CS 540 Introduction to Artificial Intelligence
Logic

Josiah Hanna
University of Wisconsin-Madison

September 23, 2021

Announcements

- **Homeworks**:
  – HW2 due Tuesday before class
- Roadmap

Date	Topic	Reading materials	Assignments
Thursday, Sept 9	Welcome and Course Overview	Slides	
Tuesday, Sept 14	Probability	Slides	HW 1 Released
Thursday, Sept 16	Linear Algebra and PCA	Slides	
Tuesday, Sept 21	Statistics and Math Review	Slides	HW 1 Due, HW 2 Released
Thursday, Sept 23	Introduction to Logic	Slides	
Tuesday, Sept 28	Natural Language Processing		HW 2 Due, HW 3 Released
Thursday, Sept 30	Machine Learning: Introduction		

Homework Review: Classification

- Expression

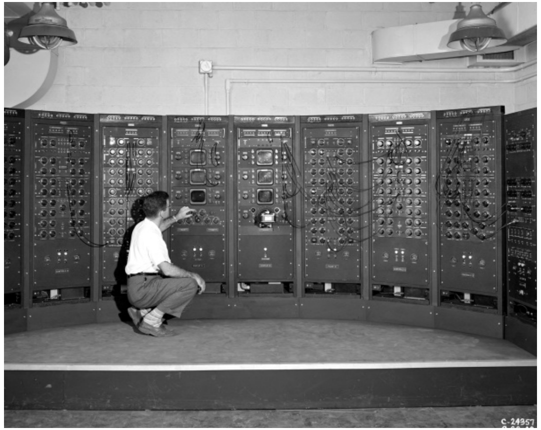
$$P(H|E_1, E_2, \ldots, E_n) = \frac{P(E_1|H)P(E_2|H)\cdots, P(E_n|H)P(H)}{P(E_1, E_2, \ldots, E_n)}$$

- *H*: some class we'd like to infer from evidence
  - Estimate prior *P(H)*
  - Estimate $P(E_i|H)$ from data!
  - Empirical count-based estimates

Logic & AI

Why are we studying logic?

- **Traditional** approach to AI ('50s-'80s)
  - "Symbolic AI"
  - The Logic Theorist – 1956
    - Proved a bunch of theorems!
- Logic also the language of:
  - Knowledge rep., databases, etc.



Symbolic Techniques in AI

Lots of systems based on symbolic approach:

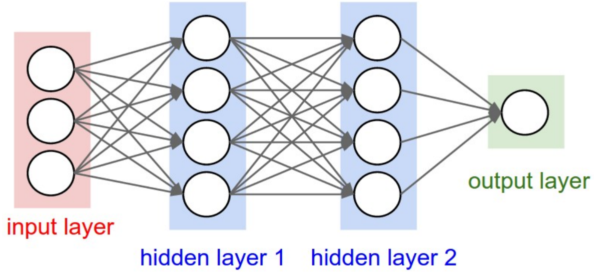
- Ex: expert systems, planning, more
- Playing great chess
- **Less popular recently**!
- “Good old fashioned AI”

J. Gardner

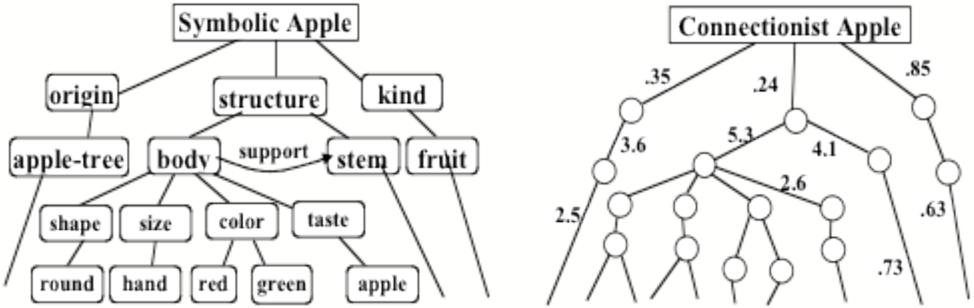
Symbolic vs Subsymbolic

Rival approach: **subsymbolic**

- Probabilistic models
- Neural networks
- **Extremely popular** last 20 years



Stanford CS231n



M. Minsky

Symbolic vs Subsymbolic

- Easier to debug
- Easier to explain
- No need for big data
- Better for abstract problems
  - Symbol grounding problem

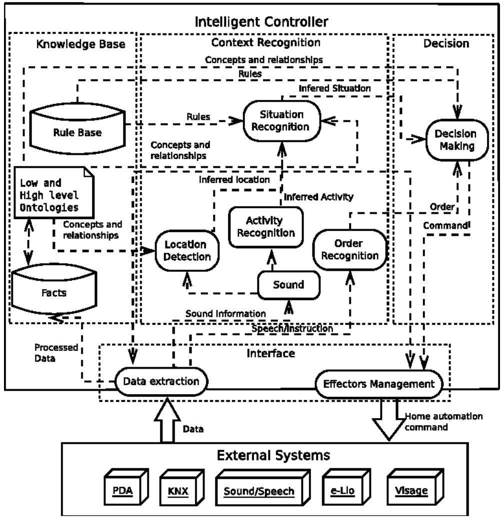
- Robust to noise
- Less built-in knowledge
- Easier to scale
- Better for perceptual problems

Credit: Henry Lieberman

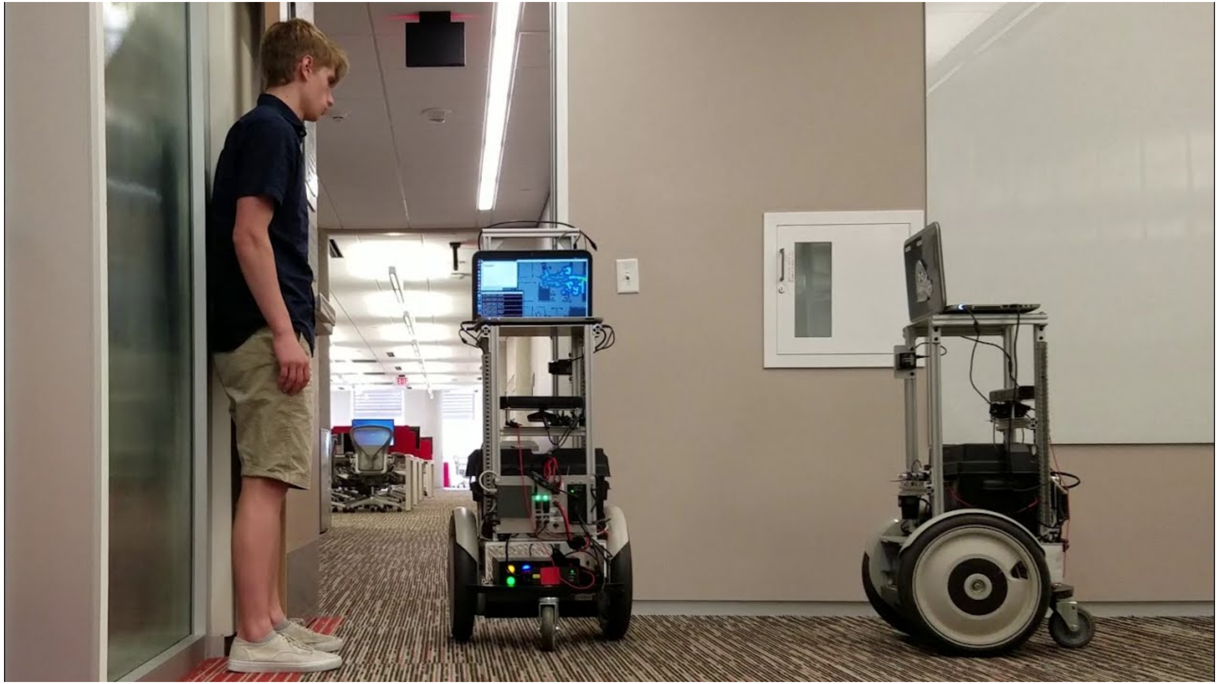
Symbolic vs Subsymbolic

Analogy: Logic versus probability

- Which is better?
- Future: combination; best-of-both-worlds
  - Actually been worked on:
  - **Example**: Markov Logic Networks



Symbolic and Connectionist



Outline

- Introduction to logic
  - Arguments, validity, soundness
- Propositional logic
  - Sentences, semantics, inference
- First order logic (FOL)
  - Predicates, objects, formulas, quantifiers


BEGRIFFSSCHRIFT,

EINE DER ARITHMETISCHEN NACHGEBILDETE

FORMELSPRACHE

DES REINEN DENKENS.

Dr. GOTTLOB FREGE,

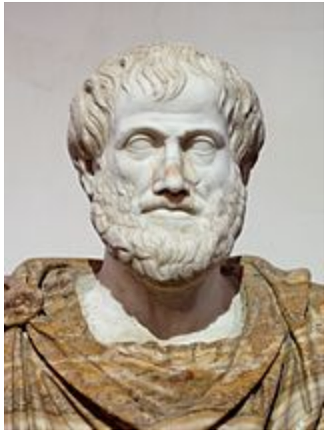
HALLE a/S.

VERLAG VON LOUIS NEBERT.

1879.

Basic Logic

- Arguments, premises, conclusions
  - Argument: a set of sentences (premises) + a sentence (a conclusion)
  - **Validity:** argument is valid iff it's necessary that if all premises are true, the conclusion is true
  - **Soundness:** argument is sound iff valid & premises true
  - **Entailment:** when valid arg., premises entail conclusion



Propositional Logic Basics

Logic Vocabulary:

- Sentences, symbols, connectives, parentheses
  - Symbols: P, Q, R, … (**atomic** sentences)
  - Connectives:

$\wedge$	and	[conjunction]
$\vee$	or	[disjunction]
$\Rightarrow$	implies	[implication]
$\Leftrightarrow$	is equivalent	[biconditional]
$\neg$	not	[negation]

  - Literal: P or negation $\neg$P

Propositional Logic Basics

Examples:

- $(P \vee Q) \Rightarrow S$
  - "If it is cold or it is raining, then I need a jacket"
- $Q \Rightarrow P$
  - "If it is raining, then it is cold"
- $\neg R$
  - "It is not hot"

Propositional Logic Basics

Several rules in place

- Precedence: ¬, ∧, ∨, ⇒, ⇔
- Use parentheses when needed
- Sentences: **well-formed** or not well-formed:
  - P ⇒ Q ⇒ S   **X (not associative!)**

Sentences & Semantics

- Think of symbols as defined by user
- Sentences: built up from symbols with connectives

  - **Interpretation**: assigning True / False to symbols
  - **Semantics**: interpretations for which sentence evaluates to True
  - **Model**: (of a set of sentences) interpretation for which all sentences are True

Evaluating a Sentence

- Example:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

- Note:
  - If P is false, P⇒Q is true regardless of Q ("5 is even implies 6 is odd" is True!)
  - Causality unneeded: "5 is odd implies the Sun is a star" is True!)

Evaluating a Sentence: Truth Table

- **Ex**:

P	Q	R	¬P	Q∧R	¬P∨Q∧R	¬P∨Q∧R⇒Q
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	0	0	0	1
1	1	0	0	0	0	1
1	1	1	0	1	1	1

- **Satisfiable**
  - There exists some interpretation where sentence true

Break & Quiz

Q 1.1: Suppose P is false, Q is true, and R is true. Does this assignment satisfy

(i) $\neg(\neg p \rightarrow \neg q) \wedge r$

(ii) $(\neg p \vee \neg q) \rightarrow (p \vee \neg r)$

- A. Both
- B. Neither
- C. Just (i)
- D. Just (ii)

Break & Quiz

Q 1.1: Suppose P is false, Q is true, and R is true. Does this assignment satisfy

(i) $\neg(\neg p \rightarrow \neg q) \wedge r$

(ii) $(\neg p \vee \neg q) \rightarrow (p \vee \neg r)$

- A. Both
- B. Neither
- **C. Just (i)**
- D. Just (ii)

Break & Quiz

Q 1.2: Let A =“Aldo is Italian” and B =“Bob is English”. Formalize “Aldo is Italian or if Aldo isn’t Italian then Bob is English”.

- a. $A \vee (\neg A \rightarrow B)$
- b. $A \vee B$
- c. $A \vee (A \rightarrow B)$
- d. $A \rightarrow B$

Break & Quiz

Q 1.2: Let A =“Aldo is Italian” and B =“Bob is English”. Formalize “Aldo is Italian or if Aldo isn’t Italian then Bob is English”.

- **a. A ∨ (¬A → B)**
- **b. A ∨ B (equivalent!)**
- c. A ∨ (A → B)
- d. A → B

Break & Quiz

Q 1.3: How many different assignments can there be to

$$(x_1 \wedge y_1) \vee (x_2 \wedge y_2) \vee \ldots \vee (x_n \wedge y_n)$$

- A. 2
- B. 2^n
- C. 2^{2n}
- D. 2n

Break & Quiz

Q 1.3: How many different assignments can there be to

$(x_1 \wedge y_1) \vee (x_2 \wedge y_2) \vee \ldots \vee (x_n \wedge y_n)$

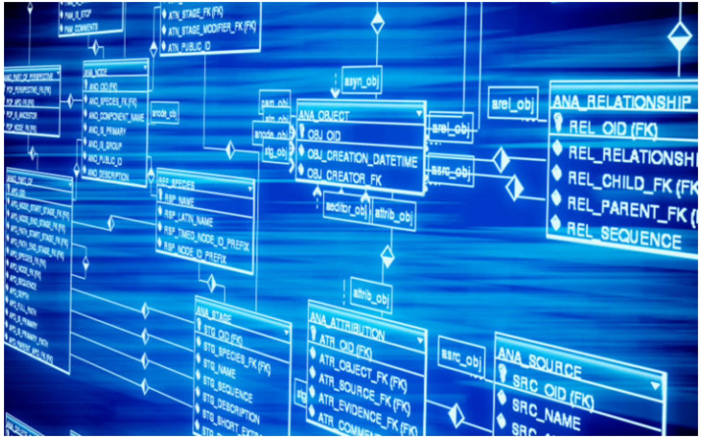
- A. 2
- B. 2^n
- **C. 2^{2n}**
- D. 2n

Knowledge Bases

- **Knowledge Base (KB)**: A set of sentences
  - Like a long sentence, connect with conjunction

Model of a KB: interpretations where all sentences are True

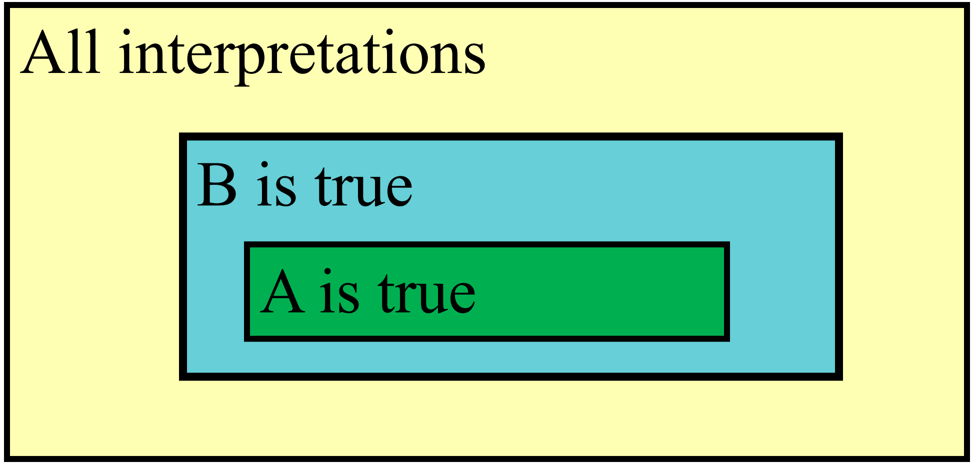
Goal: inference to discover new sentences



Entailment

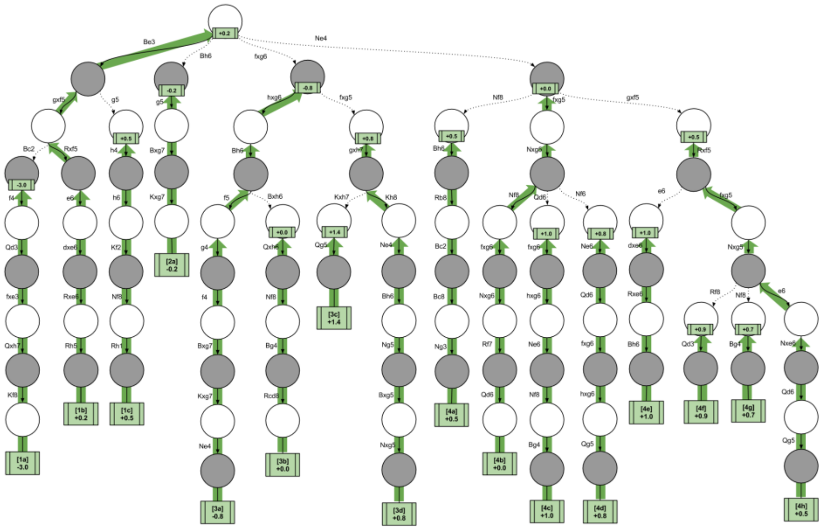
Entailment: a sentence logically follows from others

- Like from a KB. Write A ⊨ B
- A ⊨ B iff in every interpretation where A is true, B is also true



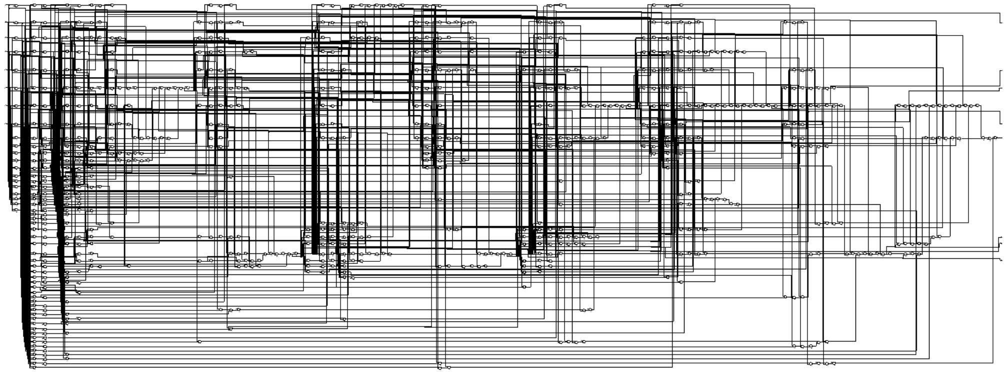
Inference

- Given a set of sentences (a KB), **logical inference** creates new sentences
  - Compare to prob. inference!
- **Challenges**:
  - Soundness
  - Completeness
  - Efficiency



Methods of Inference: **1. Enumeration**

- Enumerate all interpretations; look at the truth table
  - "Model checking"
- Downside: 2^n interpretations for n symbols



S. Leadley

Methods of Inference: **2. Using Rules**

- *Modus Ponens*: $(A \Rightarrow B, A) \models B$
- And-elimination
- Many other rules
  - Commutativity, associativity, de Morgan's laws, distribution for conjunction/disjunction

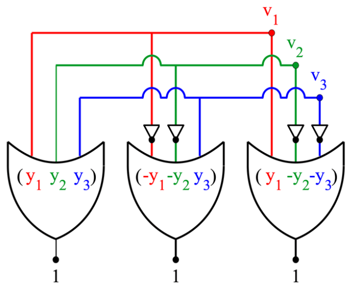
Methods of Inference: **3. Resolution**

- Convert to special form and use a single rule
- **Conjunctive Normal Form** (CNF)

$$\underbrace{(\neg A \lor B \lor C)}_{\text{a clause}} \land (\neg B \lor A) \land (\neg C \lor A)$$

Conjunction of clauses; each clause disjunction of literals

- Simple rules for converting to CNF



Methods of Inference: **3. Resolution**

Start with our KB and **query** B

- Add ¬B
- Show that this leads to a contradiction
- Take clauses with a symbol and its complement
  - Merge, throw away symbol: PvQvR, ¬QvSvT: PvRvSvT
  - If no symbol left, KB entails B
  - No new clauses, KB does not entail B

Break & Quiz

Q 2.1: What is the CNF for $(\neg p \wedge \neg(p \Rightarrow q))$

- A. $(\neg p \wedge \neg(p \Rightarrow q))$
- B. $(\neg p) \wedge (\neg p \vee \neg q)$
- C. $(\neg p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$
- D. $(\neg p \vee \neg q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$

Break & Quiz

Q 2.1: What is the CNF for $(\neg p \wedge \neg(p \Rightarrow q))$

- A. $(\neg p \wedge \neg(p \Rightarrow q))$
- B. $(\neg p) \wedge (\neg p \vee \neg q)$
- C. $(\neg p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$
- **D. $(\neg p \vee \neg q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$**

Break & Quiz

Q 2.2: Which has more rows: a truth table on *n* symbols, or a joint distribution table on *n* binary random variables?

- A. Truth table
- B. Distribution
- C. Same size
- D. It depends

Break & Quiz

Q 2.2: Which has more rows: a truth table on *n* symbols, or a joint distribution table on *n* binary random variables?

- A. Truth table
- B. Distribution
- **C. Same size**
- D. It depends

Finding CNFs

- CNF:

$$(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$$

- Automating transformation
  - Use equivalences for connectives we don't use (i.e., ⇒)
  - Move negatives inside (DeMorgan's laws)
  - Push ∨ inside ∧ by distributing
- Not guaranteed to be satisfiable

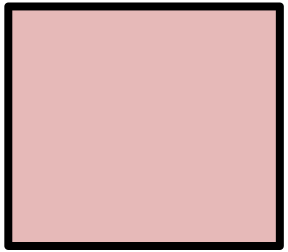
First Order Logic (FOL)

Propositional logic has some limitations

- Ex: how to say “all squares have four sides”
- No context, hard to generalize; express facts

FOL is a more expressive logic; works over

- Facts, Objects, Relations, Functions



First Order Logic (FOL)

Basics:

- Constants: “16”, “Green”, “Bob”
- Functions: map objects to objects
- Predicates: map objects to T/F:
  - Greater(5,3)
  - Color(grass, green)

First Order Logic (FOL)

Basics:

- Variables: x, y, z
- Connectives: Same as propositional logic
- Quantifiers:
  - ∀ universal quantifier: $\forall x\ \mathbf{human(x)} \Rightarrow \mathbf{mammal(x)}$
  - ∃ existential quantifier