Advanced Topics in Reinforcement Learning

Lecture 13: Linear Function Approximation and On-policy Control

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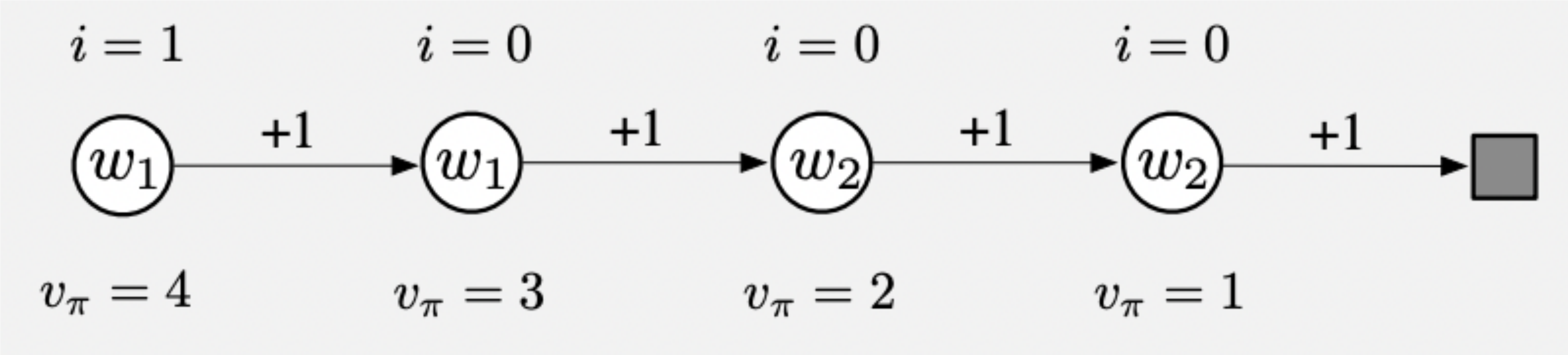
Announcements

- Homework 3 due Thursday of next week.
- Begin reading chapter 11 for next week.
- Midterm survey and evaluation.
- Looking ahead: https://pages.cs.wisc.edu/~jphanna/teaching/2022fall_cs839/schedule.html

Interest and Emphasis

- So far, assumed we are updating states equally (same learning rate) but according to the on-policy state distribution, μ.
- We may wish to emphasize some states more.
- State interest, I_t, represents how much we care about accurate estimation in state S_t.
- Emphasis is a learned multiplier on the learning rate.
  - $M_t \leftarrow I_t + \gamma M_{t-1}$
  - $\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + \alpha M_t [R_t - \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w})] \nabla \hat{v}(S_t, \mathbf{w})$

Interest and Emphasis



- Interest is (1, 0, 1, 0)
- Semi-gradient 2-step TD converges to weight vector (3.5, 1.5)
- Emphatic 2-step TD converges to weight vector (4, 2)

On-Policy Control

- As usual, for control we will estimate action-values, $\hat{q}(s, a, \mathbf{w})$.
- For linear function approximation, features are now a function of (s,a) pairs, $\mathbf{x}(s, a)$.
- Function approximation often inherently means that making $\hat{q}(s, a, \mathbf{w})$ more accurate at one state will make it less accurate at another state.
- Now making π greedy w.r.t. $\hat{q}(s, a, \mathbf{w})$ is no longer guaranteed to improve π — no more policy improvement theorem.

Semi-Gradient Sarsa

Episodic Semi-gradient Sarsa for Estimating $\hat{q} \approx q_*$

Input: a differentiable action-value function parameterization $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameters: step size $\alpha > 0$, small $\varepsilon > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
  $S, A \leftarrow$ initial state and action of episode (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    If S' is terminal:
      $\mathbf{w} \leftarrow \mathbf{w} + \alpha \boxed{\left[R - \hat{q}(S, A, \mathbf{w})\right]} \nabla \hat{q}(S, A, \mathbf{w})$
      Go to next episode
    Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha \left[R + \gamma \hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})\right] \nabla \hat{q}(S, A, \mathbf{w})$
    $S \leftarrow S'$
    $A \leftarrow A'$

Handle termination

Linear Function Approximation

- Assume value estimate is a linear function of state features.
- $$\hat{v}(s, \mathbf{w}) = \mathbf{w}^\top x(s) = \sum_{i=1}^{d} w_i x_i(s)$$
- The features, $x_i(s)$, can be non-linear functions of state variables.
  - Expressive choices for $\mathbf{x}(s)$ make linear methods more powerful than they first appear.

1-Hot Features / State Aggregation

- For a finite state-space, partition state-space into d mutually exclusive groups.
- Let i be the group to which state s belongs.
- The 1-Hot feature encoding sets $x_i(s) = 1$ and $x_j(s) = 0$ for $j \neq i$.
- What does generalization look like?
- Special case is $d = |\mathcal{S}|$ in which case we recover the tabular setting.
  - Useful tip for debugging RL implementations!
  - Easily switch between easy to understand tabular experiments and more complex function approximation within same implementation.

Polynomial Features

- Suppose the state is represented as $(s_1, s_2) \in \mathbb{R}^2$.
- Polynomial representation: $(1, s_1, s_2, s_1 s_2, s_1^2, s_2^2, \ldots)$.
- What is the advantage of the polynomial representation?
    - Can represent any function with sufficiently high order polynomials.

Fourier Features

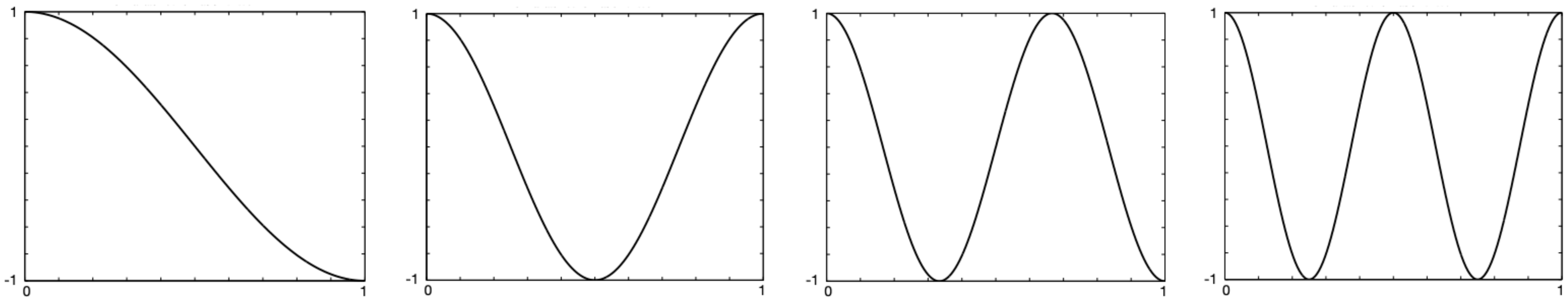
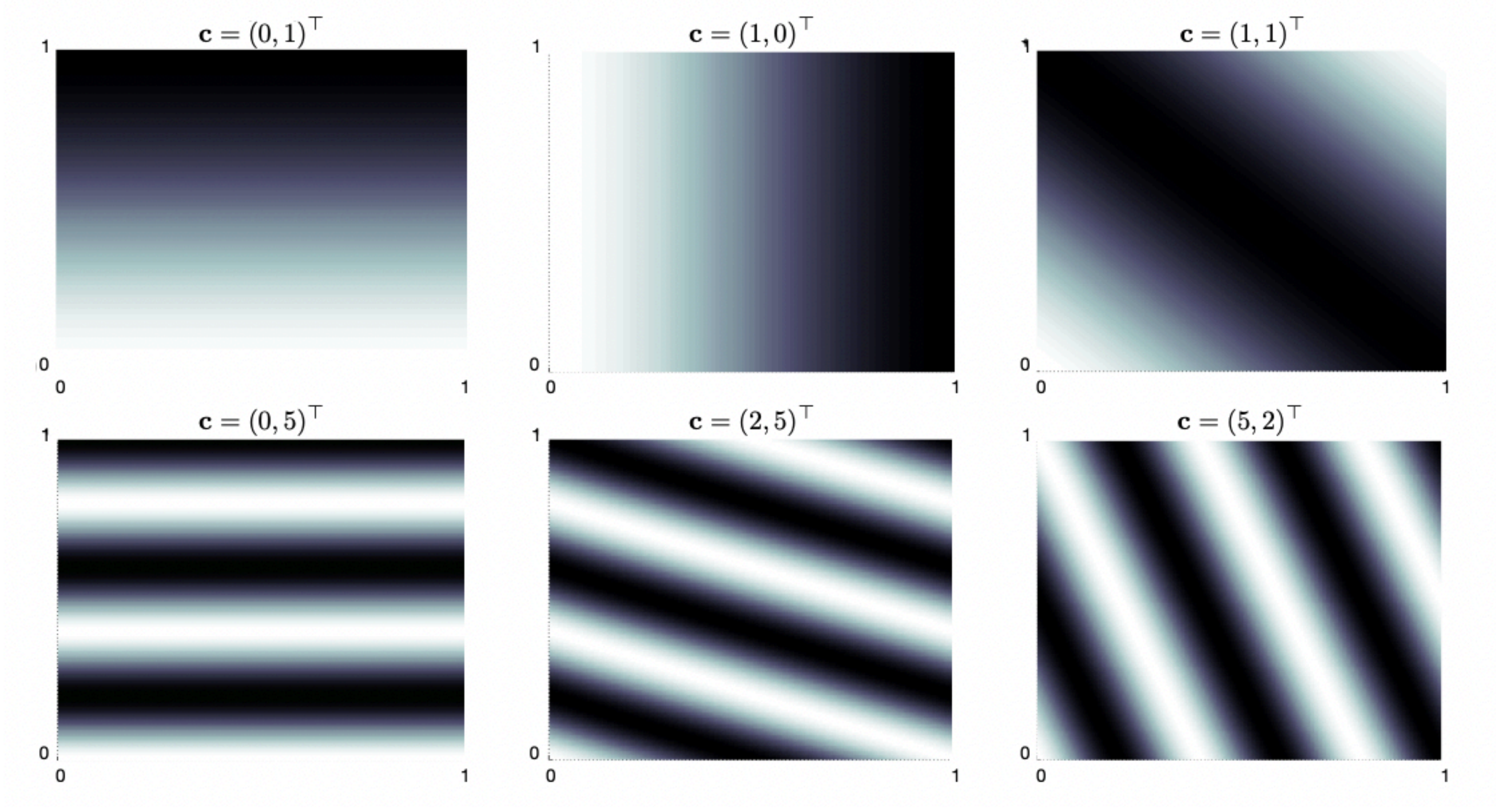


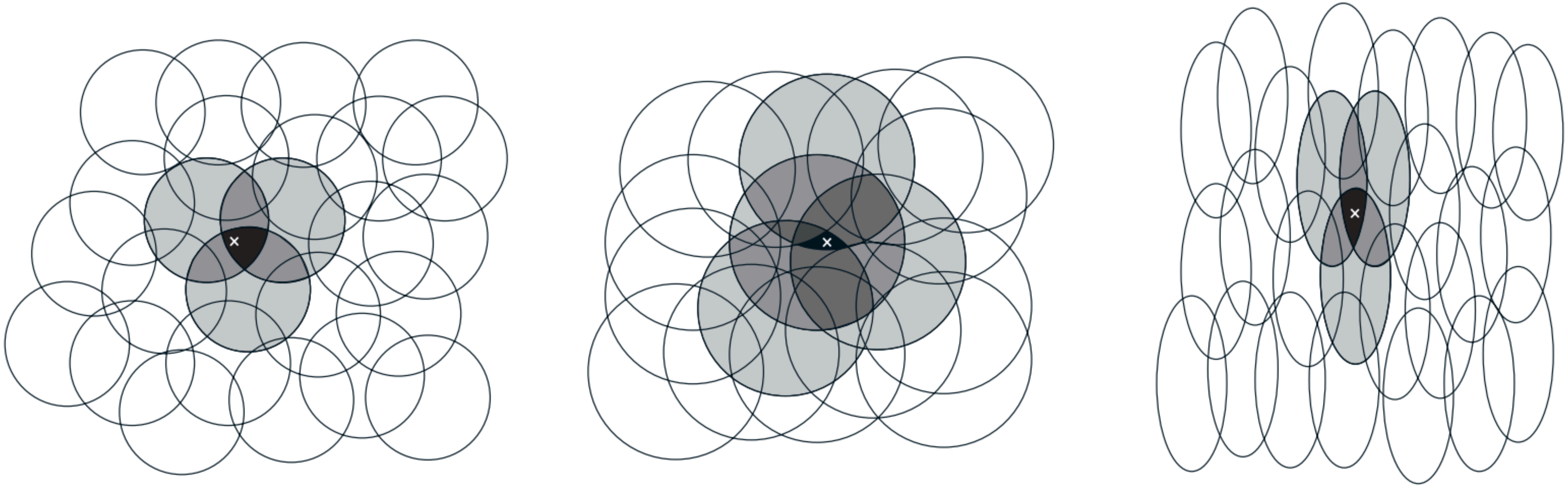
Figure 9.3: One-dimensional Fourier cosine-basis features x_i, $i = 1, 2, 3, 4$, for approximating functions over the interval $[0, 1]$. After Konidaris et al. (2011).

$$x_i(s) = \cos(\pi s^\top c^i)$$

Fourier Features



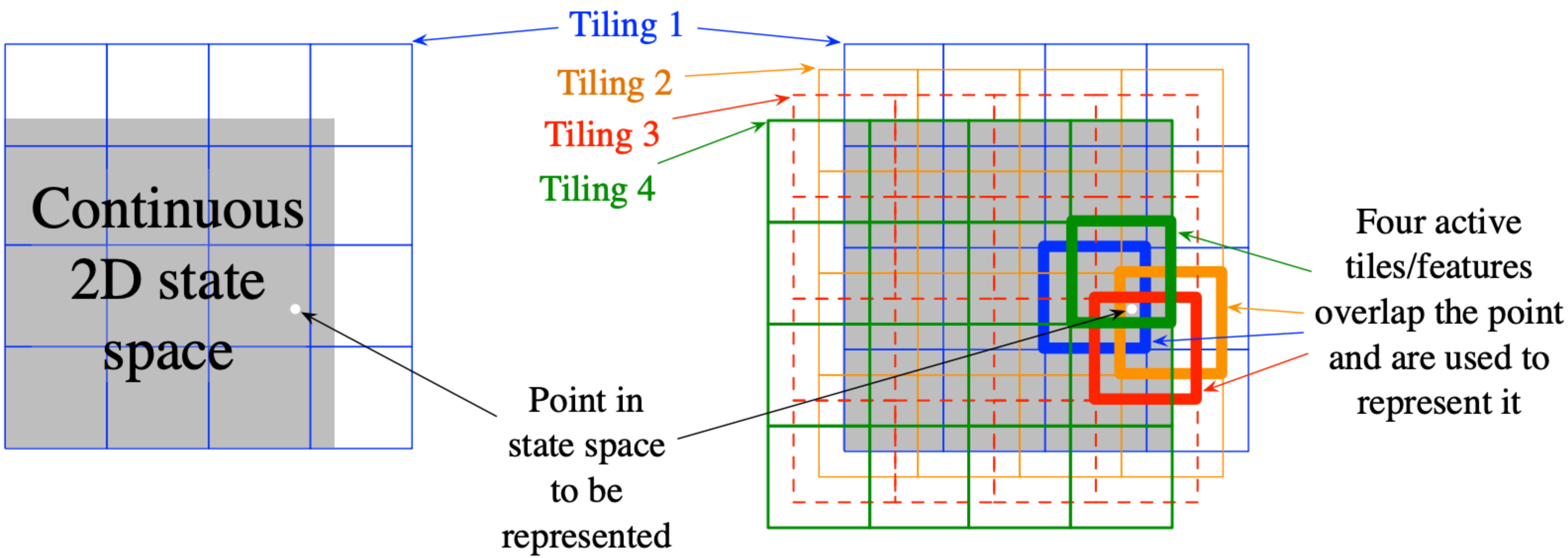
Coarse Coding



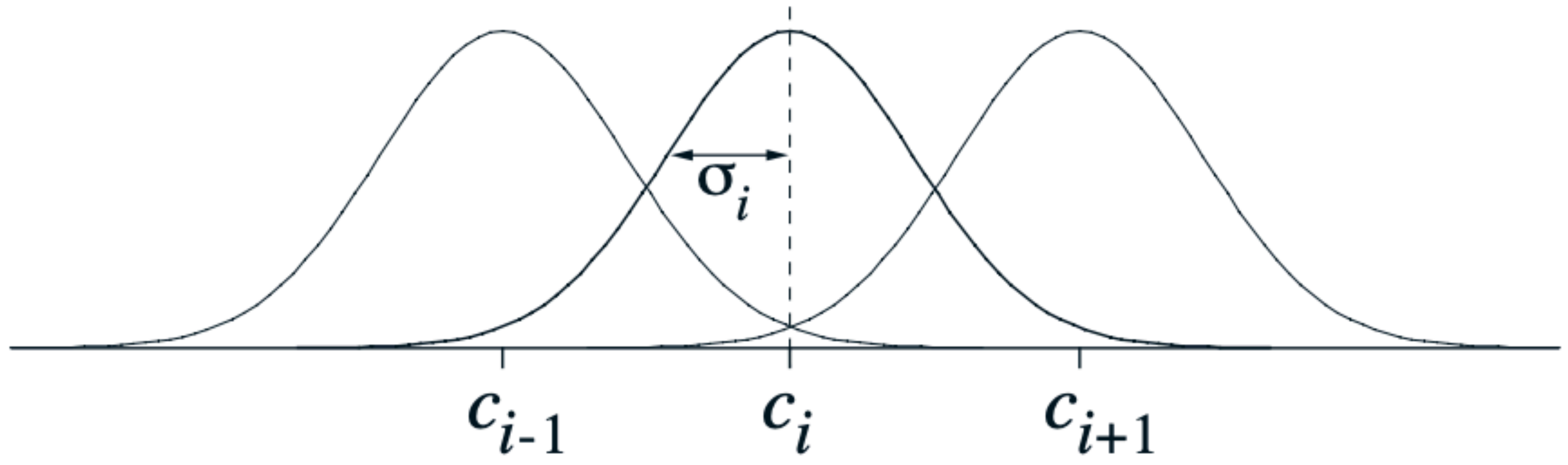
Narrow generalization   Broad generalization   Asymmetric generalization

Tile Coding

- Intuitively, multiple state aggregation mappings at the same time.

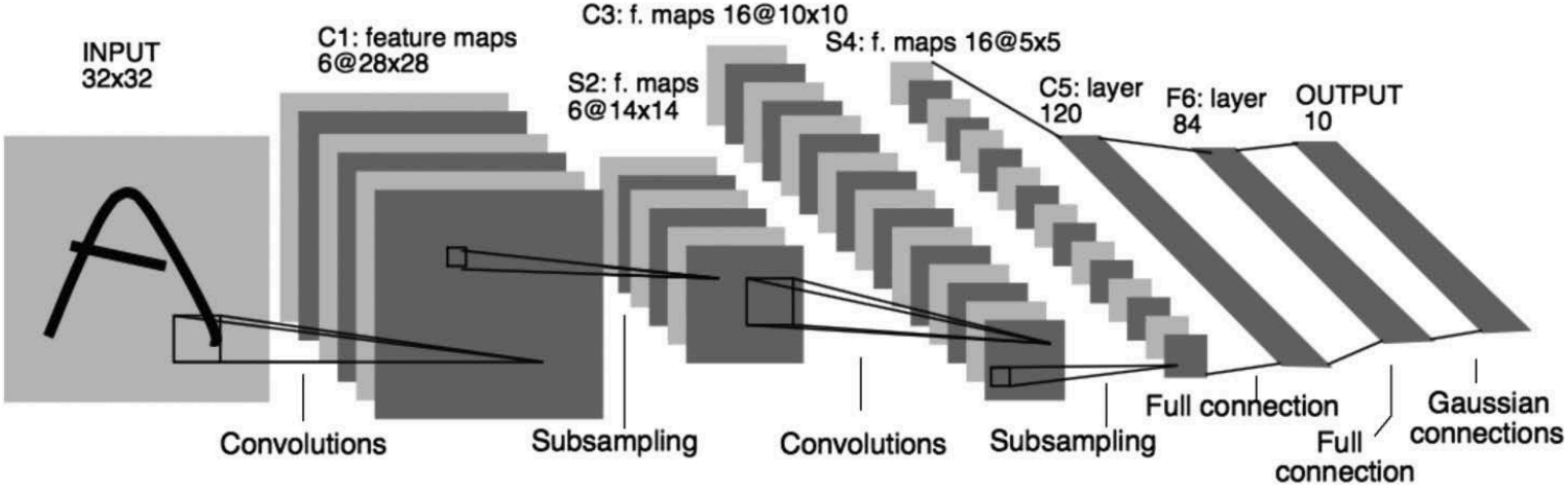


Radial Basis Functions



- Coarse coding with continuous features.
- $$x_i(s) = \exp\left(-\frac{||s - c_i||^2}{2\sigma_i^2}\right)$$

Neural Networks



Memory-Based Learning

- Non-parametric methods avoid need to fix a functional form for $\hat{v}(s, \mathbf{w})$.
- Instead, keep around all observed states and their value estimates.
- When need to compute $\hat{v}(s, \mathbf{w})$, find closest previously seen states to s and use their value estimates.
- (+) Capacity grows with amount of data, focus approximation resources on states the agent is actually visiting.
- (-) Computationally expensive to find closest states, notion of “closest” is problem dependent.

Summary

- Approximate on-policy control with semi-gradient Sarsa parallels tabular Sarsa *but we no longer have guaranteed policy improvement.*

- Linear function approximation can be powerful with the right choice of features.

- Many good options to choose from but the most practical might be to simply learn the features with a neural network.

Action Items

- Homework 3.
- Begin literature review.
- Begin reading Chapter 11.
- Midterm survey and evaluation.