

CS540 Introduction to Artificial Intelligence (Deep) Neural Networks Summary

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University of Wisconsin-Madison

November 9, 2021

Slides created by Sharon Li [modified by Josiah Hanna]

Announcements

- 1. Midterm released; let me know by Thursday if you have questions on your grade.
- 2. Homeworks 1-5 grades; regrades in progress.

Tuesday, Nov 9	Review and discussion on Neural Networks and Deep Learning		
Thursday, Nov 11	Search I: Un-Informed search		
Tuesday, Nov 16	Search II: Informed search		HW7 Due; HW8 Released
Thursday, Nov 18	Advanced Search and Review on Search		
Everything below here is tentative and subject to change.			
Tuesday, Nov 23	Games - Part I		HW8 Due; HW9 Released
Thursday, Nov 25	Happy Thanksgiving! (No class)		
Tuesday, Nov 30	Games - Part II		
Thursday, Dec 2	Reinforcement Learning I		HW9 Due; HW10 Released
Tuesday, Dec 7	Reinforcement Learning II		
Thursday, Dec 9	Review on Games and Reinforcement Learning		
Tuesday, Dec 14	Ethics and Trust in AI		HW10 Due

How to classify





How to classify Cats vs. dogs?

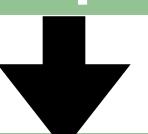




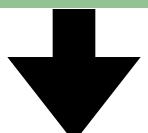
Single-layer Perceptron



Multi-layer Perceptron



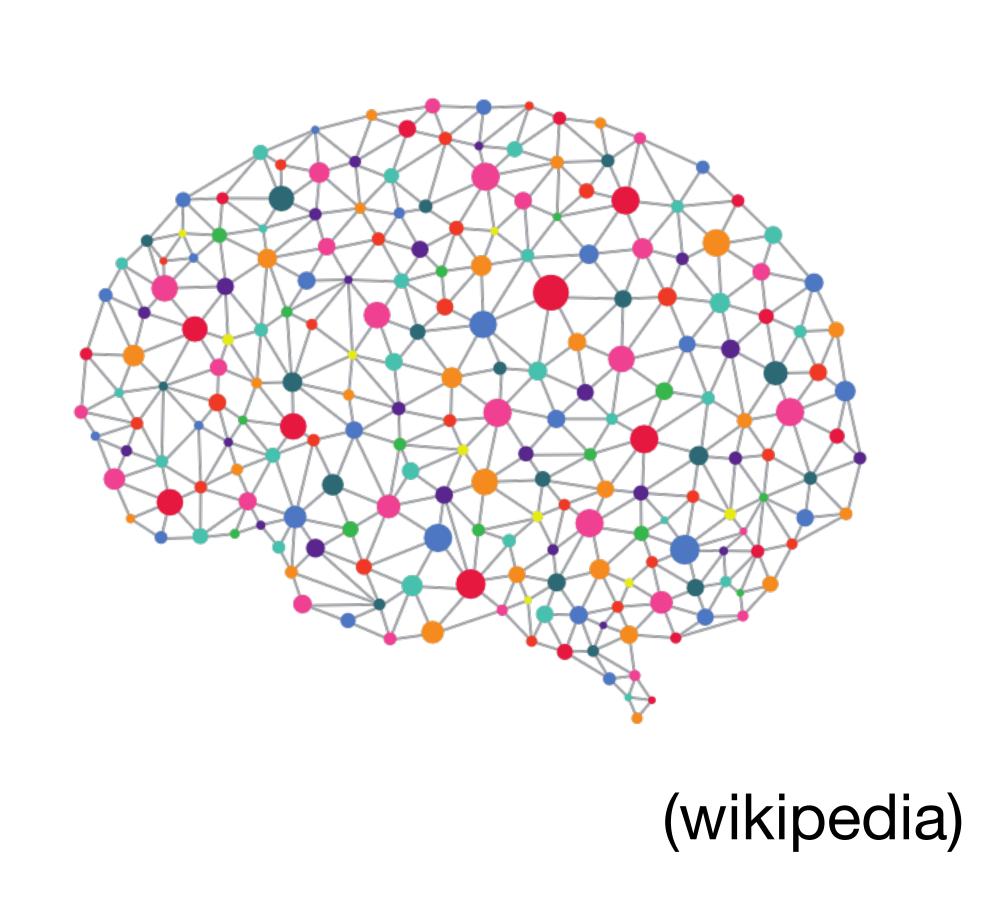
Training of neural networks

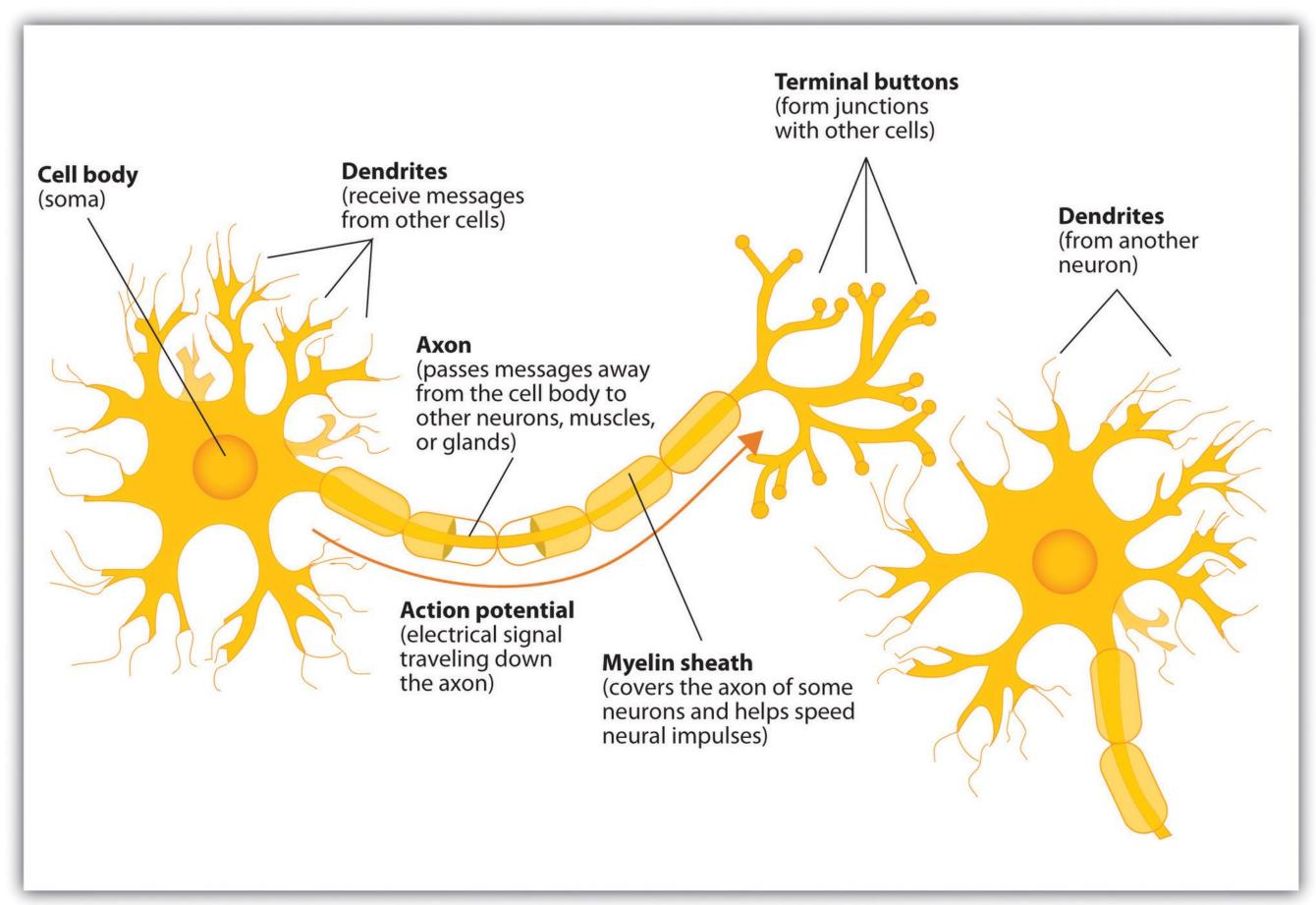


Convolutional neural networks

Inspiration from neuroscience

- Inspirations from human brains
- Networks of simple and homogenous units (a.k.a neuron)



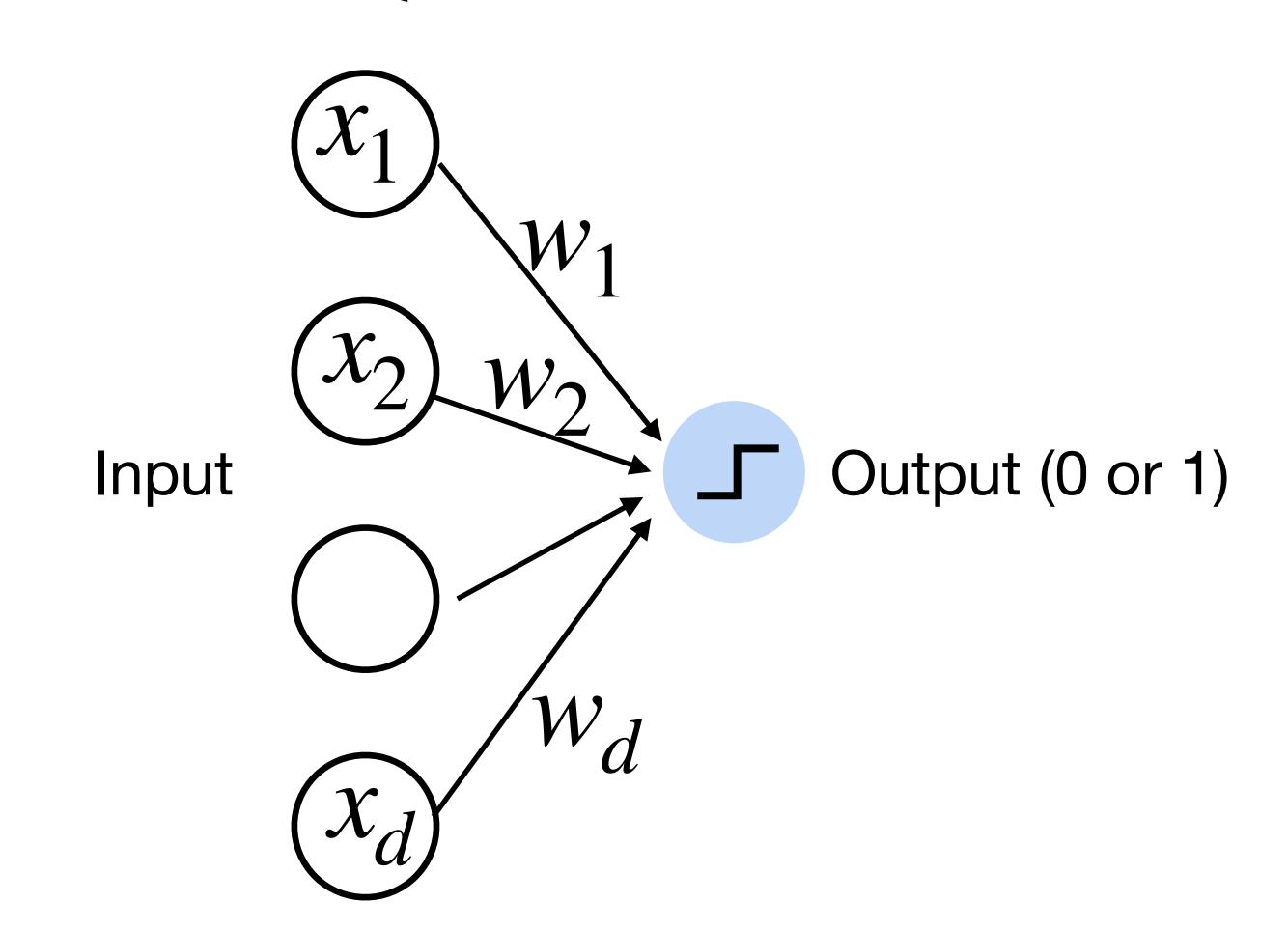


Perceptron

Given input x, weight w and bias b, perceptron outputs:

$$o = \sigma \left(\mathbf{w}^{\mathsf{T}} \mathbf{x} + b \right) \qquad \sigma(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$





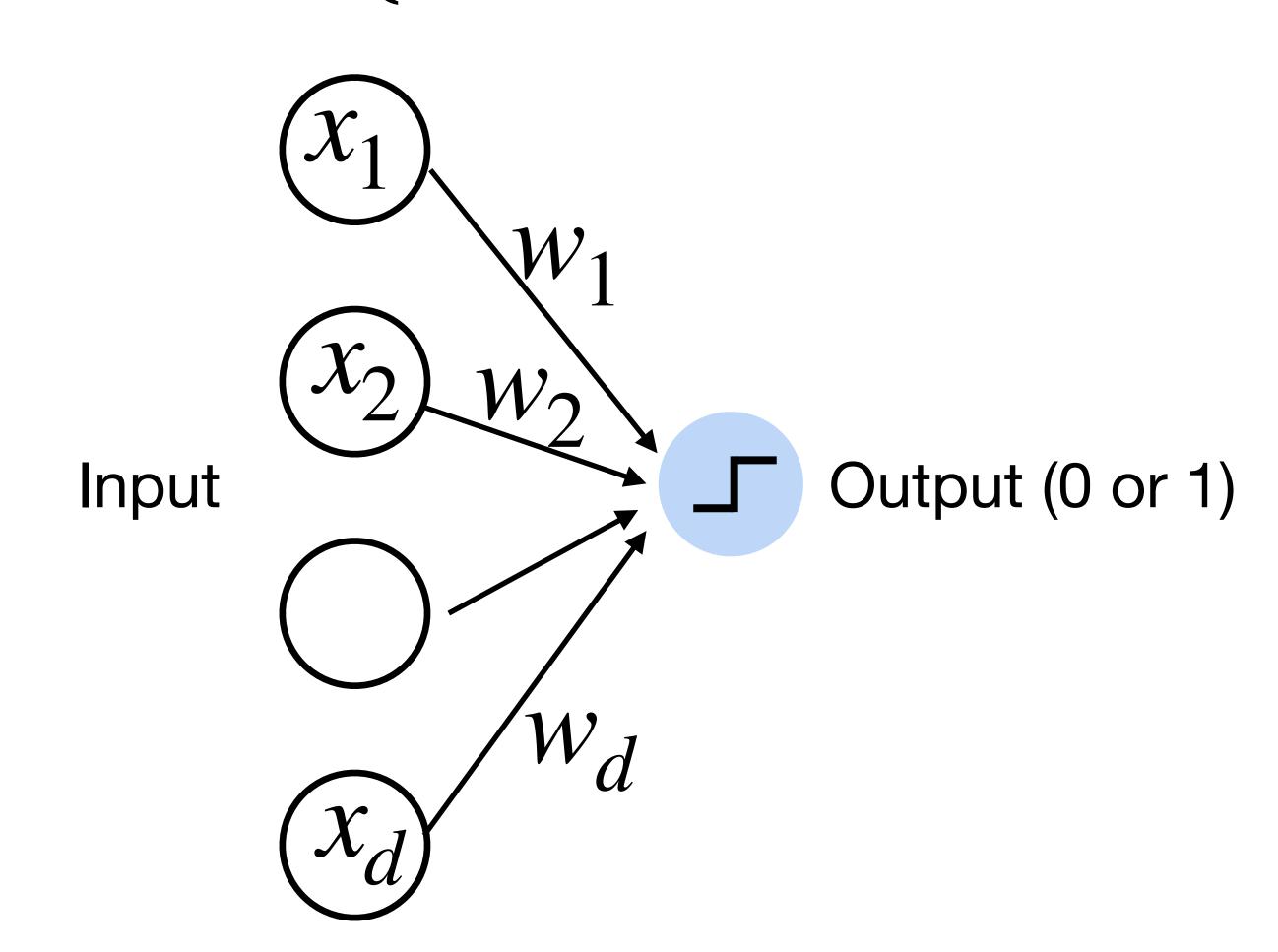
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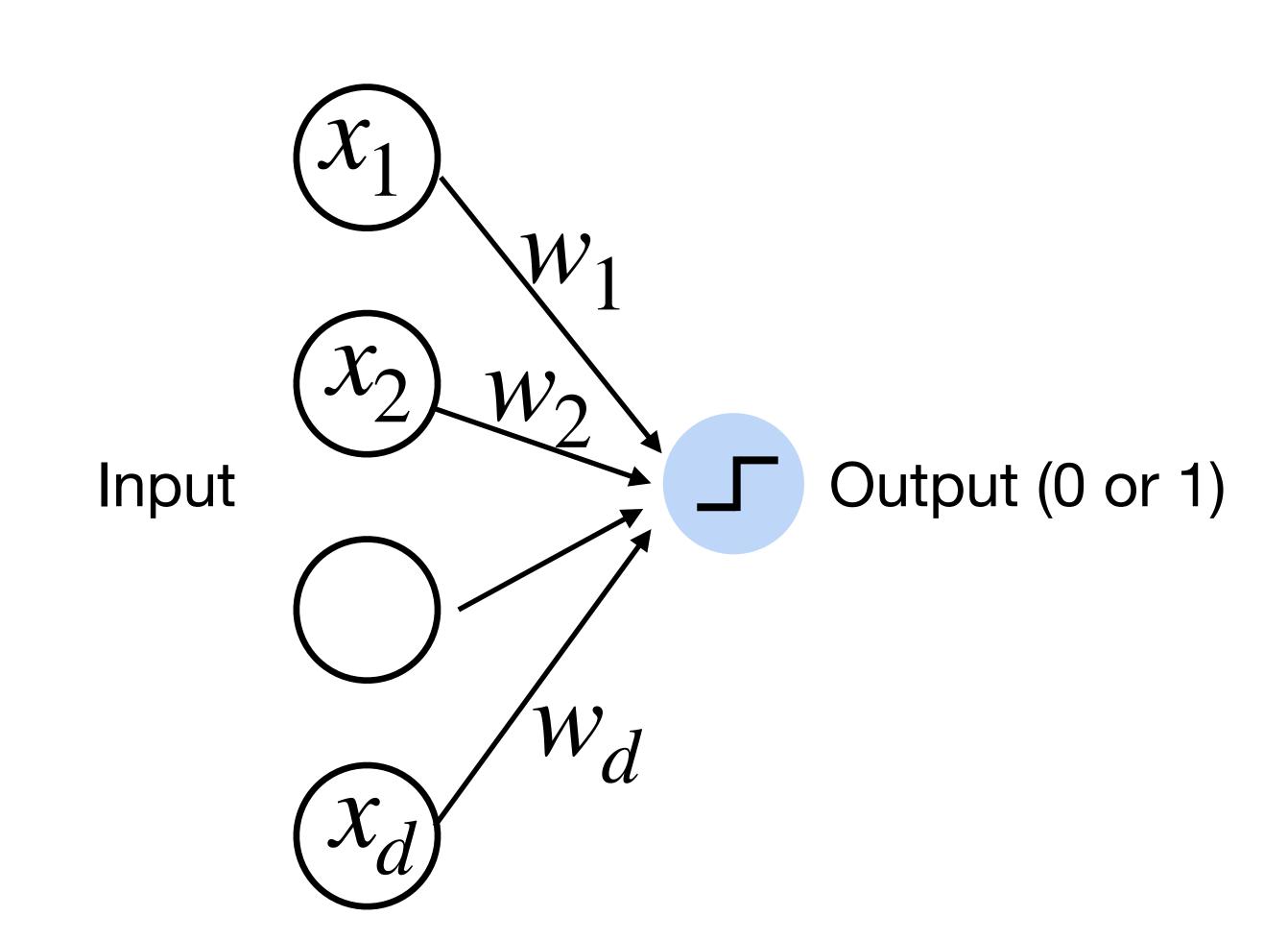




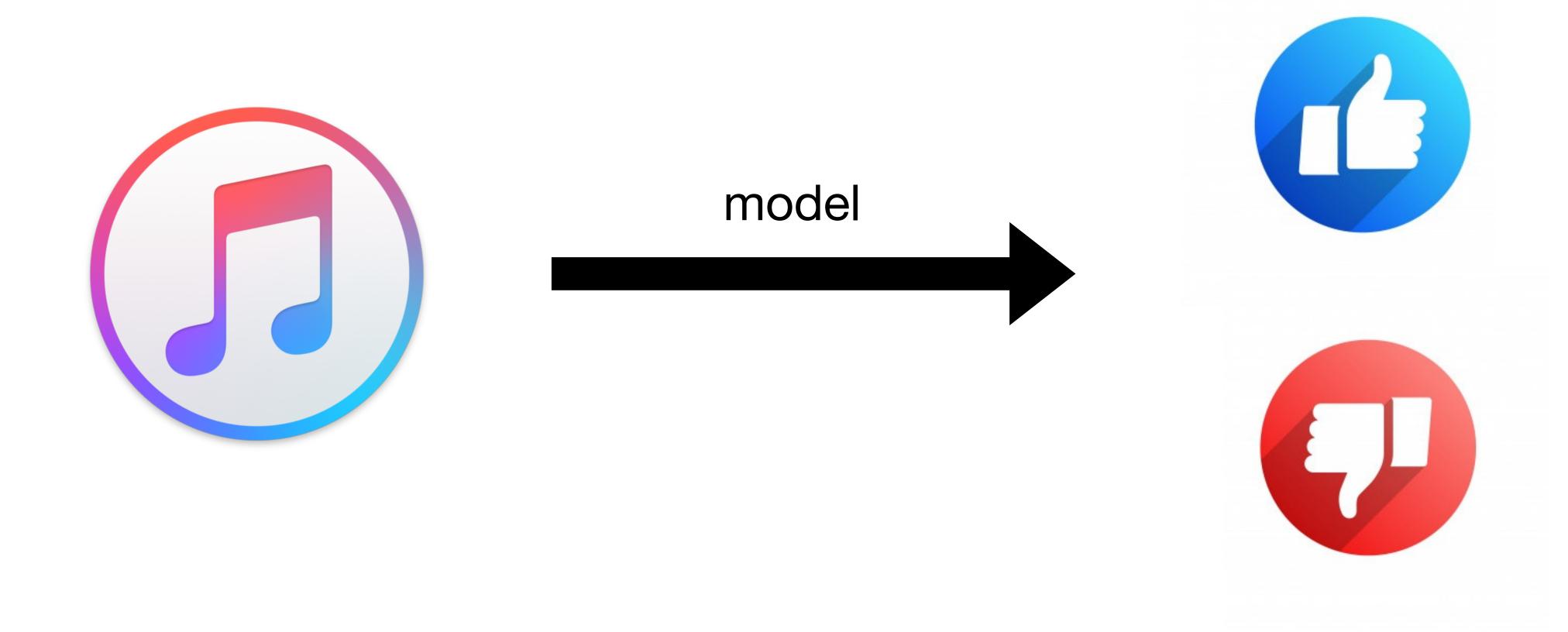
Perceptron

• Goal: learn parameters $\mathbf{w} = \{w_1, w_2, \dots, w_d\}$ and b to minimize the classification error

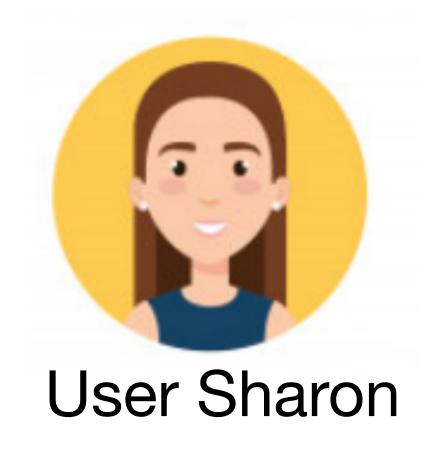




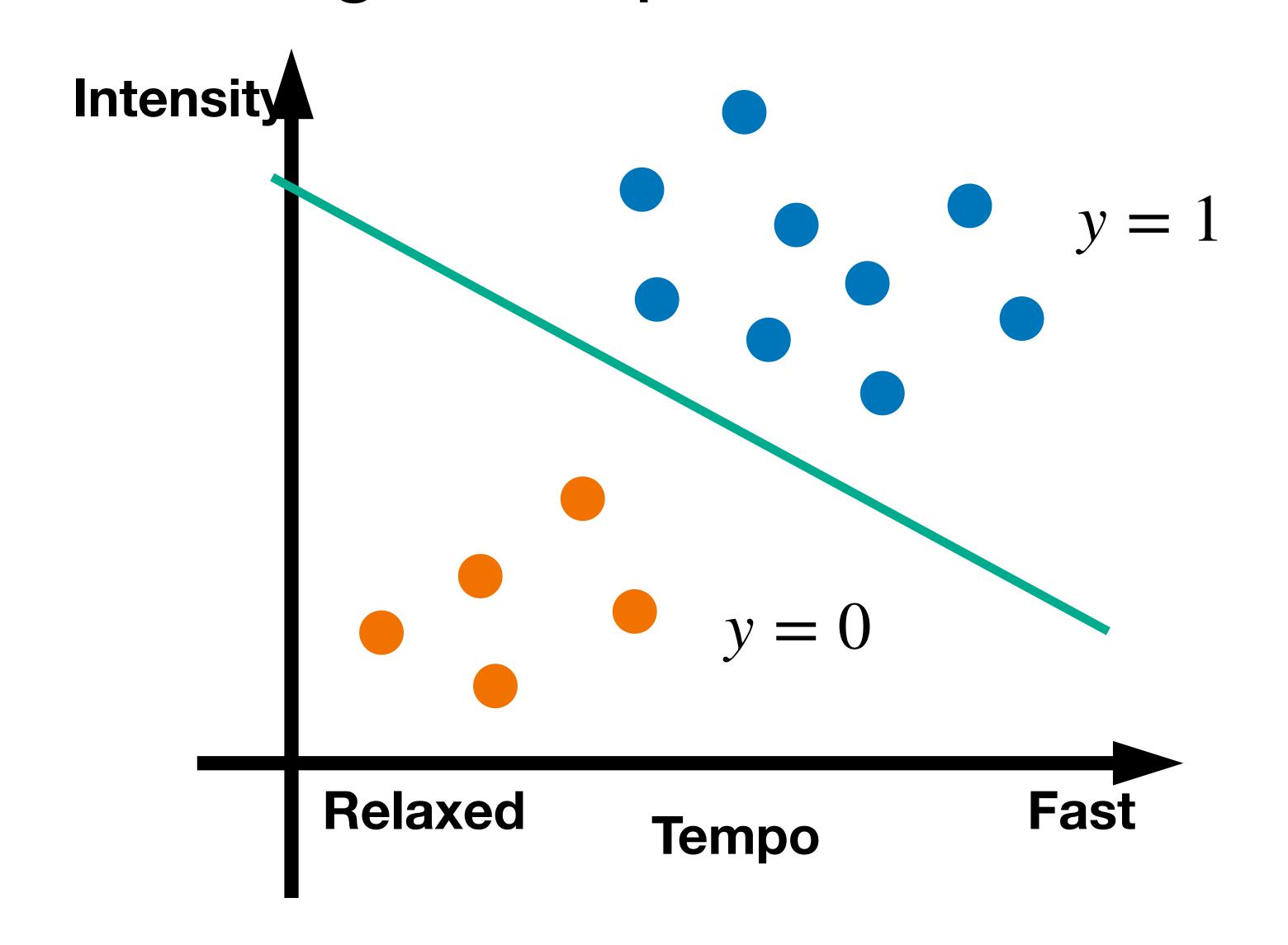
Example: Predict whether a user likes a song or not



Example: Predict whether a user likes a song or not Using Perceptron

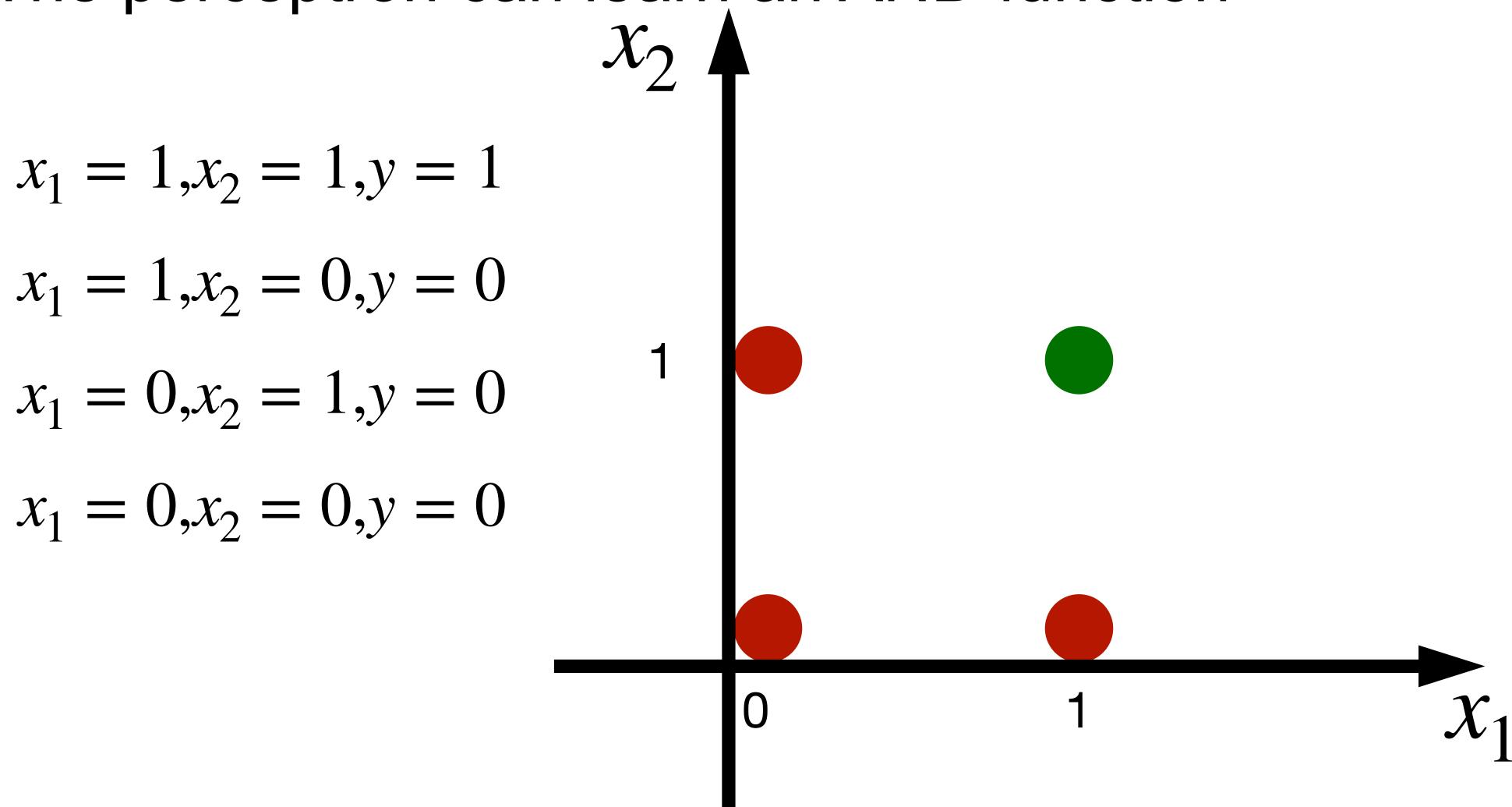


- DisLike
- Like



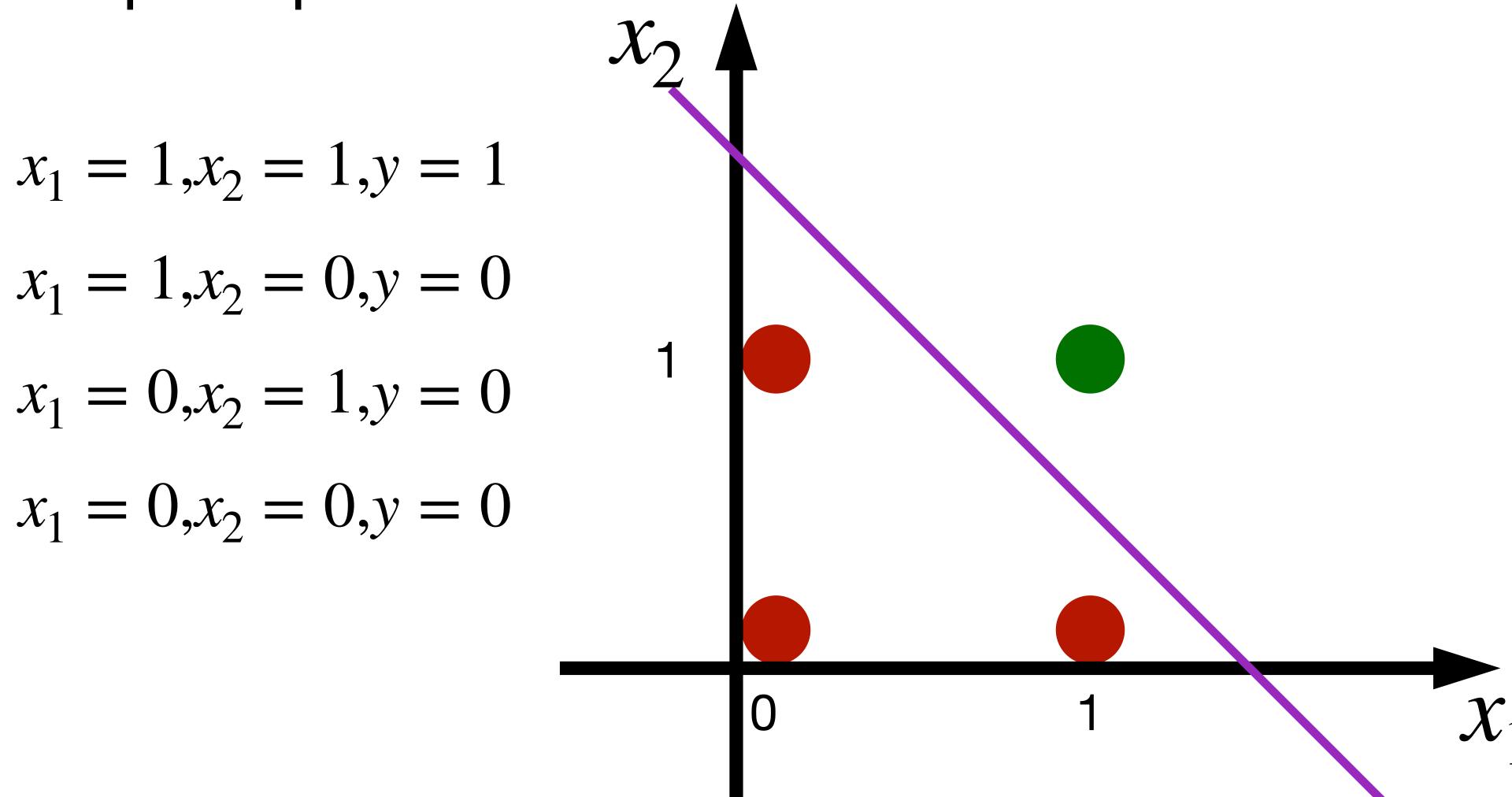
Learning logic functions using perceptron

The perceptron can learn an AND function



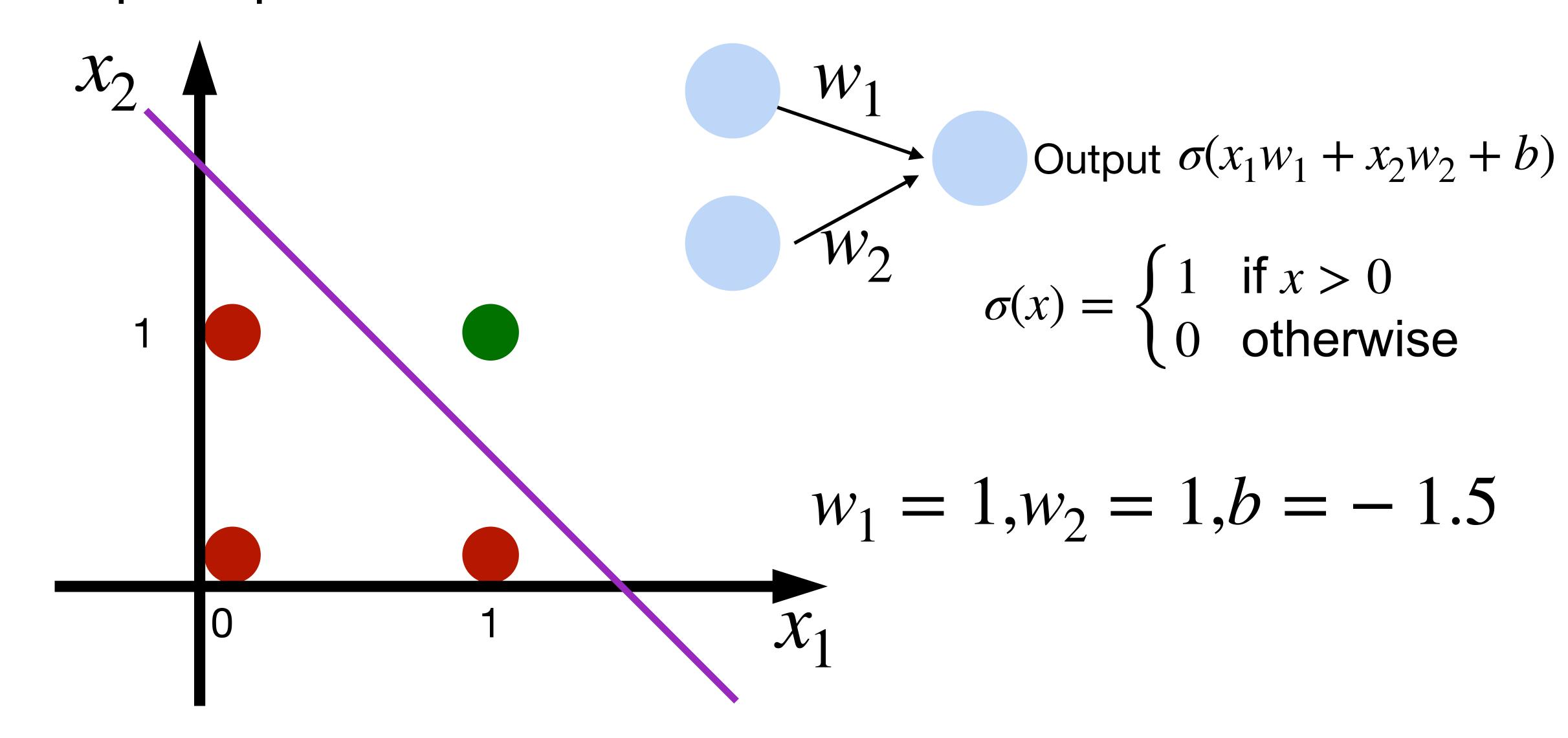
Learning logic functions using perceptron

The perceptron can learn an AND function



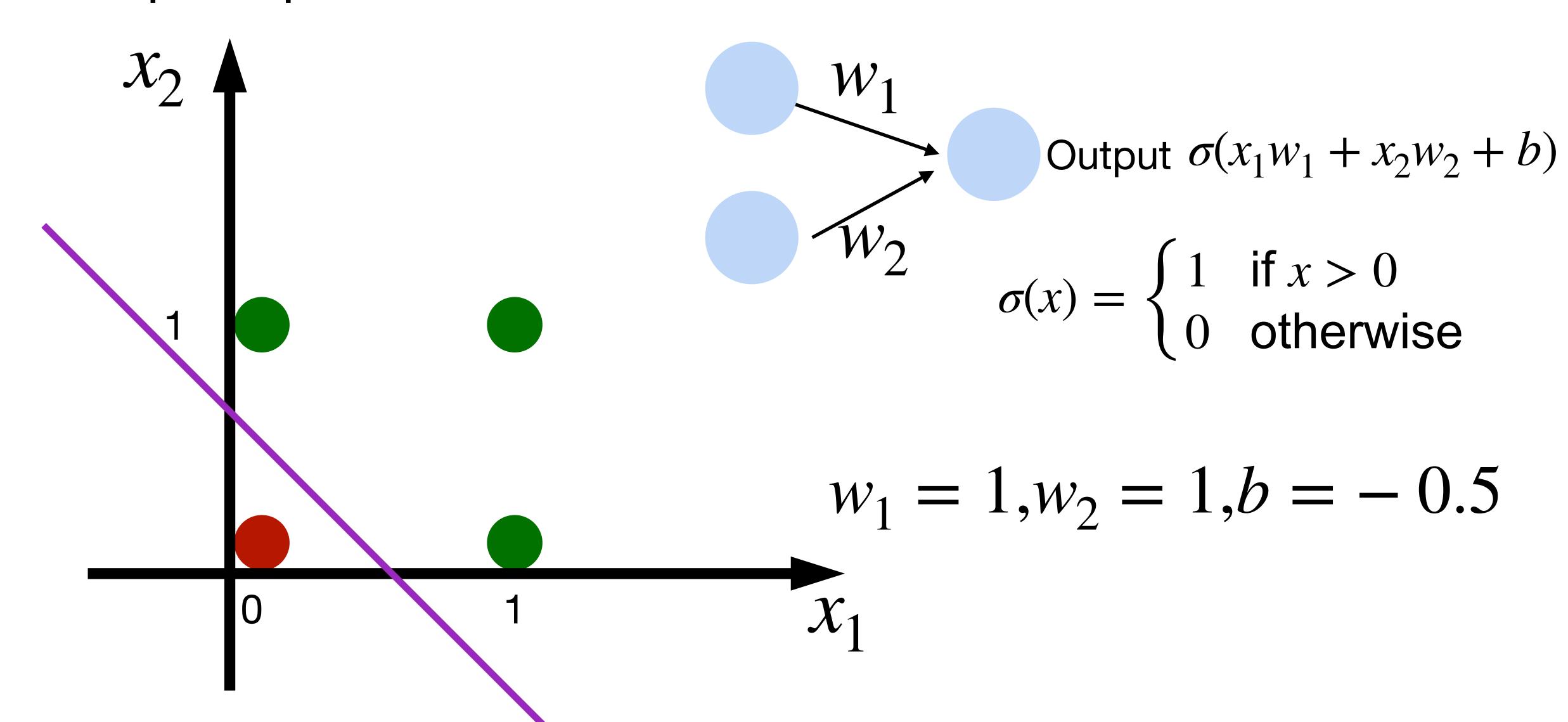
Learning logic functions using perceptron

The perceptron can learn an AND function



Learning OR function using perceptron

The perceptron can learn an OR function



XOR Problem (Minsky & Papert, 1969)

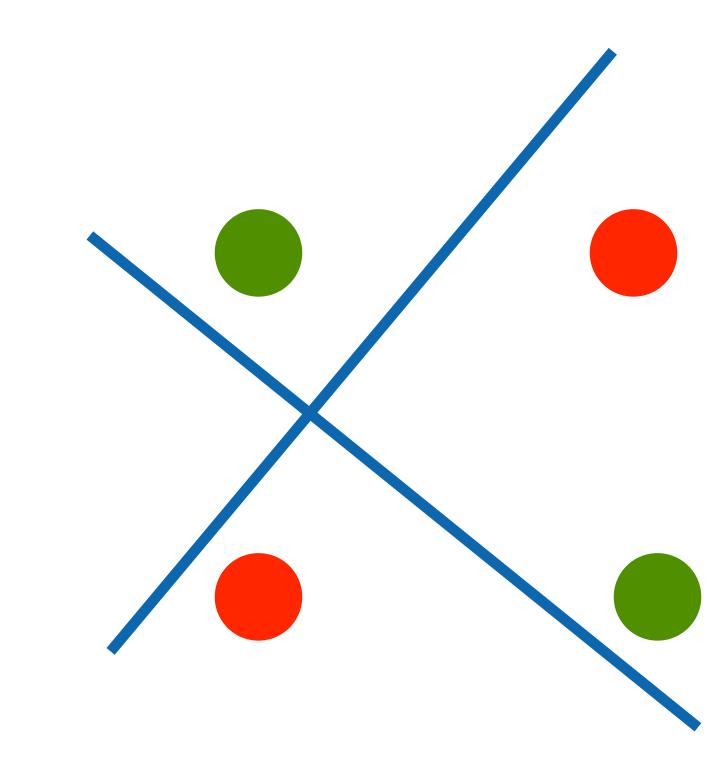
The perceptron cannot learn an XOR function (neurons can only generate linear separators)

$$x_1 = 1, x_2 = 1, y = 0$$

 $x_1 = 1, x_2 = 0, y = 1$

$$x_1 = 0, x_2 = 1, y = 1$$

$$x_1 = 0, x_2 = 0, y = 0$$



XOR Problem (Minsky & Papert, 1969)

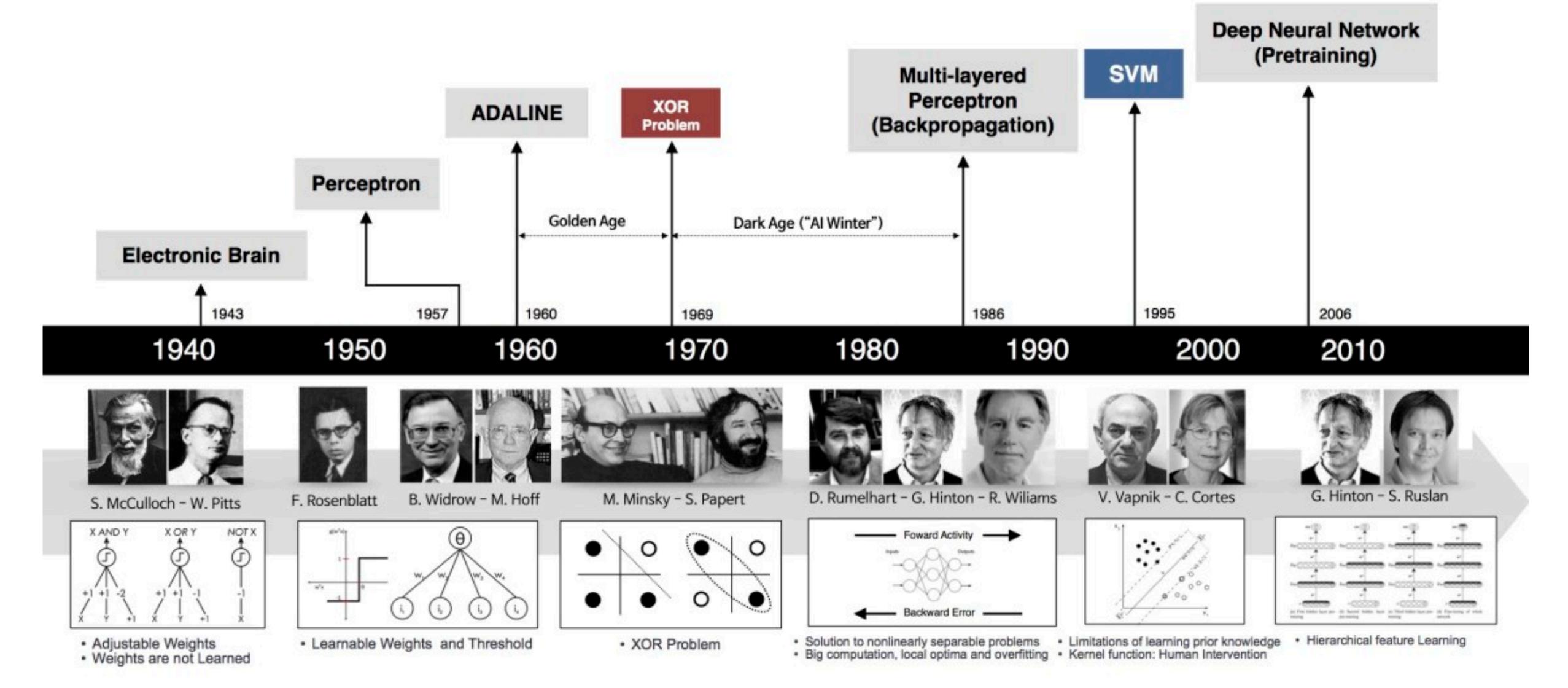
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$$x_1 = 1, x_2 = 1, y = 0$$

 $x_1 = 1, x_2 = 0, y = 1$
 $x_1 = 0, x_2 = 1, y = 1$
 $x_1 = 0, x_2 = 0, y = 0$

This contributed to the first Al winter

Brief history of neural networks



Quiz break

Which one of the following is NOT true about perceptron?

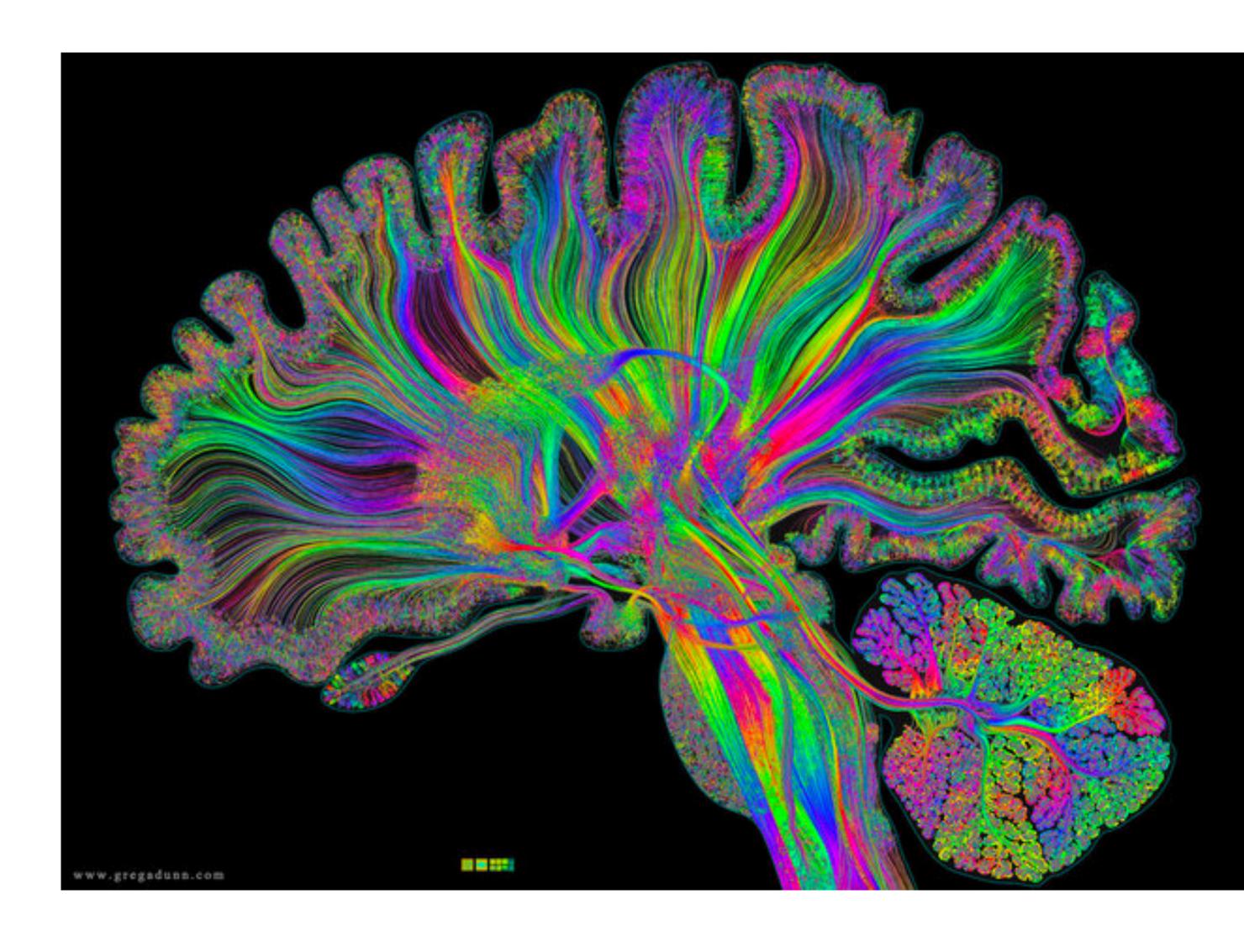
- A. Perceptron only works if the data is linearly separable.
- B. Perceptron can learn AND function
- C. Perceptron can learn XOR function
- D. Perceptron is a supervised learning algorithm

Quiz break

Which one of the following is NOT true about perceptron?

- A. Perceptron only works if the data is linearly separable.
- B. Perceptron can learn AND function
- C. Perceptron can learn XOR function
- D. Perceptron is a supervised learning algorithm

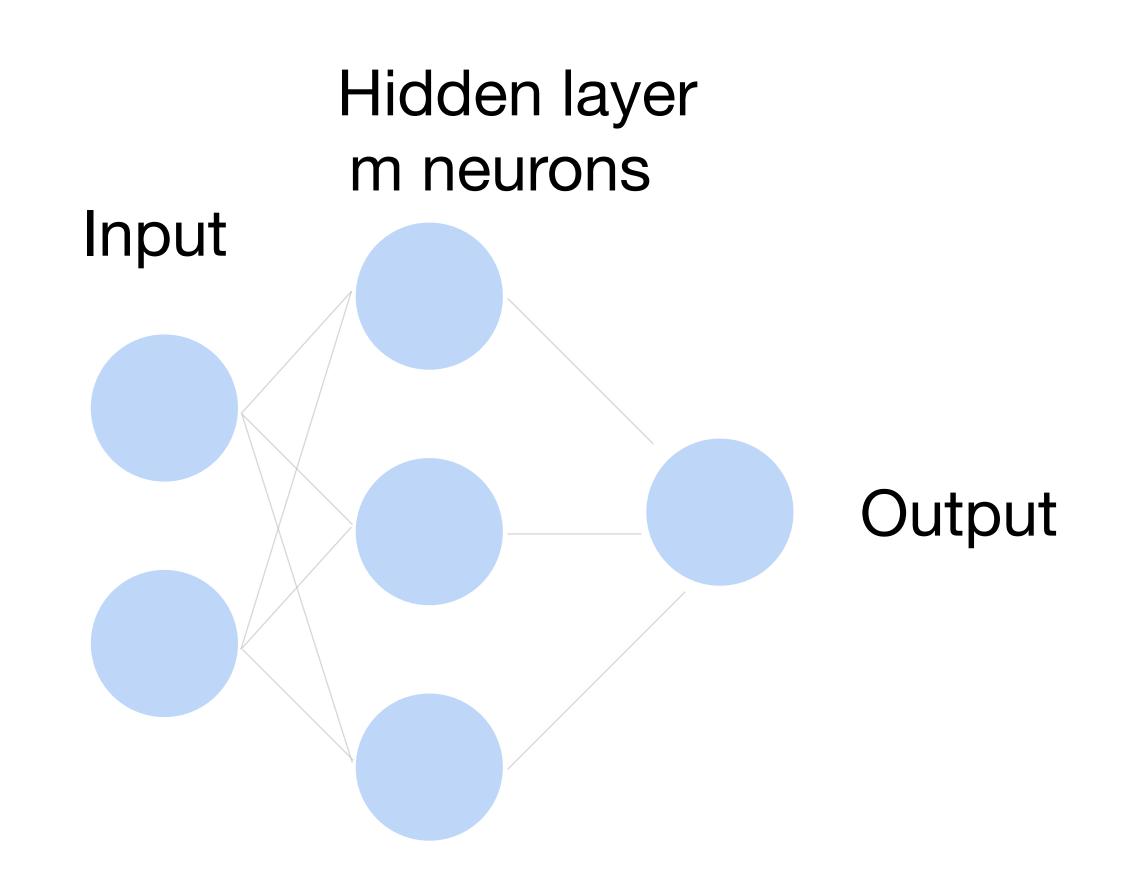
Multilayer Perceptron



Single Hidden Layer

How to classify Cats vs. dogs?



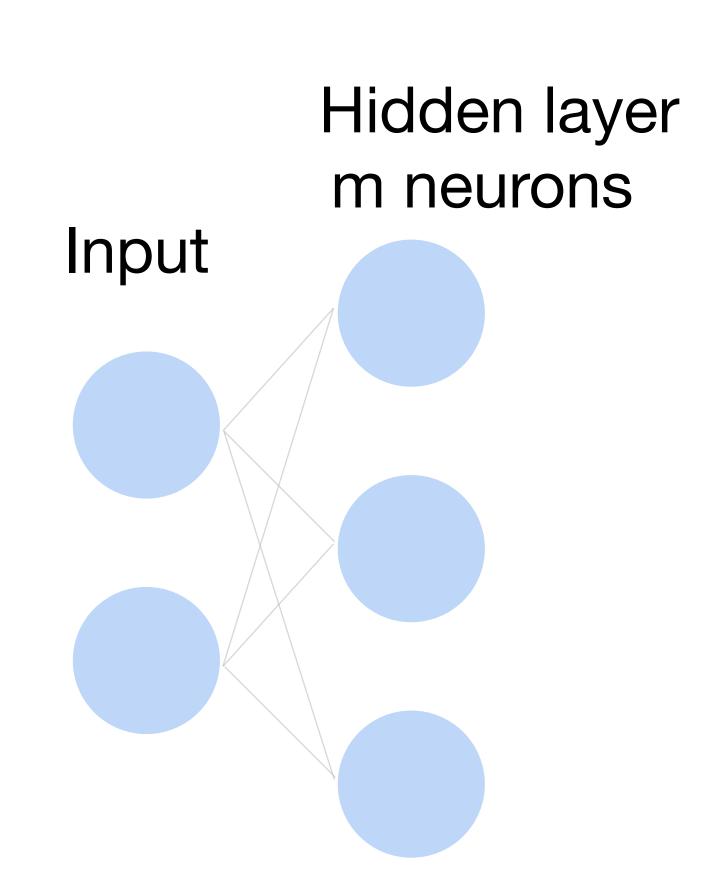


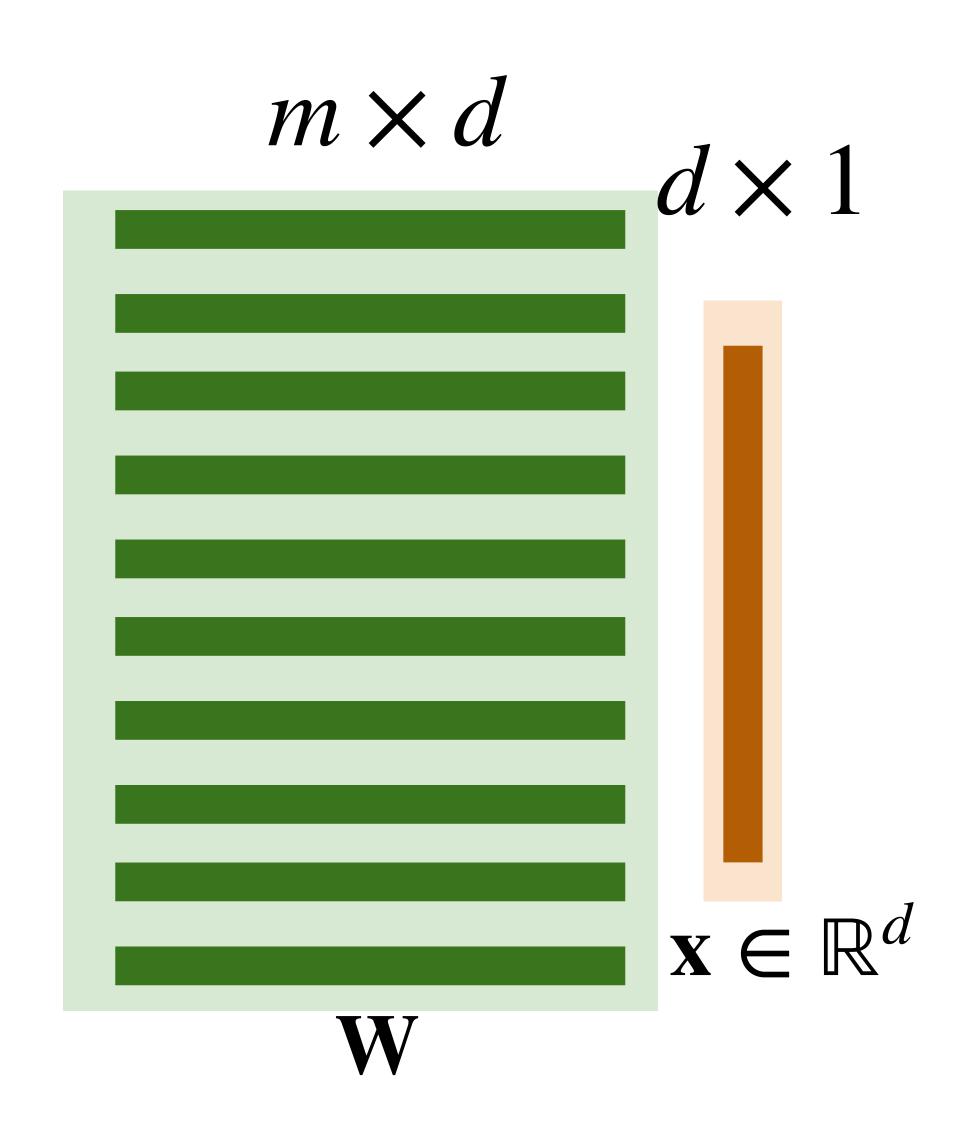
Single Hidden Layer

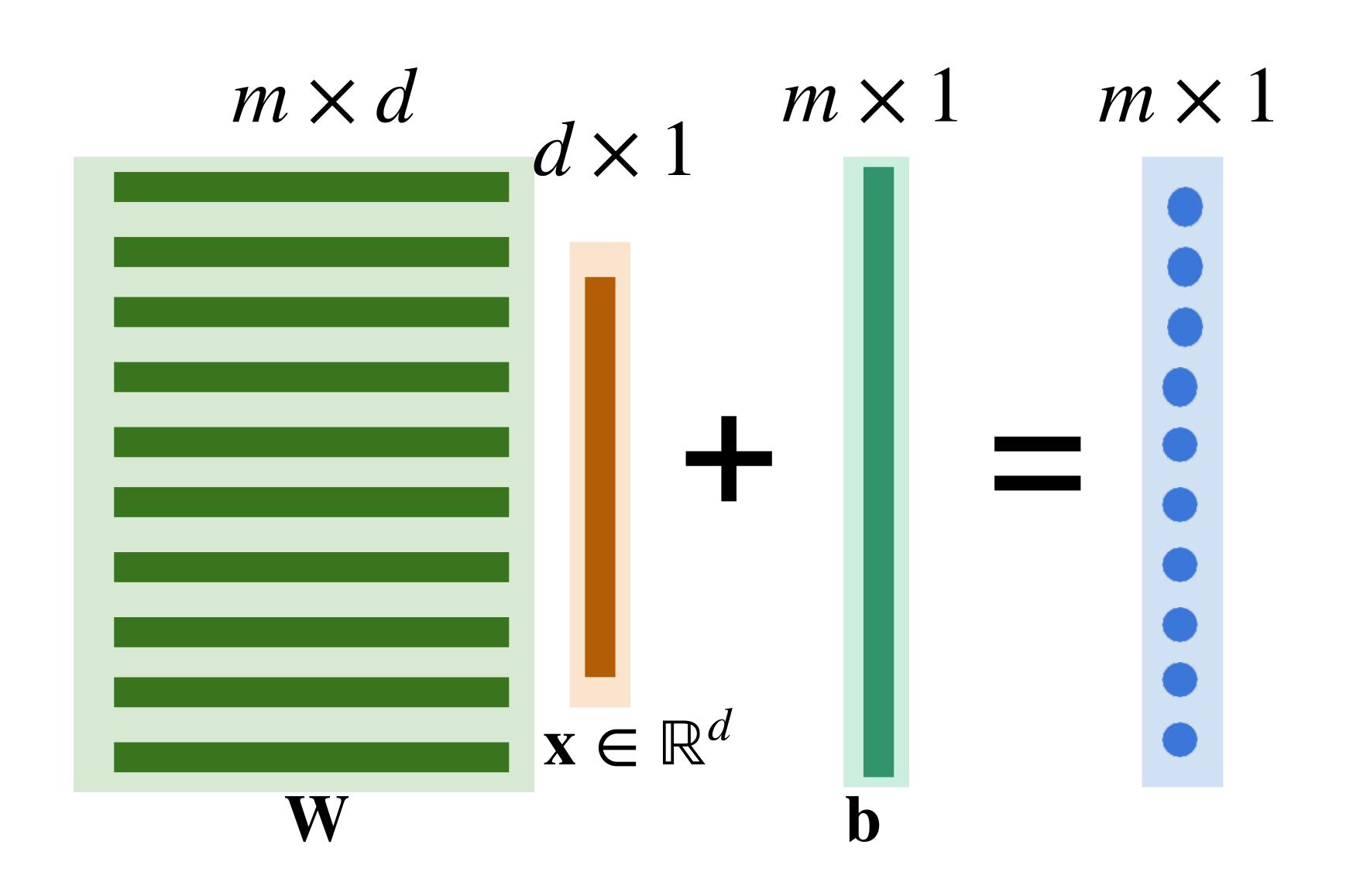
- Input $\mathbf{x} \in \mathbb{R}^d$
- Hidden $\mathbf{W} \in \mathbb{R}^{m \times d}, \mathbf{b} \in \mathbb{R}^m$
- Intermediate output

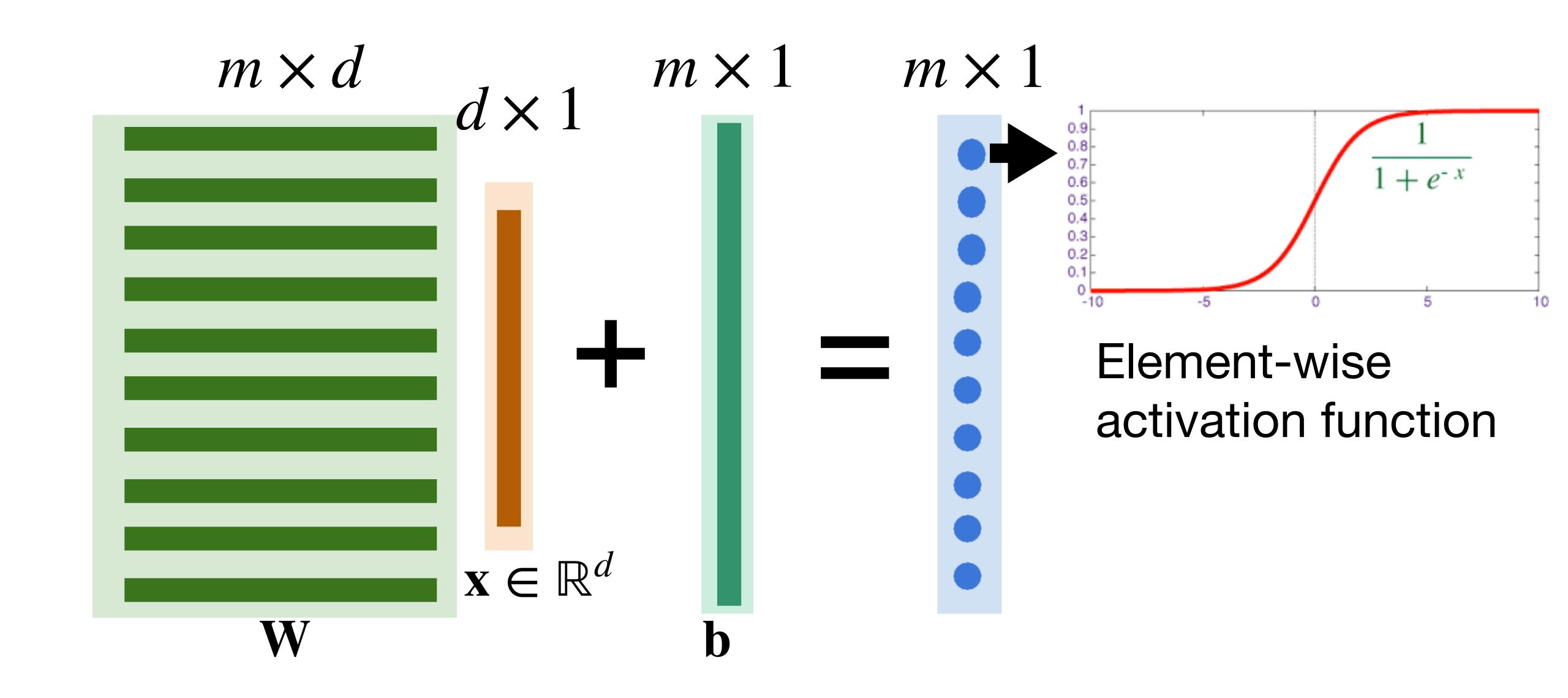
$$\mathbf{h} = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$$

 σ is an element-wise activation function

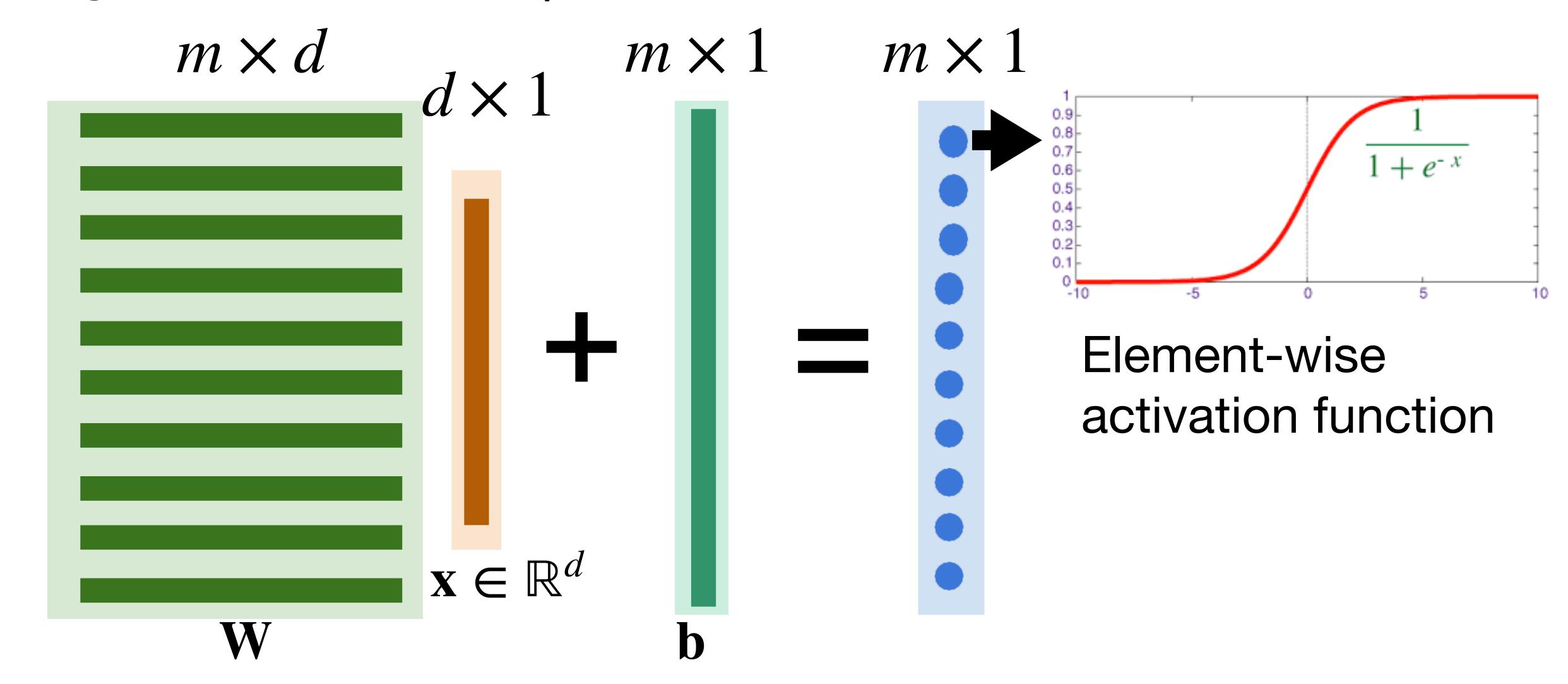








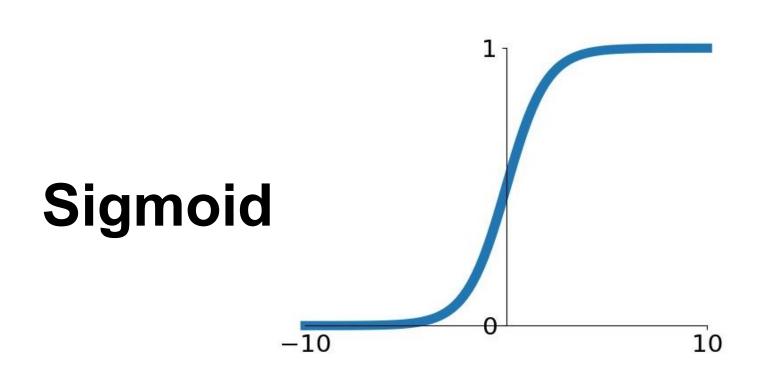
Key elements: linear operations + Nonlinear activations

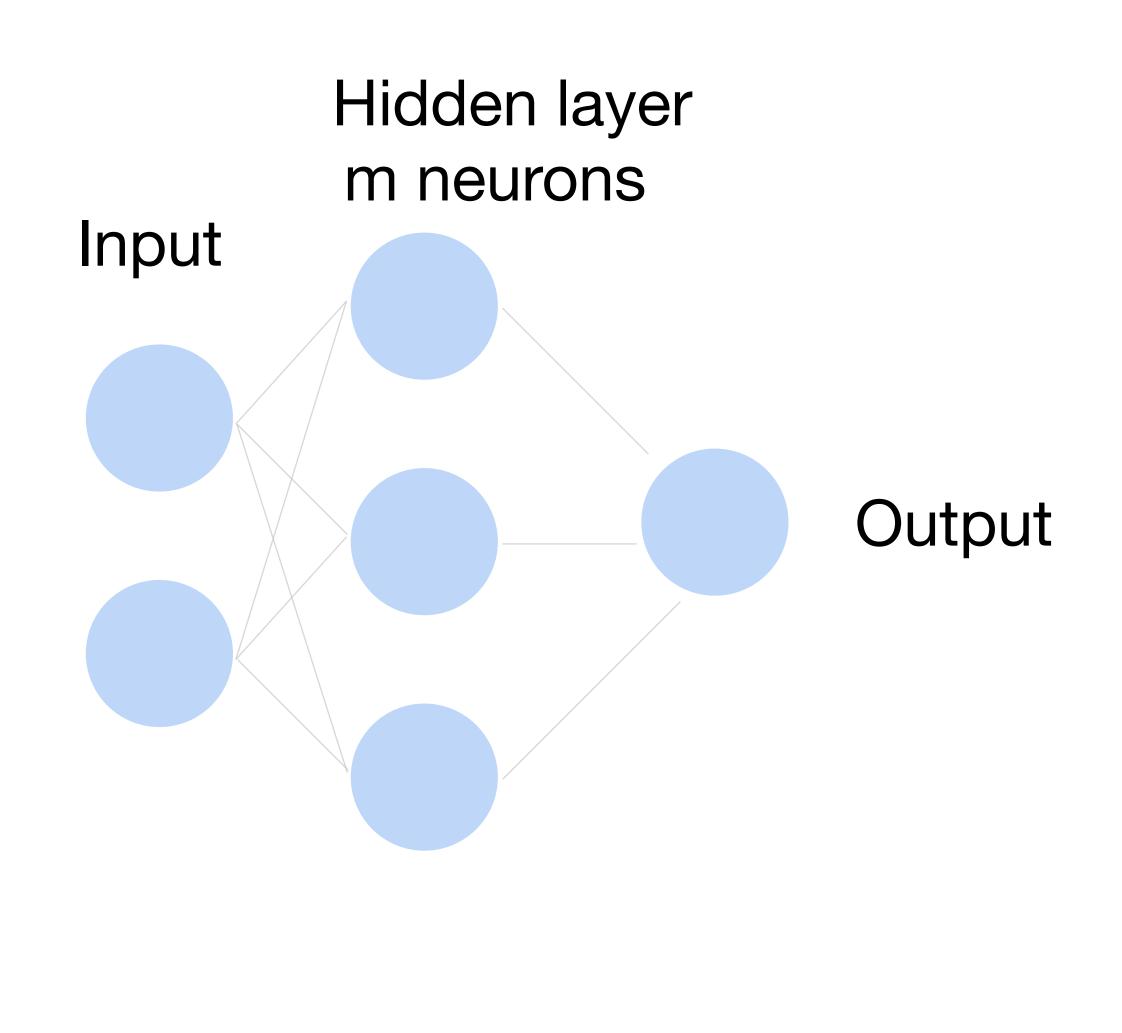


Single Hidden Layer

- Output $f = \mathbf{w}_2^\mathsf{T} \mathbf{h} + b_2$
- Normalize the output into probability using sigmoid

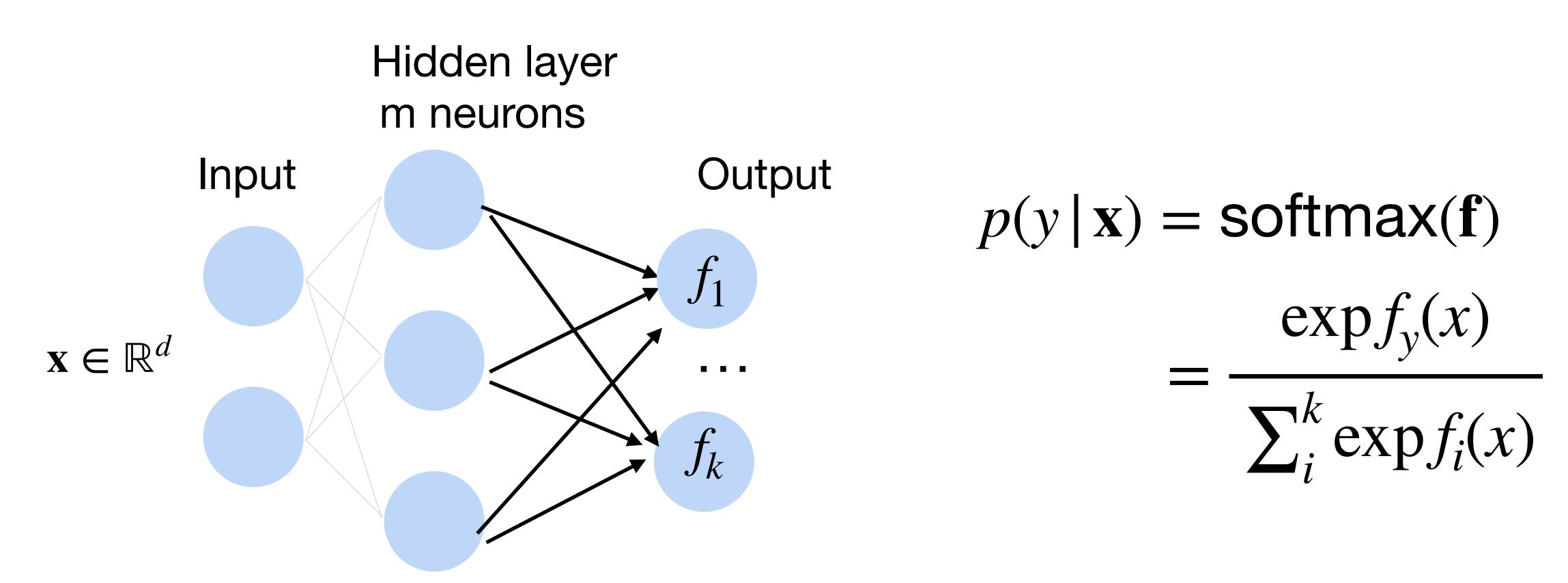
$$p(\mathbf{y} = 1 \mid \mathbf{x}) = \frac{1}{1 + e^{-f}}$$



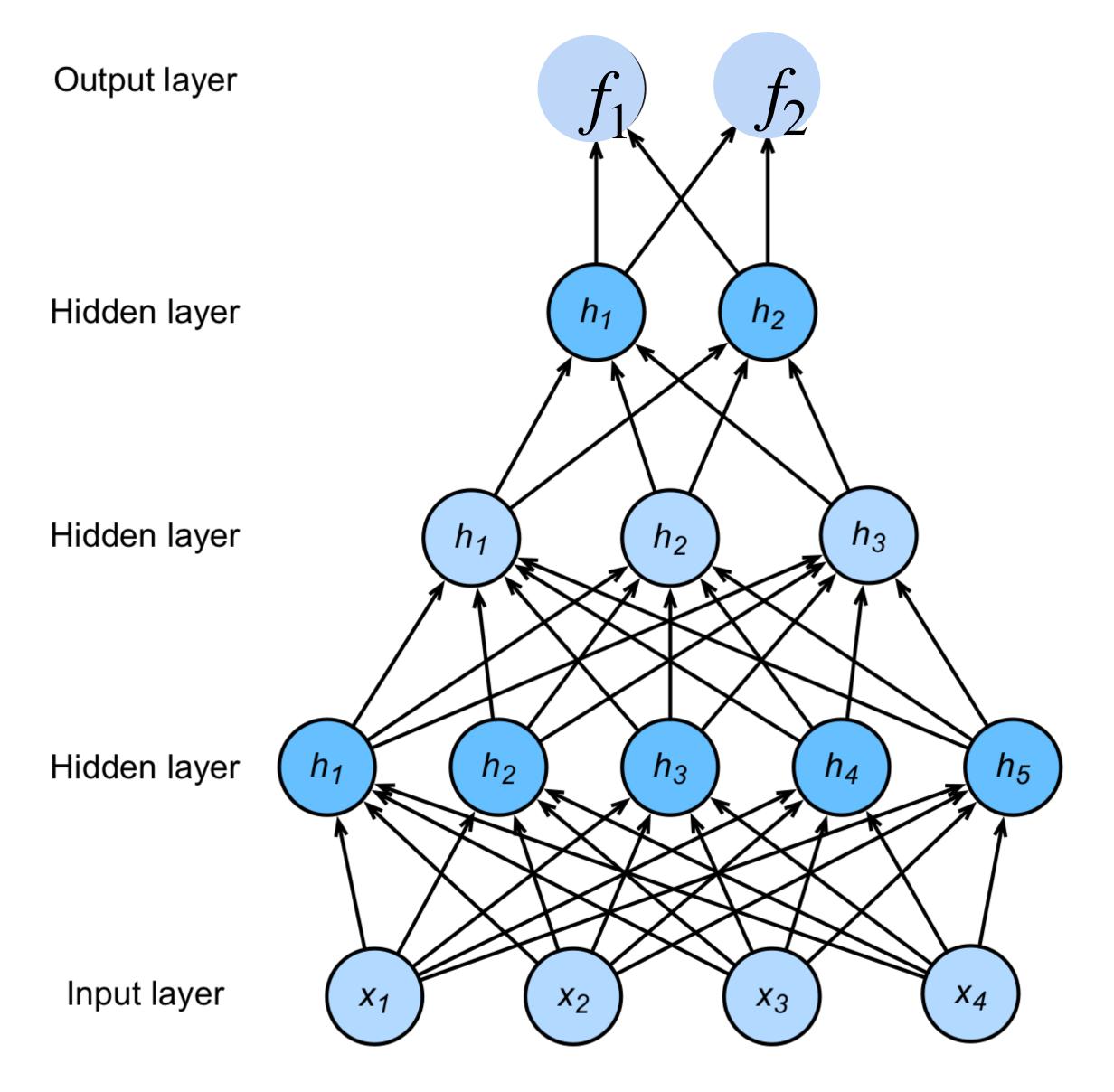


Multi-class classification

Turns outputs f into k probabilities (sum up to 1 across k classes)



Deep neural networks (DNNs)



$$\mathbf{h}_1 = \sigma(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1)$$

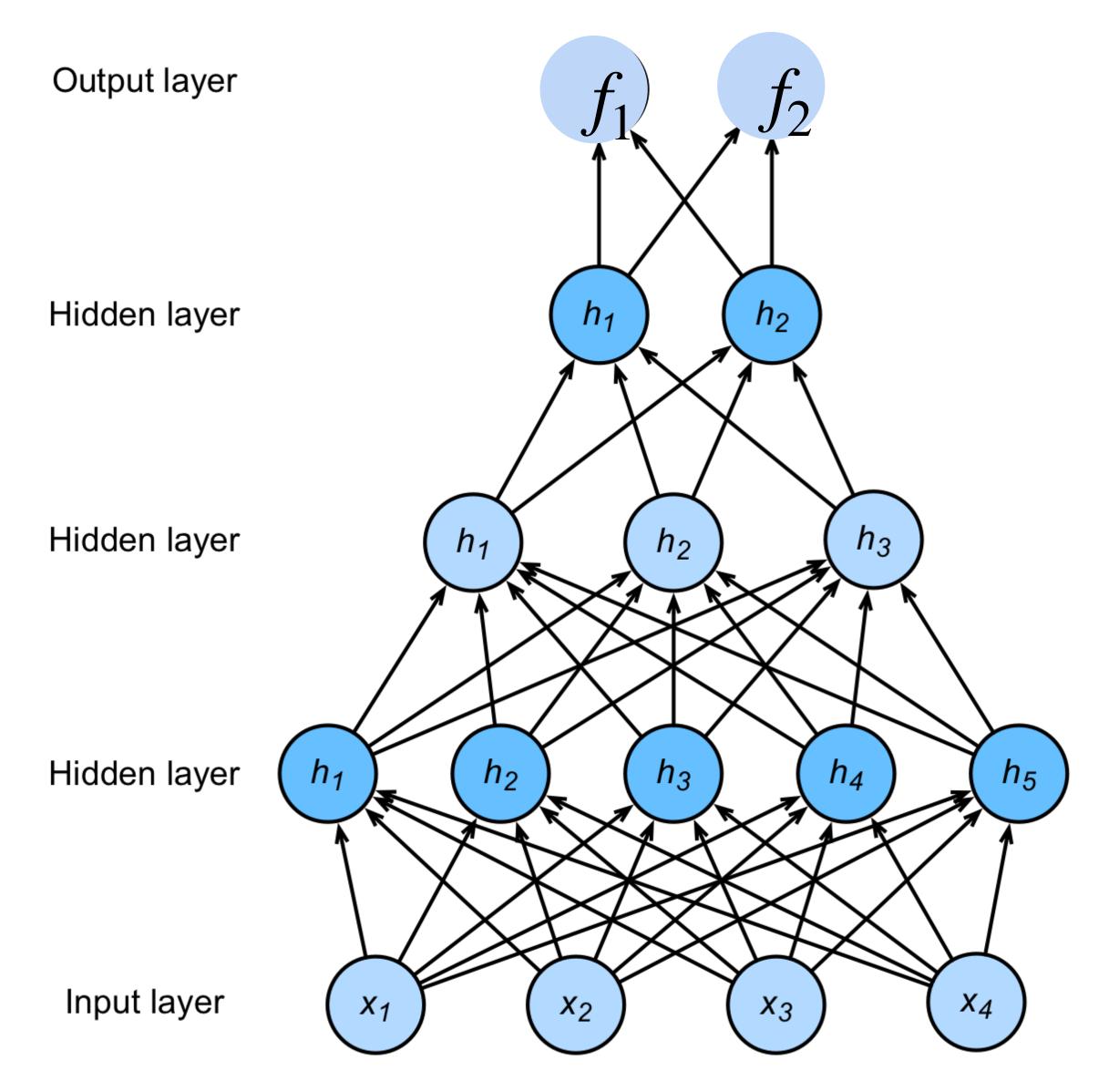
$$\mathbf{h}_2 = \sigma(\mathbf{W}_2 \mathbf{h}_1 + \mathbf{b}_2)$$

$$\mathbf{h}_3 = \sigma(\mathbf{W}_3 \mathbf{h}_2 + \mathbf{b}_3)$$

$$\mathbf{f} = \mathbf{W}_4 \mathbf{h}_3 + \mathbf{b}_4$$

$$\mathbf{y} = \text{softmax}(\mathbf{f})$$

Deep neural networks (DNNs)



$$\mathbf{h}_1 = \sigma(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1)$$

$$\mathbf{h}_2 = \sigma(\mathbf{W}_2 \mathbf{h}_1 + \mathbf{b}_2)$$

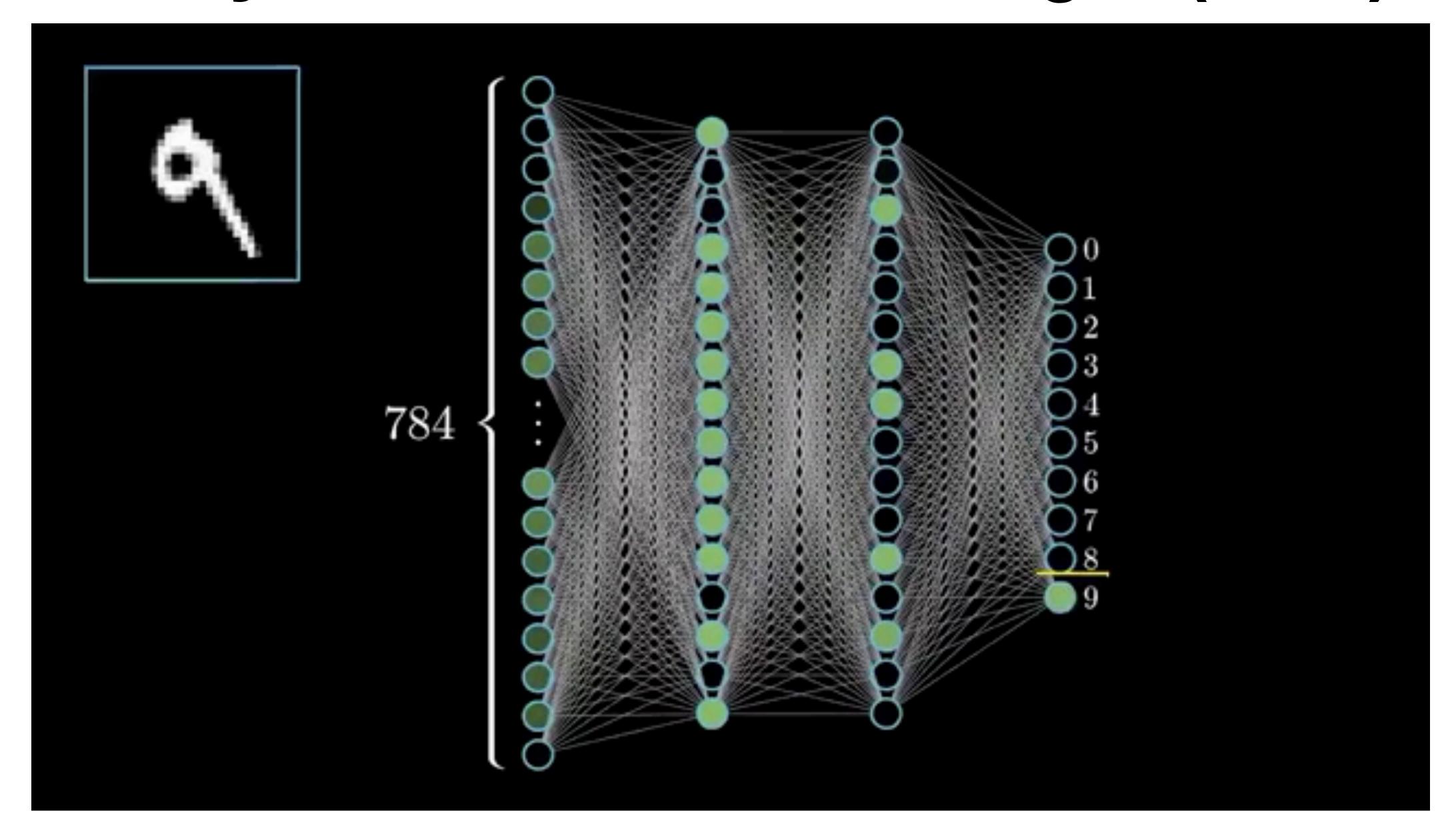
$$\mathbf{h}_3 = \sigma(\mathbf{W}_3 \mathbf{h}_2 + \mathbf{b}_3)$$

$$\mathbf{f} = \mathbf{W}_4 \mathbf{h}_3 + \mathbf{b}_4$$

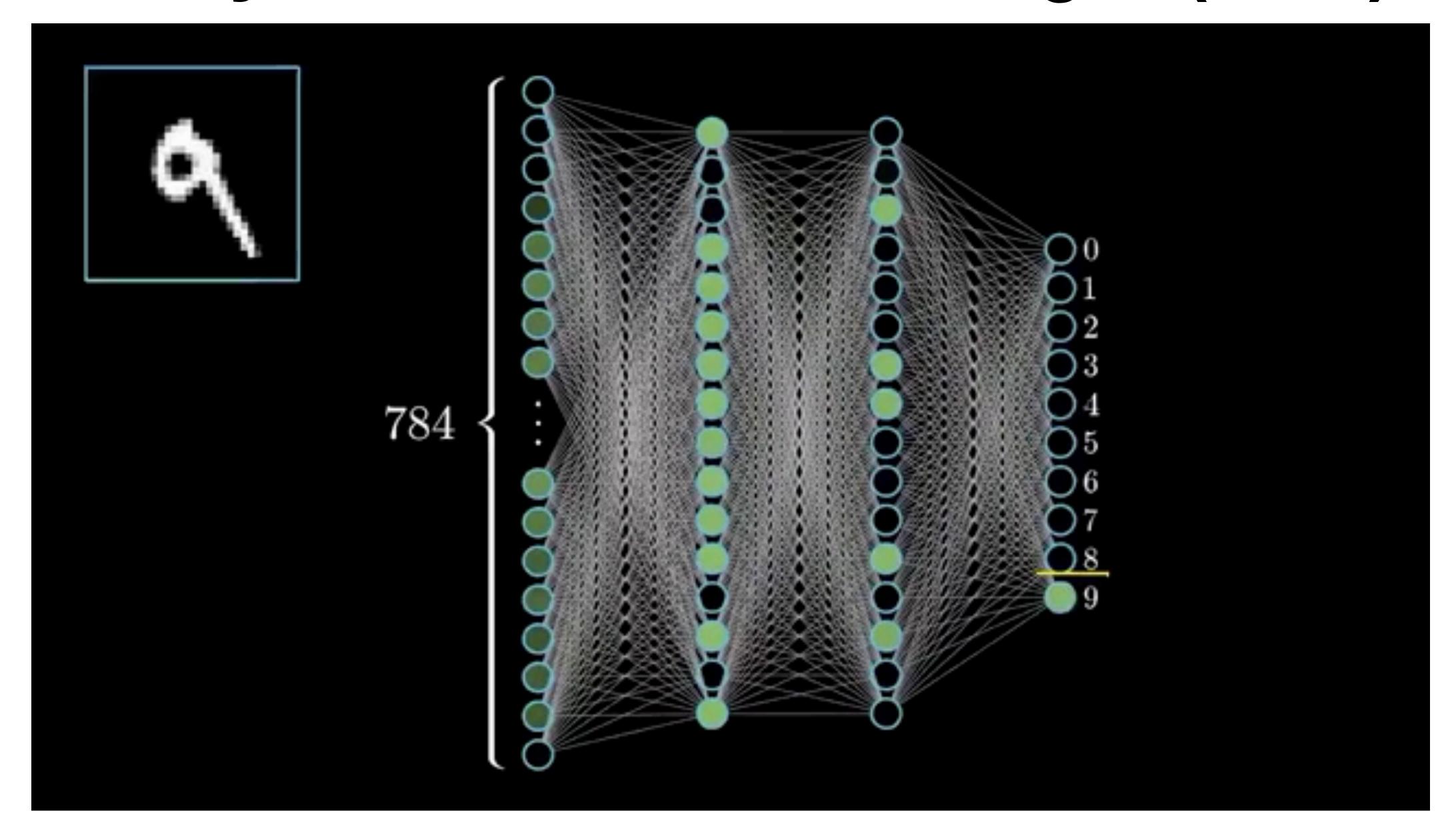
$$\mathbf{y} = \text{softmax}(\mathbf{f})$$

NNs are composition of nonlinear functions

Classify MNIST handwritten digits (HW6)



Classify MNIST handwritten digits (HW6)



Quiz break

You have a two-layer neural network with 2 units in the hidden layer. All weights have value 1 and all biases have value -1. The hidden layer has relu activations. What value does the network predict for x = [1, -1]?

- A. [-3, -3]
- B. [-1, -1]
- C. [0, 0]
- D. [1, 1]

Quiz break

You have a two-layer neural network with 2 units in the hidden layer. All weights have value 1 and all biases have value -1. The hidden layer has relu activations. What value does the network predict for x = [1, -1]?

$$Wx = [0, 0]$$

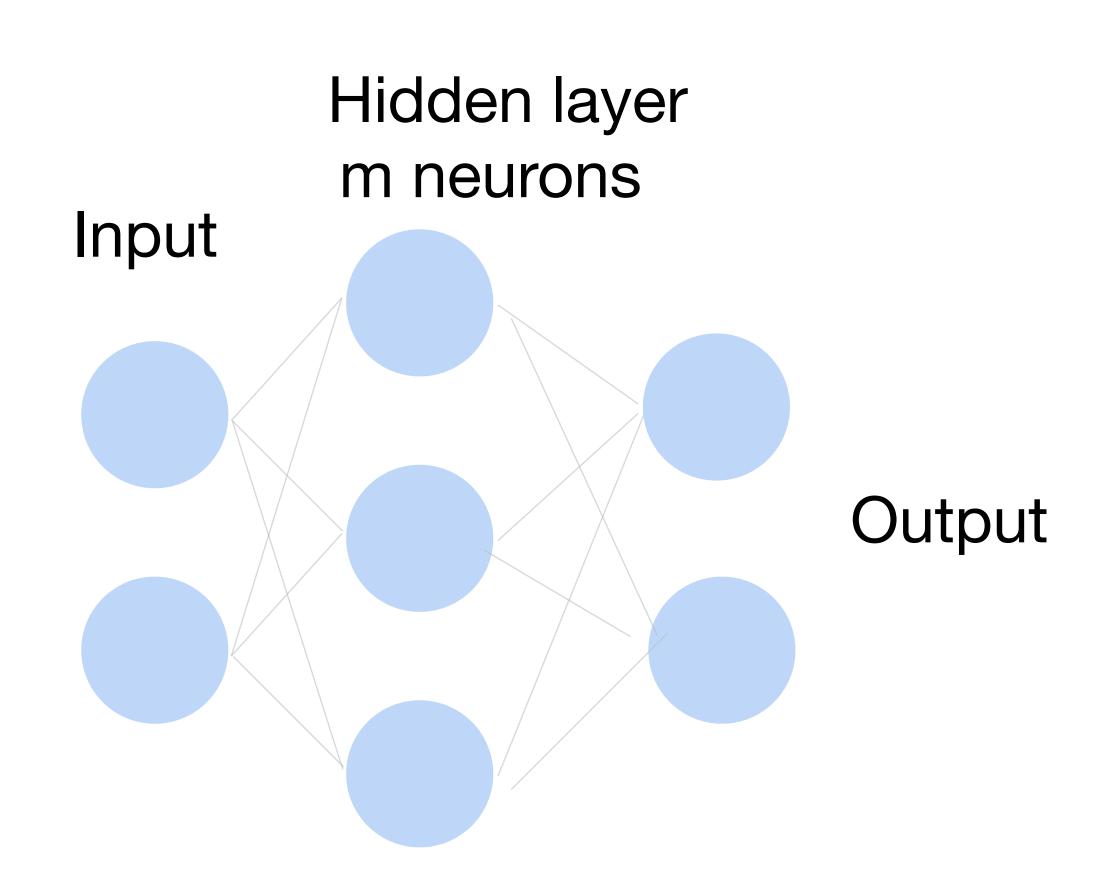
$$Wx + b = [-1, -1]$$

$$h = relu([-1, -1]) = 0$$

Wh + b =
$$[-1, -1]$$

How to train a neural network?

Loss function:
$$\frac{1}{|D|} \sum_{i} \mathcal{E}(\mathbf{x}_{i}, y_{i})$$

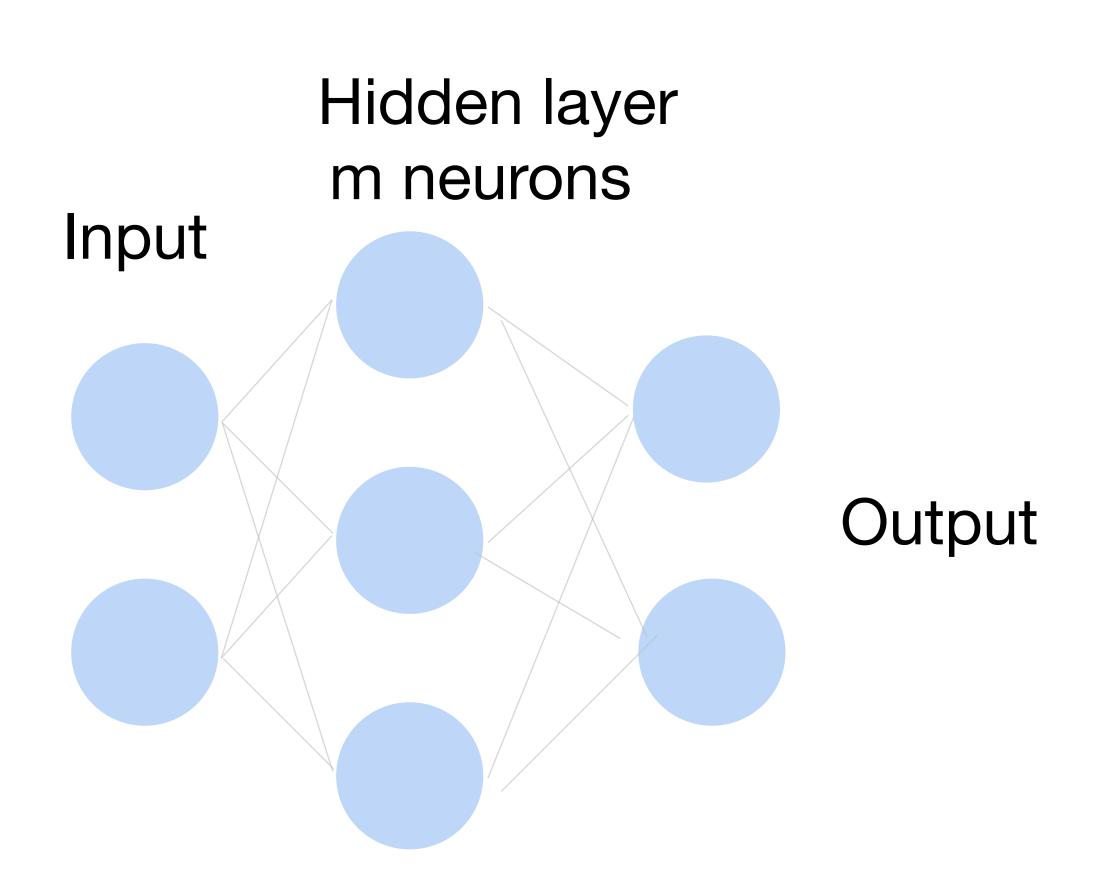


How to train a neural network?

Loss function:
$$\frac{1}{|D|} \sum_{i} \mathcal{L}(\mathbf{x}_{i}, y_{i})$$

Per-sample loss:

$$\mathcal{E}(\mathbf{x}, y) = \sum_{j=1}^{K} -y_j \log p_j$$



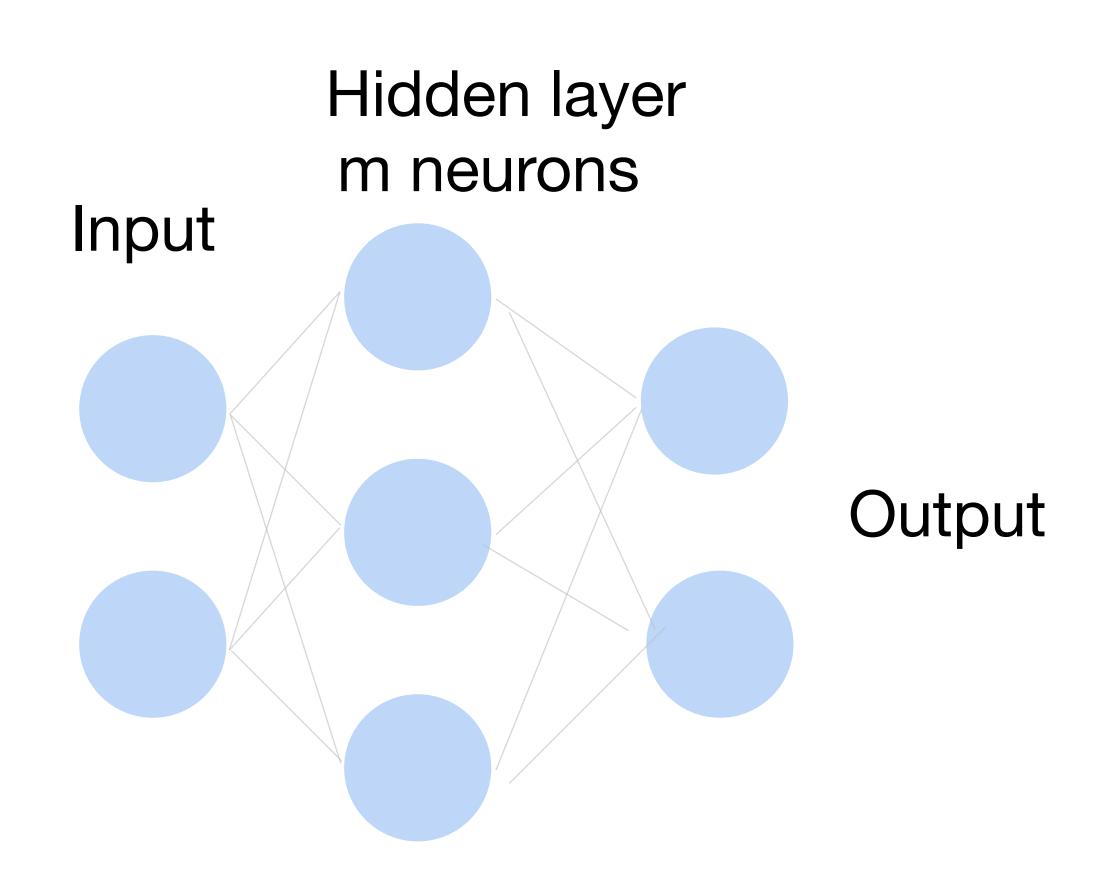
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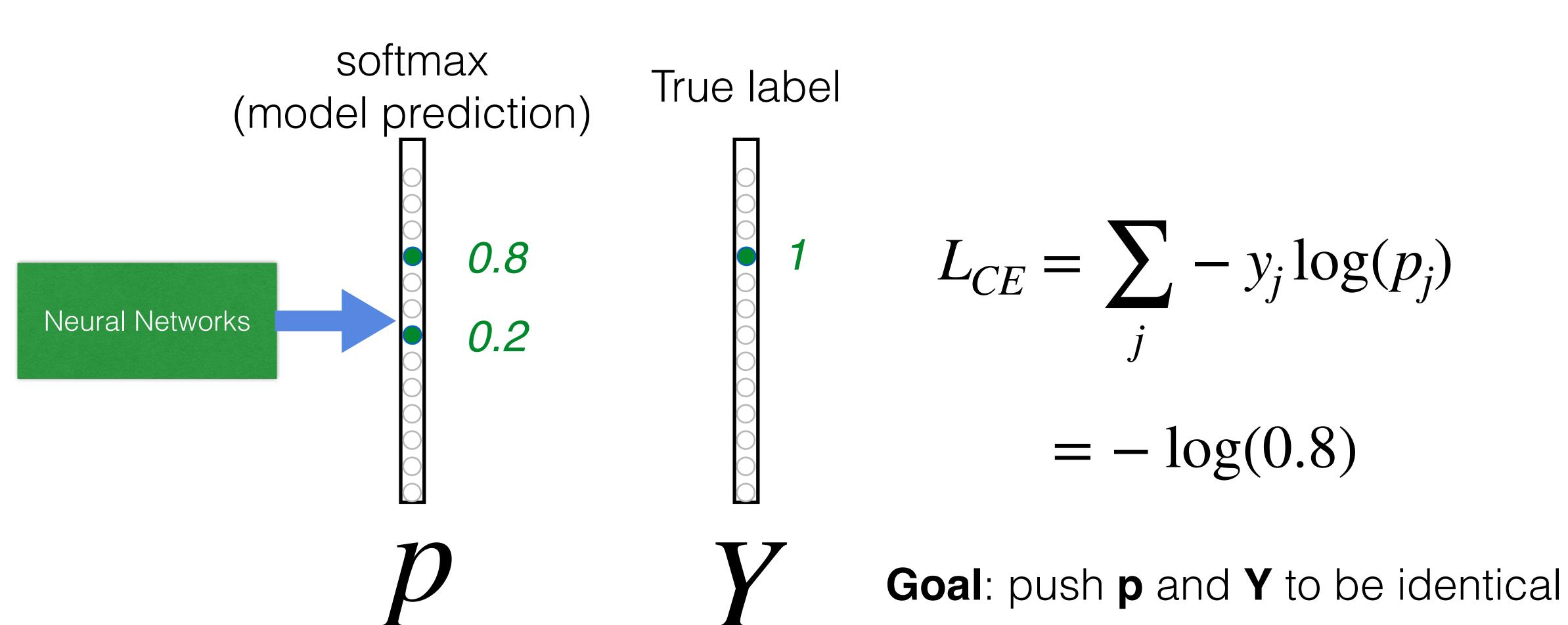
Per-sample loss:

$$\mathcal{E}(\mathbf{x}, y) = \sum_{j=1}^{K} -y_j \log p_j$$

Also known as cross-entropy loss or softmax loss



Cross-Entropy Loss

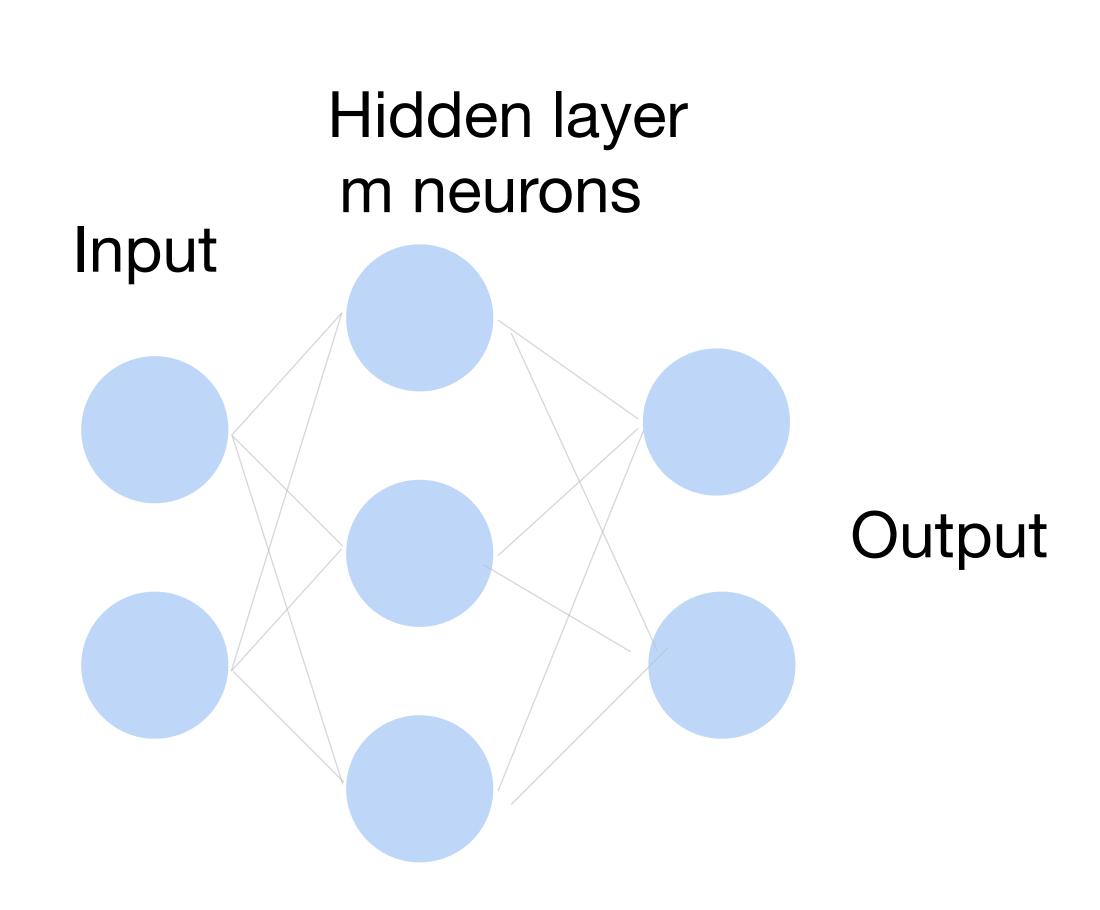


How to train a neural network?

Update the weights W to minimize the loss function

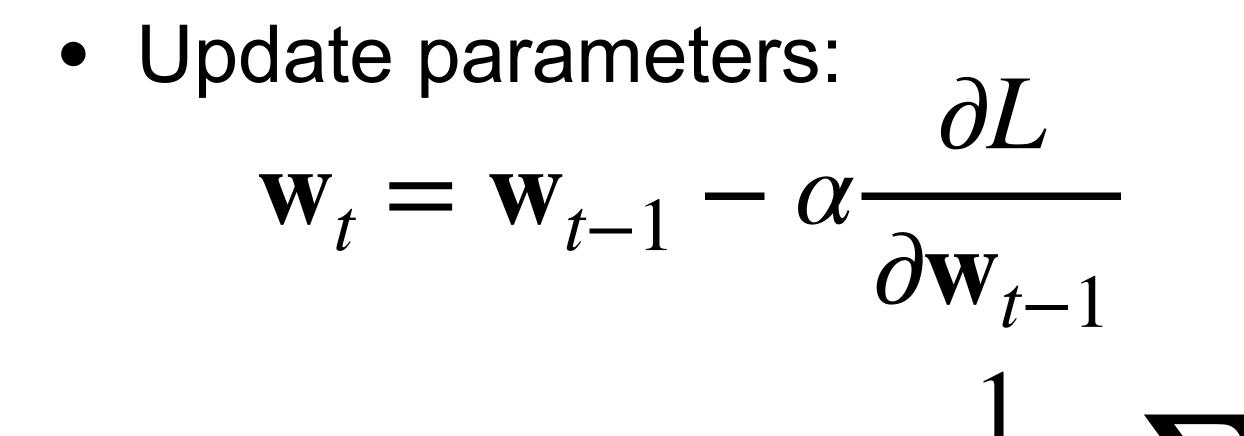
$$L = \frac{1}{|D|} \sum_{i} \ell(\mathbf{x}_{i}, y_{i})$$

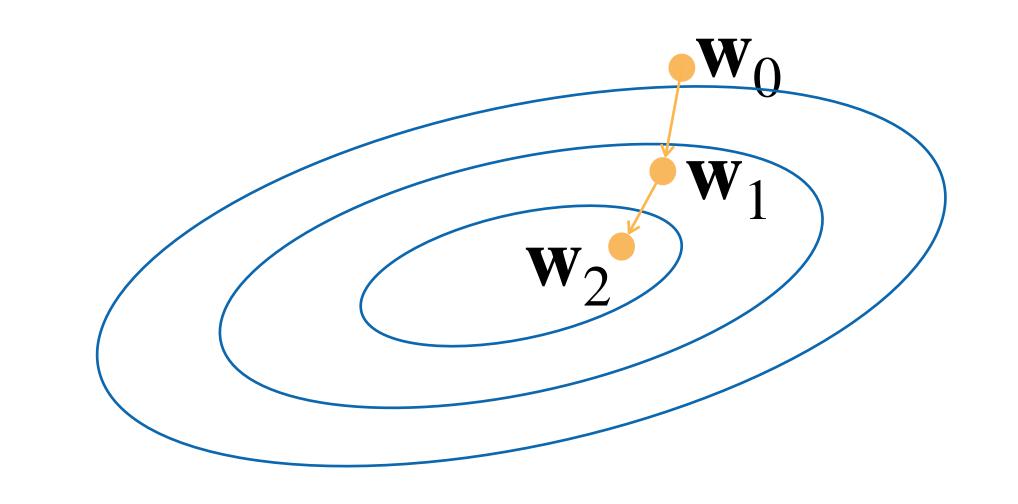
Use gradient descent!



Gradient Descent

- Choose a learning rate $\alpha > 0$
- Initialize the model parameters w_0
- For t = 1, 2, ...

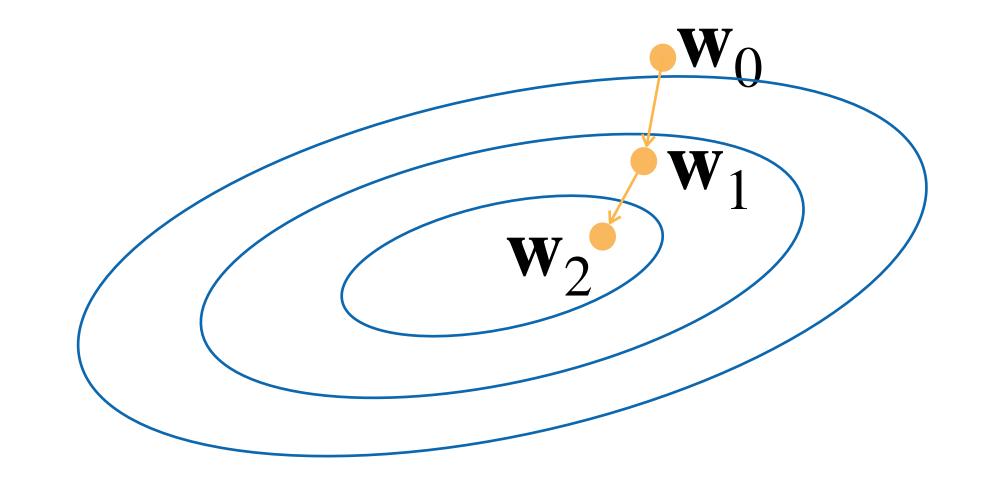




Repeat until converges

Gradient Descent

- Choose a learning rate $\alpha > 0$
- Initialize the model parameters w_0
- For t = 1, 2, ...



$$\mathbf{w}_{t} = \mathbf{w}_{t-1} - \alpha \frac{\partial L}{\partial \mathbf{w}_{t-1}}$$

D can be very large. Expensive

$$= \mathbf{w}_{t-1} - \alpha \frac{1}{|D|} \sum_{\mathbf{x} \in D} \frac{\partial \mathcal{E}(\mathbf{x}_i, y_i)}{\partial \mathbf{w}_{t-1}}$$

Repeat until converges

Minibatch Stochastic Gradient Descent

- Choose a learning rate $\alpha > 0$
- Initialize the model parameters w_0
- For t = 1, 2, ...
 - Randomly sample a subset (mini-batch) $B \subset D$ Update parameters:

$$\mathbf{w}_{t} = \mathbf{w}_{t-1} - \alpha \frac{1}{|B|} \sum_{\mathbf{x} \in B} \frac{\partial \mathcal{E}(\mathbf{x}_{i}, y_{i})}{\partial \mathbf{w}_{t-1}}$$

Repeat

Calculate gradient: backpropagation with chain rule

- Define a loss function L
- Gradient with respect to a variable =

gradient of the top x gradient from the current operation

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial z_1} \frac{\partial z_1}{\partial W}$$

$$x$$

$$x$$

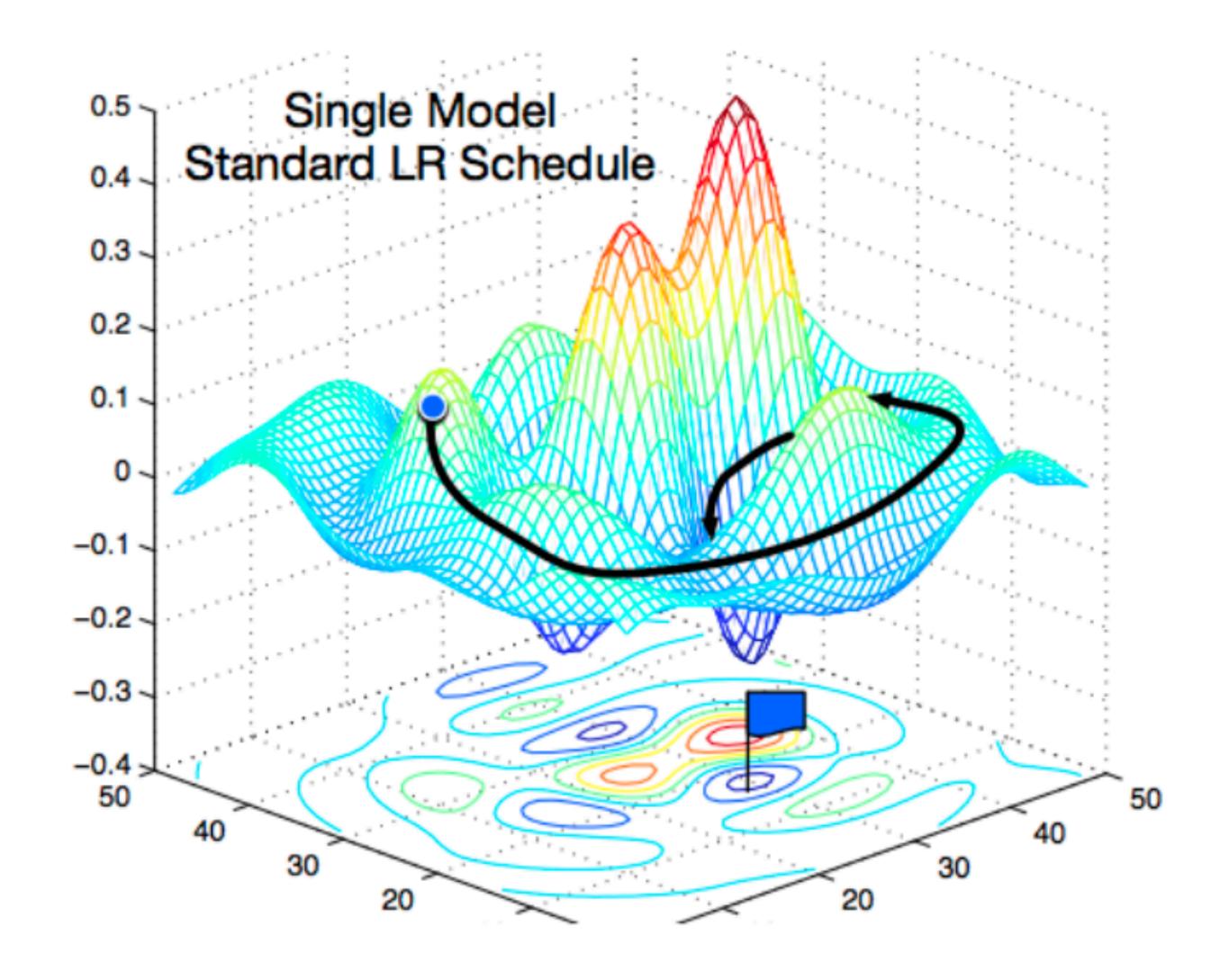
$$z_1$$

$$+$$

$$a$$

$$a = sigmoid(Wx + b)$$

Non-convex Optimization

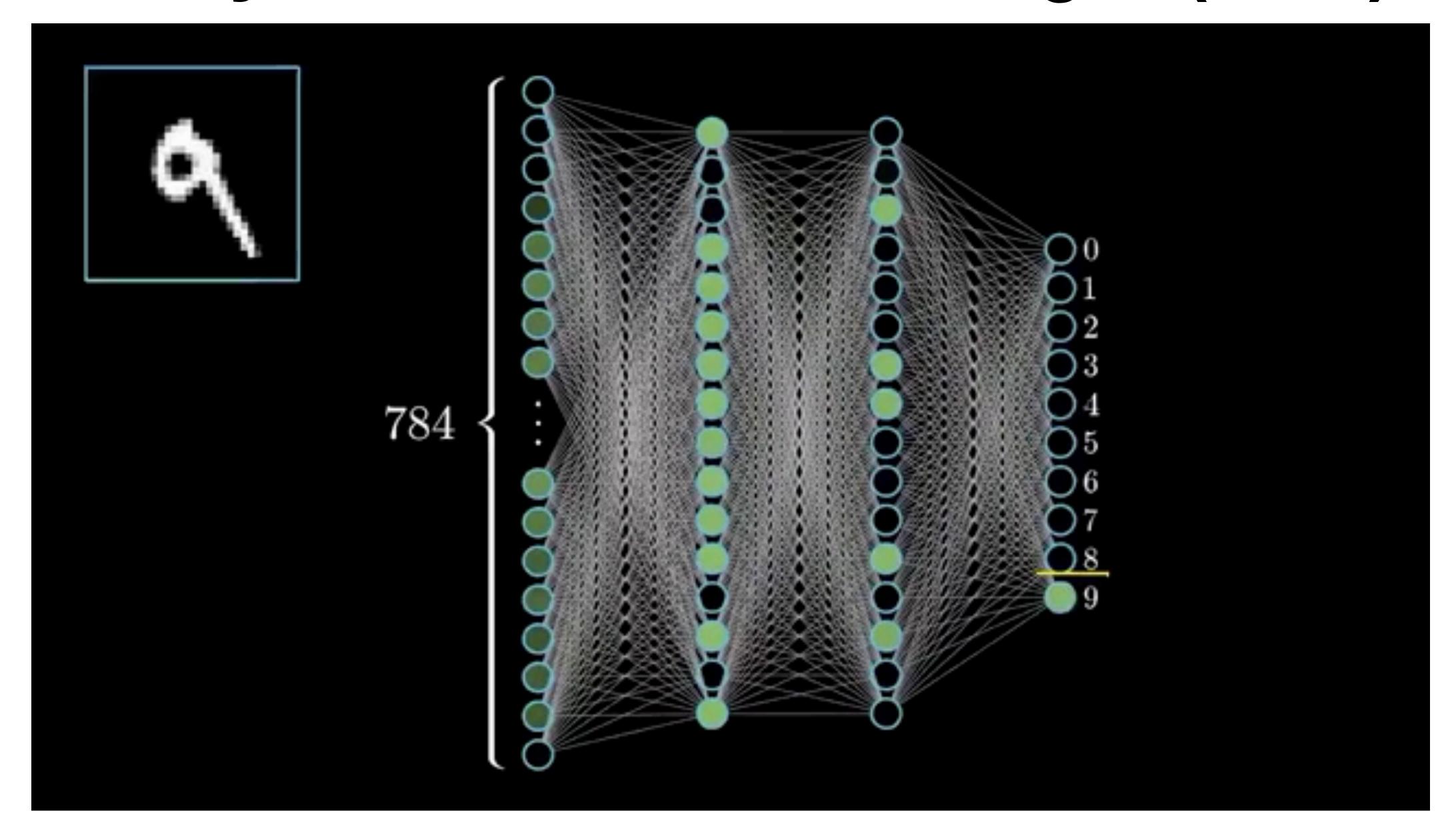


[Gao and Li et al., 2018]

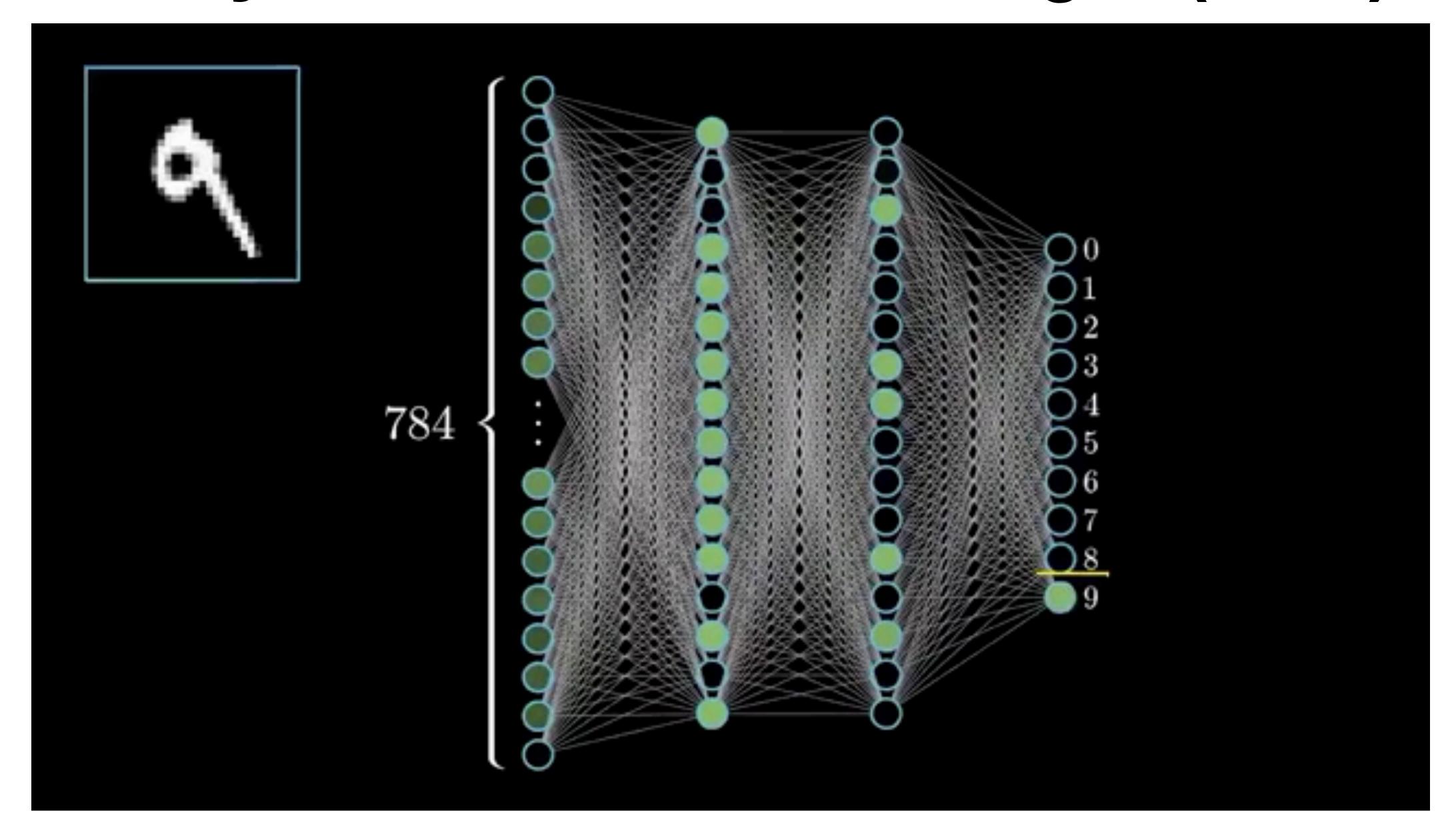
Using SGD in PyTorch (code demo)

https://colab.research.google.com/github/pytorch/tutorials/blob/gh-pages/_downloads/17a7c7cb80916fcdf921097825a0f562/cifar10_tutorial.ipynb#scrollTo=9eTWywELV7kr

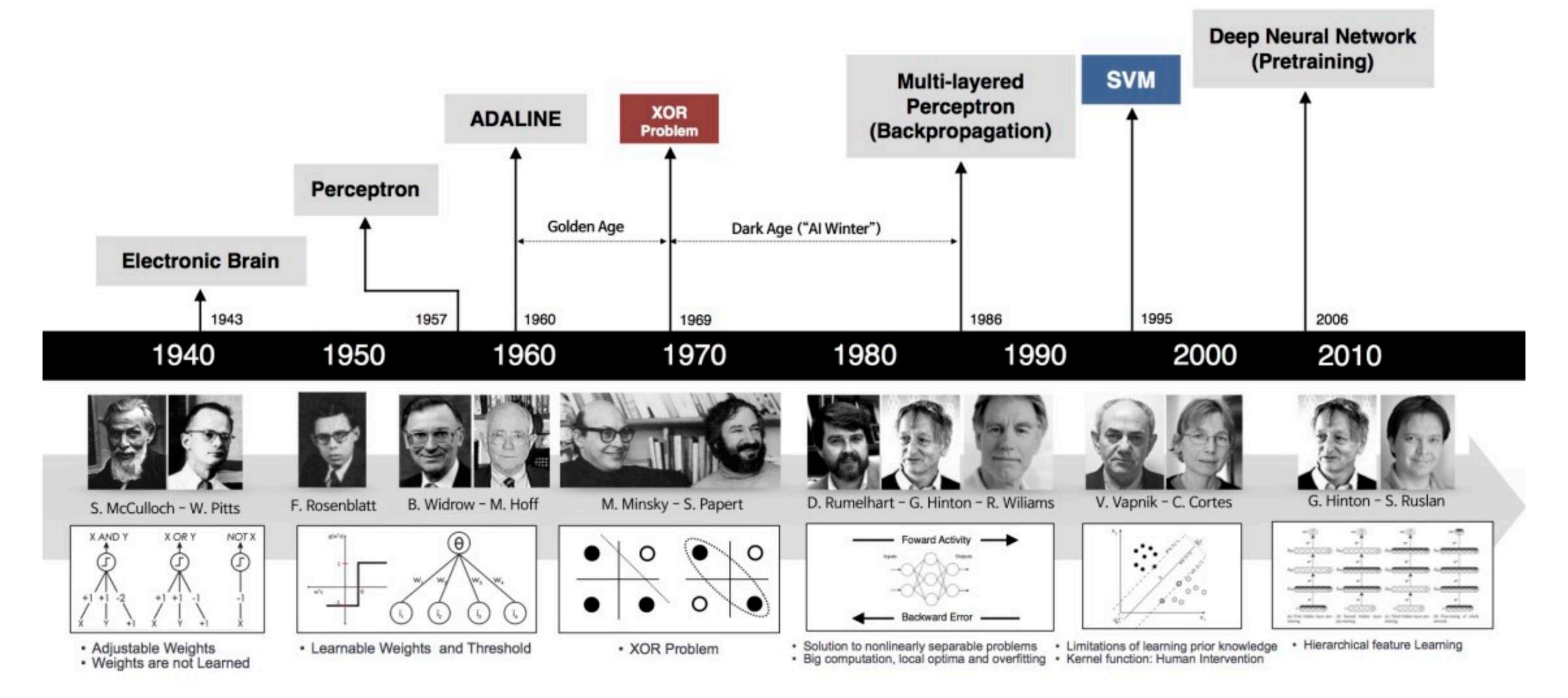
Classify MNIST handwritten digits (HW6)



Classify MNIST handwritten digits (HW6)



Brief history of neural networks



How to classify

Cats vs. dogs?





How to classify Cats vs. dogs?







12MP

wide-angle and telephoto cameras

36M floats in a RGB image!

Fully Connected Networks

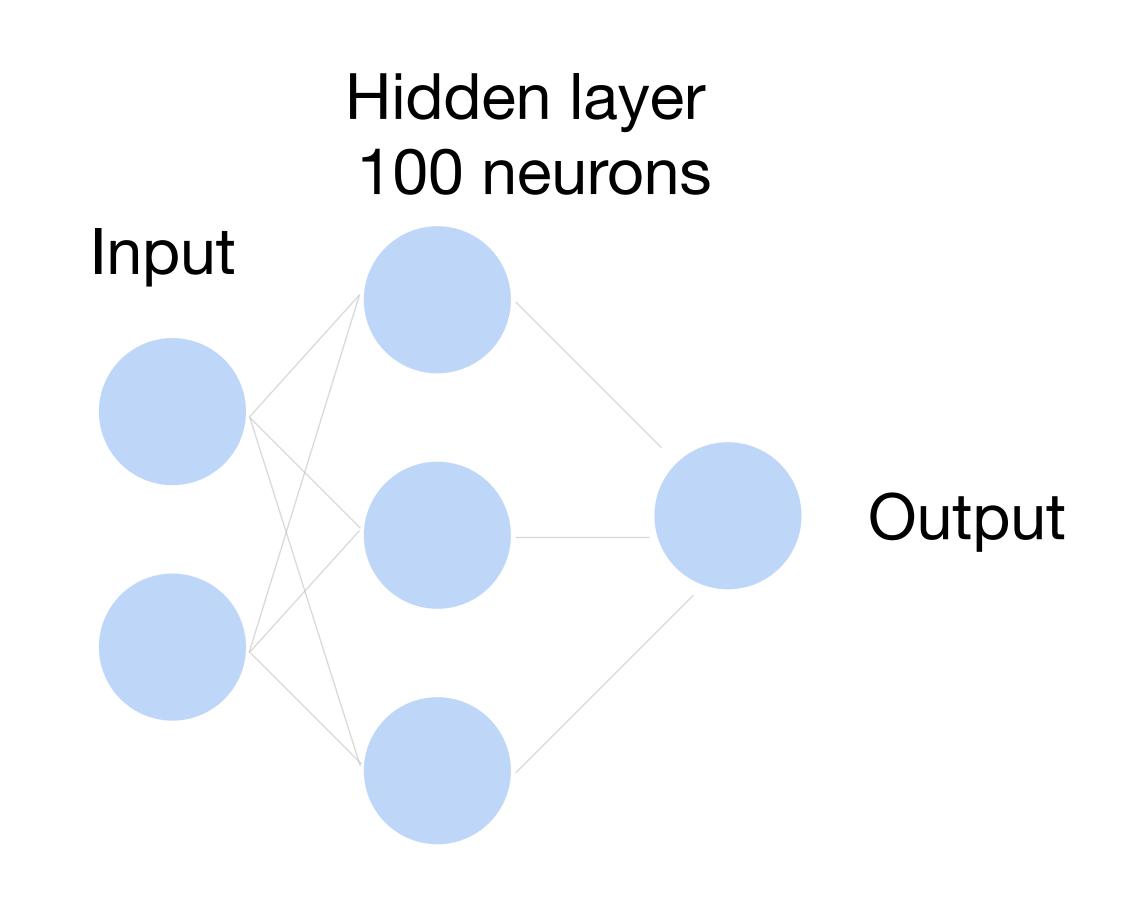
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Fully Connected Networks

Cats vs. dogs?

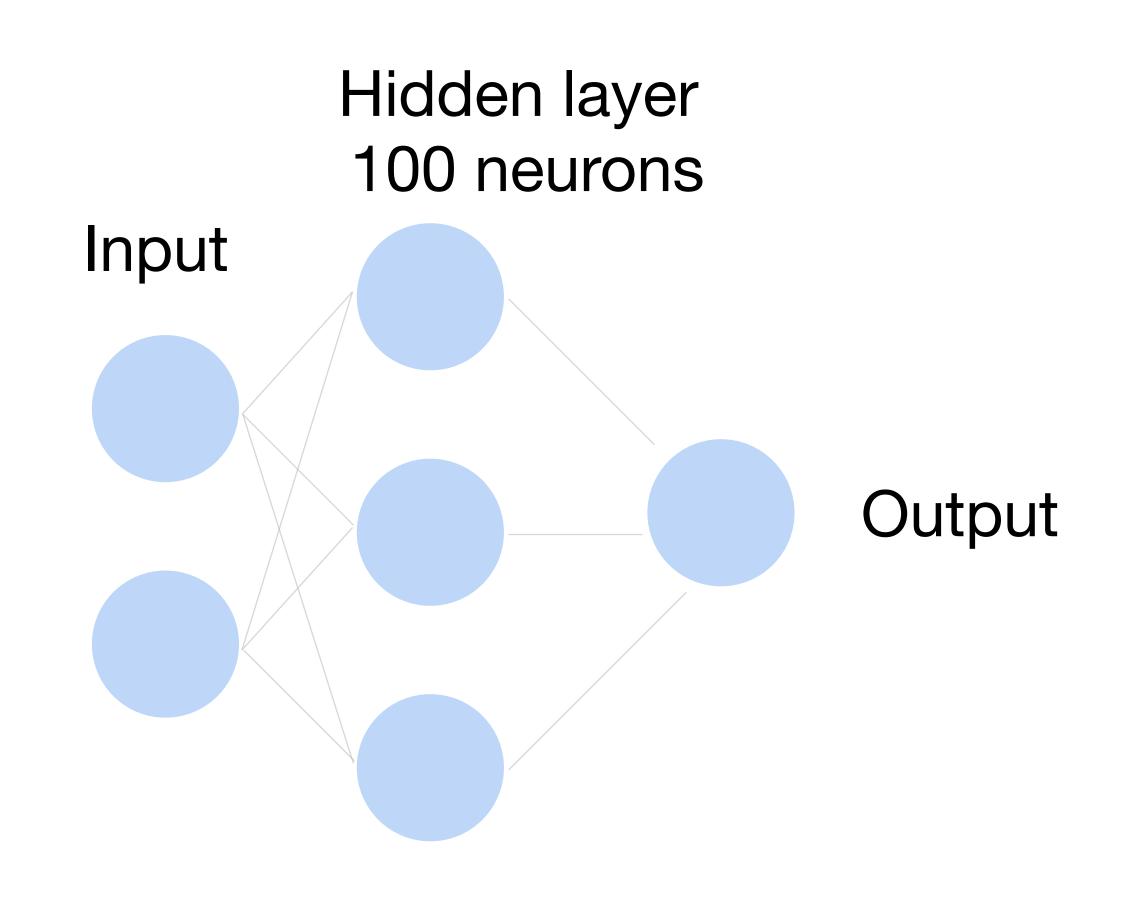




Fully Connected Networks

Cats vs. dogs?





~ 36M elements x 100 = ~3.6B parameters!

Convolutions come to rescue!

Where is Waldo?





Why Convolution?

- Translation
 Invariance
- Locality



Input Kernel Output

O 1 2
3 4 5
6 7 8

Kernel Output

- 19 25
37 43

Input

Kernel

Output

0	1	2
3	4	5
6	7	8

023

 19
 25

 37
 43

$$0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19,$$

 $1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25,$
 $3 \times 0 + 4 \times 1 + 6 \times 2 + 7 \times 3 = 37,$
 $4 \times 0 + 5 \times 1 + 7 \times 2 + 8 \times 3 = 43.$

Input

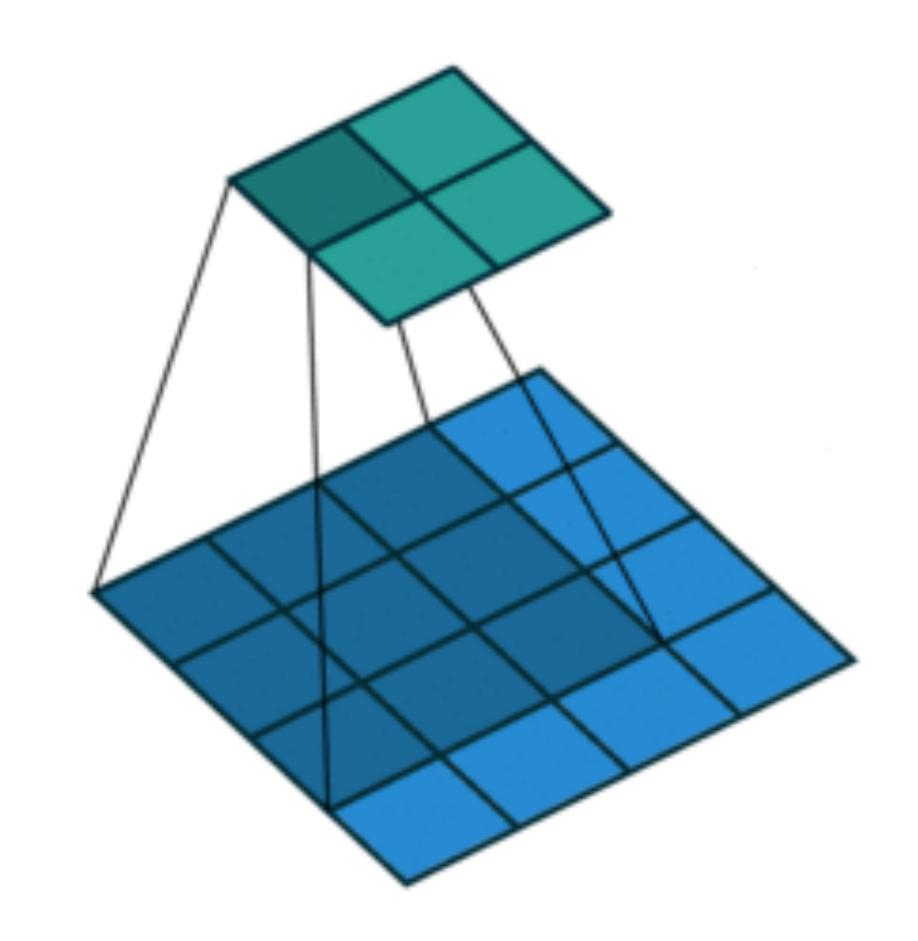
0	1	2
3	4	5
6	7	8

Kernel

Output

$$0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19,$$

 $1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25,$
 $3 \times 0 + 4 \times 1 + 6 \times 2 + 7 \times 3 = 37,$
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(vdumoulin@ Github)

2-D Convolution Layer

	0	1	2					
L	U	ı			0	1	19	25
ı	3	4	5	*		•		
l	O	-		**	2	3	37	43
	6	7	8		_			
		'						

- $\mathbf{X}: n_h \times n_w$ input matrix
- $\mathbf{W}: k_h \times k_w$ kernel matrix
- b: scalar bias
- $\mathbf{Y}: (n_h k_h + 1) \times (n_w k_w + 1)$ output matrix

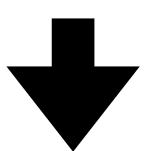
$$Y = X \star W + b$$

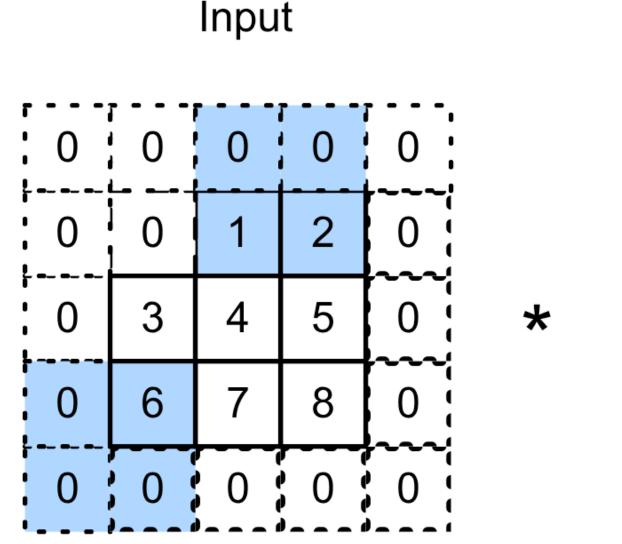
• W and b are learnable parameters

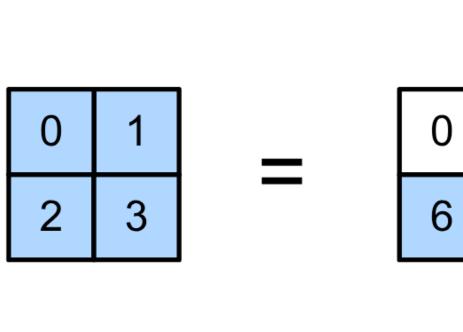
2-D Convolution Layer with Stride and Padding

- Stride is the #rows/#columns per slide
- Padding adds rows/columns around input
- Output shape

Kernel/filter size



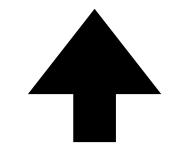


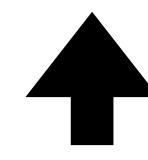


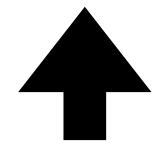
Output

Kernel

$[(n_h - k_h + p_h + s_h)/s_h] \times [(n_w - k_w + p_w + s_w)/s_h]$
--







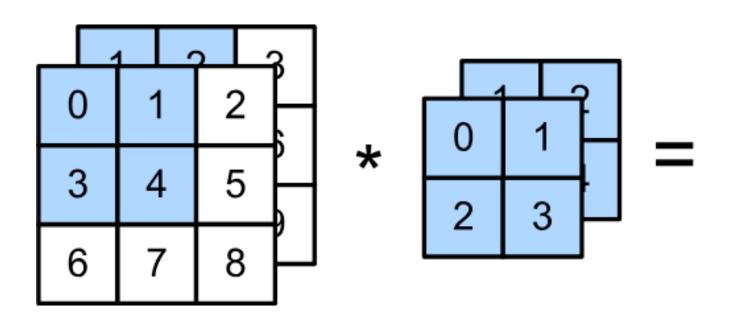
Input size

Pad

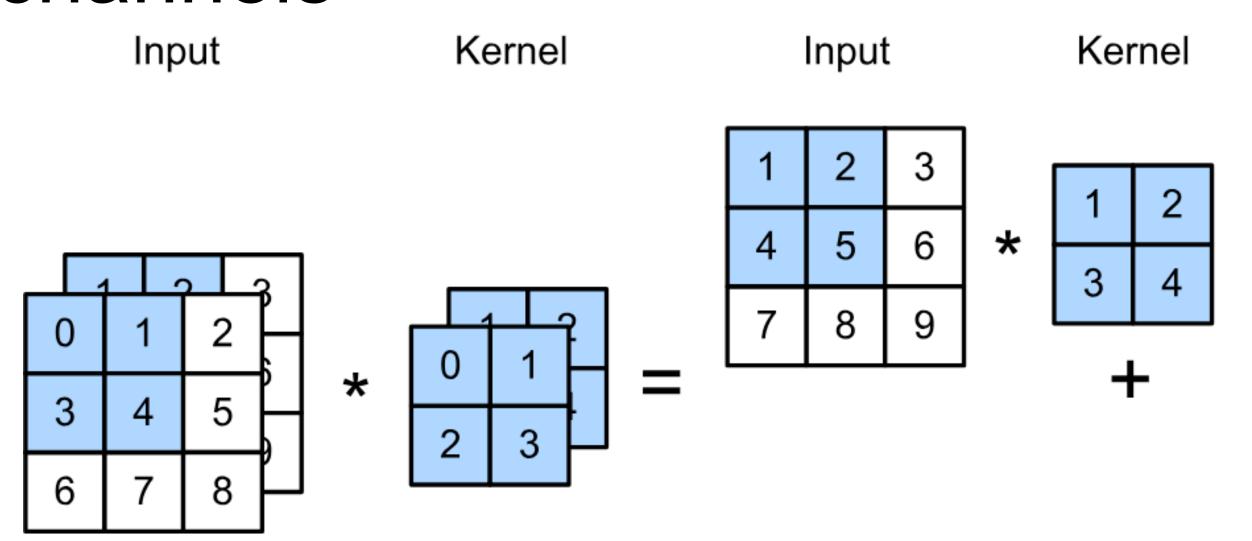
Stride

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a kernel for each channel, and then sum results over channels

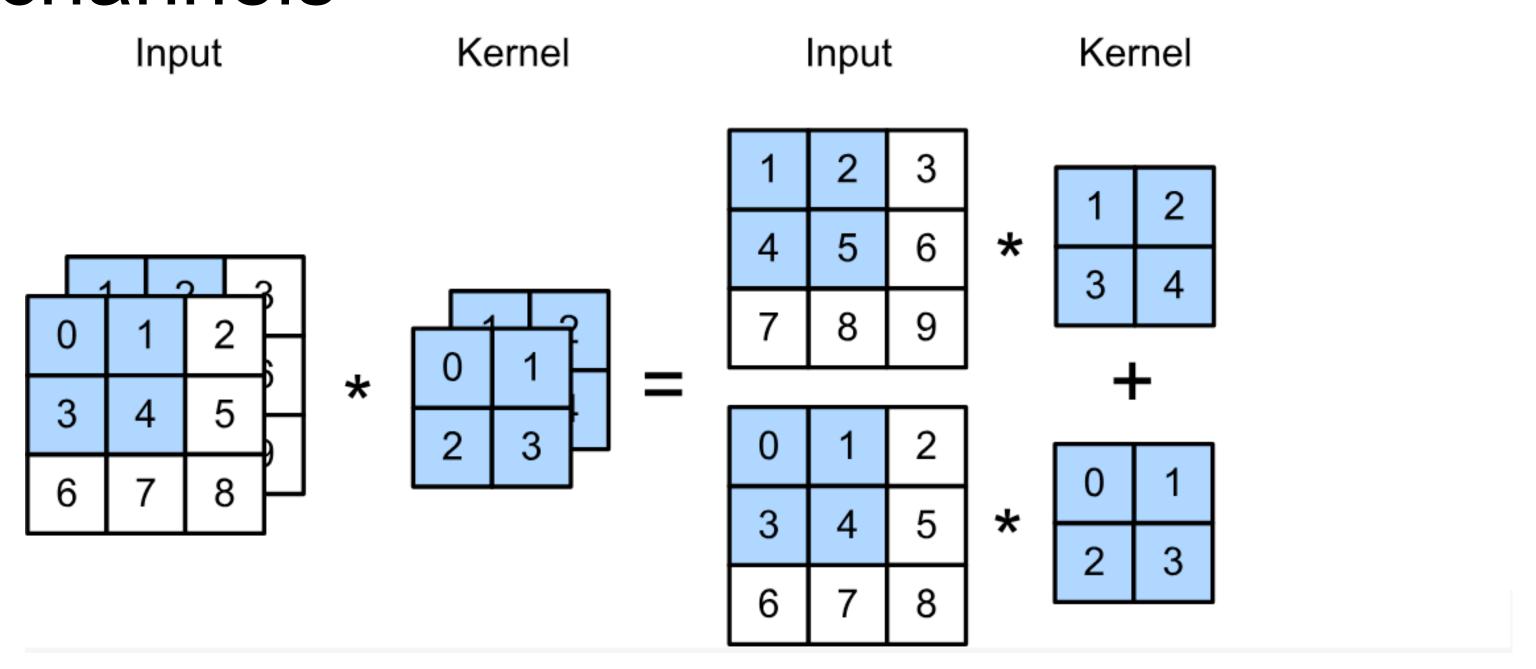
Input Kernel



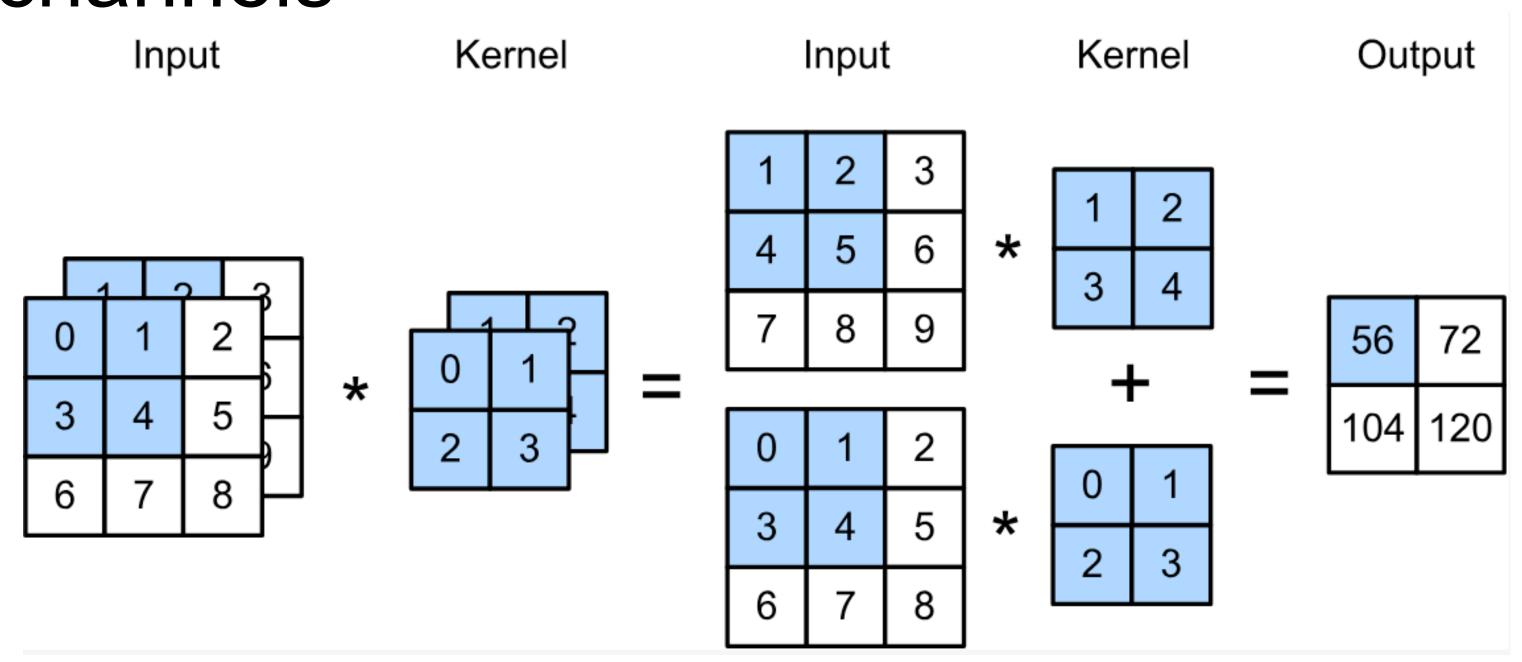
- Input and kernel can be 3D, e.g., an RGB image have 3 channels
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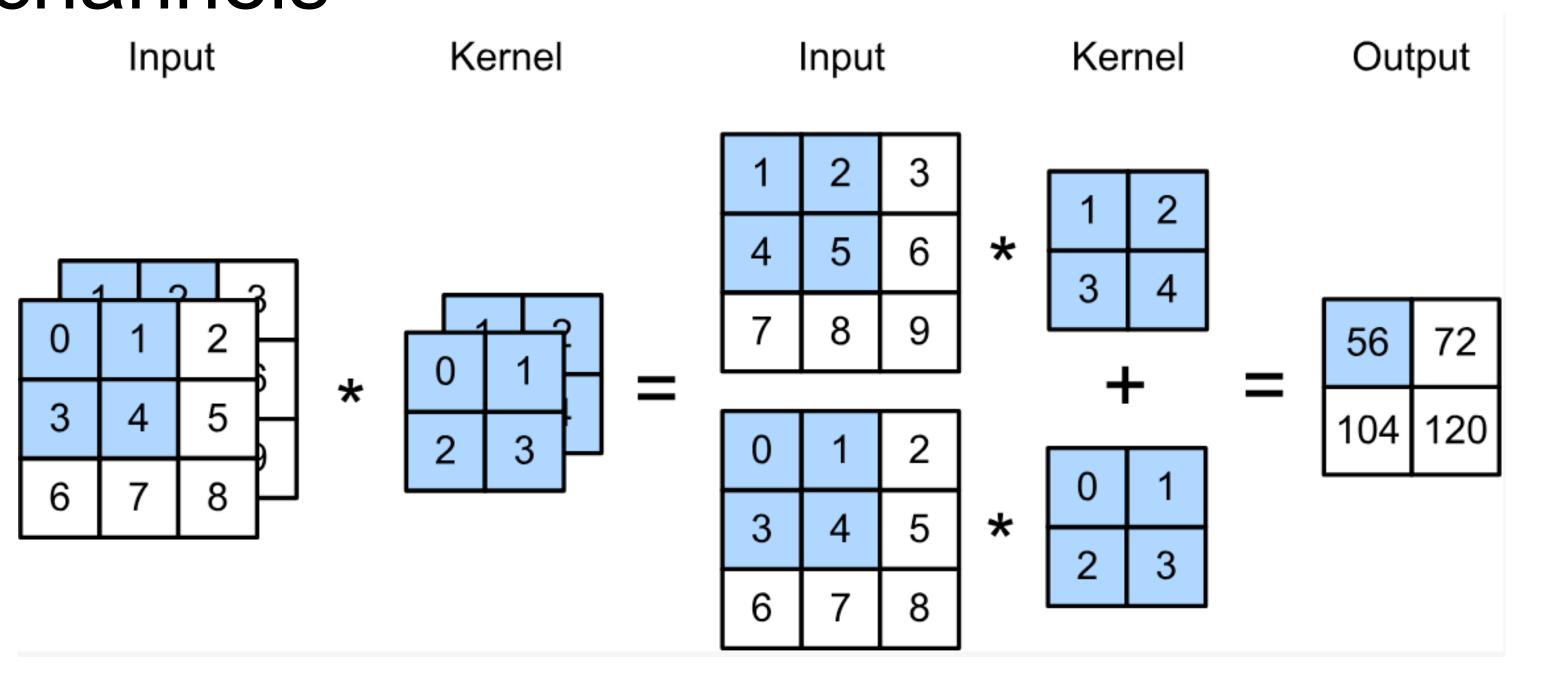
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- Have a kernel for each channel, and then sum results over channels



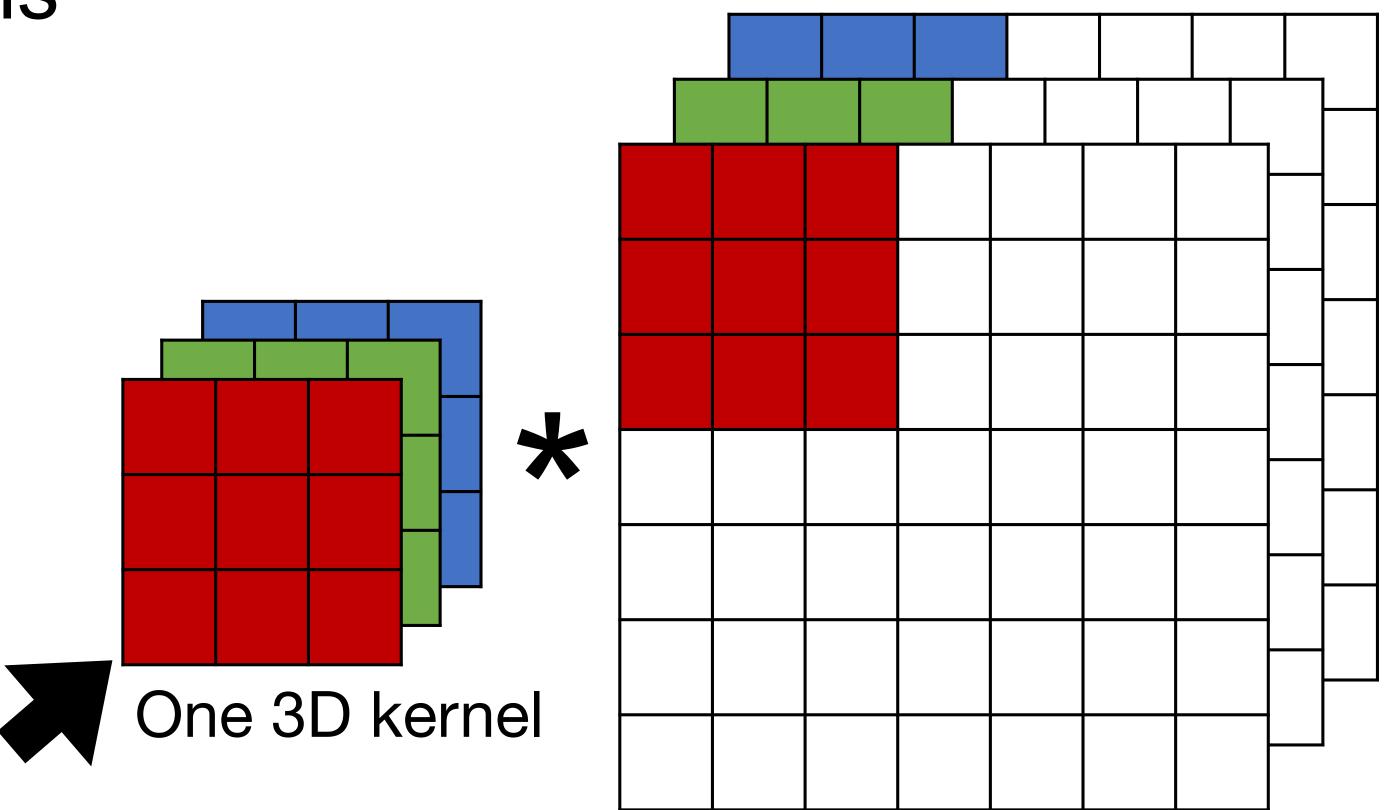
$$(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4)$$

+ $(0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3)$
= 56

Input and kernel can be 3D, e.g., an RGB image have 3 channels

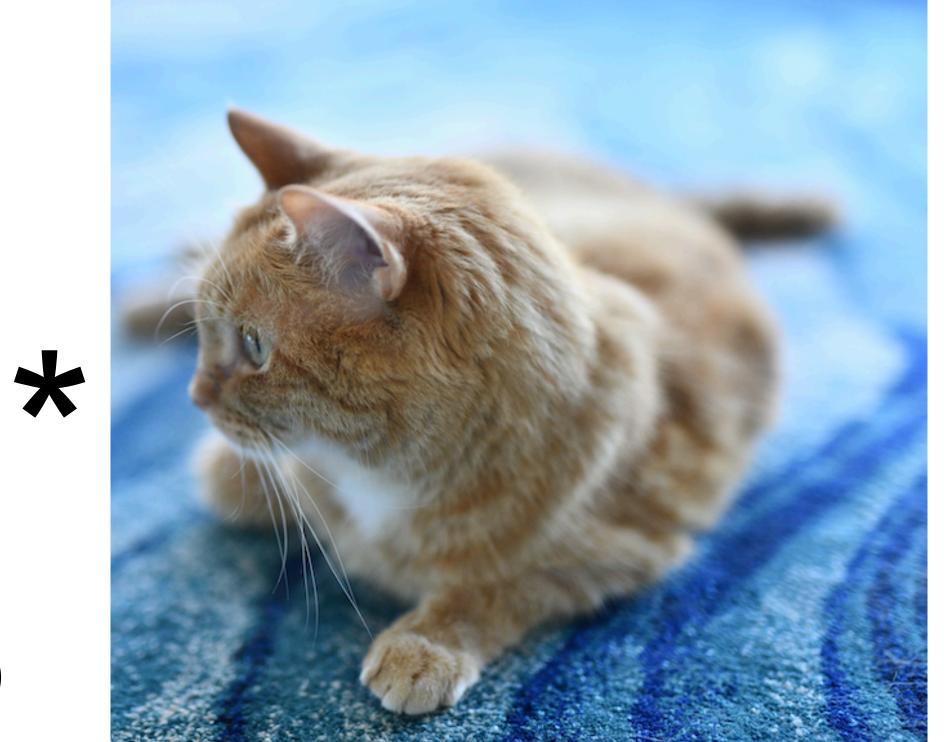
Have a 2D kernel for each channel, and then sum results over

channels



Input and kernel can be 3D, e.g., an RGB image have 3 channels

 Also call each 3D kernel a "filter", which produce only one output channel (due to summation over channels)

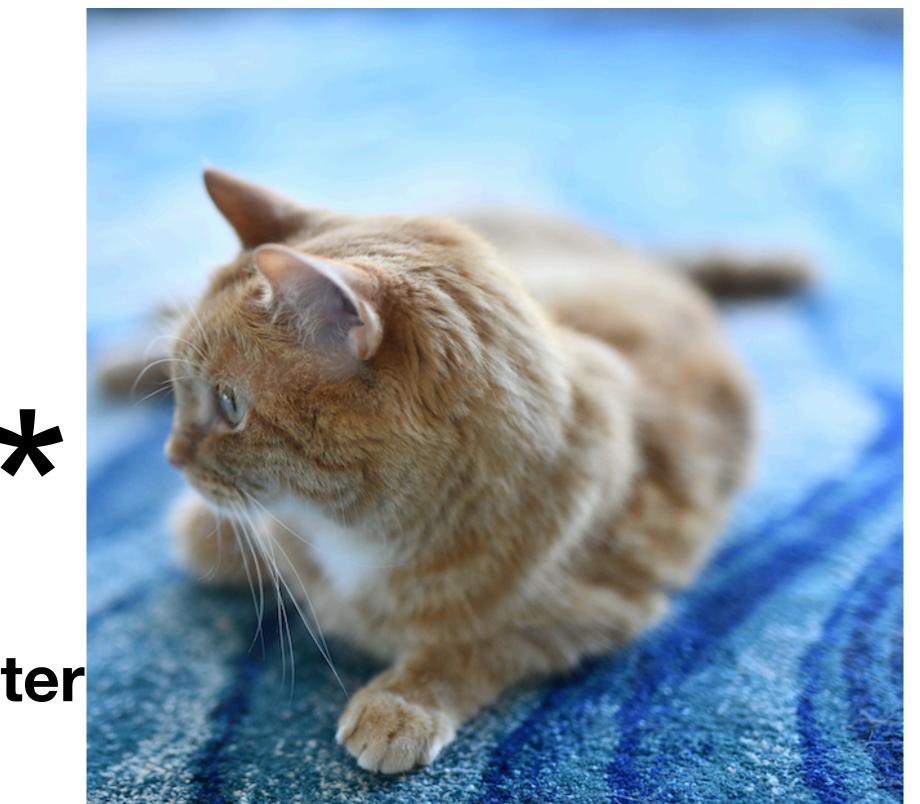




RGB (3 input channels)

Multiple filters (in one layer)

- Apply multiple filters on the input
- Each filter may learn different features about the input
- Each filter (3D kernel) produces one output channel



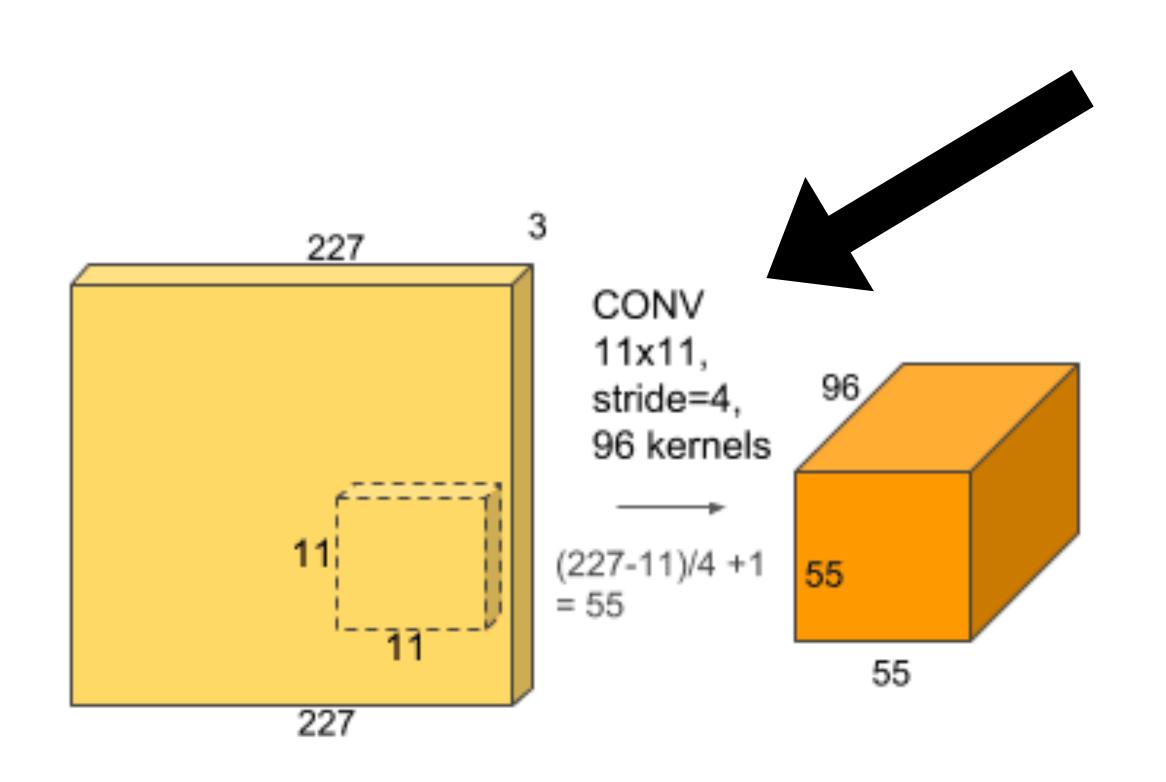


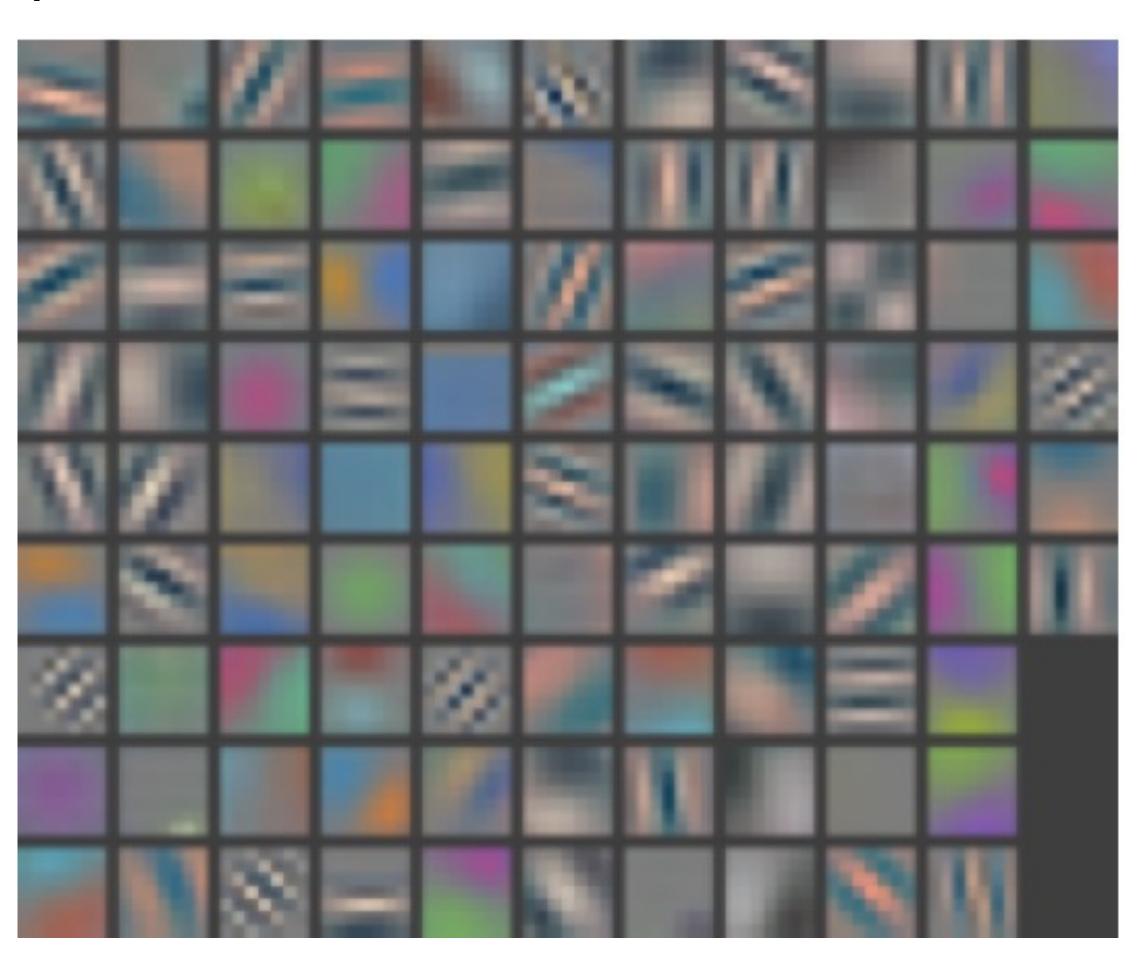
RGB (3 input channels)

Conv1 Filters in AlexNet

• 96 filters (each of size 11x11x3)

Gabor filters





Figures from Visualizing and Understanding Convolutional Networks by *M. Zeiler and R. Fergus*

Multiple Output Channels

- The # of output channels = # of filters
- Input $\mathbf{X}: c_i \times n_h \times n_w$
- Kernel W: $c_o \times c_i \times k_h \times k_w$
- Output $\mathbf{Y}: c_o \times m_h \times m_w$

$$\mathbf{Y}_{i,:,:} = \mathbf{X} \star \mathbf{W}_{i,:,:,:}$$

for
$$i = 1, ..., c_o$$

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Multiple Output Channels

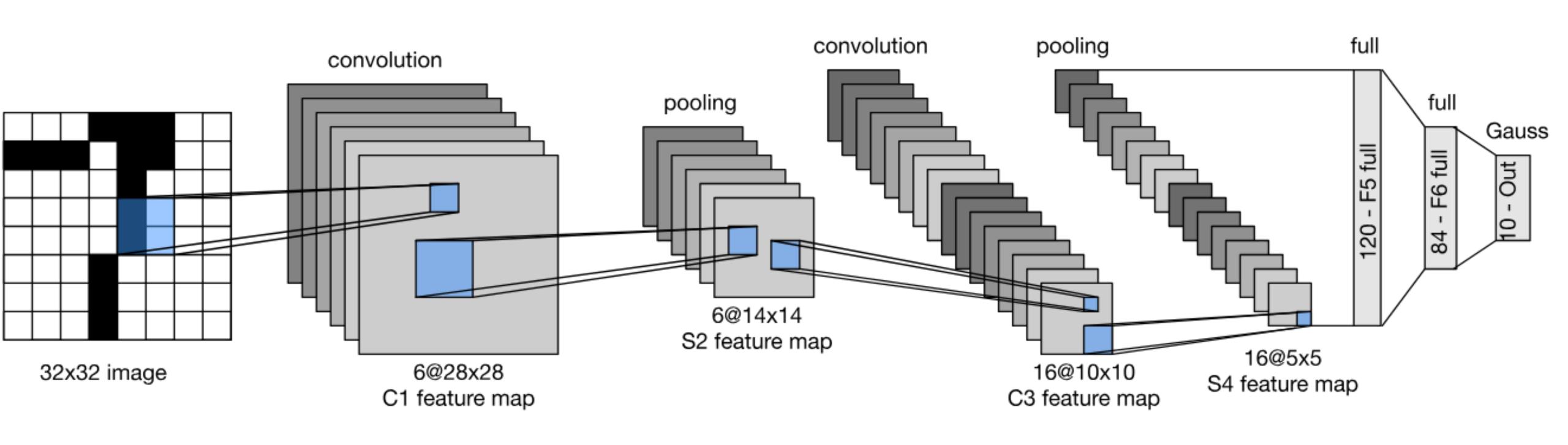
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Convolutional Neural Networks

LeNet Architecture











Y. LeCun, L.
Bottou, Y. Bengio,
P. Haffner, 1998
Gradient-based
learning applied to
document
recognition









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LeNet in Pytorch (HW7)

Connect theory and practice

```
def __init__(self):
    super(LeNet5, self).__init__()
    # Convolution (In LeNet-5, 32x32 images are given as input. Hence padding of 2 is done below)
    self.conv1 = torch.nn.Conv2d(in_channels=1, out_channels=6, kernel_size=5, stride=1, padding=2, bias=True)
    # Max-pooling
    self.max_pool_1 = torch.nn.MaxPool2d(kernel_size=2)
    # Convolution
    self.conv2 = torch.nn.Conv2d(in_channels=6, out_channels=16, kernel_size=5, stride=1, padding=0, bias=True)
    # Max-pooling
    self.max_pool_2 = torch.nn.MaxPool2d(kernel_size=2)
    # Fully connected layer
    self.fc1 = torch.nn.Linear(16*5*5, 120) # convert matrix with 16*5*5 (= 400) features to a matrix of 120 features (col
    self.fc2 = torch.nn.Linear(120, 84)
                                              # convert matrix with 120 features to a matrix of 84 features (columns)
    self.fc3 = torch.nn.Linear(84, 10)
                                              # convert matrix with 84 features to a matrix of 10 features (columns)
```

Which one of the following is NOT true?

- A. LeNet has two convolutional layers
- B. The first convolutional layer in LeNet has 5x5x6x3 parameters, in case of RGB input
- C. Pooling is performed right after convolution
- D. Pooling layer does not have learnable parameters

Which one of the following is NOT true?

- A. LeNet has two convolutional layers
- B. The first convolutional layer in LeNet has 5x5x6x3 parameters, in case of RGB input
- C. Pooling is performed right after convolution
- D. Pooling layer does not have learnable parameters

Pooling is performed after ReLU: conv->relu->pooling

You have a kernel (filter) and a given image. What filter map (output) is obtained from applying the kernel to the image without padding and zero bias?

3	2	1
3	3	1
1	2	1

Image

1	2
2	1

Kernel

You have a kernel (filter) and a given image. What filter map (output) is obtained from applying the kernel to the image without padding and zero bias?

3	2	1
3	3	1
1	2	1

Image

$$3*1 + 2*2 + 3*2 + 3*1 = 16$$

 $2*1 + 1*2 + 3*2 + 1*1 = 11$
 $3*1 + 3*2 + 1*2 + 2*1 = 13$
 $3*1 + 1*2 + 2*2 + 1*1 = 10$

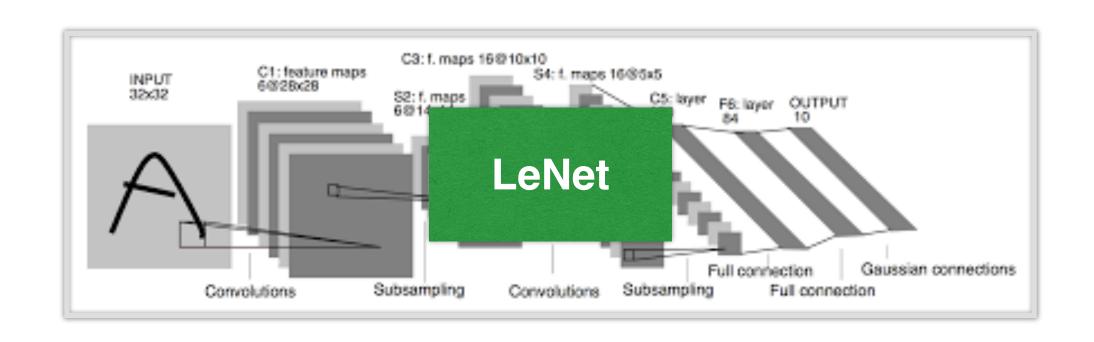
1	2
2	1

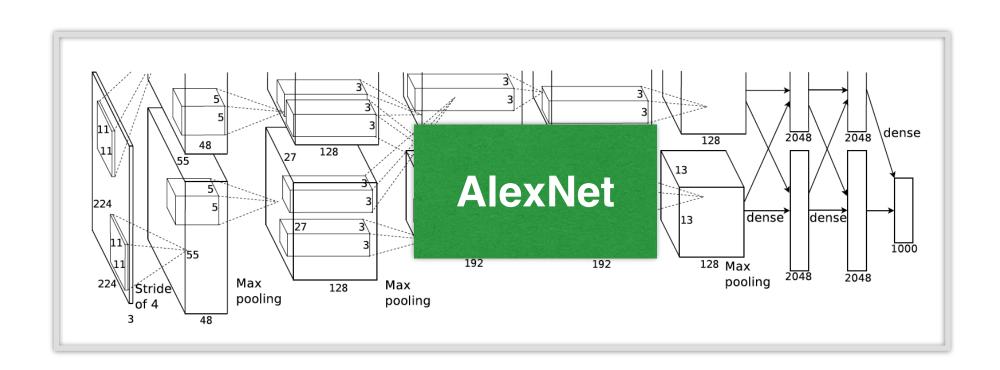
Kernel

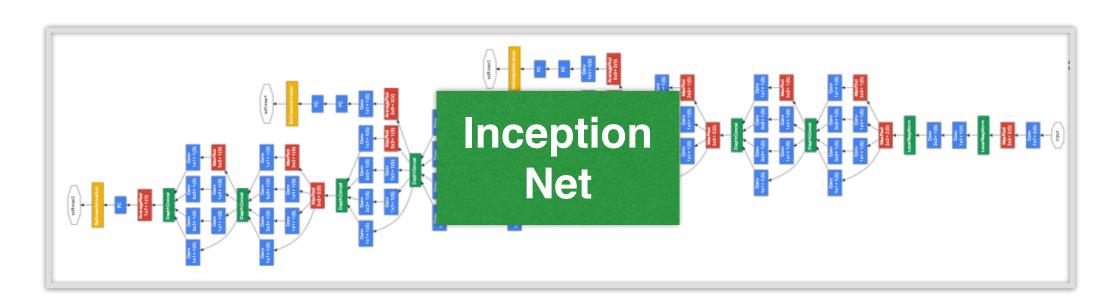
16	11
13	10

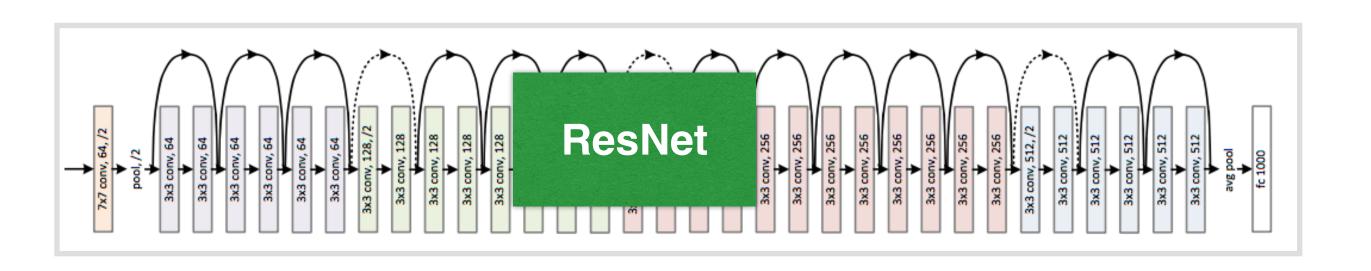
Evolution of neural net architectures

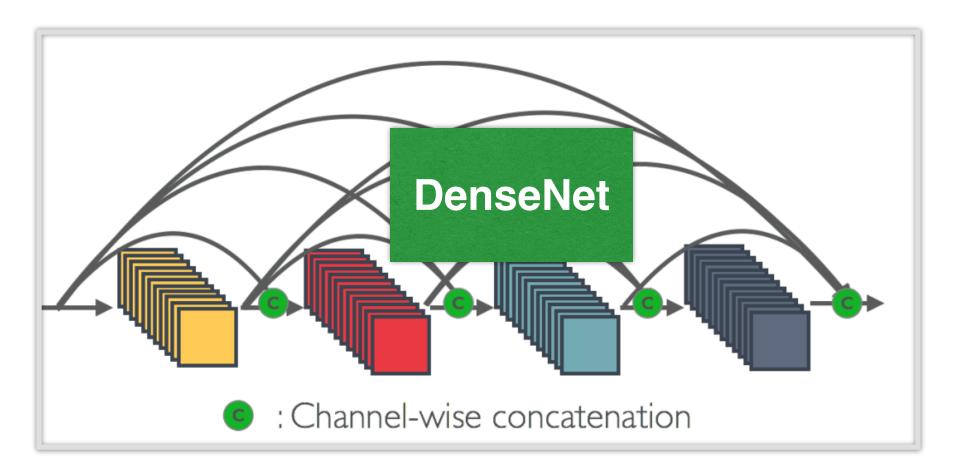
Evolution of neural net architectures







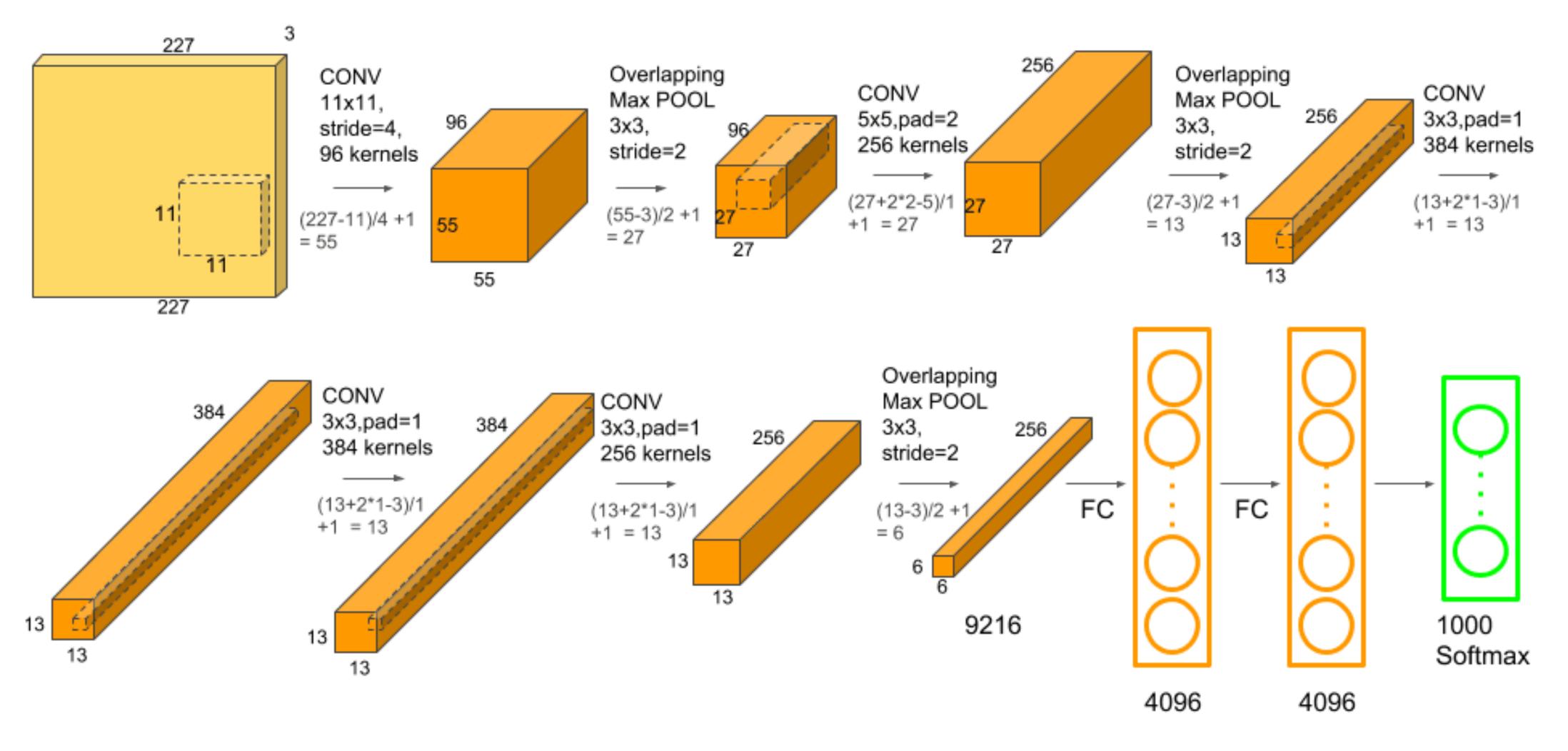






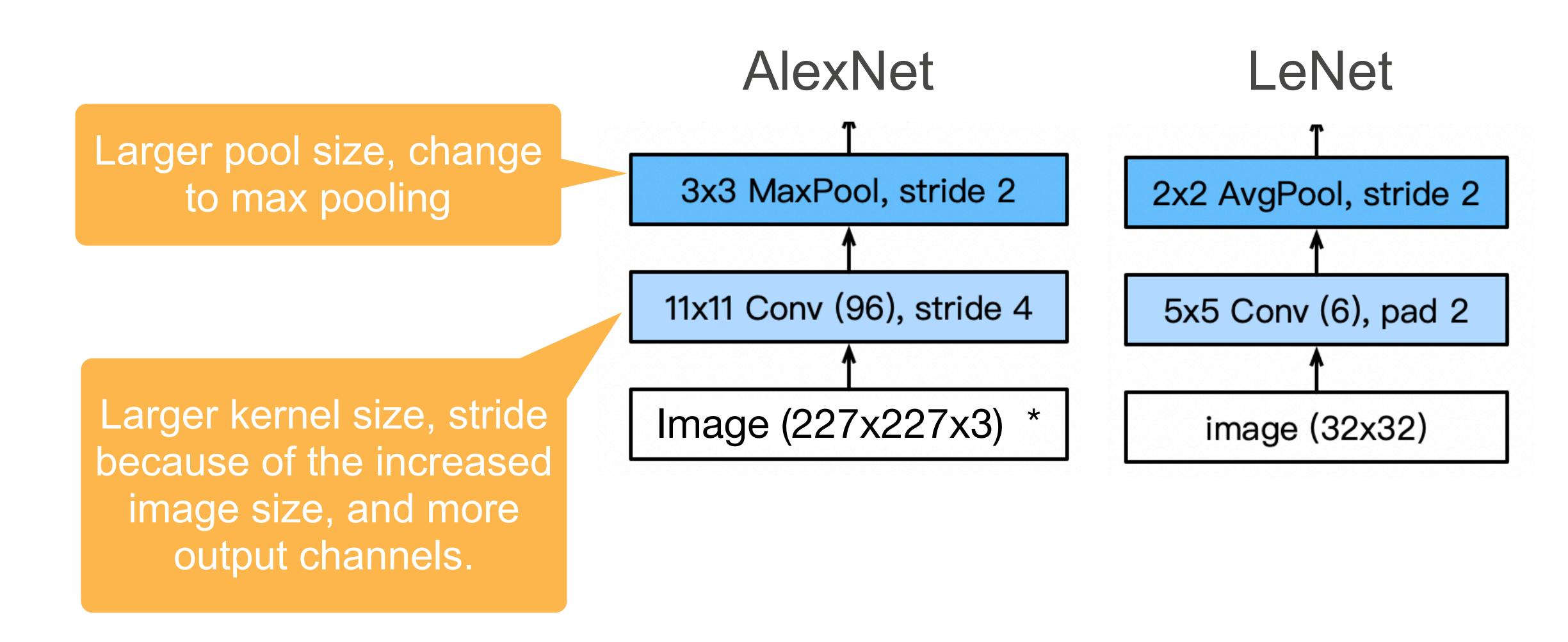
Deng et al. 2009

AlexNet



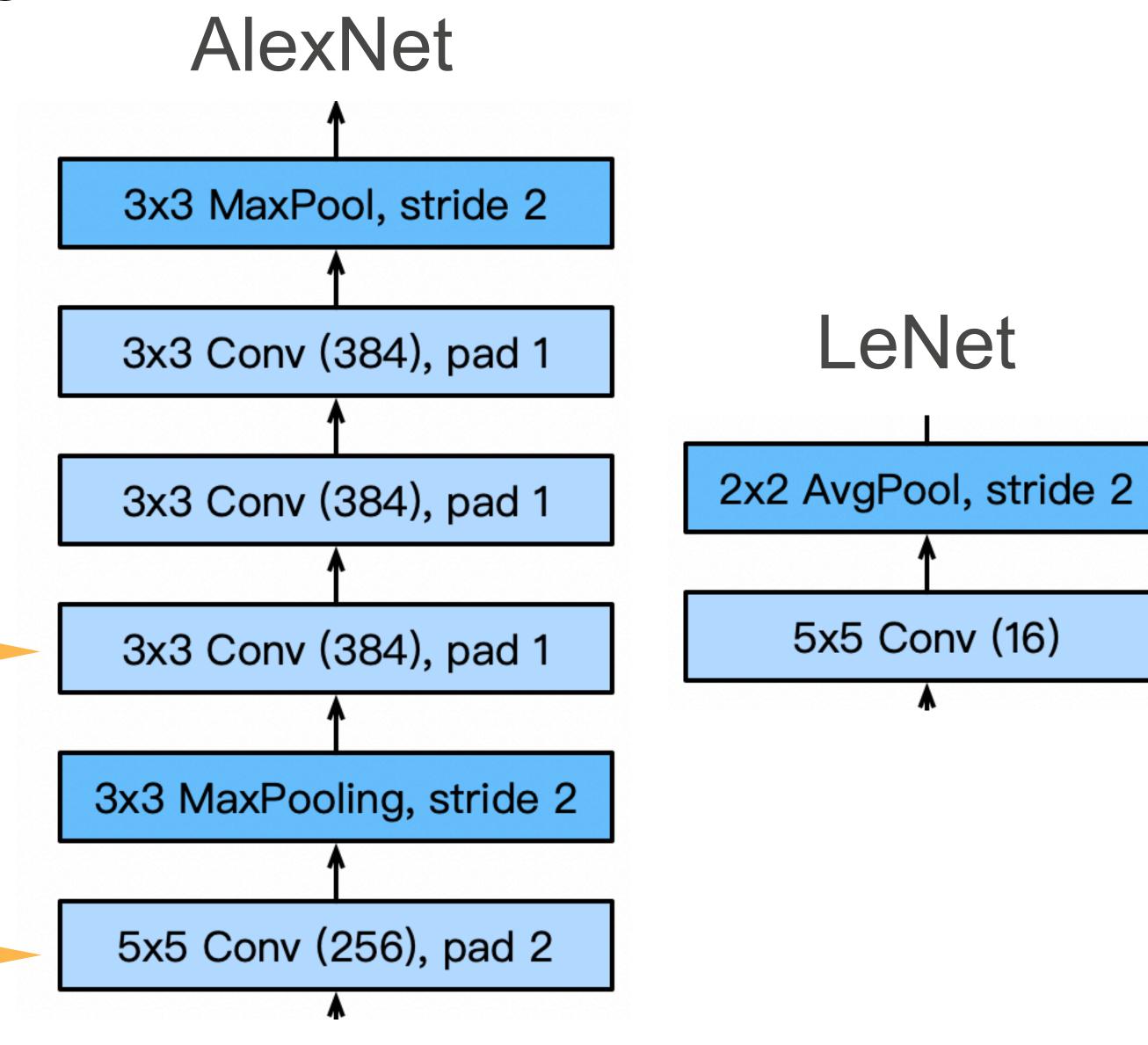
[Krizhevsky et al. 2012]

AlexNet vs LeNet Architecture



*Note that the original paper used 224x224x3, which was incorrect

AlexNet Architecture

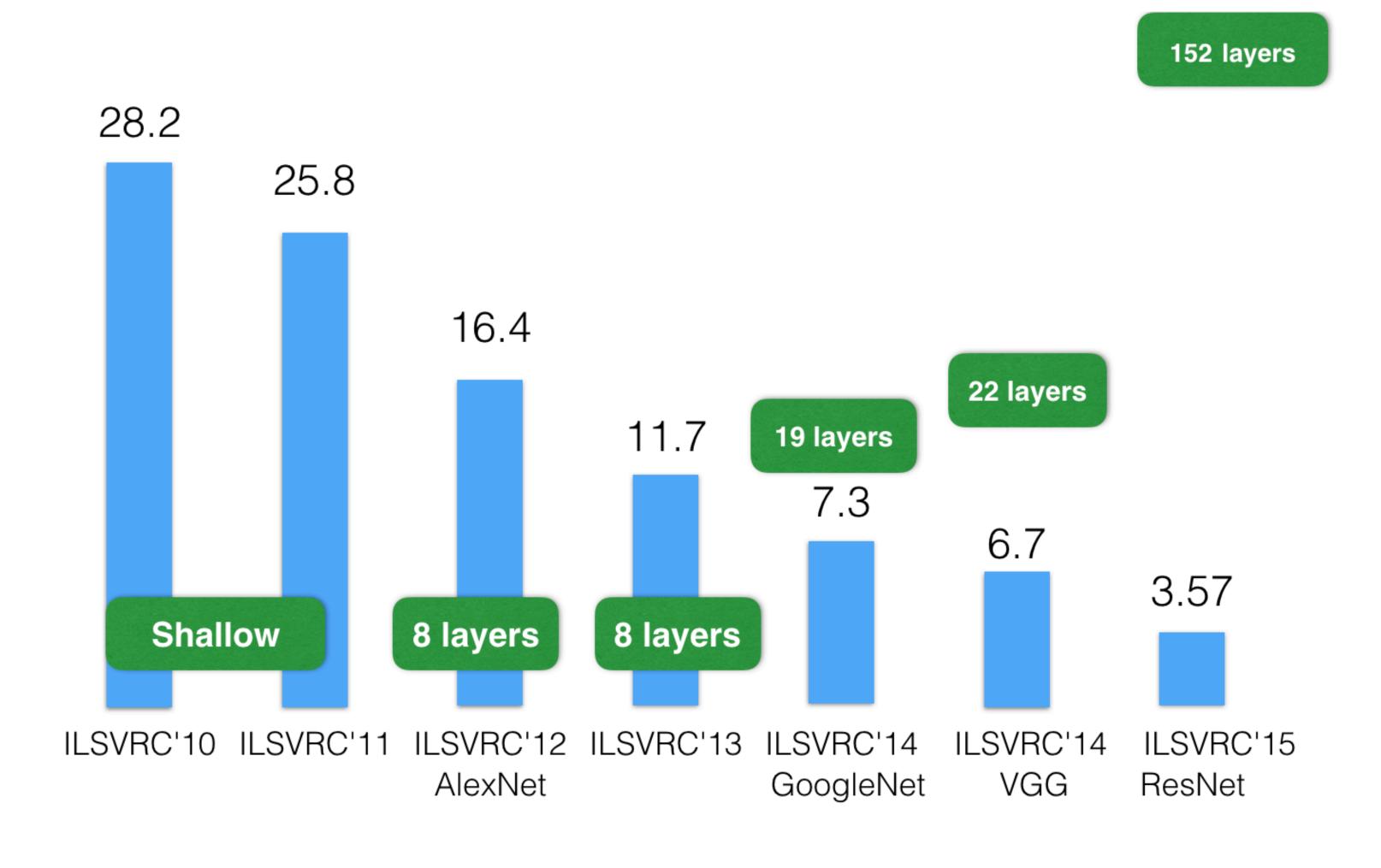


More output channels.

3 additional

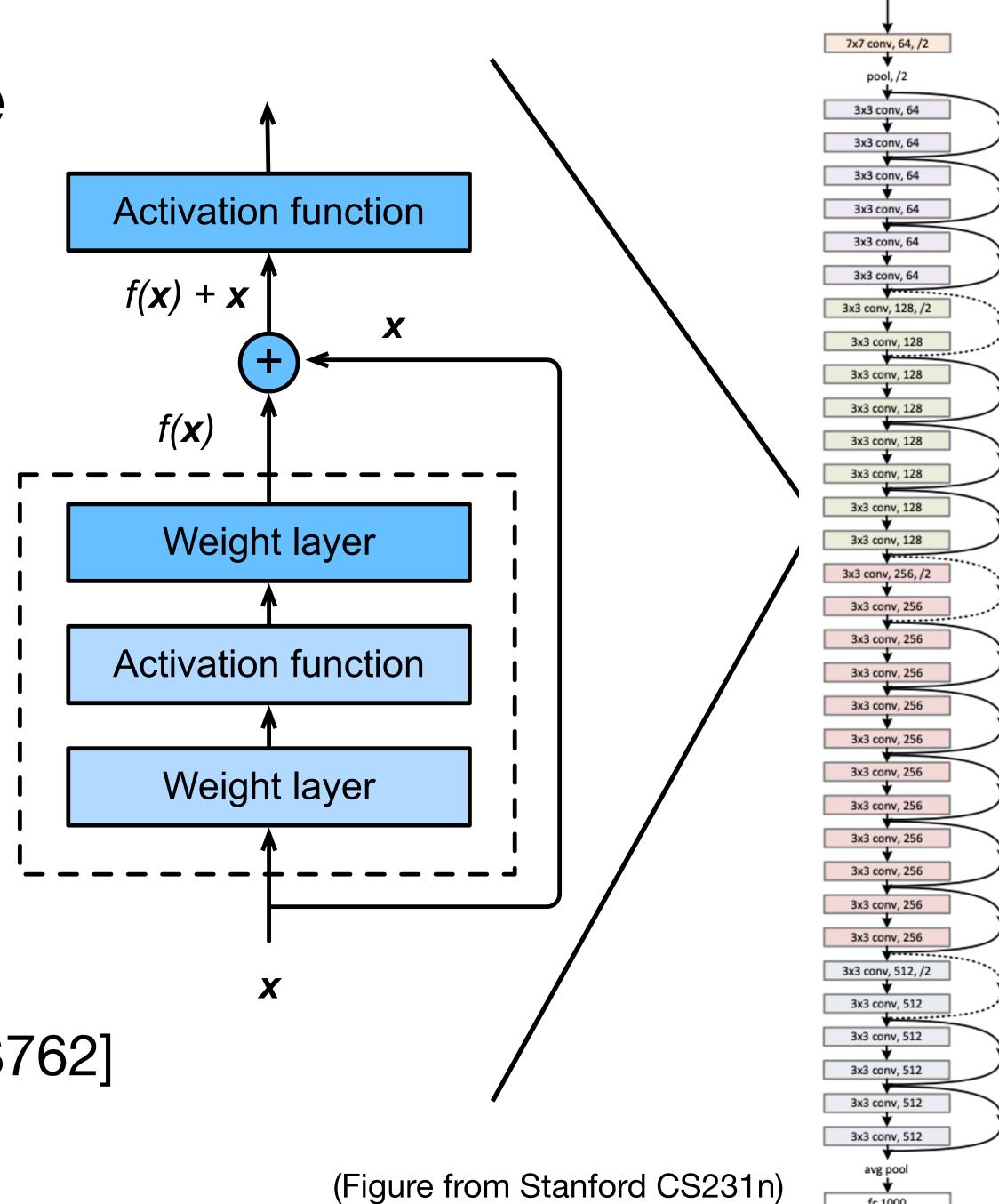
convolutional layers

ResNet: Going deeper in depth



ImageNet Top-5 error%

[He et al. 2015]

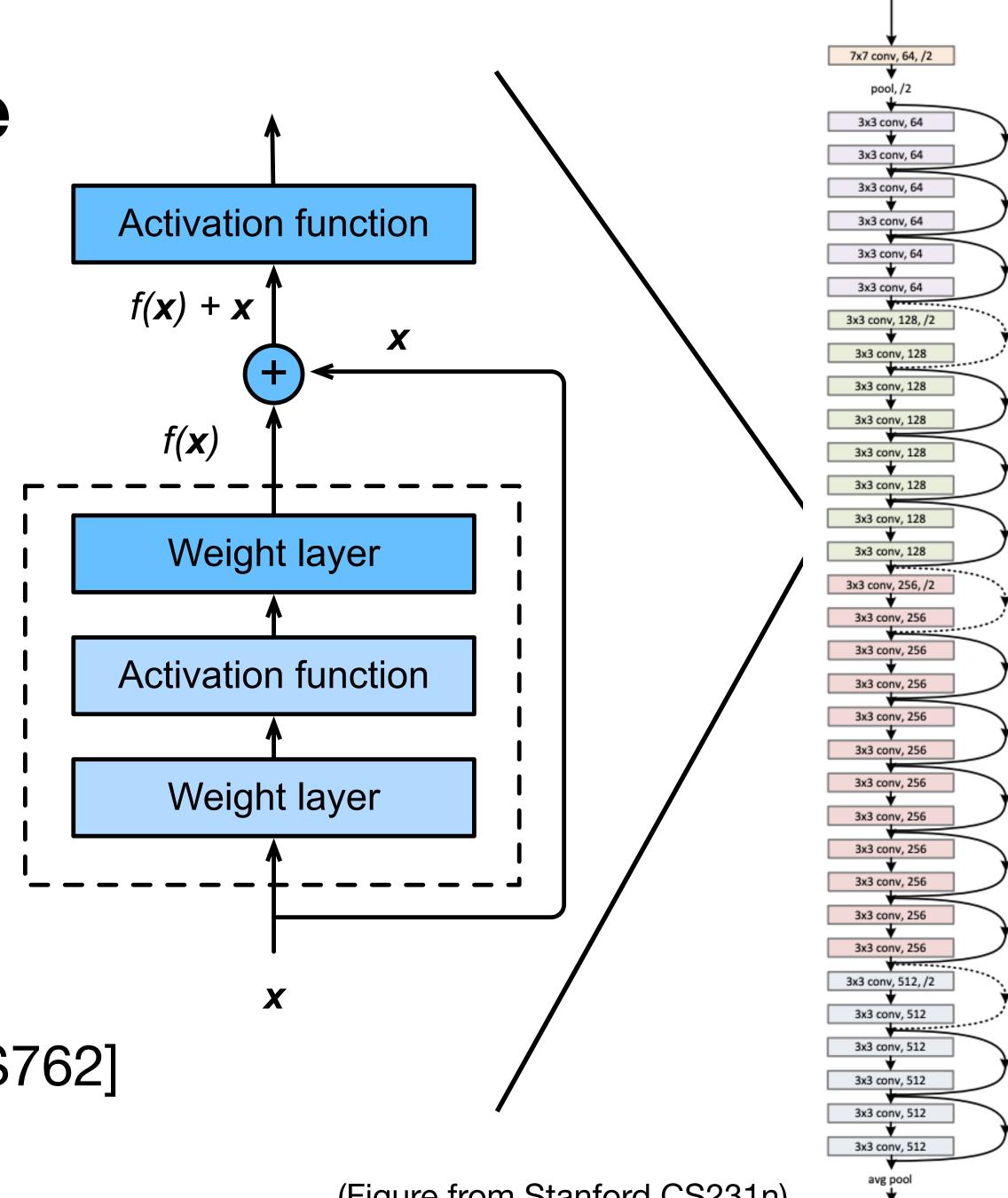


fc 1000

[More advanced topics covered in CS762]

[He et al. 2015]

Stack residual blocks



[More advanced topics covered in CS762]

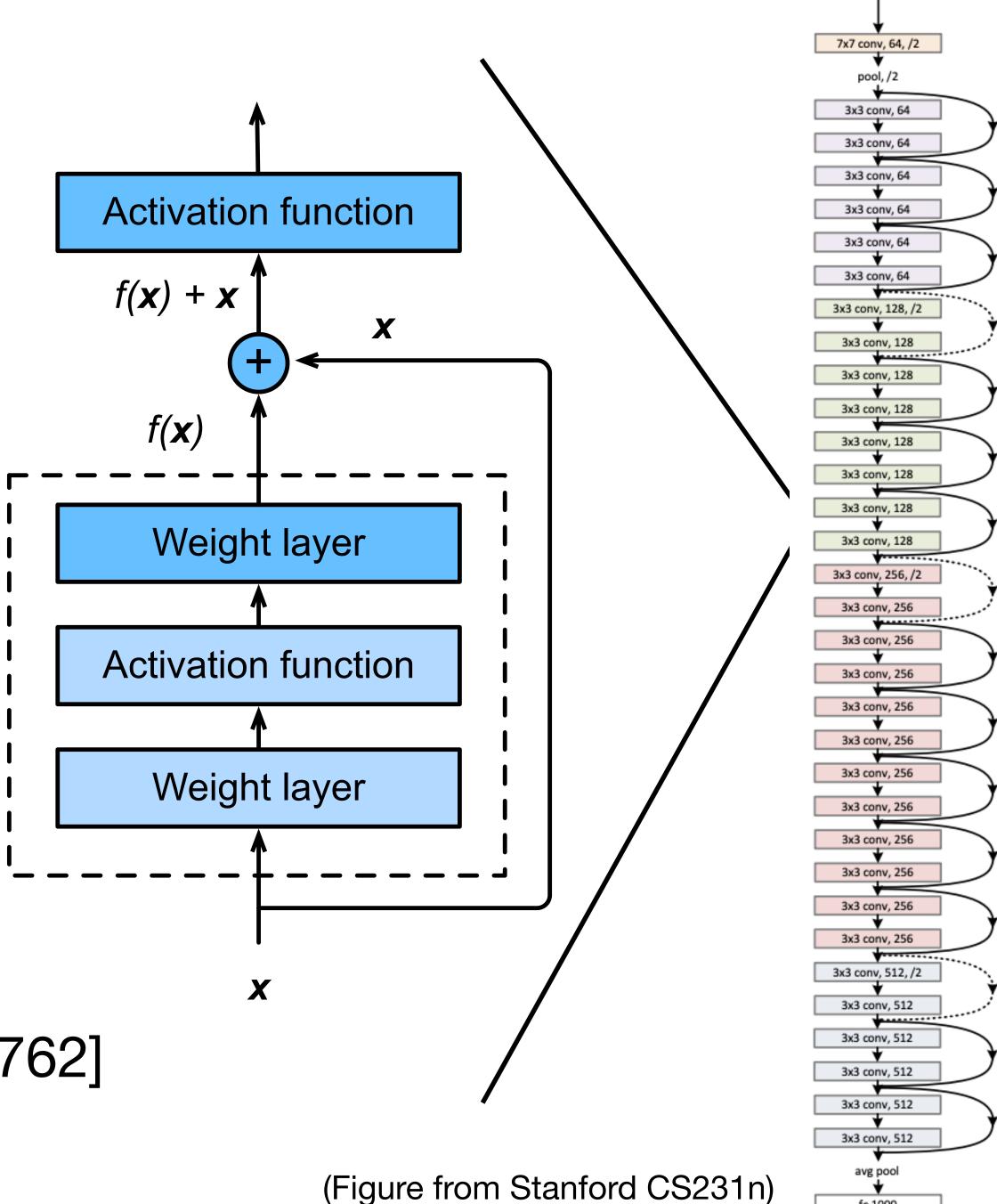
(Figure from Stanford CS231n)

fc 1000

[He et al. 2015]

Stack residual blocks

 Every residual block has two 3x3 [conv layers



fc 1000

[More advanced topics covered in CS762]

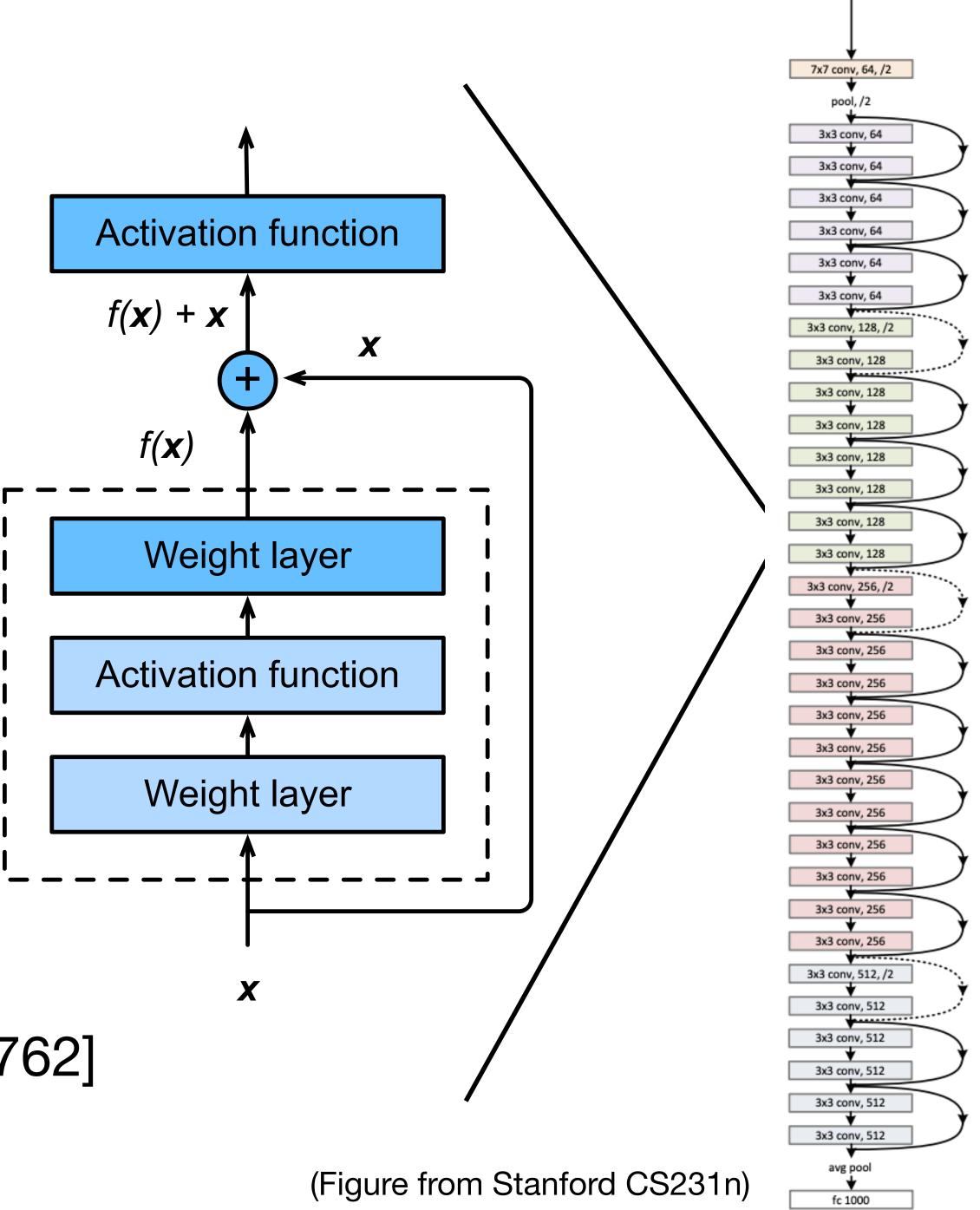
[He et al. 2015]

Stack residual blocks

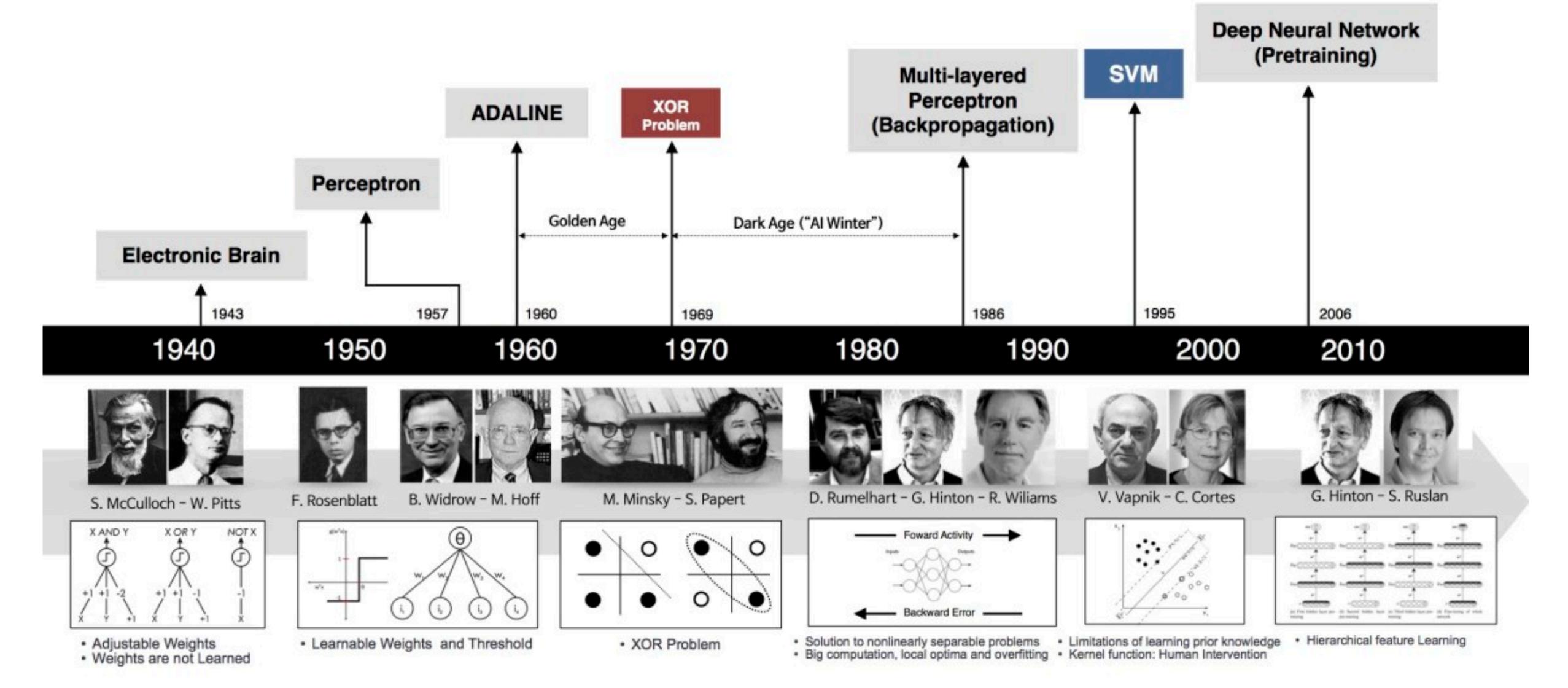
 Every residual block has two 3x3 { conv layers

• Periodically, double # of filters and downsample spatially using stride of 2 (/2 in each dimension)

[More advanced topics covered in CS762]



Brief history of neural networks



Modeling a single neuron

- Modeling a single neuron
 - Linear perceptron

- Modeling a single neuron
 - Linear perceptron
 - Limited power of a single neuron

- Modeling a single neuron
 - Linear perceptron
 - Limited power of a single neuron
- Multi-layer perceptron

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 - Linear perceptron
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- Training of neural networks

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 - Loss function (cross entropy)

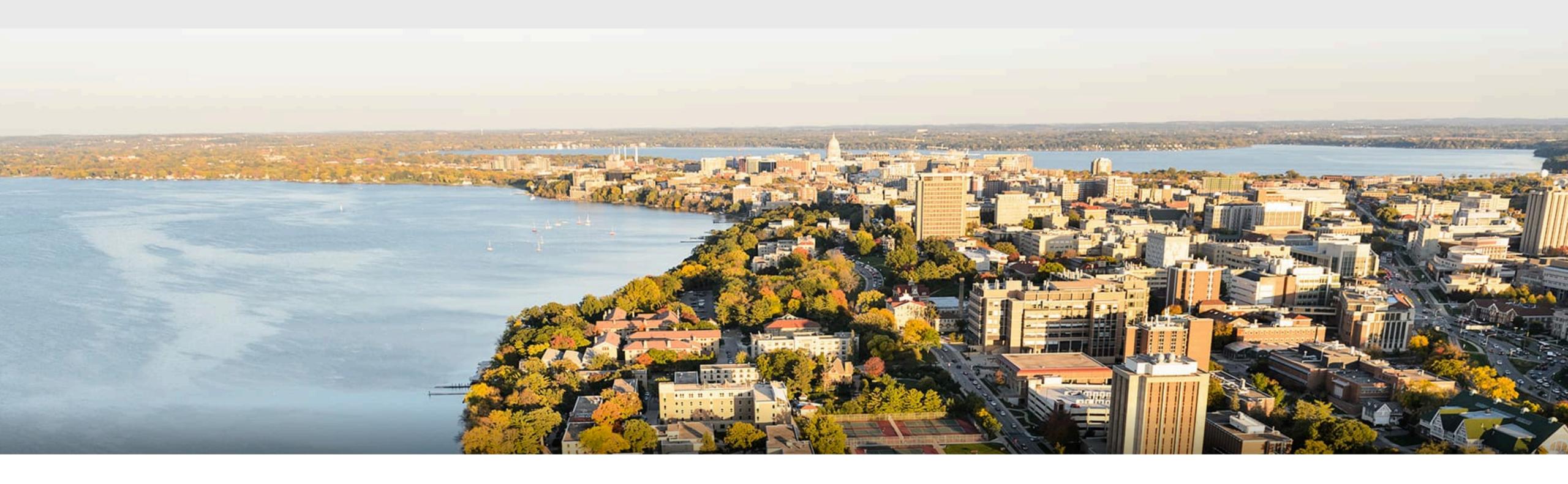
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- Convolutional neural networks
 - Convolution, pooling, stride, padding
 - Basic architectures (LeNet etc.)
 - More advanced architectures (AlexNet, ResNet etc)



Thank you!

Some of the slides in these lectures have been adapted from materials developed by Alex Smola and Mu Li: https://courses.d2l.ai/berkeley-stat-157/index.html