Advanced Topics in Reinforcement Learning

Lecture 7: Off-Policy Monte Carlo Methods

Josiah Hanna
University of Wisconsin — Madison

Announcements

- Homework 1 due 1 minute ago.
- Homework 2 released this evening.
- Start reading chapter 6 for next week.
- Project proposals due in **1 week.**

Today

- On-policy Exploration
- Off-policy Monte Carlo Prediction
- Off-policy Monte Carlo Control

Ensuring Exploration

- Exploring starts are restrictive. What else to do?
  - ϵ-greedy policies: select $a^\star = \arg\max_a q(s, a)$ with probability $1 - \epsilon$; else random action.
  - Hard policy $\equiv$ Deterministic policy, Soft policy $\equiv$ All actions have some probability.
- Do ϵ-greedy methods converge? If so, to what?
- Can we still reach $\pi^\star$?
  - What if we decay epsilon?

Off-Policy Motivation

- What is the difference between off-policy and on-policy learning?
    - Trajectories generated by behavior policy, used to evaluate target policy.
    - If behavior = target ($\forall s, a, \pi(a \mid s) = b(a \mid s)$), then on-policy. Otherwise, off-policy.
- Why do we need off-policy learning?
    - Behavior policy explores, target policy exploits.
    - Learn for many reward functions at the same time.
    - Behavior policy is a known and safe policy.
- What is the main challenge in off-policy learning?
    - Distribution shift! Behavior policy and target policy induce different trajectory distributions. Thus, $q_b(s, a) \neq q_\pi(s, a)$ in general.

Importance Sampling Methods

- Given distribution $d(X)$ and real-valued function $f(X)$, estimate:

$$\mathbf{E}_d[f(X)] = \sum_x d(x)f(x)$$

- The distribution d is unknown but we can sample $X \sim b$.

- Monte Carlo approximation:

If we set $b \leftarrow d$ then reduces to standard Monte Carlo.

$$\sum_x d(x)f(x) = \sum_x b(x)\frac{d(x)}{b(x)}f(x) \approx \frac{1}{n}\sum_{i=1}^{n}\frac{d(X_i)}{b(X_i)}f(X_i) \qquad X_i \sim b$$

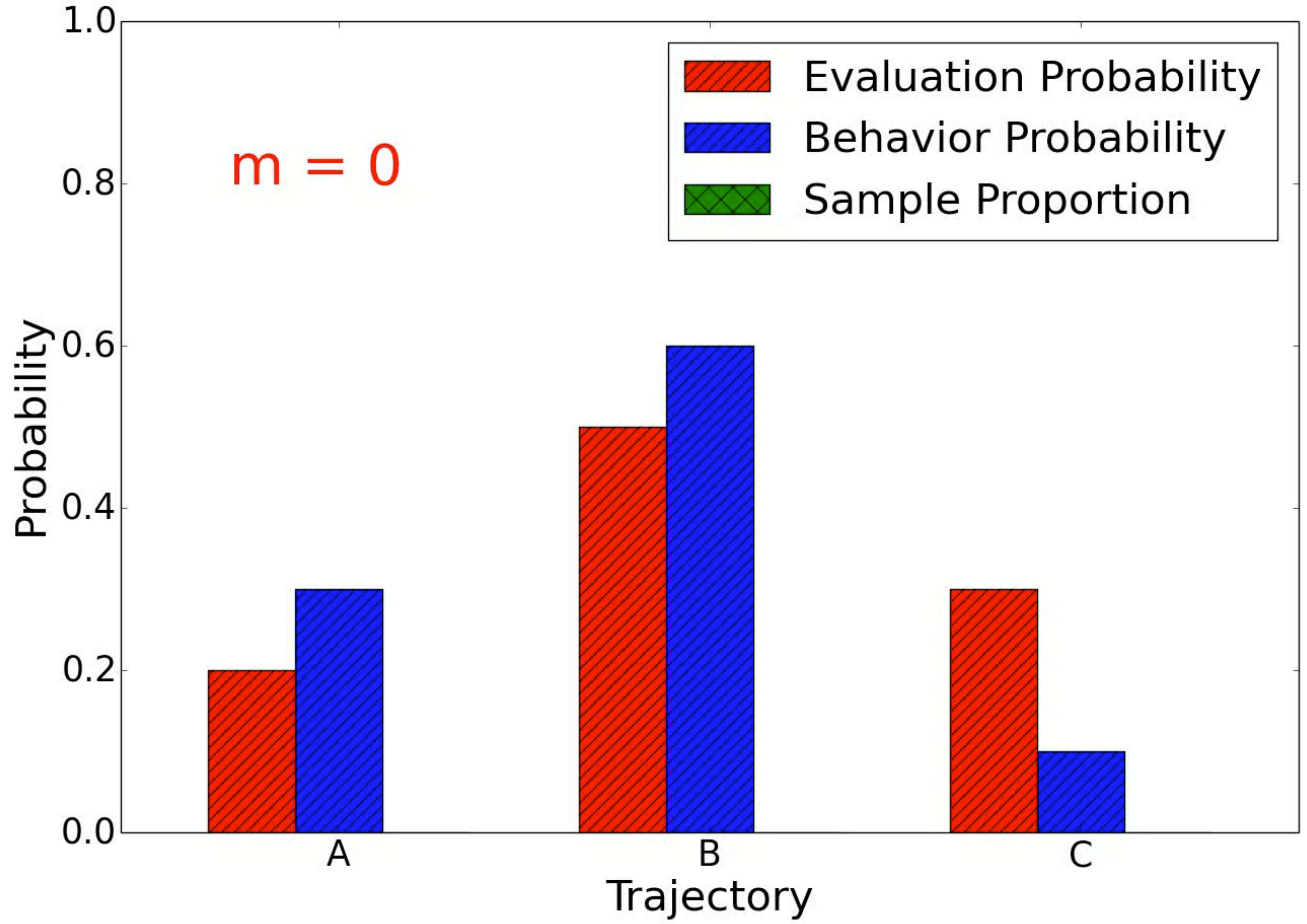
- Law of large numbers tells us that as $n \rightarrow \infty$ that error in the approximation goes to zero.

- Error is order $1/\sqrt{n}$ (assuming $\frac{d(x)}{b(x)}$ is bounded).

Importance Sampling

$$\sum_x d(x)f(x) = \approx \frac{1}{n}\sum_{i=1}^{n}\frac{d(X_i)}{b(X_i)}f(X_i)$$

$$X_i \sim b$$



Off-Policy Monte Carlo in RL

- Key idea: correct return distribution with importance sampling.
- Trajectory distribution that is induced from MDP and behavior policy.

$$\Pr(s_0, a_0, r_1, s_1, \ldots r_T, s_T) = \prod_{t=0}^{T-1} b(a_t \mid s_t) p(s_{t+1}, r_{t+1} \mid s_t, a_t)$$

- Desired trajectory distribution induced from MDP and target policy.

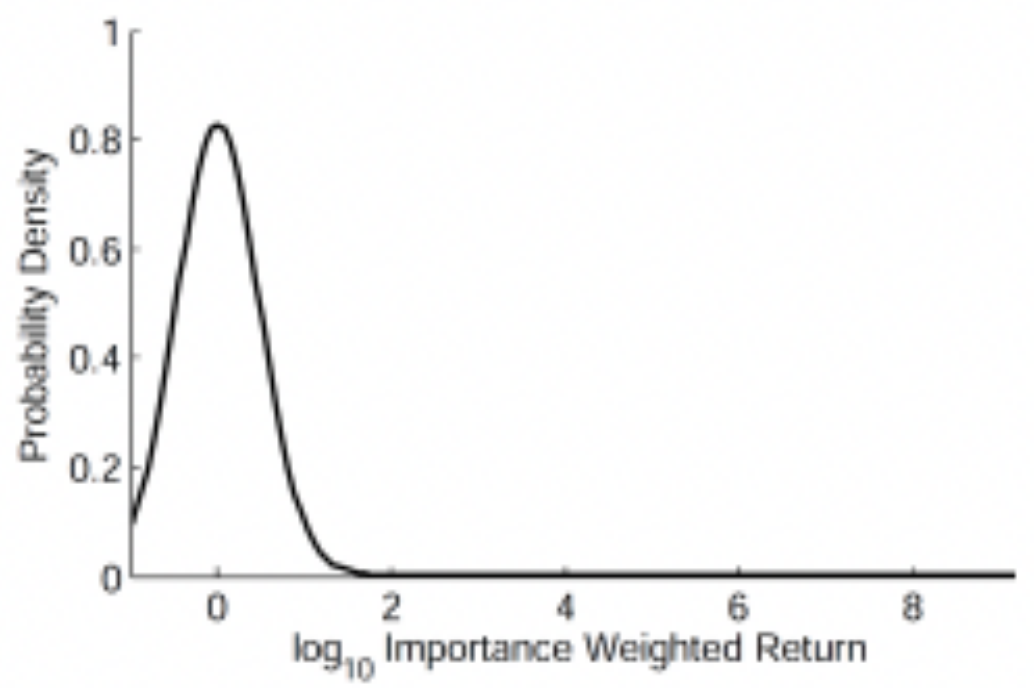
$$\Pr(s_0, a_0, r_1, s_1, \ldots r_T, s_T) = \prod_{t=0}^{T-1} \pi(a_t \mid s_t) p(s_{t+1}, r_{t+1} \mid s_t, a_t)$$

- Importance weighted returns:

$$\rho_{t:T} := \prod_{i=t}^{T-1} \frac{\pi(a_i \mid s_i) p(s_{i+1}, r_{i+1} \mid s_i, a_i)}{b(a_i \mid s_i) p(s_{i+1}, r_{i+1} \mid s_i, a_i)} = \prod_{i=t}^{T-1} \frac{\pi(a_i \mid s_i)}{b(a_i \mid s_i)} \qquad v_\pi(s_t) \approx \rho_{t:T} G_t$$

Importance Sampling Variance

- Importance sampling provides unbiased estimates of $v_\pi(s)$ using returns sampled by running the behavior policy.
    - Assuming that, if $\pi(a \mid s) > 0$, then $b(a \mid s) > 0$.
- In practice:
    - Can have infinite variance.
    - Most of the time, importance sampling severely under-estimates and then rarely, massively over-estimates.
    - Can return implausible estimates.
    - Ex: Suppose you *know* G_t is bounded and hence $v_\pi(s)$ is bounded. Importance sampling may estimate a value much greater than the bound.



Thomas et al. 2015

Importance Sampling V

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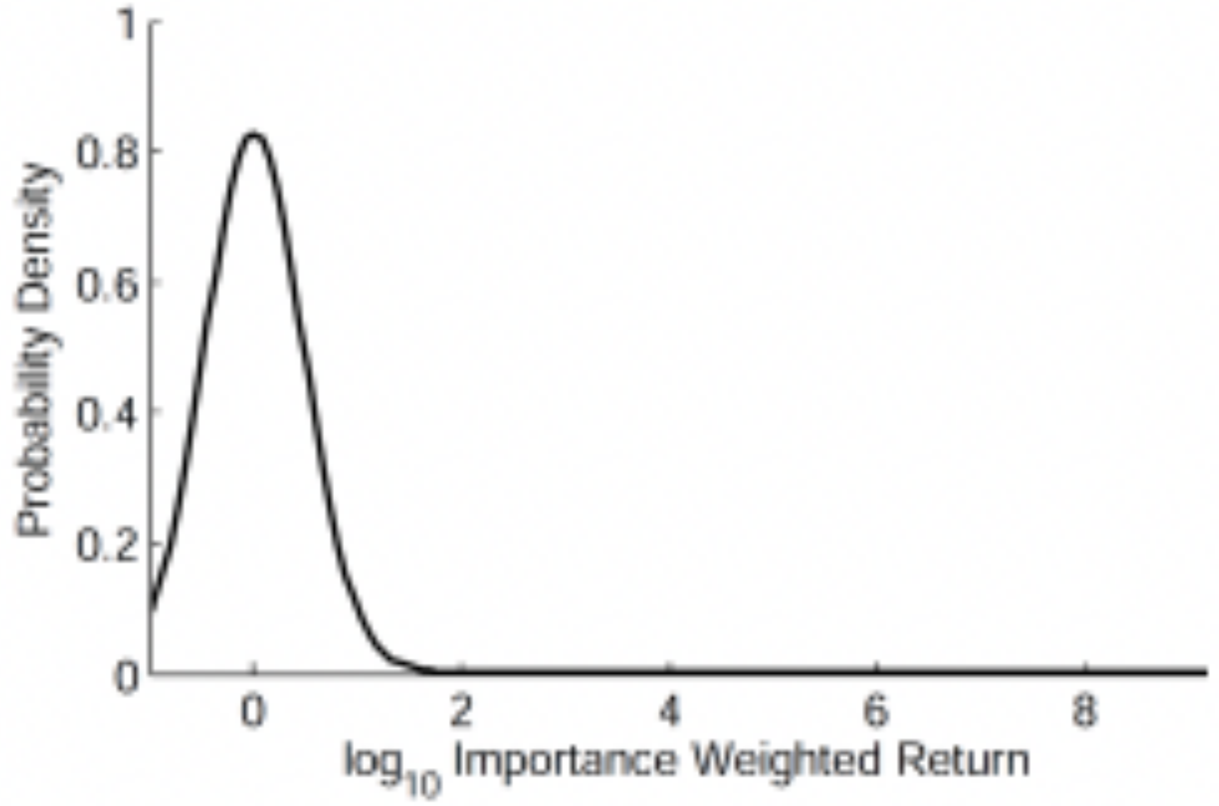
    - In most cases, importance sampling over-estimates.



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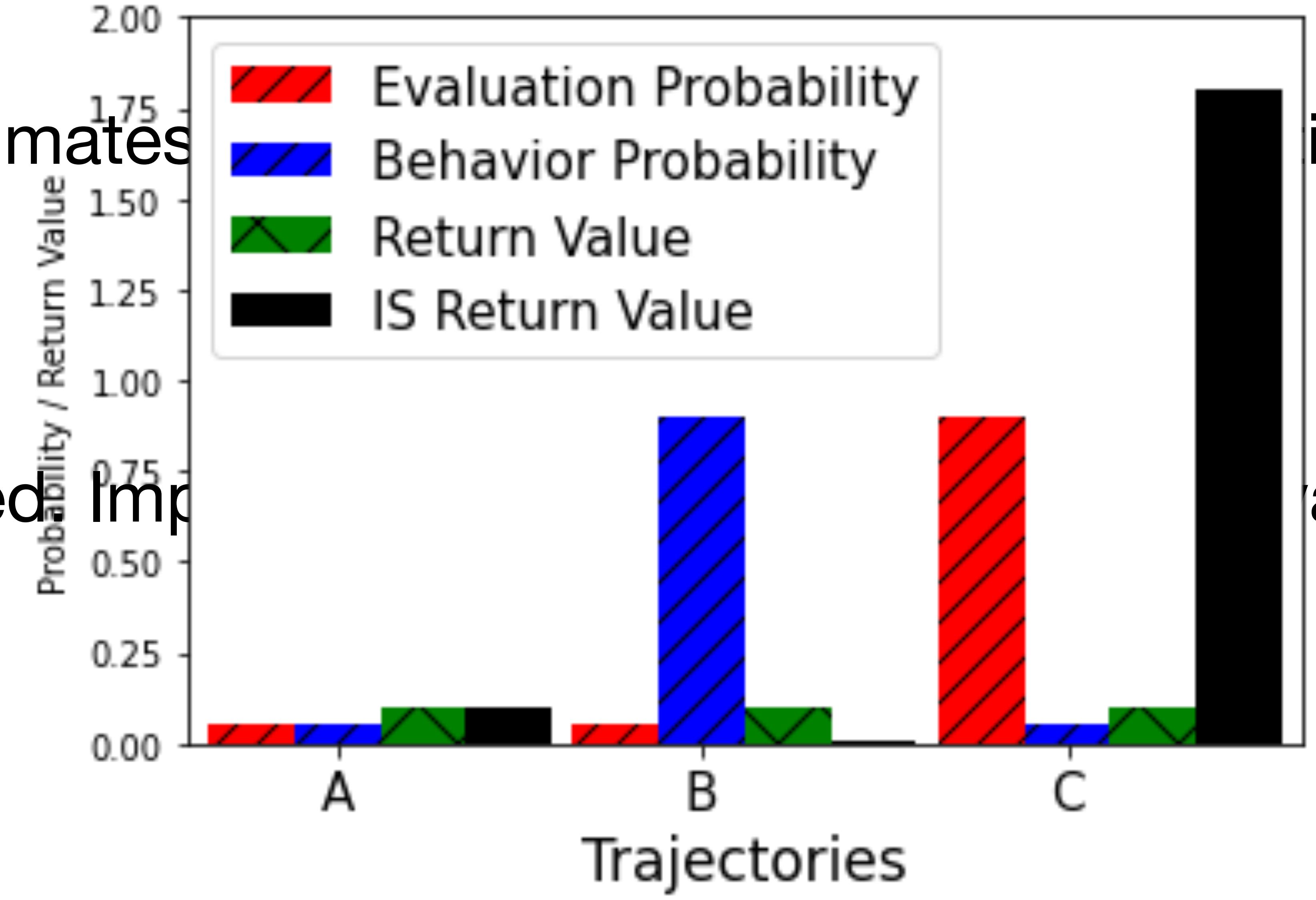
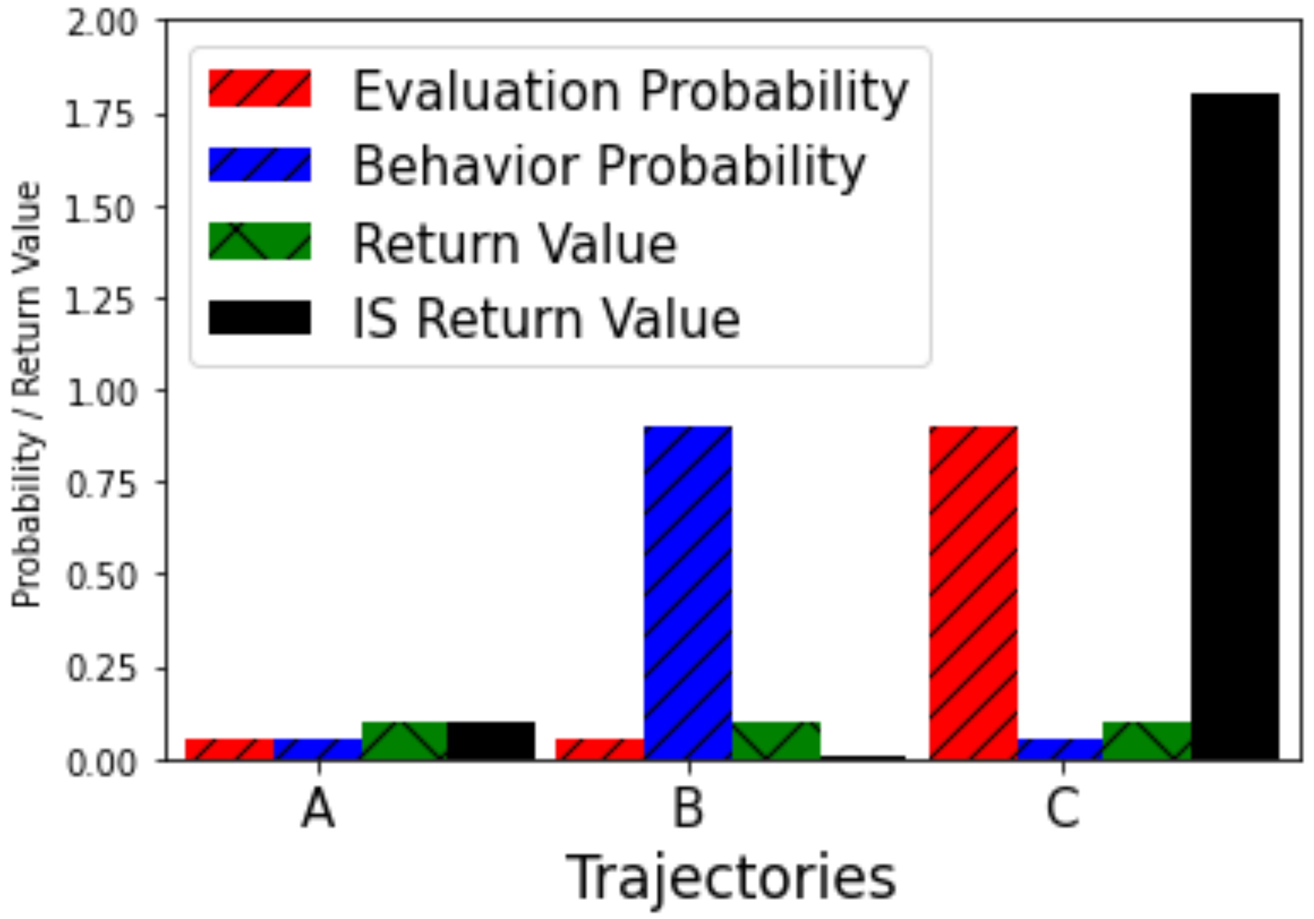
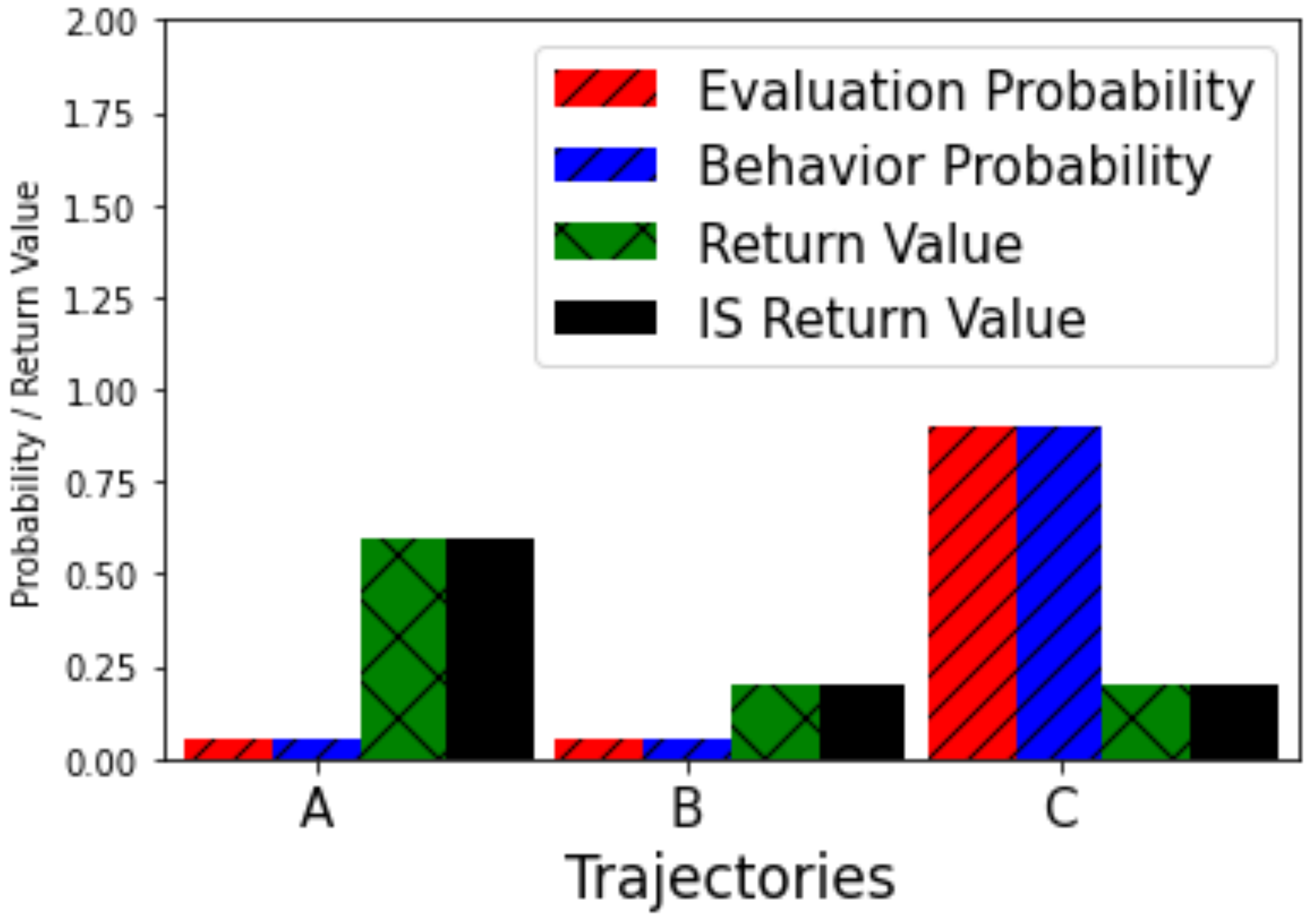


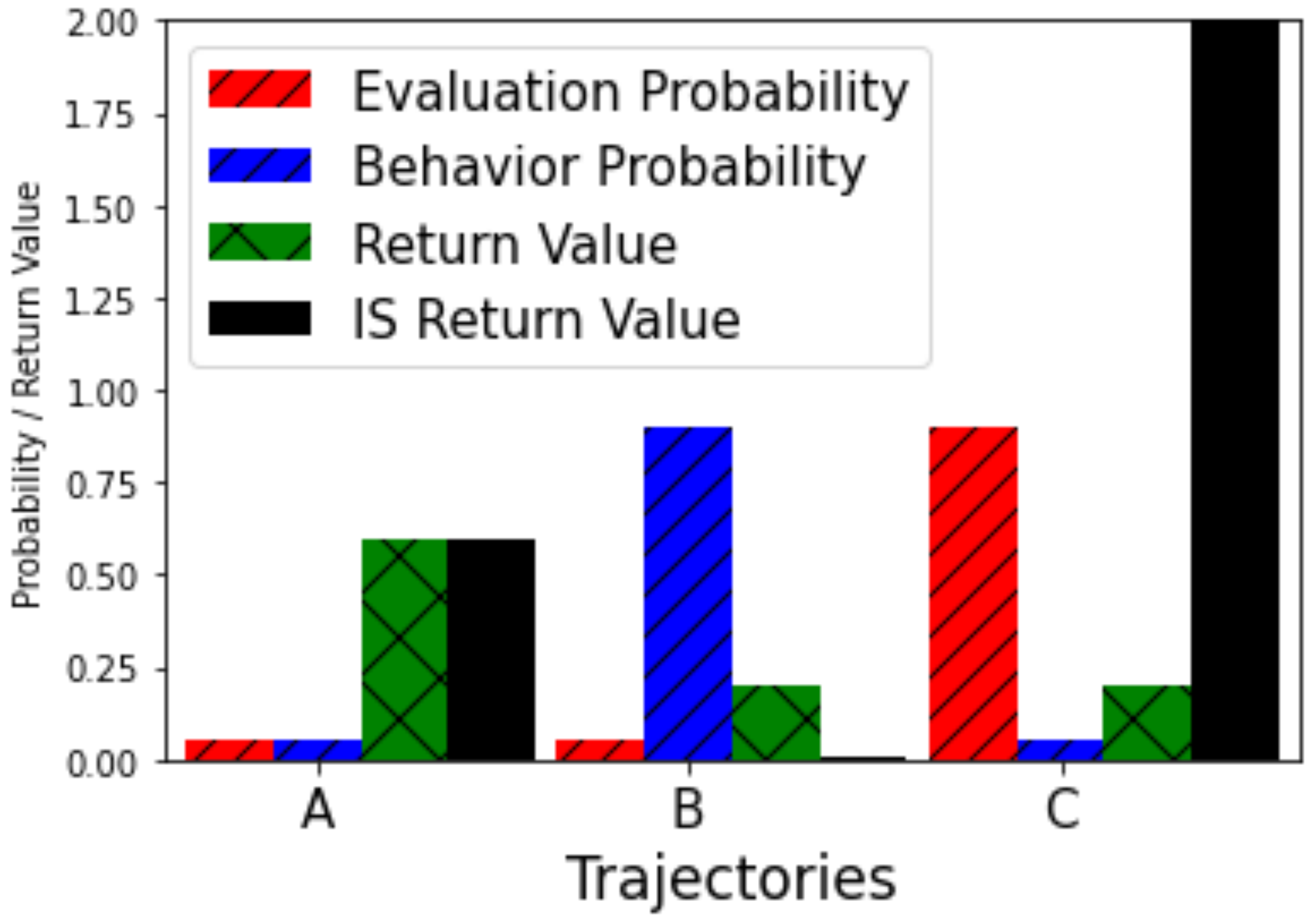
Variance of Importance Sampling



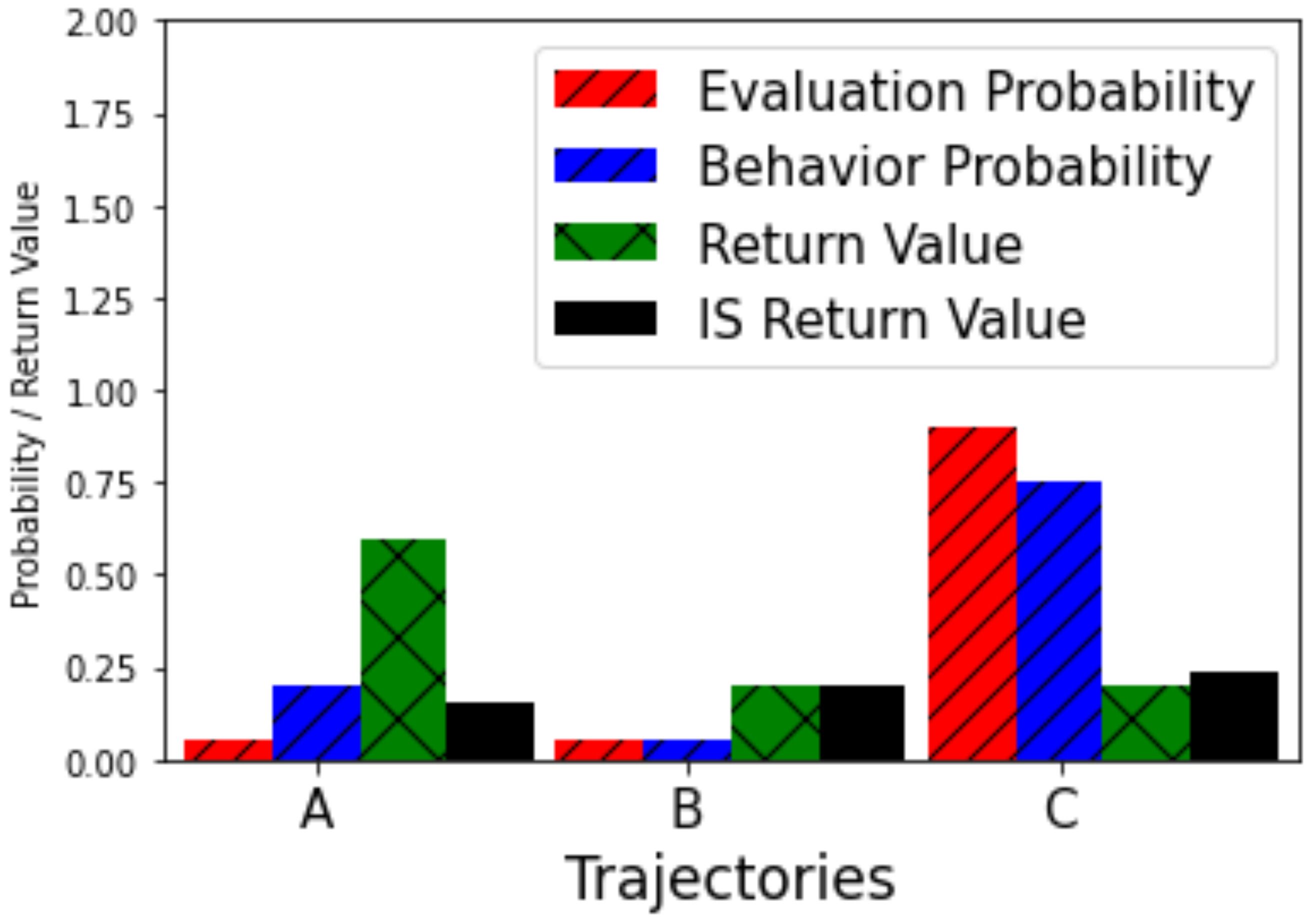
Extreme High Variance Off-Policy



On-Policy



High Variance Off-Policy



Low Variance Off-Policy

Choice of Behavior Policy

- In RL, importance sampling often has high variance.
- Outside of RL, importance sampling is a technique for lowering variance.
- In RL, the behavior policy is typically dictated by circumstances. Can we do better if we get to choose the behavior policy?
- After a single return is observed, our state-value estimate is:

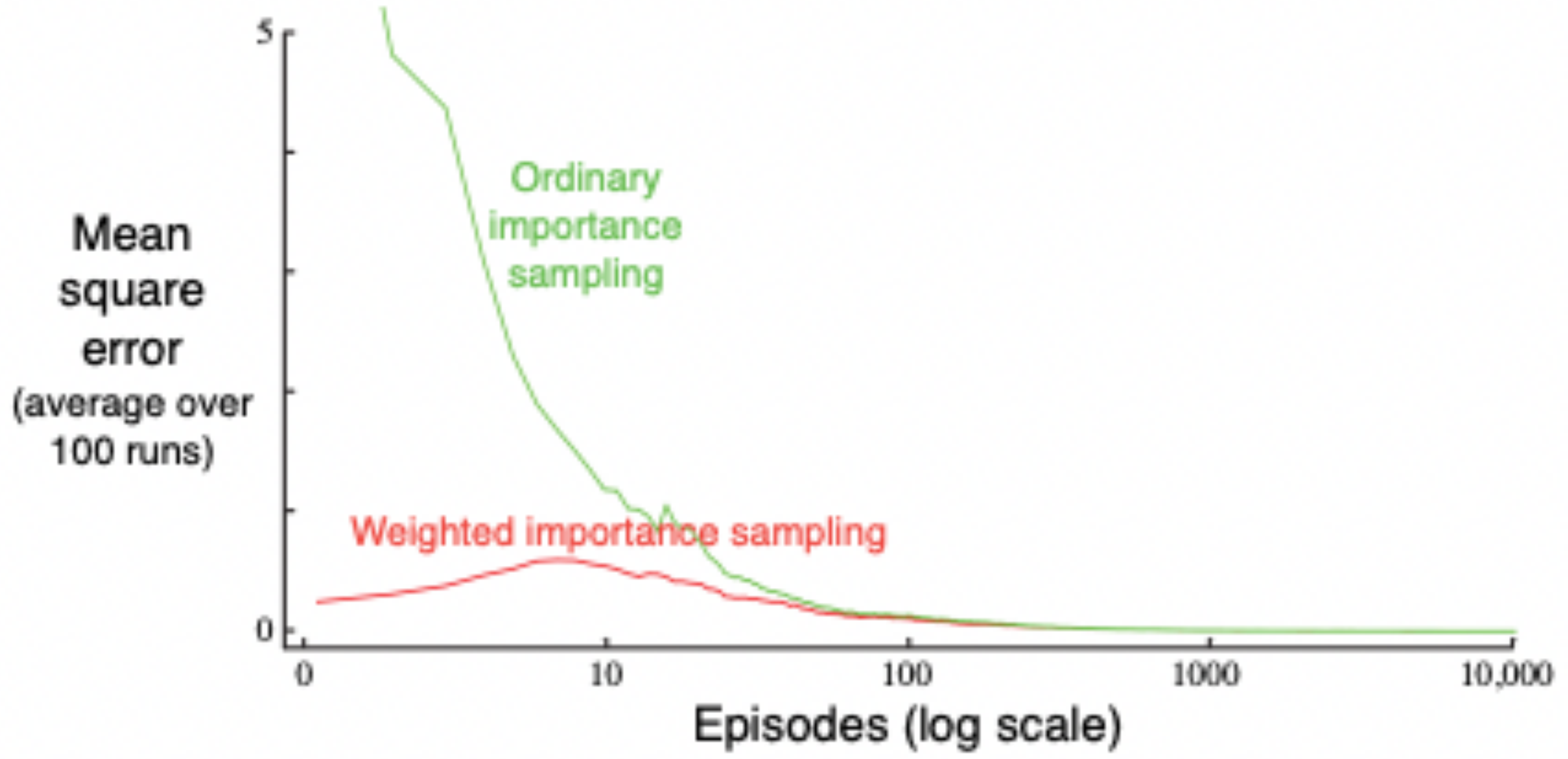
$$V(s_t, a_t) = \rho_{1:T} G_t = \frac{w_\pi}{w_b} G_t \qquad w_\pi = \prod_{t=0}^{T} \pi(A_t \mid S_t)$$

$$V(s_t, a_t) = v_\pi(s_t, a_t) \qquad w_\star = \frac{w_\pi G_t}{v_\pi(s_t)}$$

Weighted Importance Sampling

- Estimation error = Variance + Bias^2. Often a trade-off: can reduce variance by introducing bias.
- Weighted Importance Sampling introduces bias but can drastically lower variance.

$$V(s) := \frac{\sum_{t \in T(s)} \rho_{t:T} G_t}{\sum_{t \in T(s)} \rho_{t:T}}$$



Per-Decision Importance Sampling

- Ordinary importance sampling re-weights all rewards the same:

$$\rho_{t:T} G_t = \rho_{t:T}(R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-t-1} R_T)$$

- Actions that follow a reward do not affect the likelihood of that reward.

$$\rho_{t:T} \gamma^k R_{t+k+1} = \rho_{1:k} \cdot \rho_{k+1:T} \gamma^k R_{t+k+1}$$

- Per-decision importance sampling takes advantage of this by dropping factors in the importance ratios.

$$\mathbf{E}_b[\rho_{t:T} \gamma^k R_{t+k+1}] = \mathbf{E}_b[\rho_{1:k} \gamma^k R_{t+k+1}]$$

Off-Policy Control

- With off-policy prediction, we can run a *soft* behavior policy to provide exploration while improving the target policy greedily.
  - Behavior policy must ensure state-action coverage.
  - Ex: Behavior policy is ϵ-greedy and target policy is greedy.
- Still follow general policy iteration scheme:
  - Evaluate target policy (i.e., estimate q_π) with off-policy Monte Carlo.
  - Make target policy greedy w.r.t. q_π.
  - Converges to $\pi^\star$.
- Is this efficient?

$$\rho_{t:T} := \prod_{i=t}^{T-1} \frac{\pi(a_i \mid s_i) p(s_{i+1}, r_{i+1} \mid s_i, a_i)}{b(a_i \mid s_i) p(s_{i+1}, r_{i+1} \mid s_i, a_i)} = \prod_{i=t}^{T-1} \frac{\pi(a_i \mid s_i)}{b(a_i \mid s_i)}$$

How to use IS in practice

- Clip or bound weights, i.e., $\rho \leftarrow \min(\frac{\pi(a \mid s)}{b(a \mid s)}, 1)$.
- Restrict policy difference.
- Baselines and doubly robust estimators.
- Bootstrap — truncate the return after k steps and use $\gamma^{t+k-1} v_\pi(S_{t+k})$ in place of the sum of the remaining rewards.

Aakarsh’s Presentation

<u>Slides</u>

Gambler’s Problem

- Why should we bet all capital when s=50 but only \$1 when s=\$51?
- Why does the value function update in a step-wise manner?

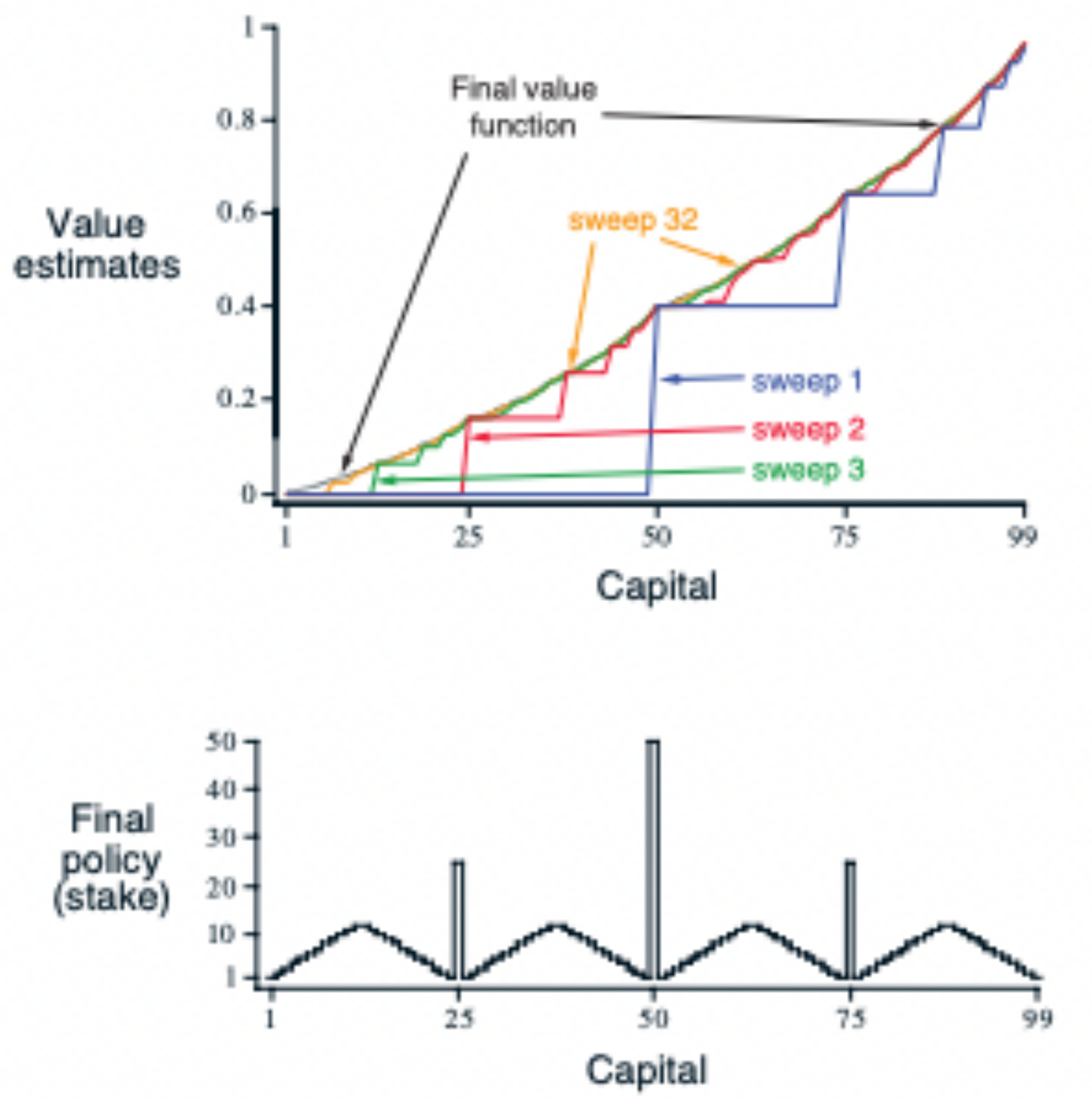


Figure 4.3: The solution to the gambler’s problem for $p_h = 0.4$. The upper graph shows the value function found by successive sweeps of value iteration. The lower graph shows the final policy. ■

Discounting Aware Importance Sampling

- Discounted return: $G_t := R_{t+1} + \gamma R^2_{t+2} + \ldots \gamma^{T-1} R_T$

- Alternatively, the discount represents the probability of not terminating. Episodes terminate with probability $1 - \gamma$.

- What is the expected *undercounted* return under this formalism:

- $(1-\gamma)R_{t+1} + (1-\gamma)\gamma(R_{t+1} + R_{t+2}) + \ldots + (1-\gamma)\gamma^{T-t-2}\sum_{k=1}^{T-1} R_{t+k} + \gamma^{T-t-1}\sum_{k=1}^{T} R_{t+k} = G_t$

- Now a very similar idea to per-decision IS; no need to importance sample actions after all rewards in a partial return have been received.

Summary

- Off-Policy Monte Carlo policy evaluation methods enable learning q_π while taking actions according to a behavior policy b.
- Importance sampling re-weights returns so that — in expectation — they are equal to q_π.
- Off-Policy Monte Carlo policy iteration uses a behavior policy for exploration while learning an optimal target policy.

Action Items

- Homework 1 due Thursday @ 9:29 am.
- Start reading chapter 6 for next week.
- Be thinking about final project — proposal due next week.
    - The more concrete your proposal is, the better guidance you will receive!