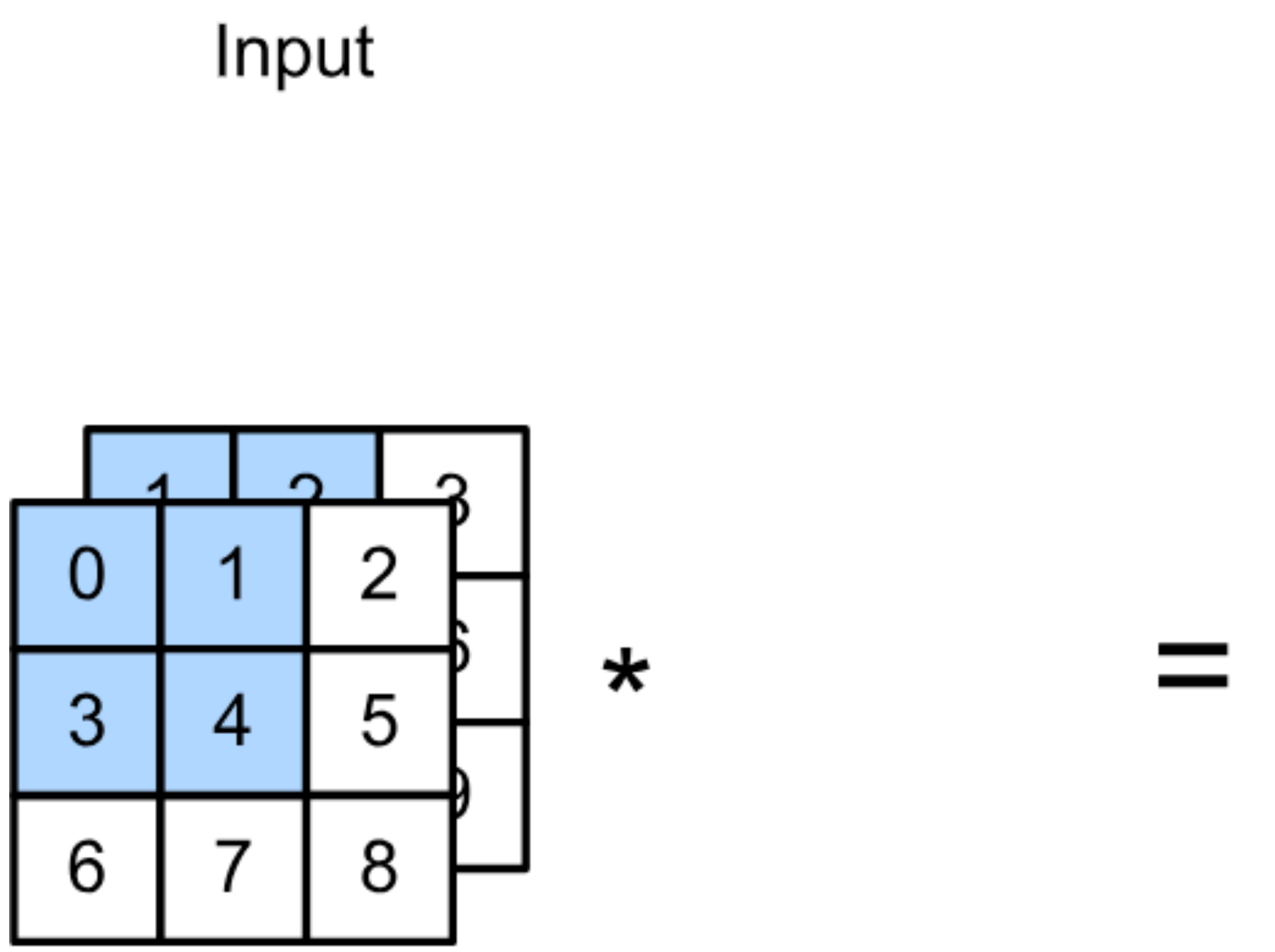
Multiple Input Channels

- Have a kernel for each channel, and then sum results over channels



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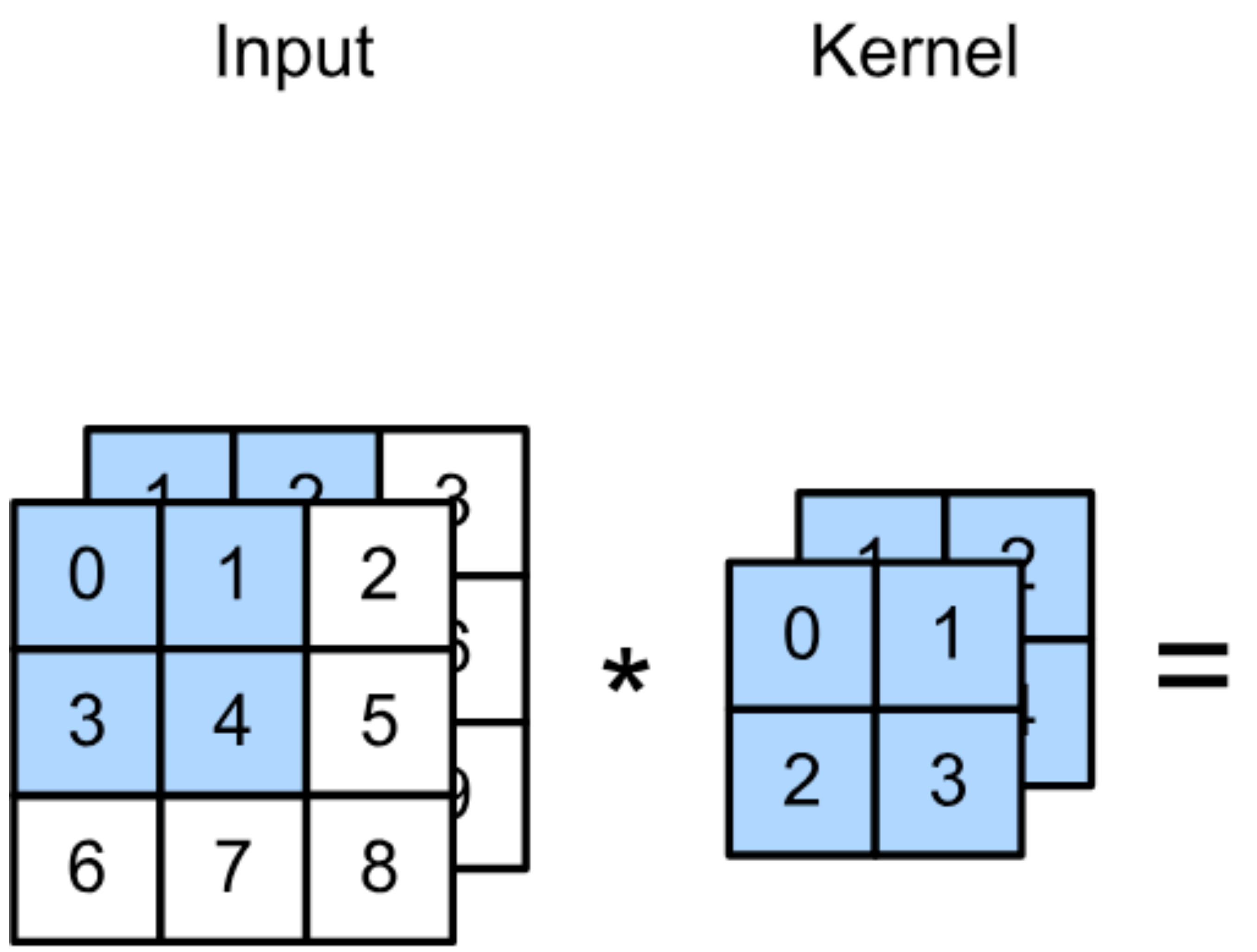

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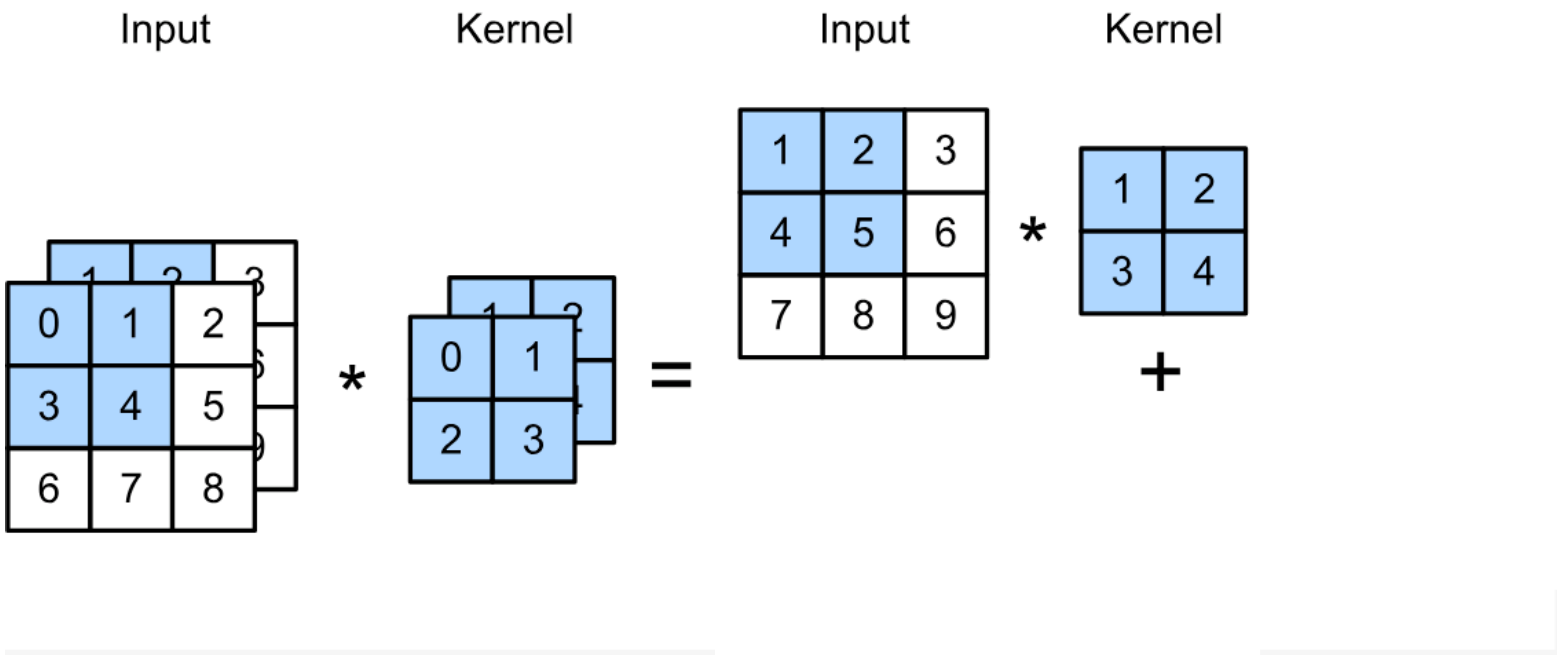

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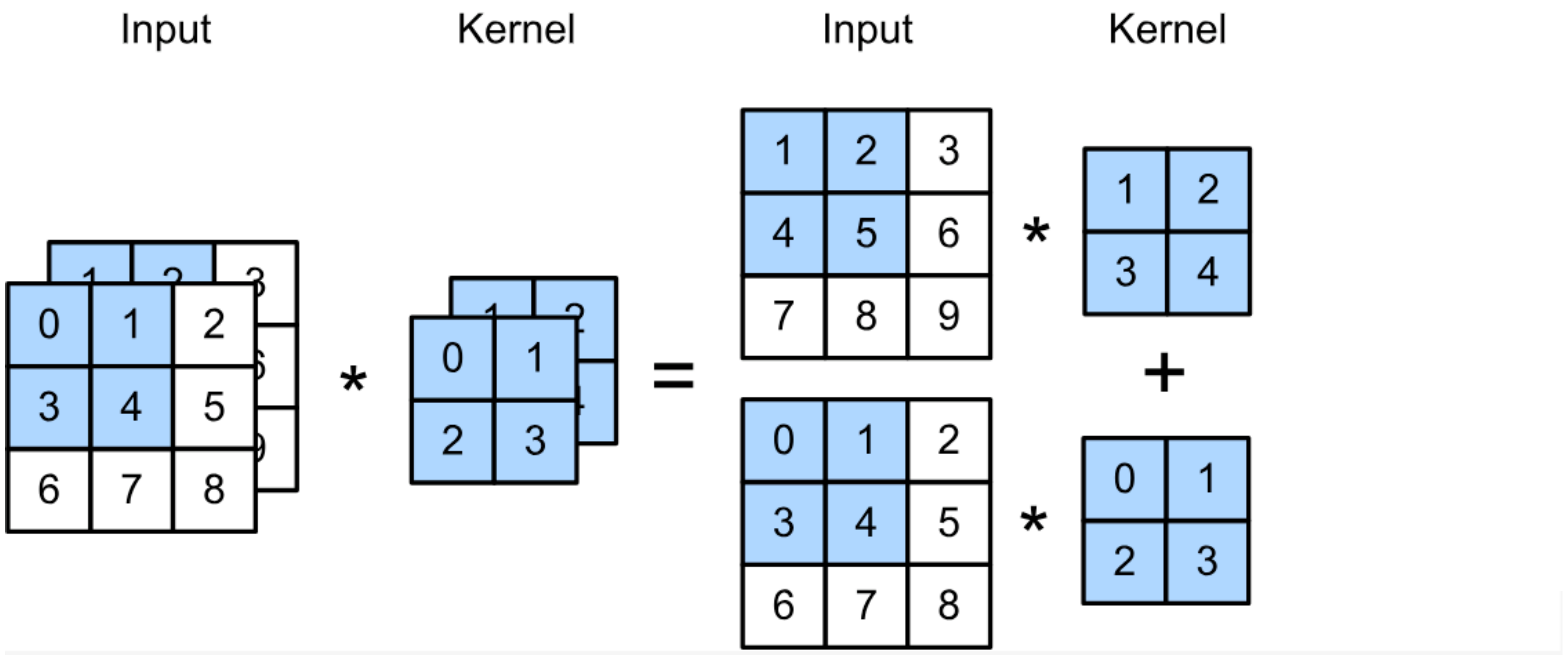

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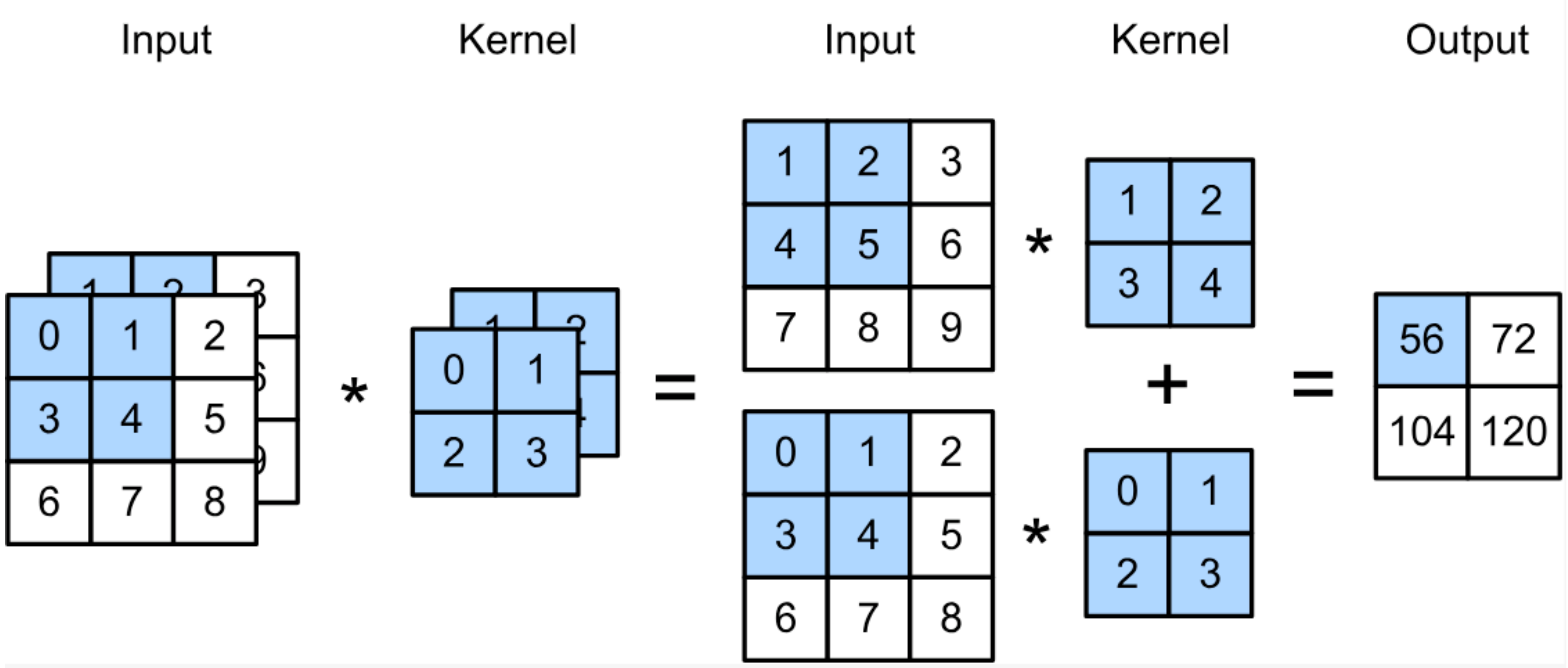

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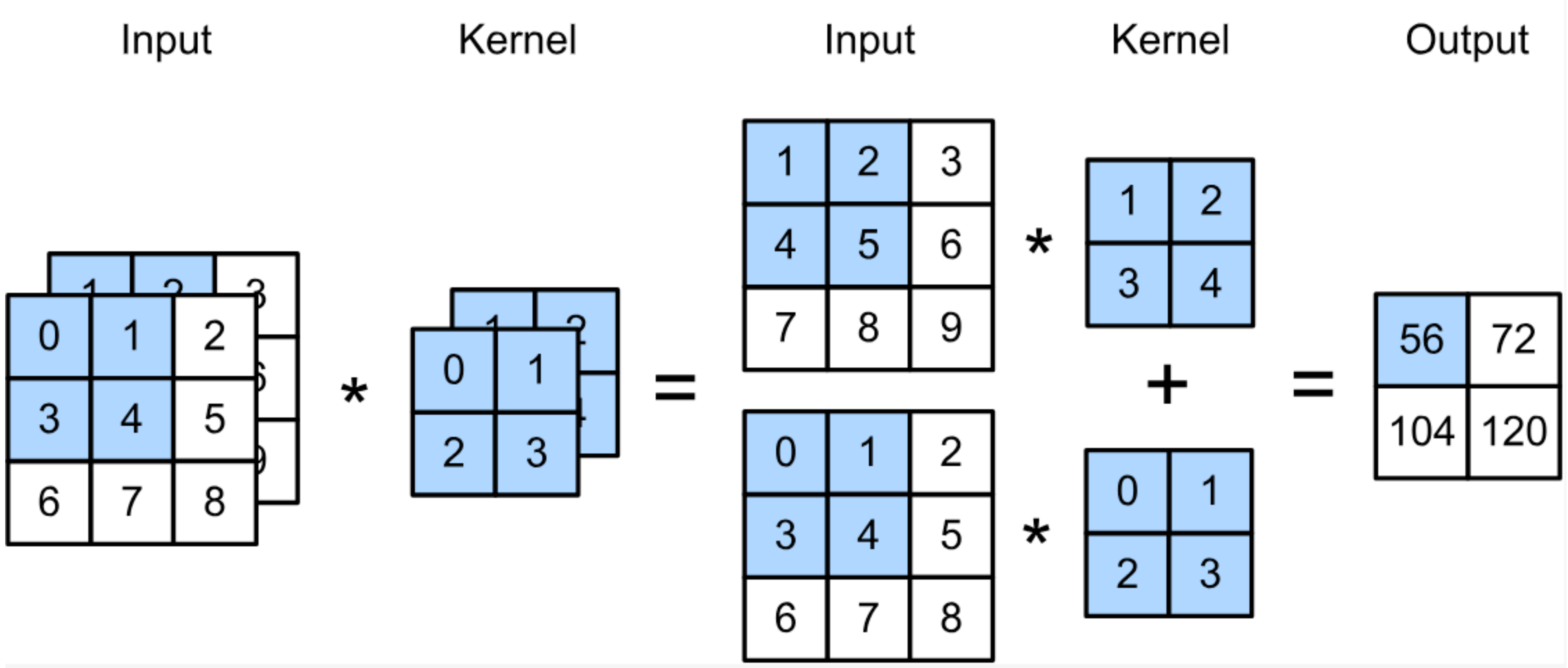

$$(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4)$$
$$+(0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3)$$
$$= 56$$

Multiple Input Channels

- $\mathbf{X} : c_i \times n_h \times n_w$ input
- $\mathbf{W} : c_i \times k_h \times k_w$ kernel
- $\mathbf{Y} : m_h \times m_w$ output

$$\mathbf{Y} = \sum_{i=0}^{c_i} \mathbf{X}_{i,:,:} \star \mathbf{W}_{i,:,:}$$

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Multiple Output Channels

- No matter how many inputs channels, so far we always get single output channel
- We can have **multiple 3-D kernels**, each one generates an output channel
- Input $\mathbf{X} : c_i \times n_h \times n_w$
- Kernel $\mathbf{W} : c_o \times c_i \times k_h \times k_w$
- Output $\mathbf{Y} : c_o \times m_h \times m_w$

$$\mathbf{Y}_{i,:,:} = \mathbf{X} \star \mathbf{W}_{i,:,:,:}$$

$$\text{for } i = 1,\ldots,c_o$$

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$$\mathbf{Y}_{i,:,:} = \mathbf{X} \star \mathbf{W}_{i,:,:,:}$$

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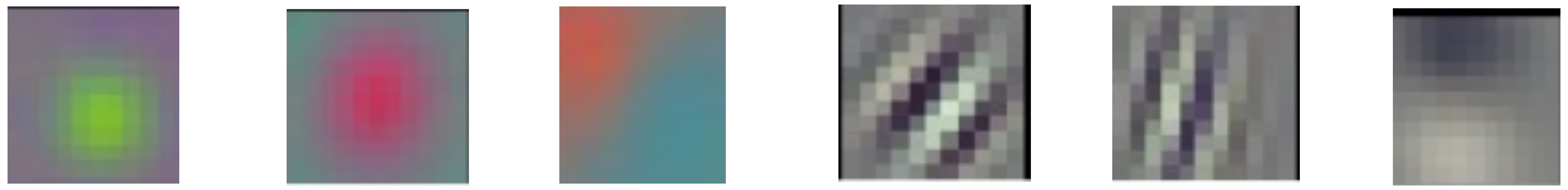
Multiple Input/Output Channels

- Each 3-D kernel may recognize a particular pattern

Multiple Input/Output Channels

- Each 3-D kernel may recognize a particular pattern



(Gabor filters)

Q3-1. Suppose we want to perform convolution on a RGB image of size 224x224 (no padding) with 64 kernels of size 3x3. Stride = 1. Which is a reasonable estimate of the total number of scalar multiplications involved in this operation (without considering any optimization in matrix multiplication)?

A. 64 x 3 x 3 x 222 x 222

B. 64 x 3 x 3 x 222

C. 3 x 3 x 222 x 222

D. 64 x 3 x 3 x 3 x 222 x 222

Q3-1. Suppose we want to perform convolution on a RGB image of size 224x224 (no padding) with 64 kernels of size 3x3. Stride = 1. Which is a reasonable estimate of the total number of scalar multiplications involved in this operation (without considering any optimization in matrix multiplication)?

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Muls per kernel   Per kernel output

# of kernels

Q 3-2. Suppose we want to perform convolution on a RGB image of size 224x224 (no padding) with 64 kernels of size 3x3. Stride = 1. Which is a reasonable estimate of the total number of learnable parameters?

A. 64 x 222 x 222

B. 64 x 3 x 3 x 222

C. 3 x 3 x 3 x 64

D. (3 x 3 x 3 + 1) x 64

Q 3-2. Suppose we want to perform convolution on a RGB image of size 224x224 (no padding) with 64 kernels of size 3x3. Stride = 1. Which is a reasonable estimate of the total number of learnable parameters?

A. 64 x 222 x 222

B. 64 x 3 x 3 x 222

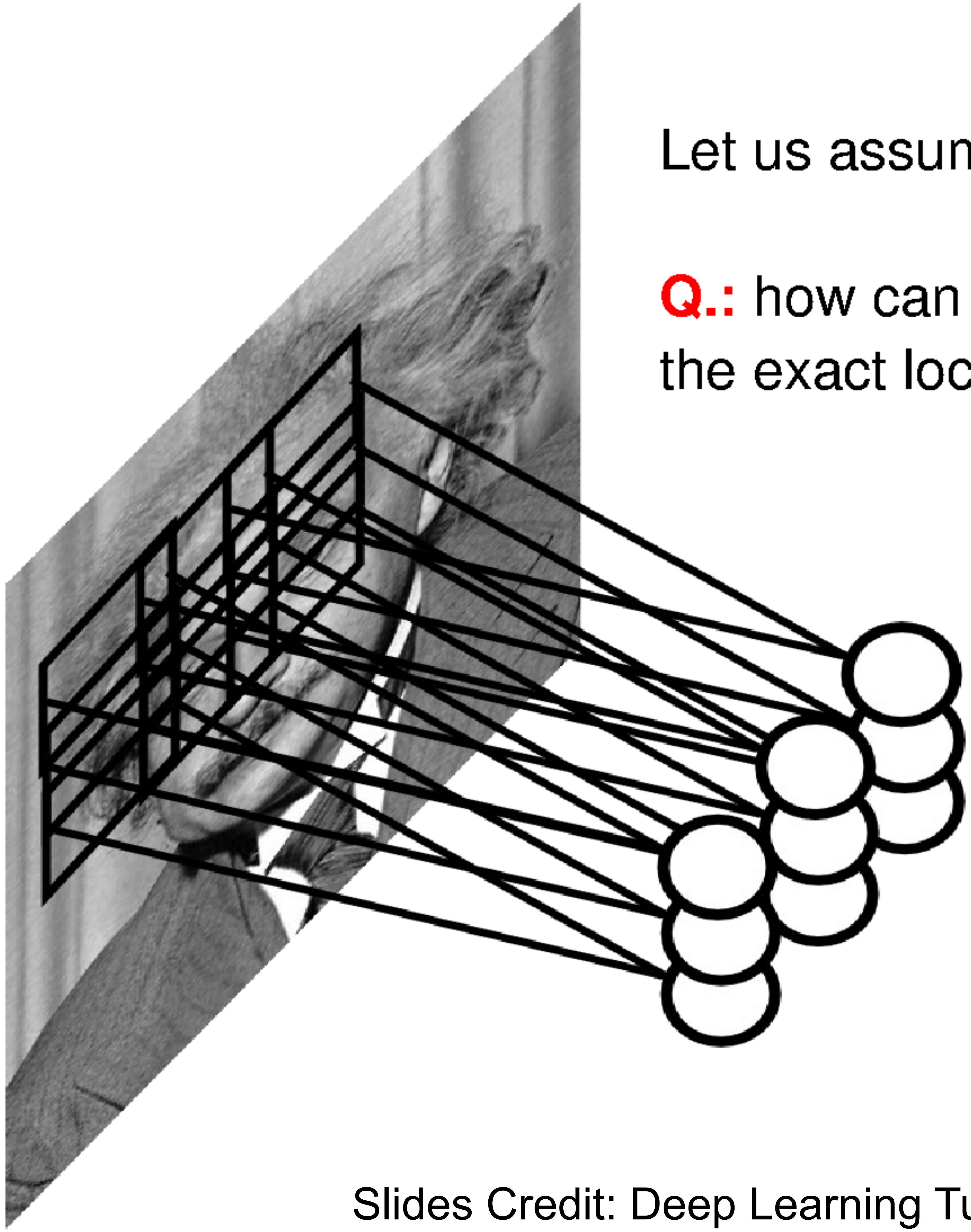
C. 3 x 3 x 3 x 64

D. (3 x 3 x 3 + 1) x 64

9 weights / kernel * 3 channels + 1 bias / kernel

Pooling Layer

Pooling

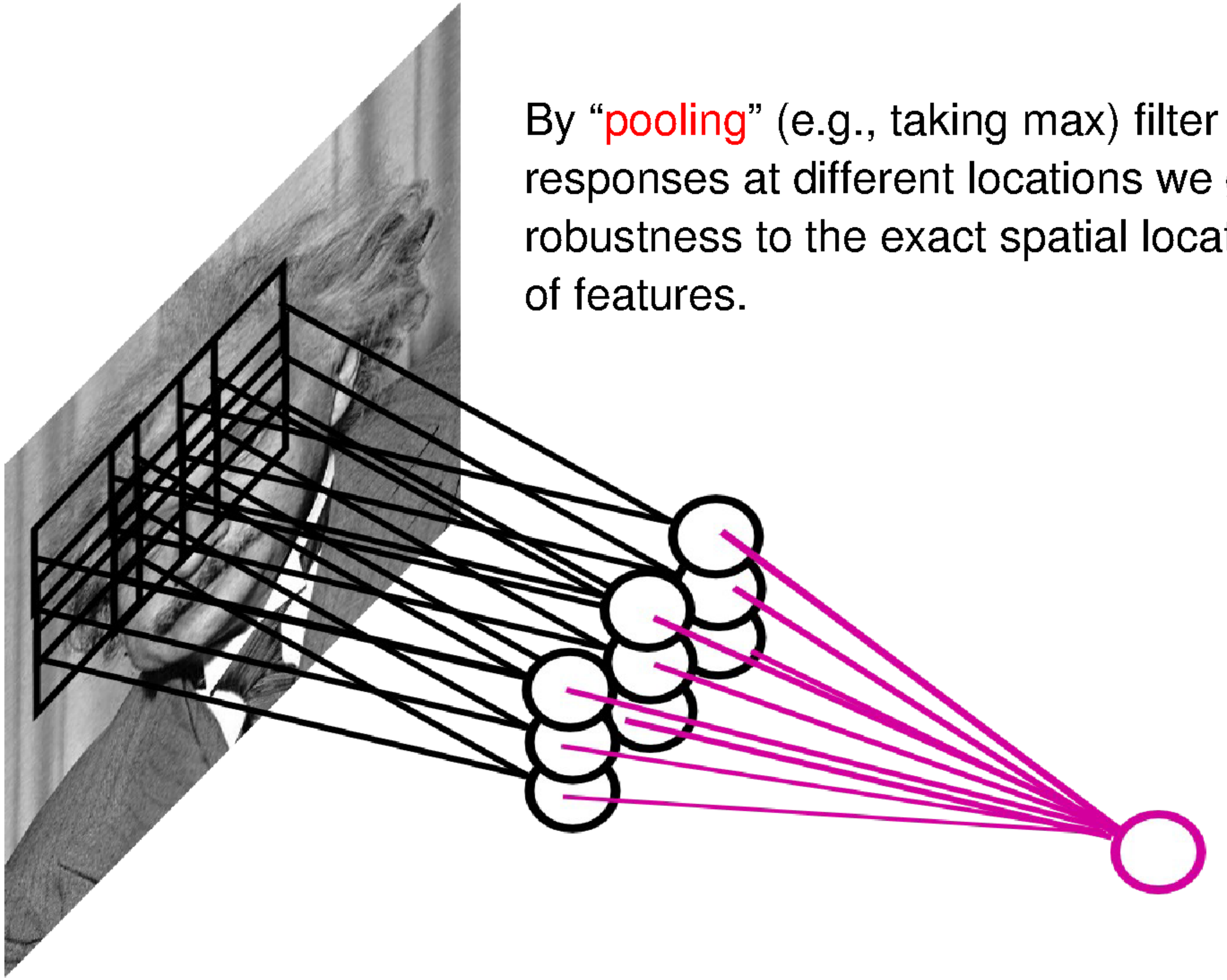


Let us assume filter is an "eye" detector.

Q.: how can we make the detection robust to the exact location of the eye?

Slides Credit: Deep Learning Tutorial by Marc'Aurelio Ranzato

Pooling

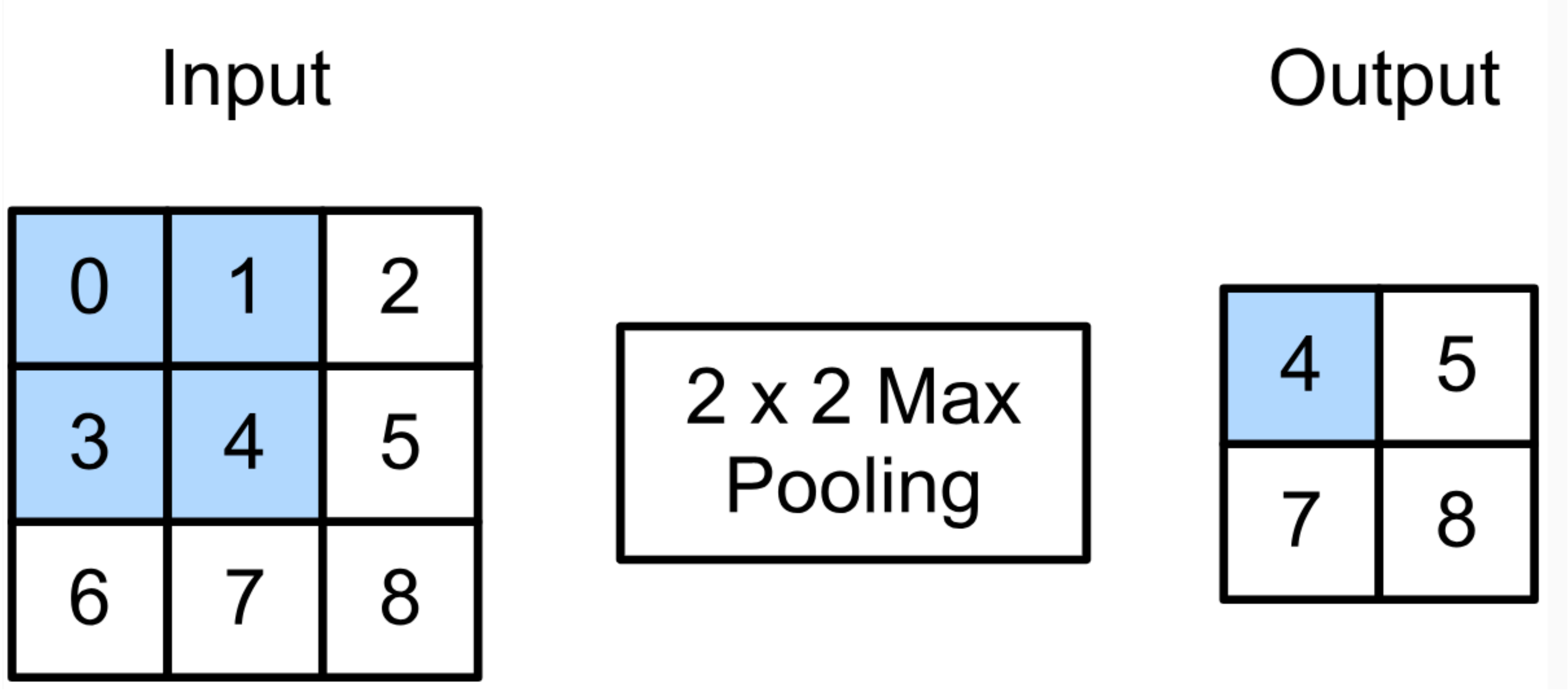


By "pooling" (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

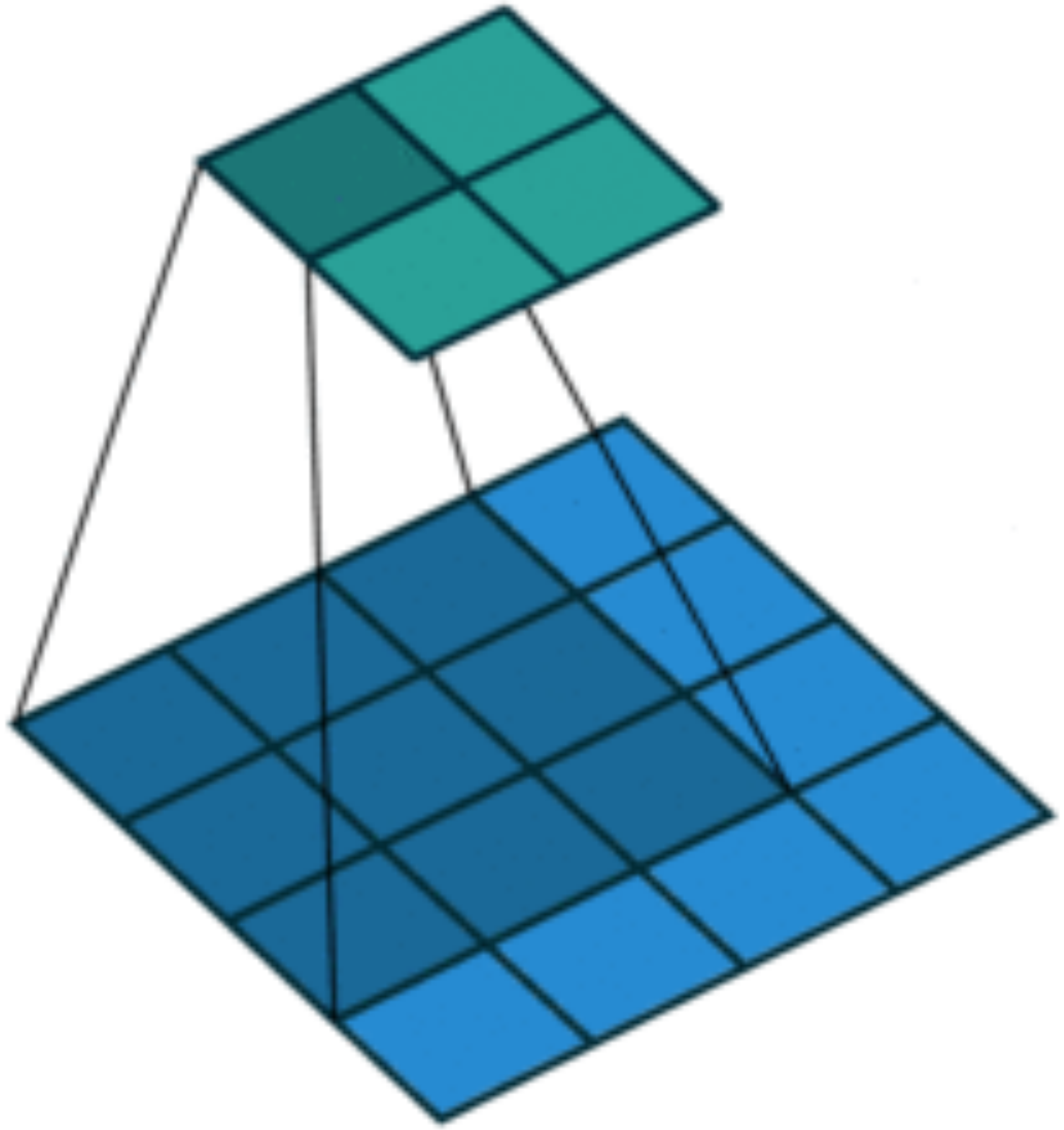
Slides Credit: Deep Learning Tutorial by Marc'Aurelio Ranzato

2-D Max Pooling

- Returns the maximal value in the sliding window



$$\max(0,1,3,4) = 4$$



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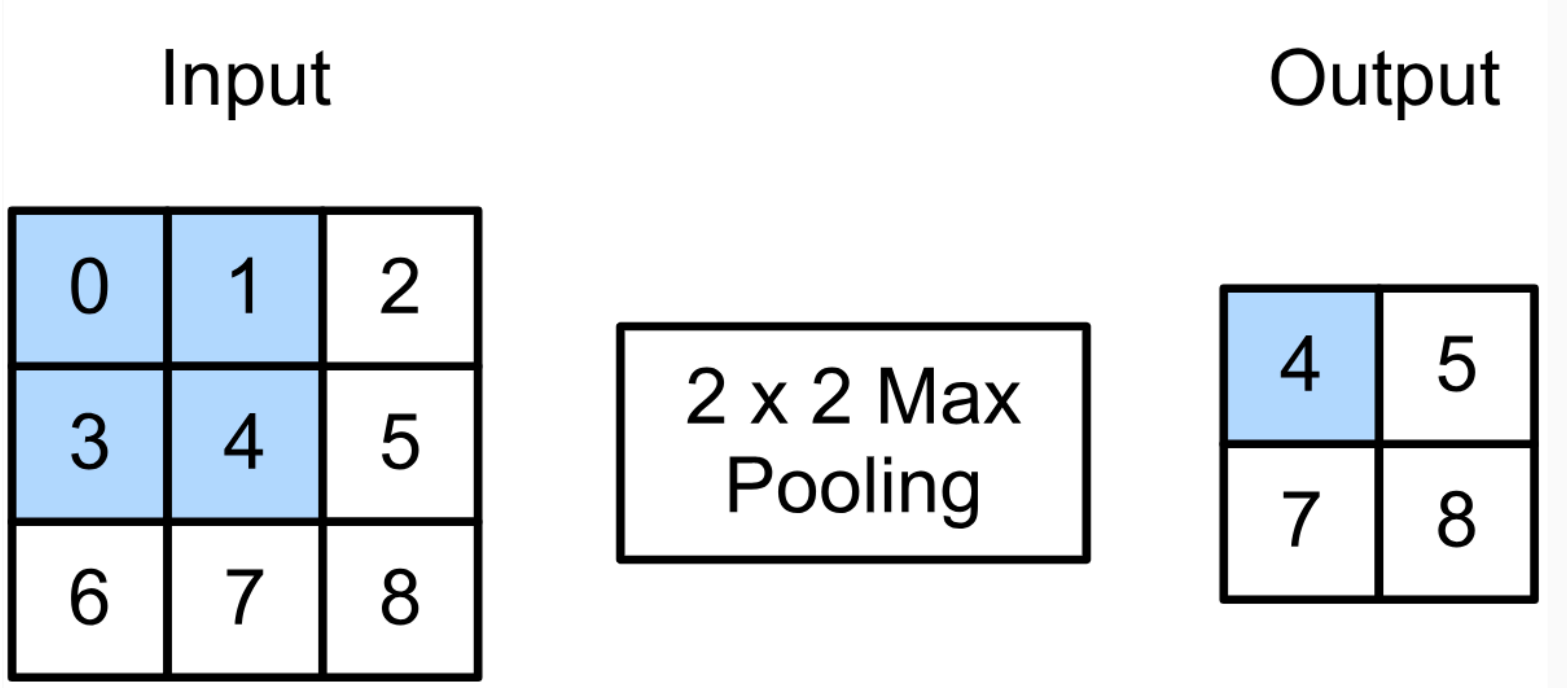

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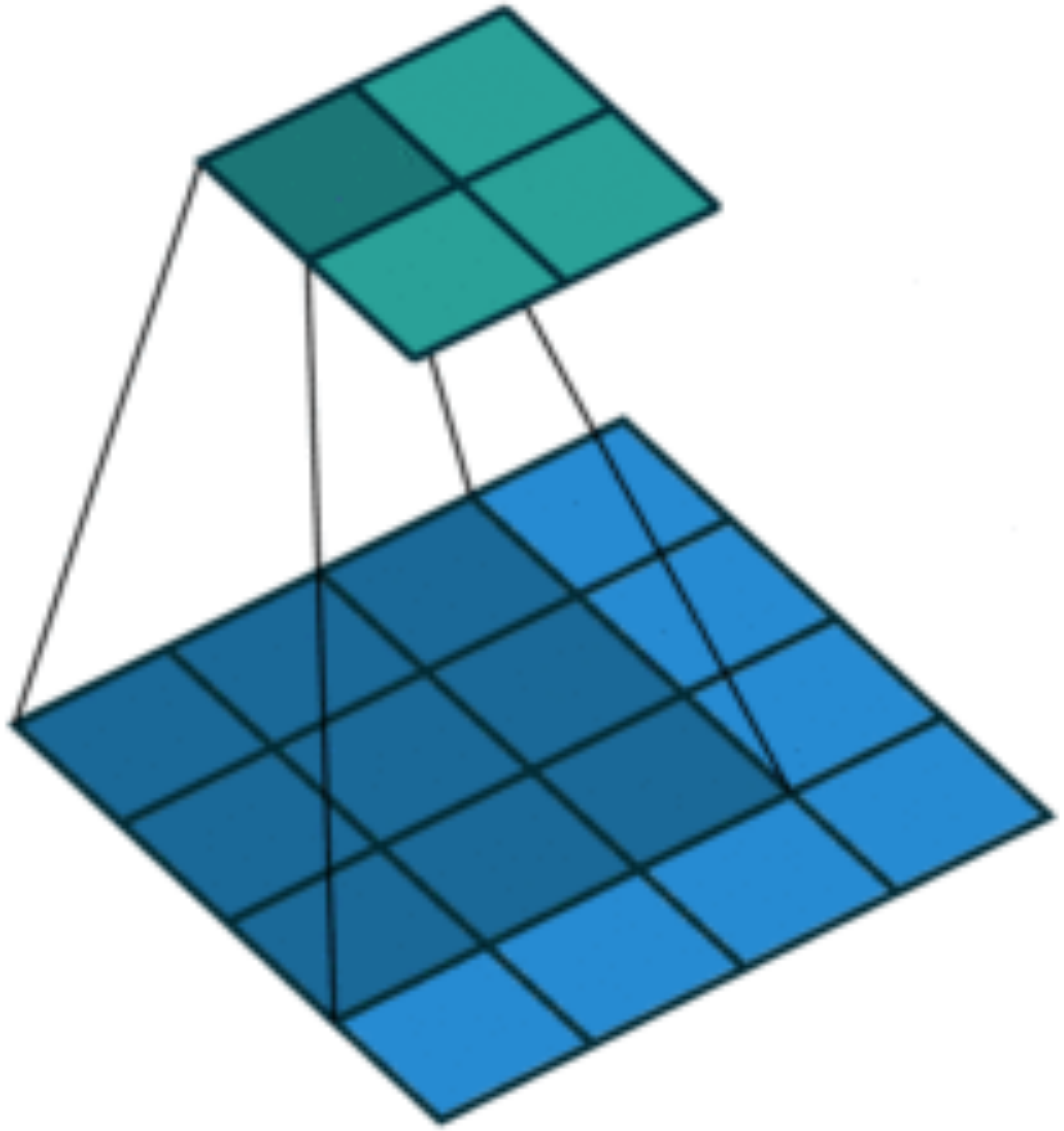
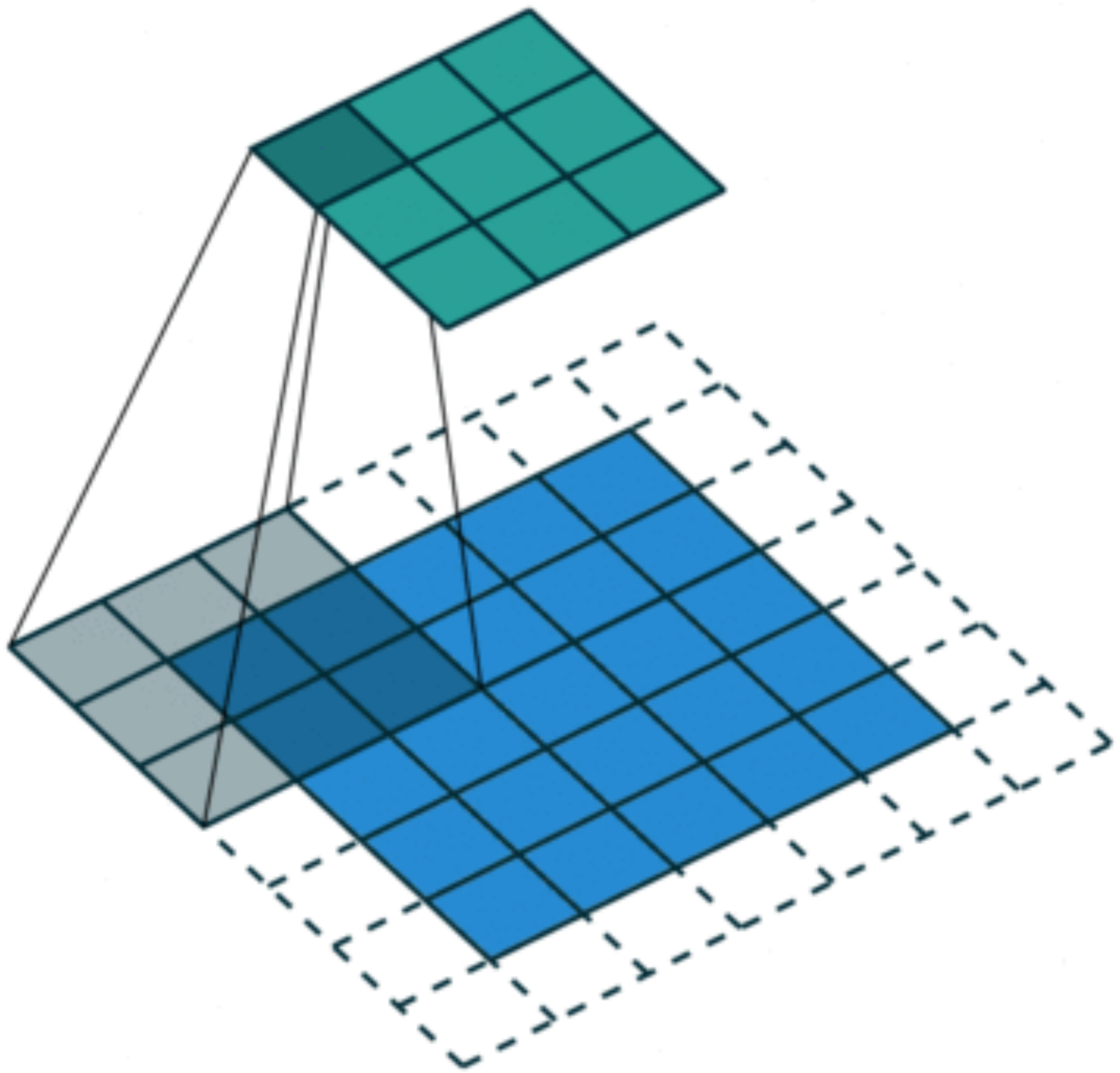


Padding, Stride, and Multiple Channels

- Pooling layers have similar padding and stride as convolutional layers
- No learnable parameters
- Apply pooling for each input channel to obtain the corresponding output channel



#output channels = #input channels

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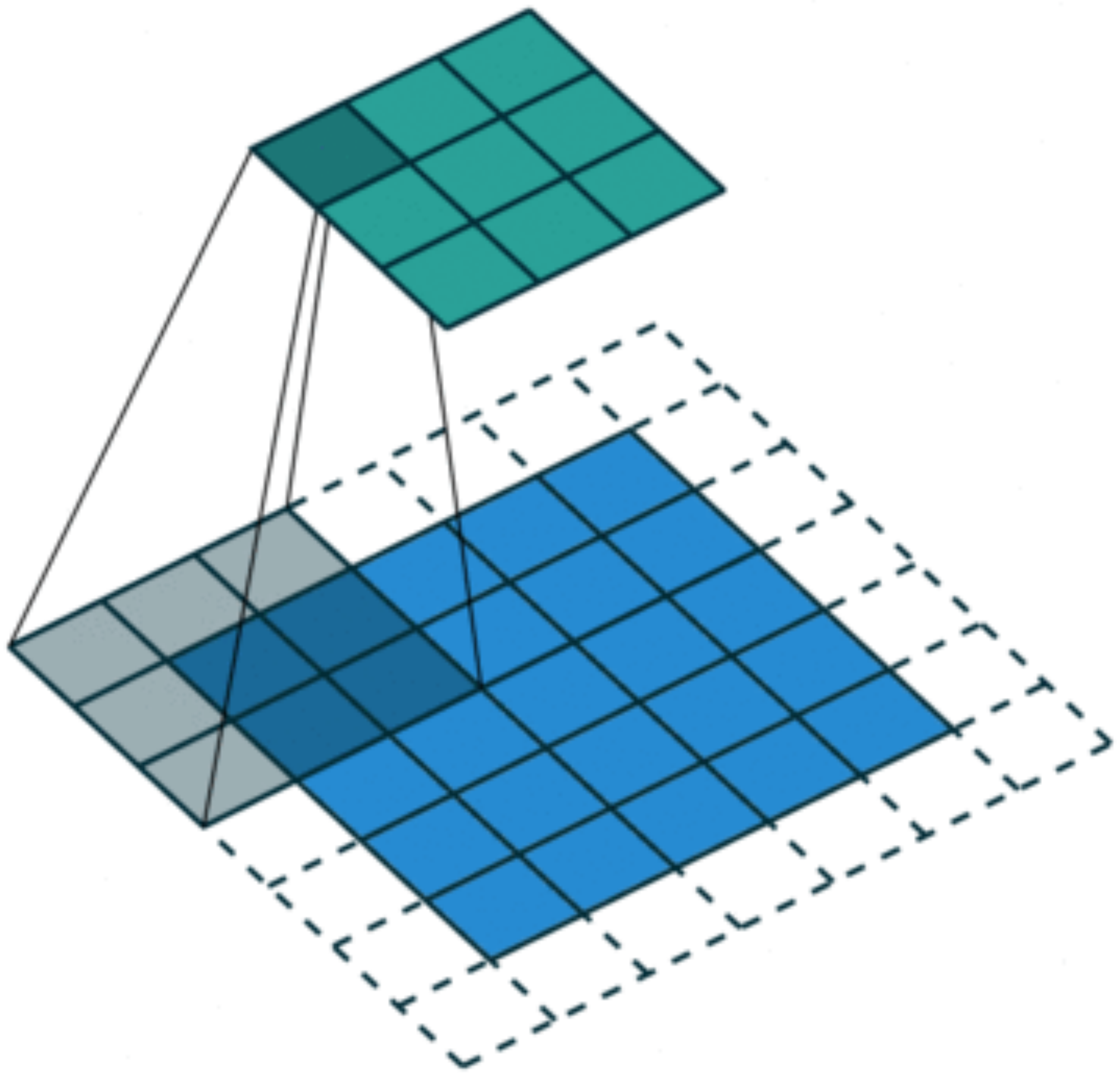
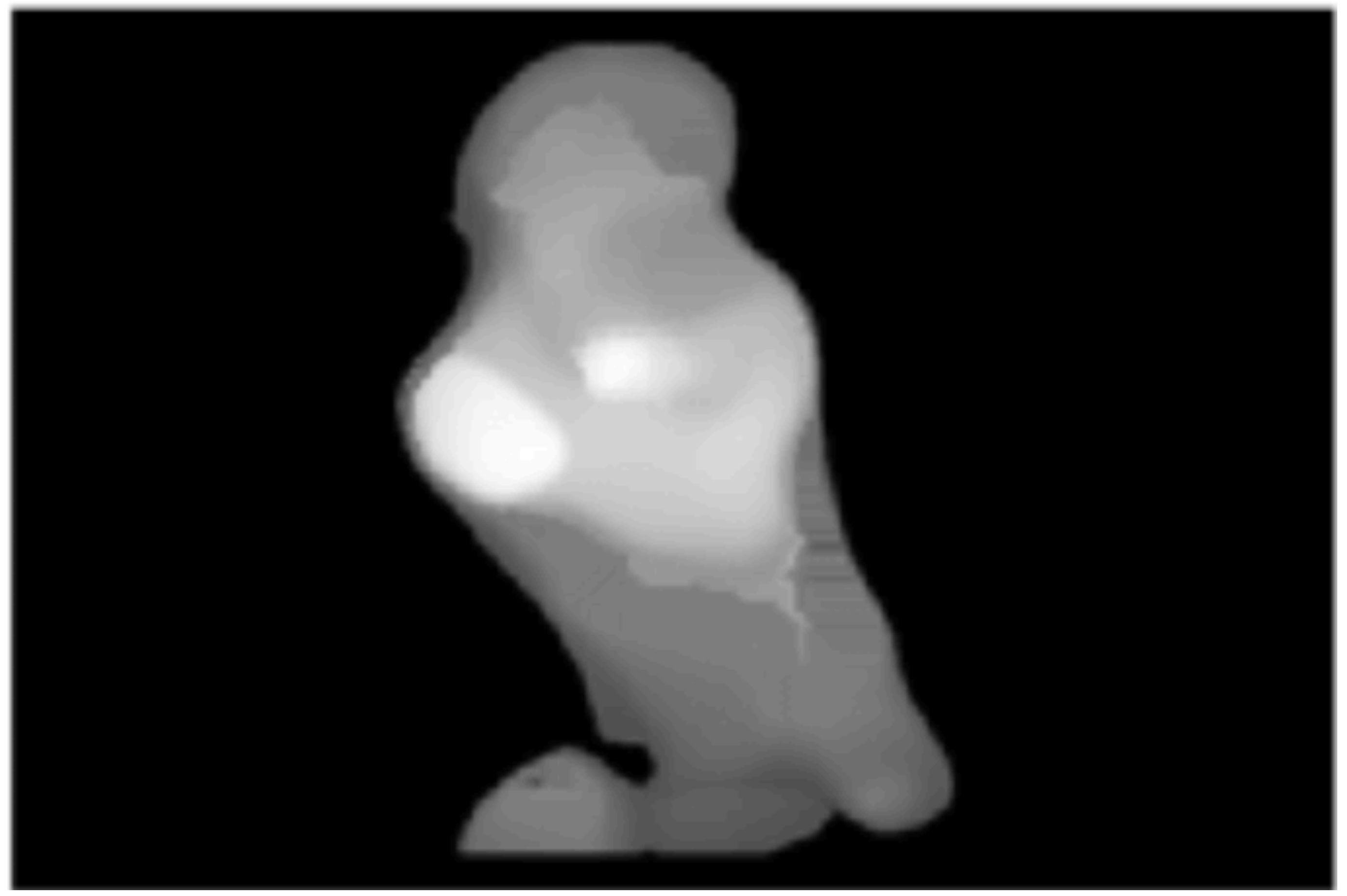


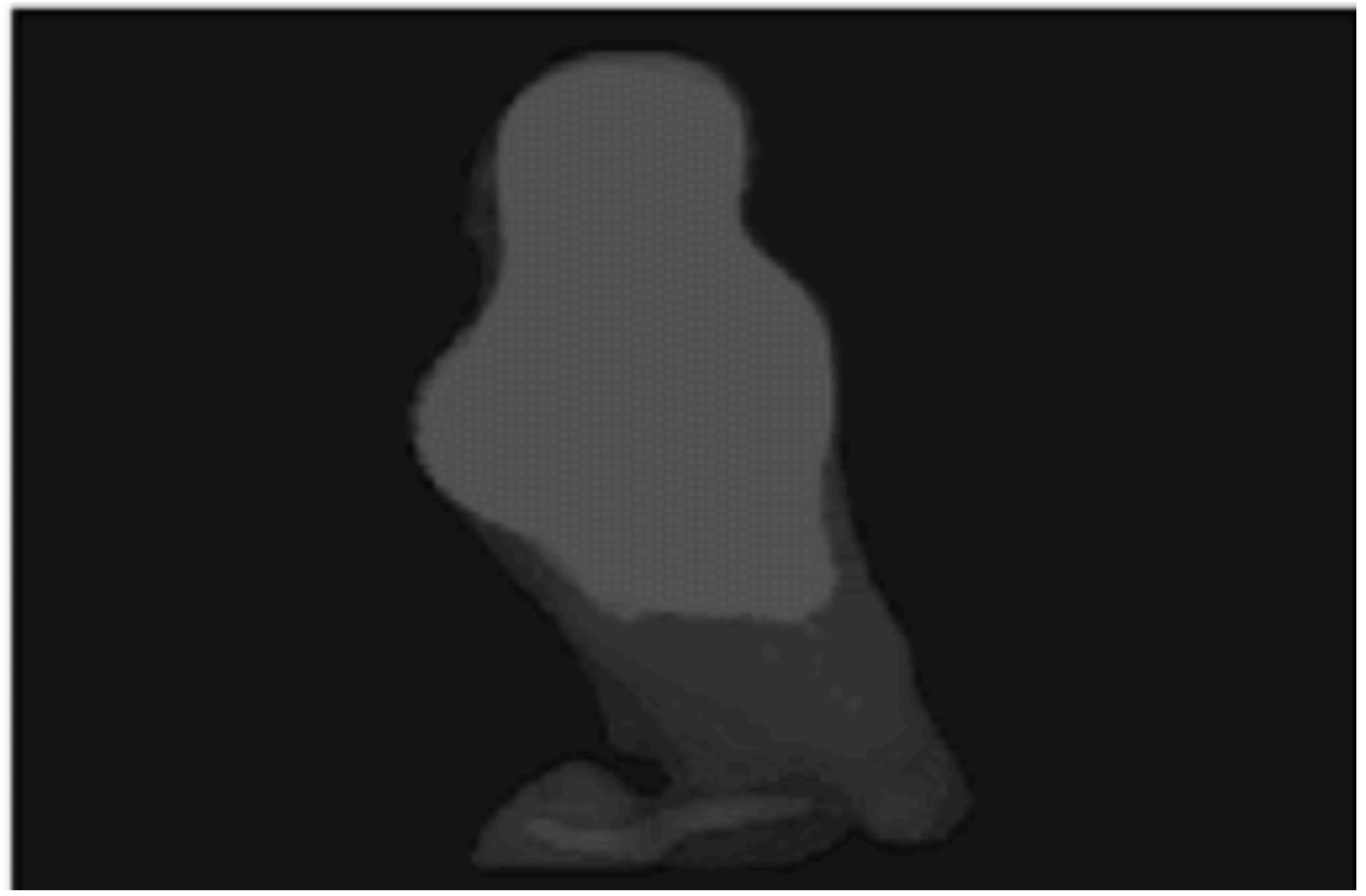
Average Pooling

- Max pooling: the strongest pattern signal in a window
- Average pooling: replace max with mean in max pooling
  - The average signal strength in a window

Max pooling



Average pooling



Q2-1. Suppose we want to perform 2x2 average pooling on the following single channel feature map of size 4x4 (no padding), and stride = 2. What is the output?

12	20	30	0
20	12	2	0
0	70	5	2
8	2	90	3

A.

20	30
70	90

B.

16	8
20	25

C.

20	30
20	25

D.

12	2
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Stride=2 w/ 2x2 input patch means output is mean of each quadrant.

Q2-2. What is the output if we replace average pooling with 2 x 2 max pooling (other settings are the same as previous question)?

12	20	30	0
20	12	2	0
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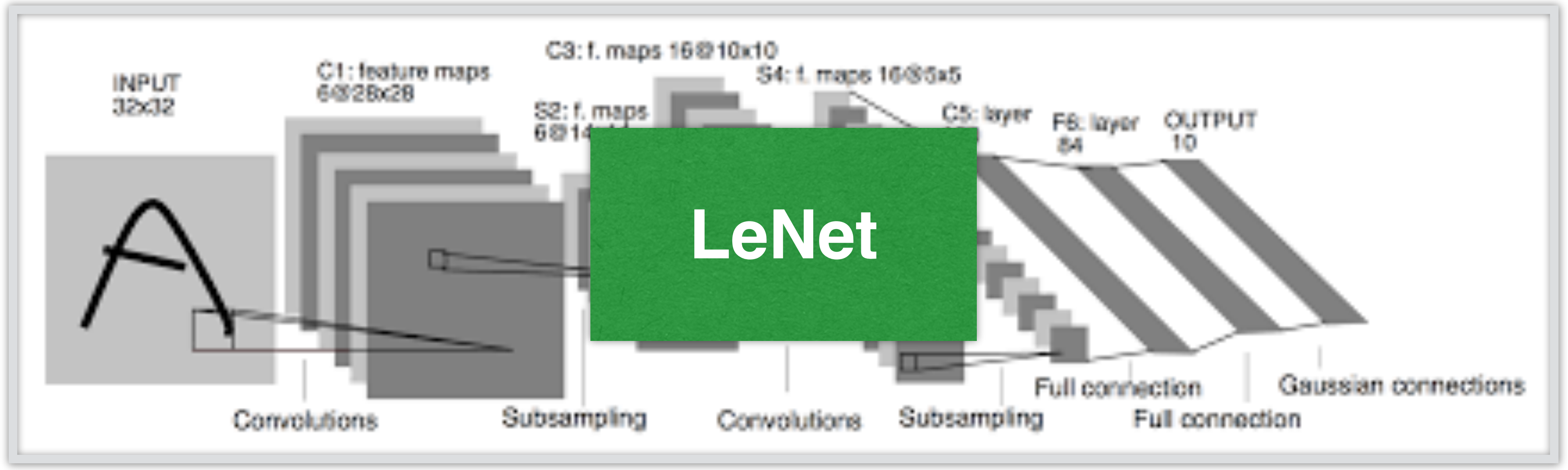
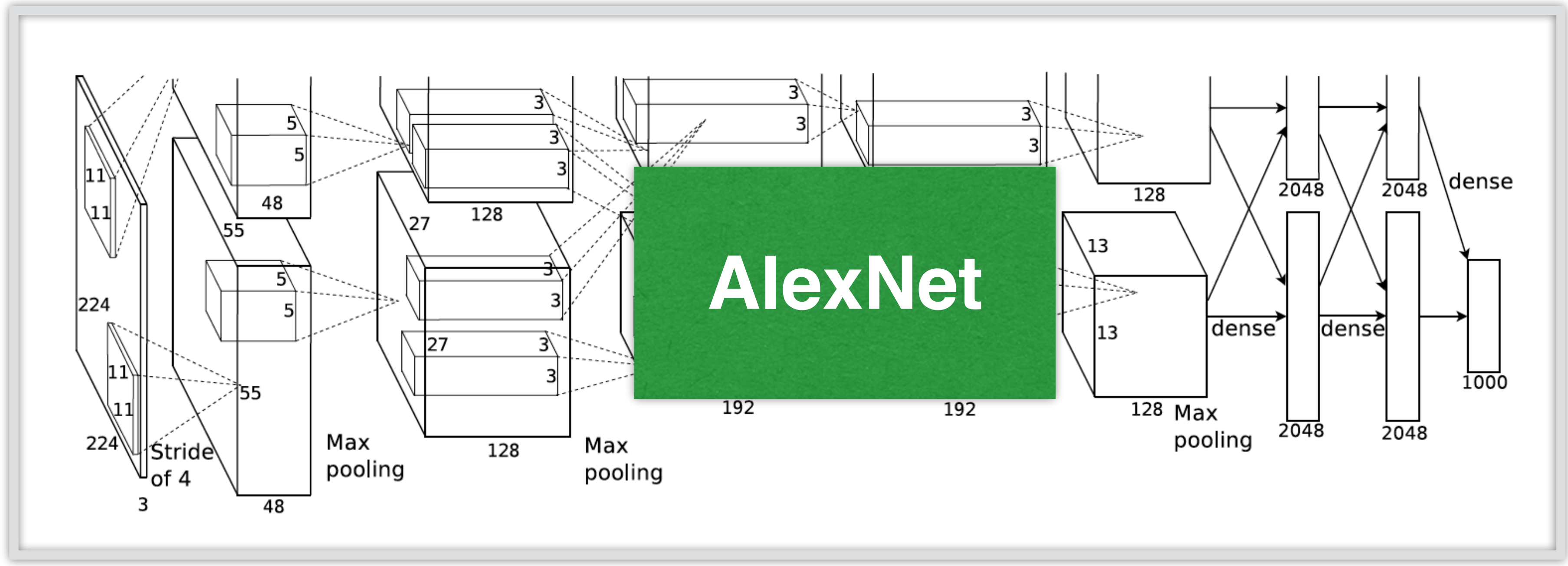
Stride=2 w/ 2x2 input patch means output is max of each quadrant.

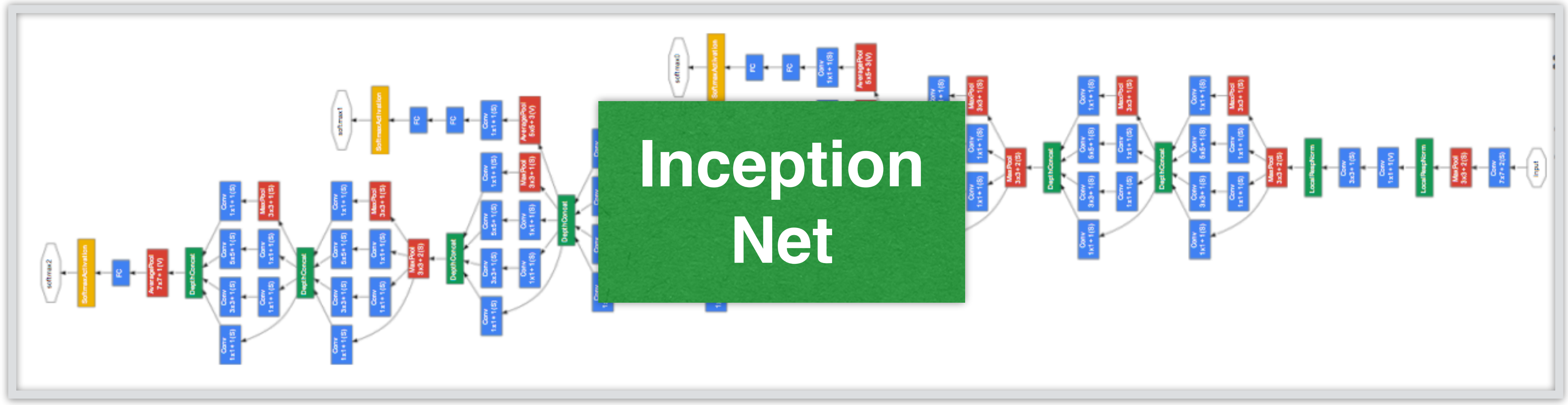
Convolutional Neural Networks

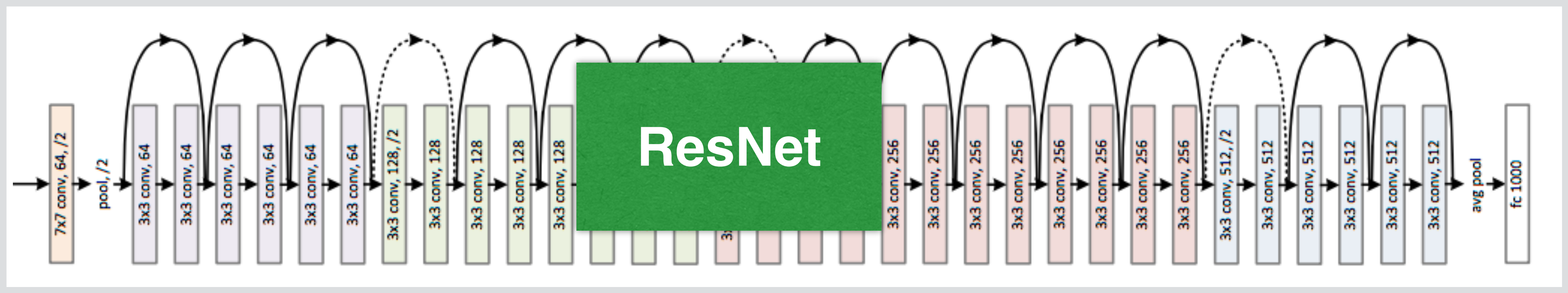
Evolution of neural net architectures

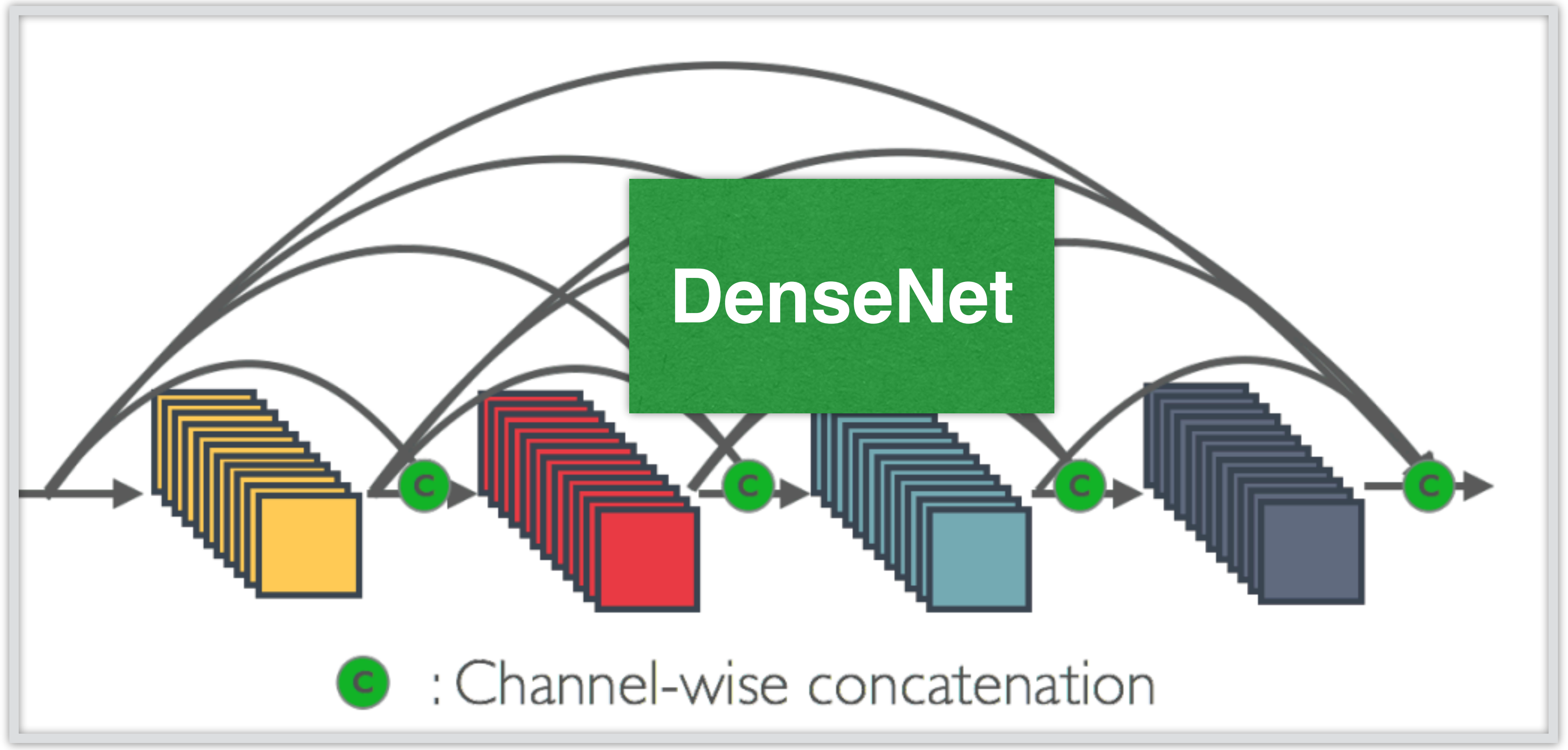
Evolution of neural net architectures





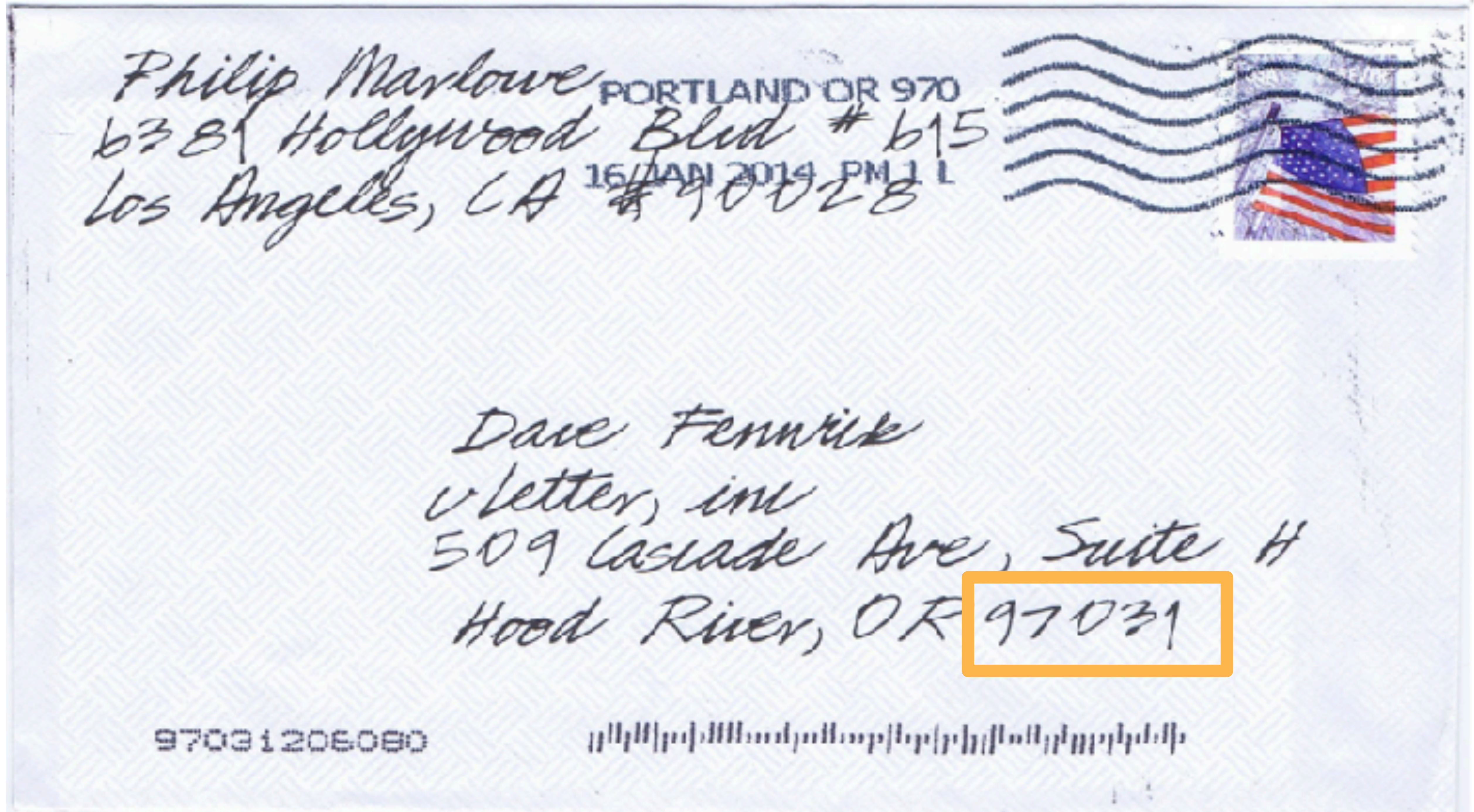


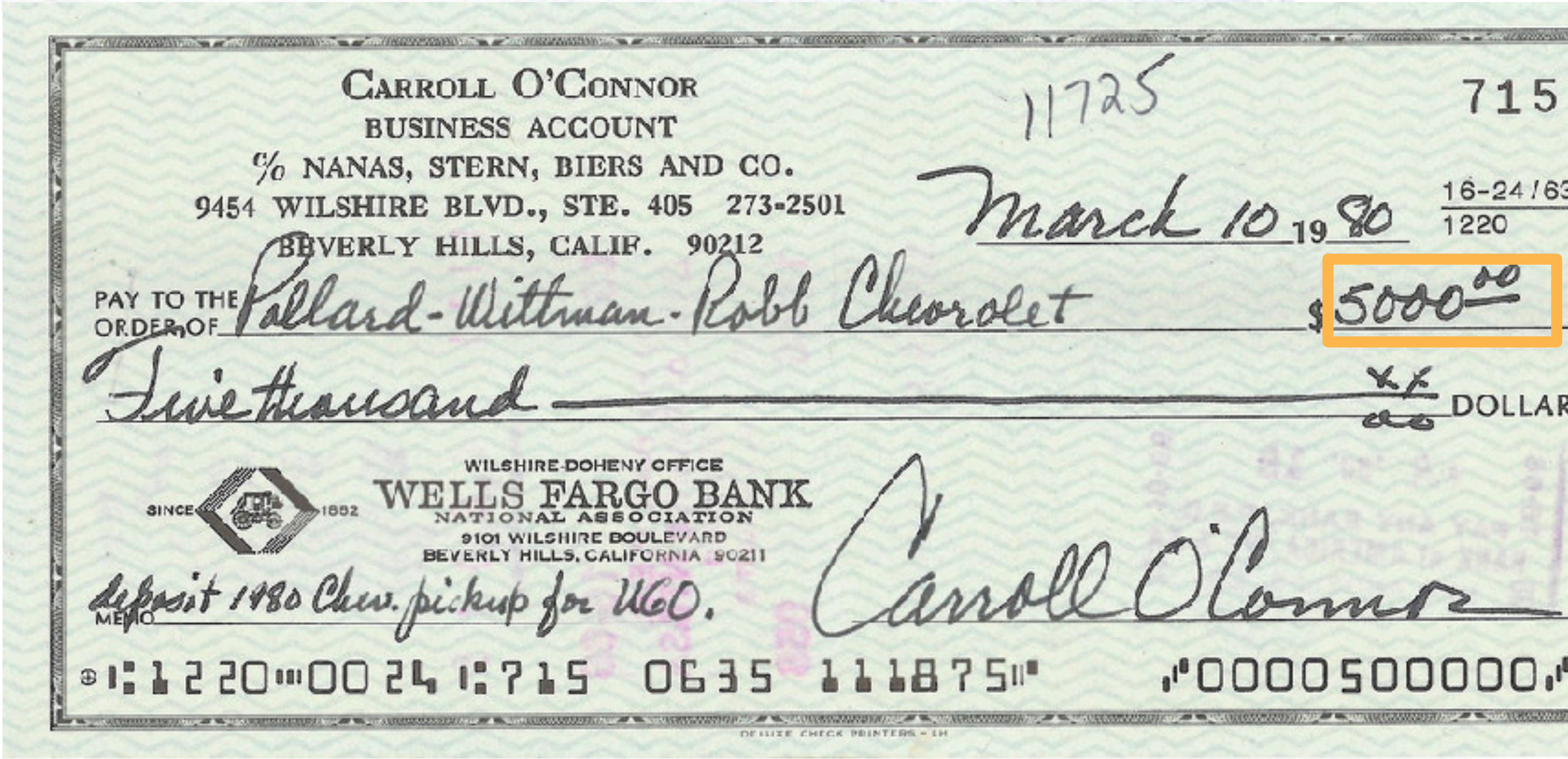




LeNet Architecture

Handwritten Digit Recognition

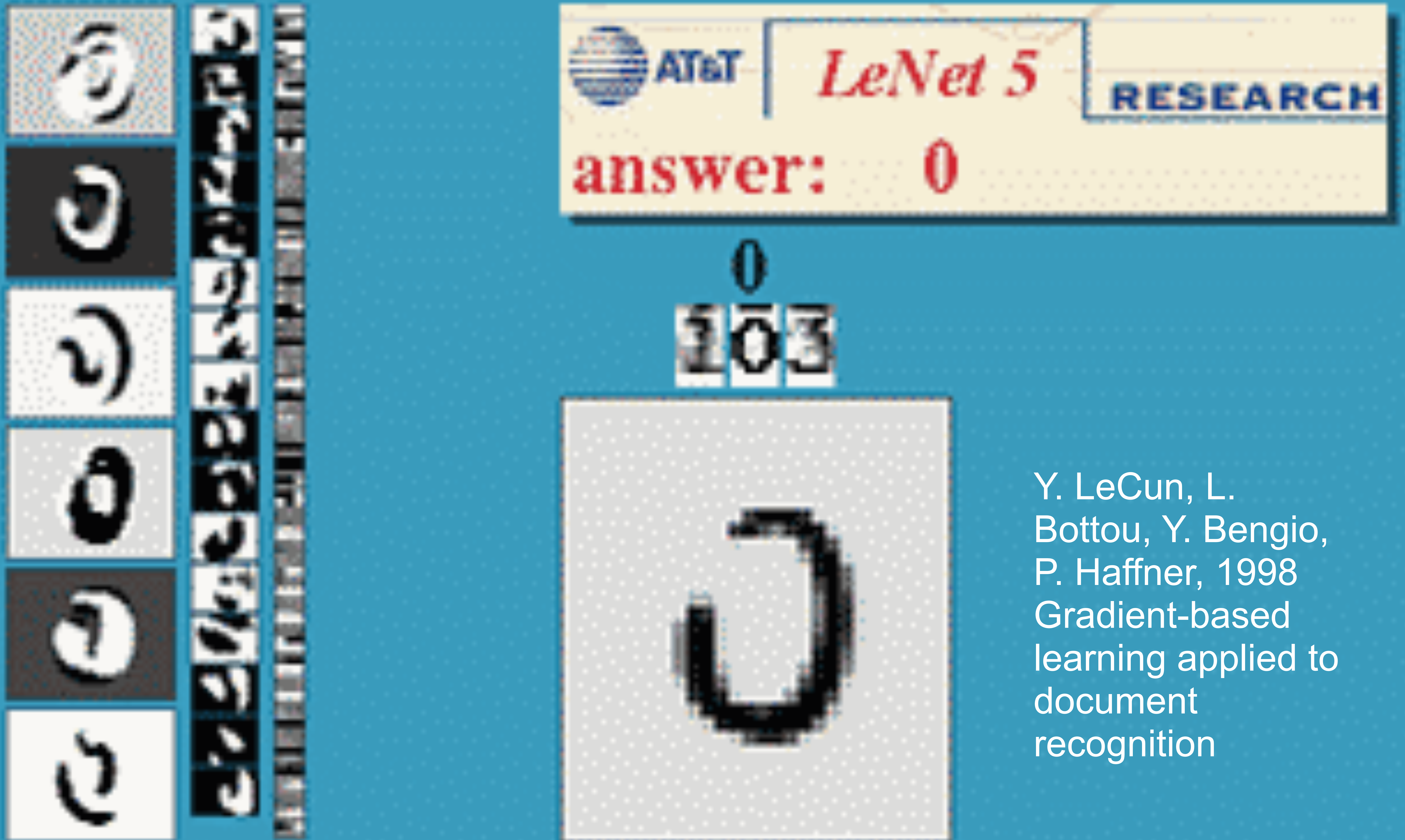




MNIST

- Centered and scaled
- 50,000 training data
- 10,000 test data
- 28 x 28 images
- 10 classes

Y. LeCun, L. Bottou, Y. Bengio, P. Haffner, 1998 Gradient-based learning applied to document recognition



Y. LeCun, L. Bottou, Y. Bengio, P. Haffner, 1998 Gradient-based learning applied to document recognition

LeNet Architecture

LeNet in Pytorch

```
def __init__(self):
    super(LeNet5, self).__init__()
    # Convolution (In LeNet-5, 32x32 images are given as input. Hence padding of 2 is done below)
    self.conv1 = torch.nn.Conv2d(in_channels=1, out_channels=6, kernel_size=5, stride=1, padding=2, bias=True)
    # Max-pooling
    self.max_pool_1 = torch.nn.MaxPool2d(kernel_size=2)
    # Convolution
    self.conv2 = torch.nn.Conv2d(in_channels=6, out_channels=16, kernel_size=5, stride=1, padding=0, bias=True)
    # Max-pooling
    self.max_pool_2 = torch.nn.MaxPool2d(kernel_size=2)
    # Fully connected layer
    self.fc1 = torch.nn.Linear(16*5*5, 120)   # convert matrix with 16*5*5 (= 400) features to a matrix of 120 features (col
    self.fc2 = torch.nn.Linear(120, 84)       # convert matrix with 120 features to a matrix of 84 features (columns)
    self.fc3 = torch.nn.Linear(84, 10)        # convert matrix with 84 features to a matrix of 10 features (columns)
```

Summary

Summary

- Intro of convolutional computations

Summary

- Intro of convolutional computations
  - 2D convolution

Summary

- Intro of convolutional computations
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  - Padding, stride etc

Summary

- Intro of convolutional computations
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    - Multiple input and output channels

Summary

- Intro of convolutional computations
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  - Pooling

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- Intro of convolutional computations
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- Basic Convolutional Neural Networks

Summary

- Intro of convolutional computations
    - 2D convolution
    - Padding, stride etc
    - Multiple input and output channels
    - Pooling
- Basic Convolutional Neural Networks
    - LeNet (first conv nets)

Acknowledgement:

Some of the slides in these lectures have been adapted from materials developed by Alex Smola and Mu Li:

https://courses.d2l.ai/berkeley-stat-157/index.html