Advanced Topics in Reinforcement Learning

Lecture 4: Dynamic Programming

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Announcements

- Homework 1 released on canvas; due Thursday, September 29.
- Start reading **chapter 5** for next week.
- Project page: https://pages.cs.wisc.edu/~jphanna/teaching/2022fall_cs839/project.html

Overview

- The Policy Improvement Theorem
- Policy Iteration
- Value Iteration
- Asynchronous Value Iteration
- Samarth’s Presentation

Policy Evaluation (Prediction)

- Given a policy, compute its state- or action-value function.

$$v_{k+1}(s) \leftarrow \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a)[r + \gamma v_k(s')]$$

$$q_{k+1}(s, a) \leftarrow \sum_{s'} \sum_r p(s', r|s, a)[r + \gamma \sum_{a'} q_k(s', a')]$$

Policy Improvement (Control)

- We have $v_\pi(s)$ for the current policy π. How can we improve π?
  - Behave greedily w.r.t. $\sum_{s',r} p(s', r \mid s, a)[r + \gamma v_\pi(s')$.
- Suggests a simple algorithm. Alternate:
  - Run policy evaluation updates to find v_π.
  - Set $\pi'(s) \leftarrow \arg\max_a \sum_{s',r} p(s', r \mid s, a)[r + \gamma v_\pi(s')]$
  - Why does this work?

Policy Improvement Theorem

- Suppose for π that $\exists s, a$ such that $q_\pi(s, a) \geq v_\pi(s)$.
- Let $\pi'(s) = a$ and $\pi'(\tilde{s}) = \pi(\tilde{s})$ for all other states.
- What is true about π'? Why?
  - As good as or better than π, i.e., $v_{\pi'}(s) \geq v_\pi(s), \forall s$
  - If $q_\pi(s, a) \geq v_\pi(s)$ then always taking action a cannot decrease expected return.
- If π is sub-optimal, does there exist s, a such that $q_\pi(s, a) > v_\pi(s)$?
  - Yes, this follows from Bellman Optimality. Must be at least one state where π is not greedy w.r.t. its action-value function.
  - Optimal value function: $v_\star(s) = \max_a q_\star(s, a) \forall s$

Policy Iteration

- First, evaluate π to obtain v_π.
- Then, update π to π' such that $\pi'(s) = \arg\max_a \sum_{s',r} p(s', r \mid s, a)[r + v_\pi(s')]$
- Policy improvement theorem guarantees that $v_{\pi'}(s) \geq v_\pi(s) \forall s$.
- Can converge quickly in practice (in terms of policy updates).
- (Fixing the subtle bug on page 80).

Policy Iteration Demo

https://cs.stanford.edu/people/karpathy/reinforcejs/gridworld_dp.html

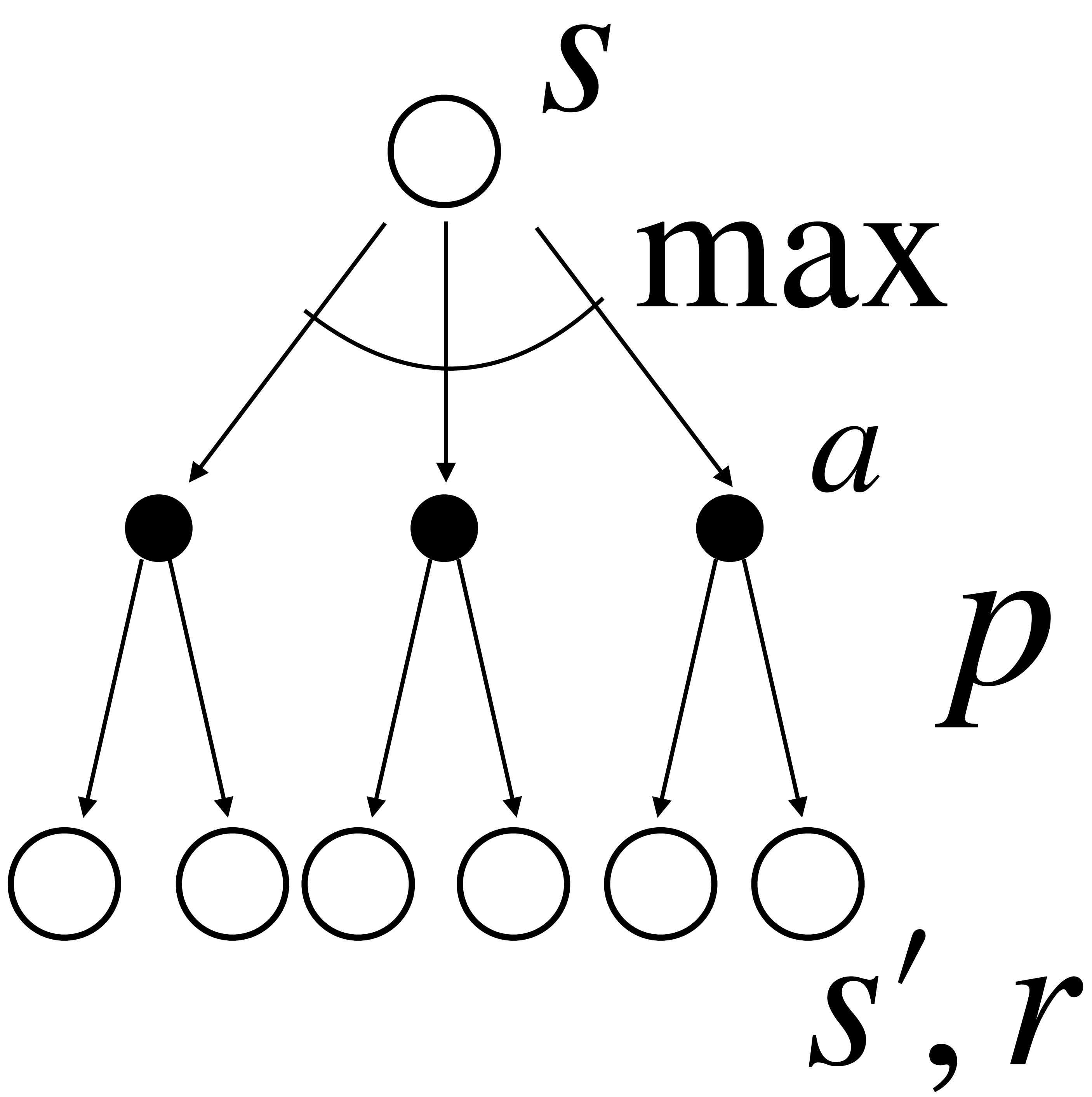
Value Iteration

- What's wrong with policy iteration?
  - Policy evaluation must converge between policy updates.
  - We don't need the exact action-values — just which action has maximal action-value.
- Value iteration combines policy evaluation and iteration in one step:

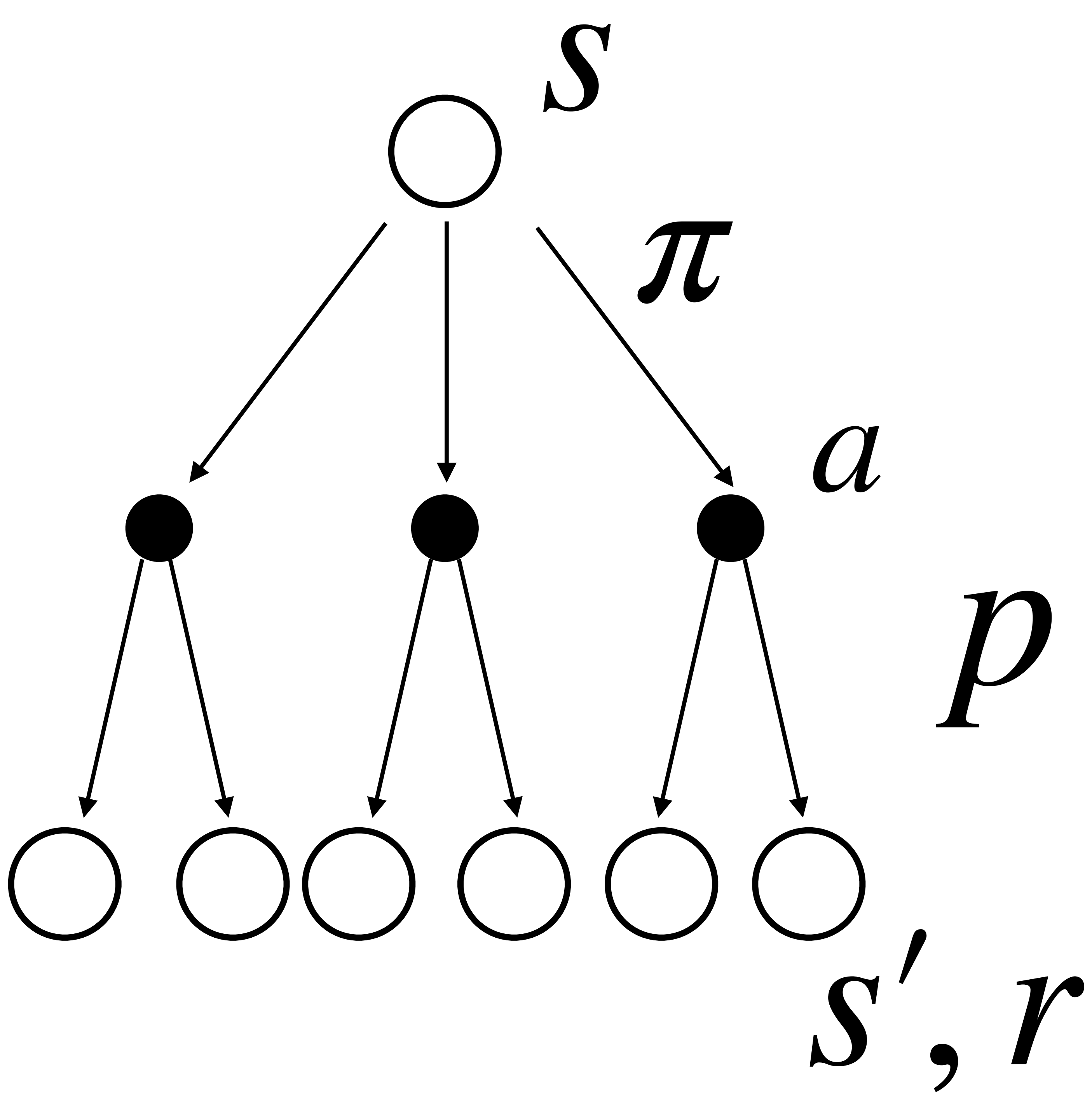
$$v_{k+1}(s) \leftarrow \max_a \sum_{s',r} p(s', r \mid s, a)[r + \gamma v_k(s')]$$

- In-place or out-of-place updates?
  - In-place propagates value updates faster.
  - Out-of-place is more amenable to analysis.

Value Iteration



Value Iteration



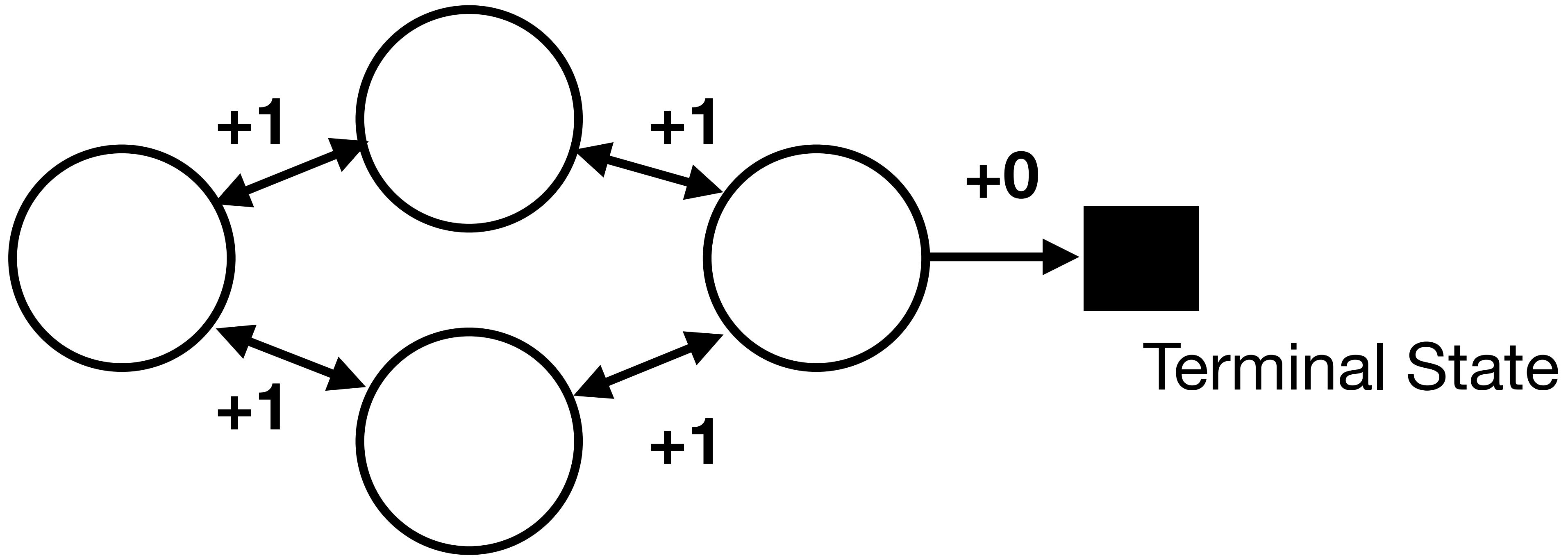
Policy Evaluation

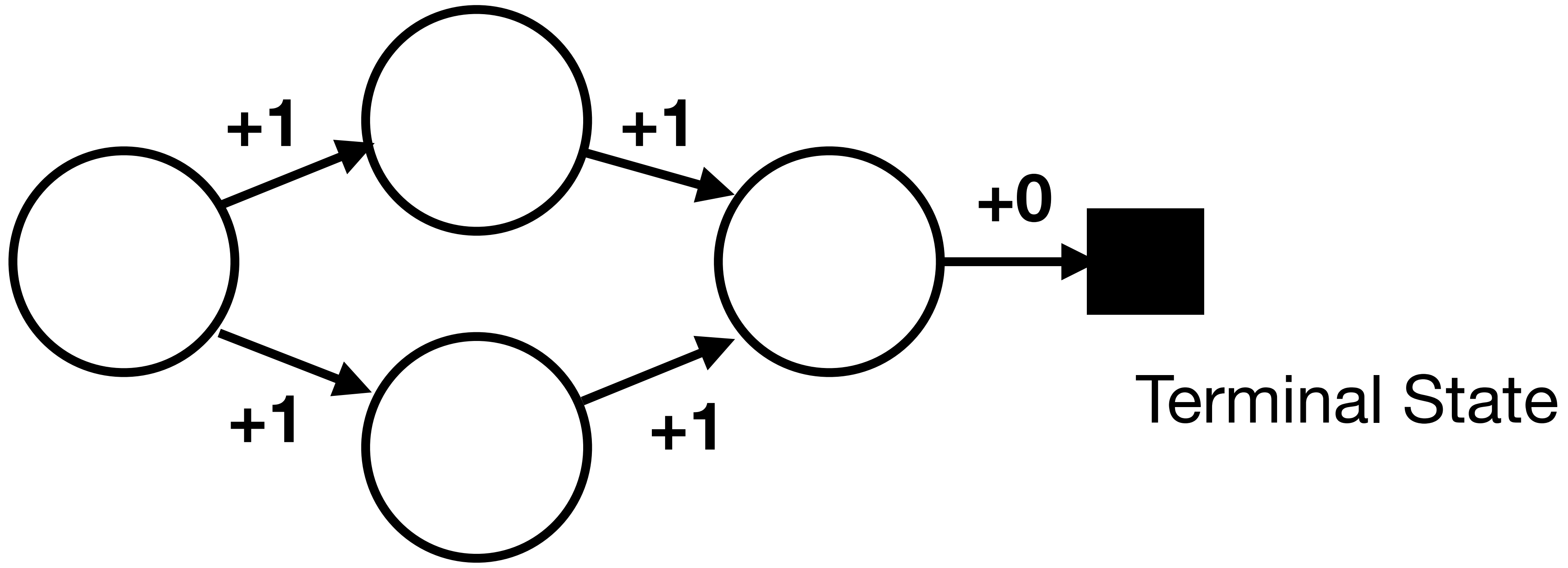
Value Iteration Demo

https://cs.stanford.edu/people/karpathy/reinforcejs/gridworld_dp.html

Asynchronous DP

- Regular DP methods require sweeps over the entire state space *per-iteration!*
    - Infeasible if we have a large state-space.
- Actually unnecessary — can update states in any order and still converge as long as all states updated infinitely often in the limit.
- Why does this help?





Generalized Policy Iteration

- What is it?
  - We can be quite permissive in how we mix evaluation and improvement.
  - As long as v becomes closer to v_π and π becomes greedy w.r.t. v we will converge to $v_\star, \pi^\star$.
- A general framework for all algorithms we will introduce in this class.
- Do you think this holds when v_π must generalize across states? I.e., increasing the value of $v_\pi(s)$ will also increase the value of $v_\pi(s')$ for s' close to s.

Samarth's Presentation

Slides

Summary

- Learning value functions allow us to compute optimal policies.
- Policy Evaluation: find value function for a fixed policy.
- Policy Iteration: compute optimal policy by iterating 1) policy evaluation and 2) greedy policy improvement.
- Value Iteration: directly learn optimal value function.
- Dynamic programming methods don’t solve the full RL problem but they are the basis for most of the methods we will see in this class.

Action Items

- Homework 1 released. Due September 29 @ 9:29 am.
- Start reading chapter 5 for next week. First learning methods!
- Be thinking about final project — proposal due in 2 weeks.
  - The more concrete your proposal is, the better guidance you will receive!