CS 540 Introduction to Artificial Intelligence

Search II: Informed Search

University of Wisconsin-Madison

Spring 2023

Announcements

Homeworks:

- Homework 8 released today; due Tuesday April 18

Class roadmap:

Tuesday, April 11	Informed Search
Thursday, April 13	Advanced Search
Tuesday, April 18	Games I
Thursday, April 20	Games II
Tuesday, April 25	Reinforcement Learning I

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Practice questions on search and neural networks on Canvas.

Today’s Goals

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- Understand the difference between uninformed and informed search.
- Introduce A* Search
  - Heuristic properties, stopping rules, analysis
- Extensions: Beyond A*
  - Iterative deepening, beam search

Breadth-First Search

Recall: expand **shallowest** node first

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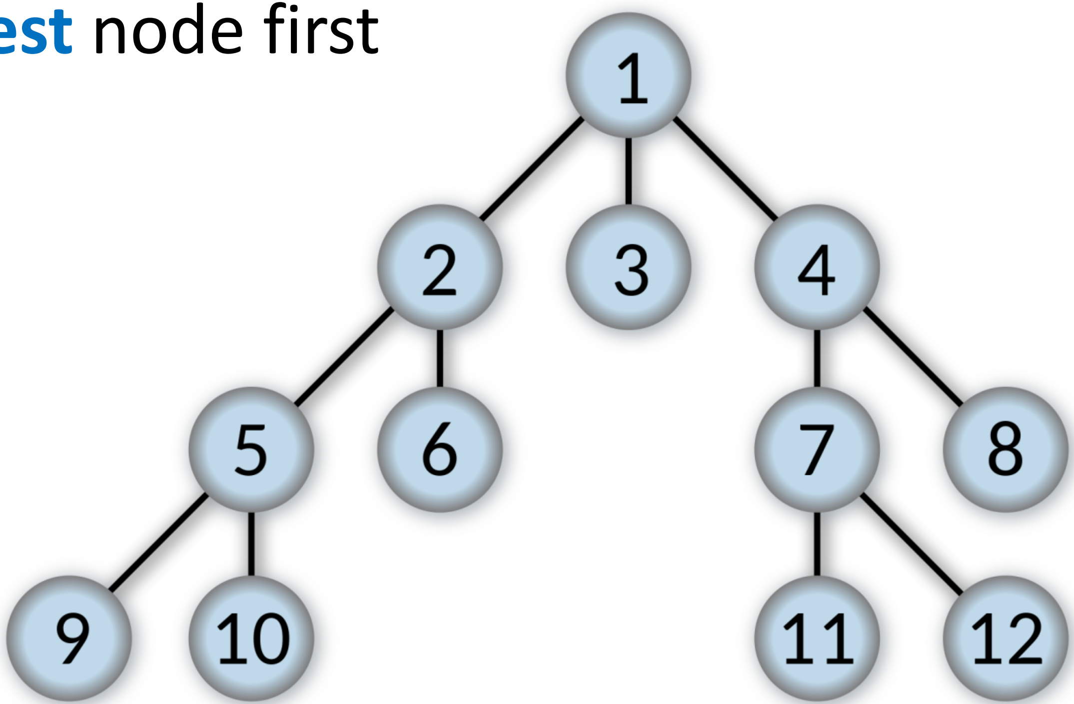

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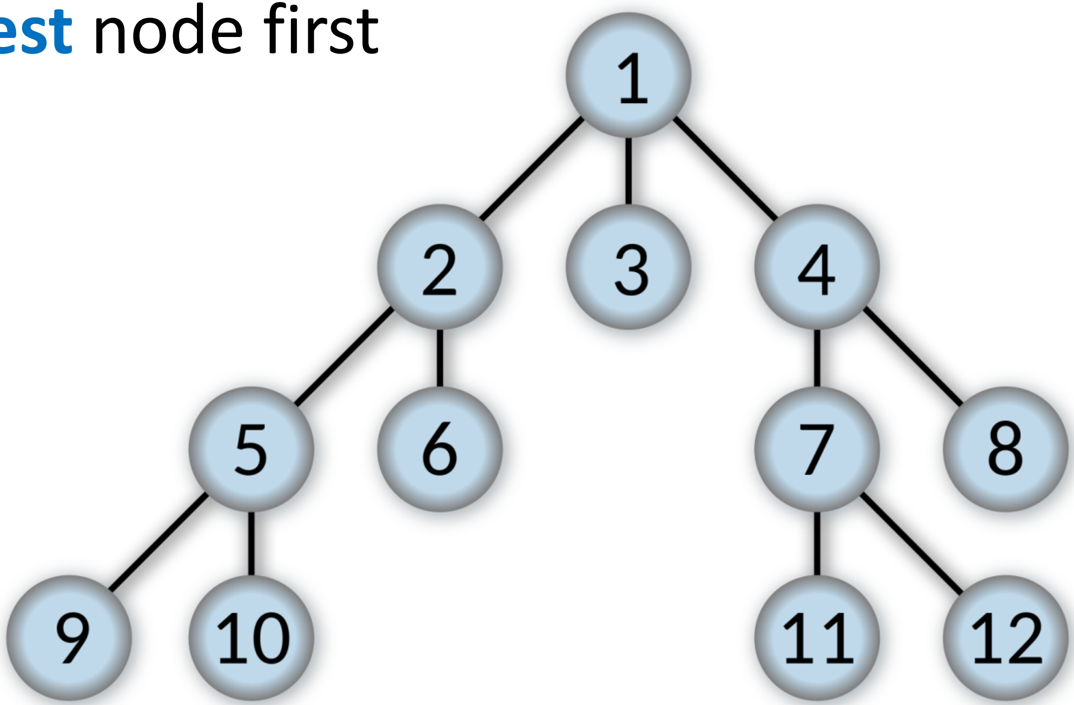

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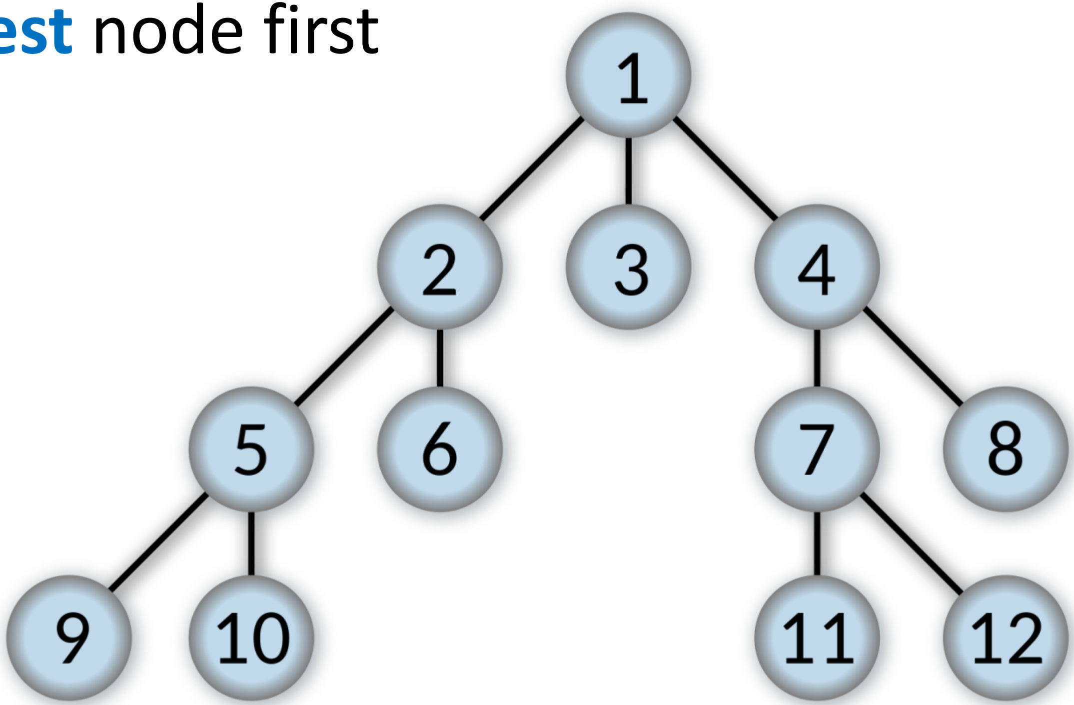

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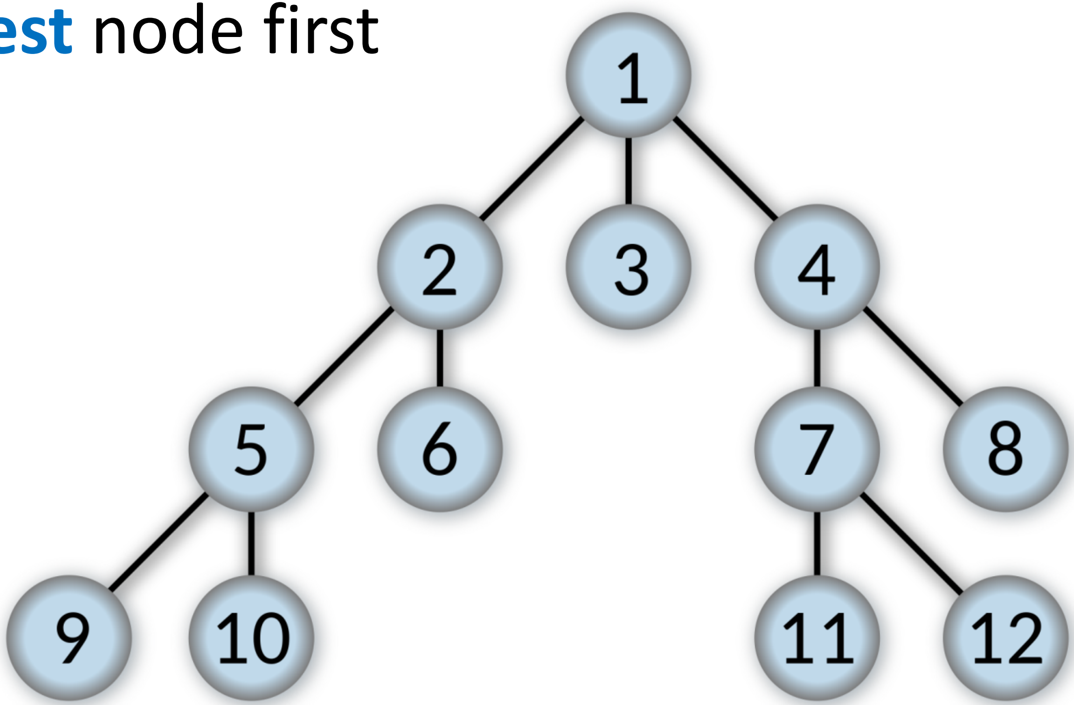

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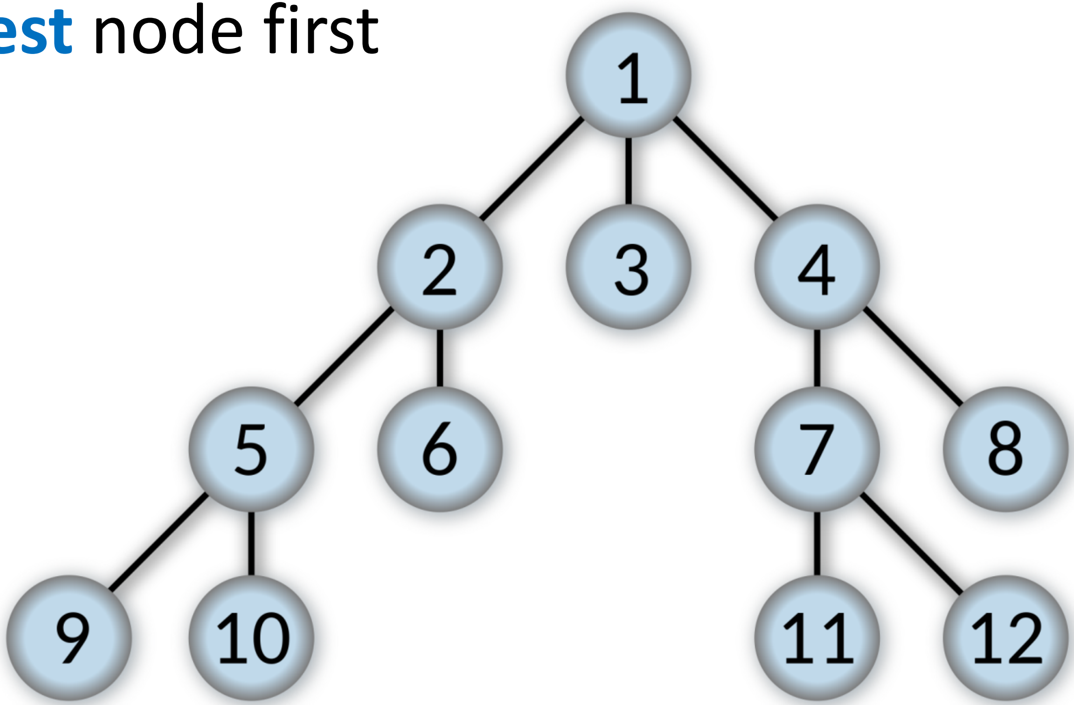

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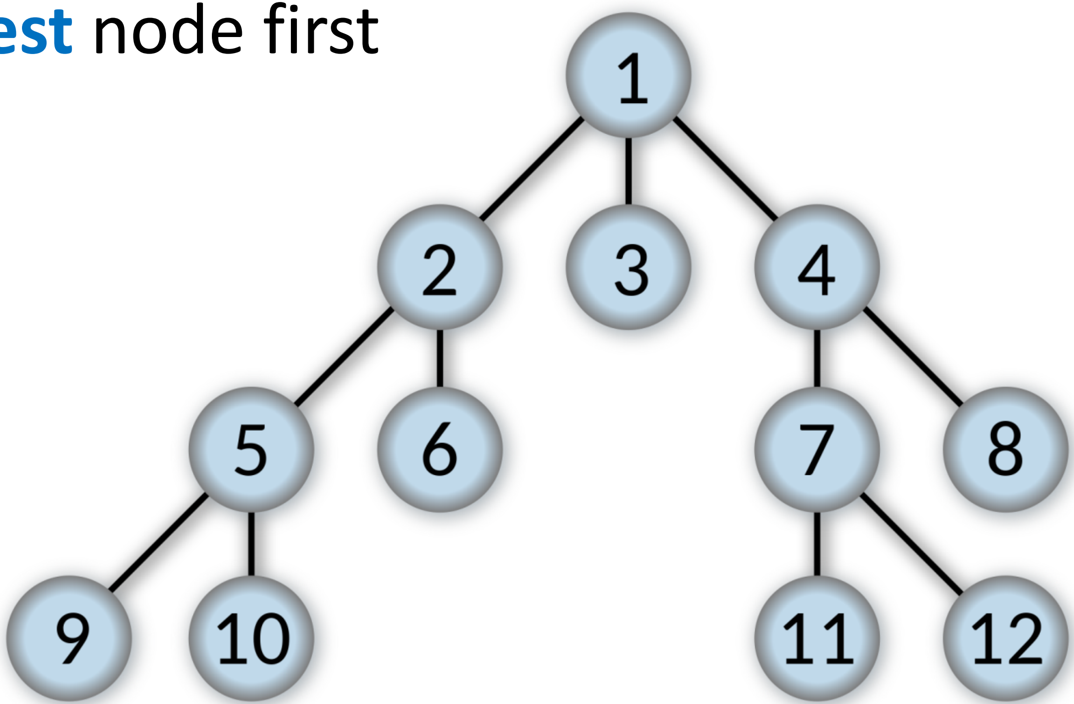

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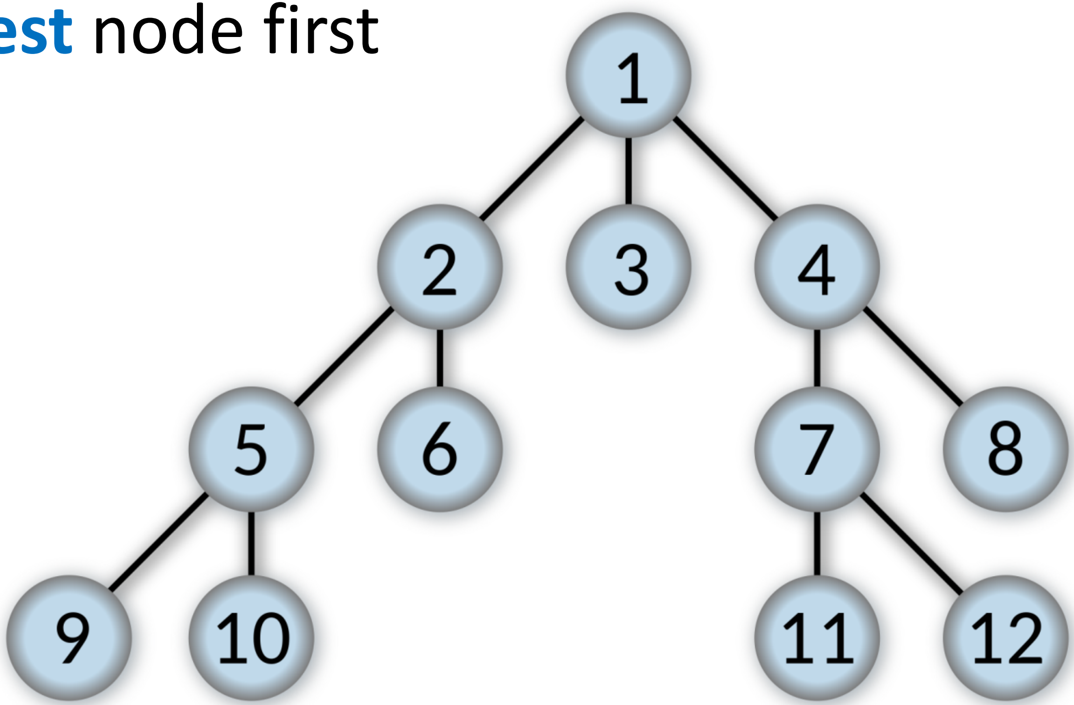

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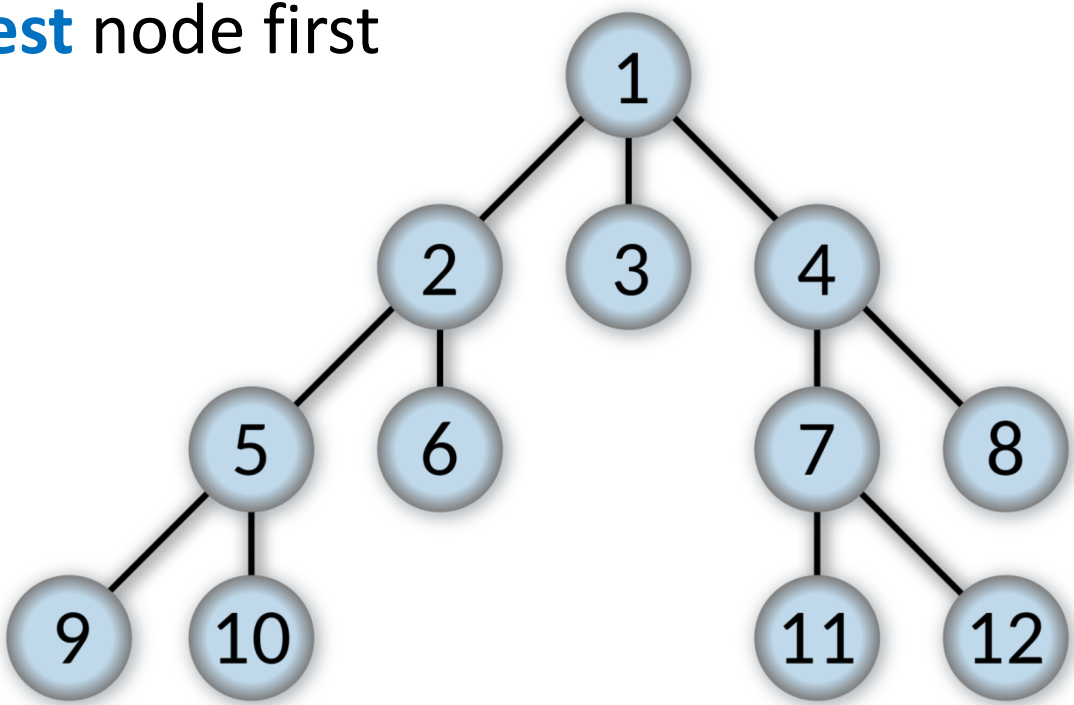

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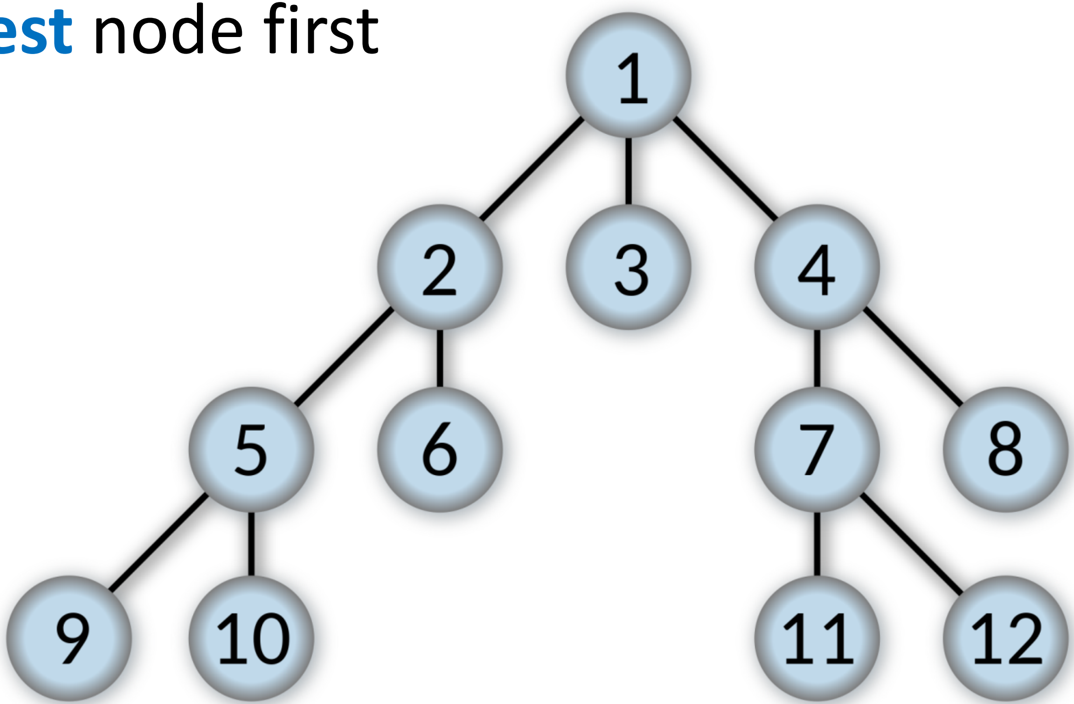

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    C* is optimal path cost to goal.

    ϵ is cost of edge with smallest cost.

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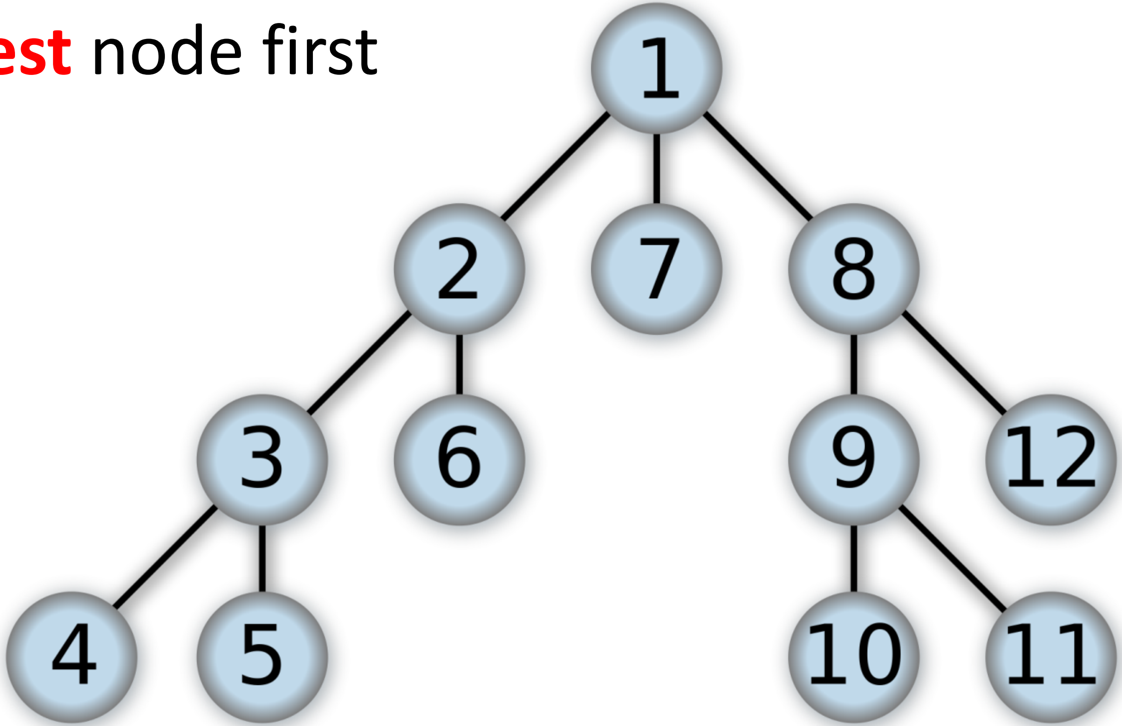

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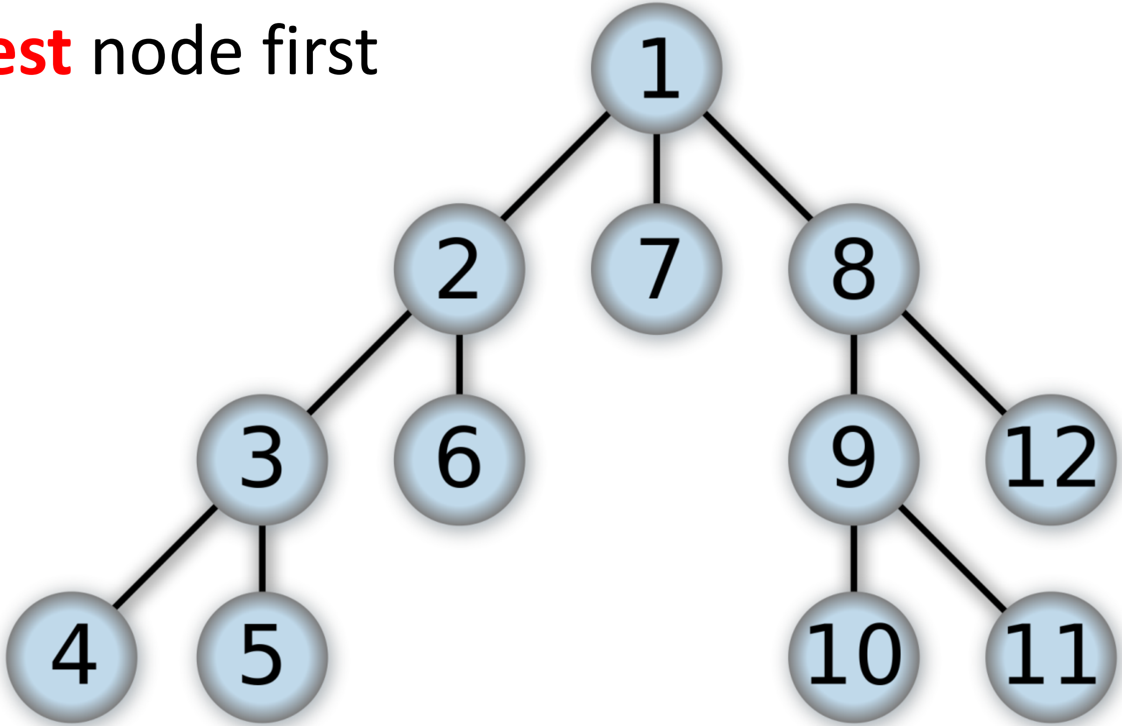

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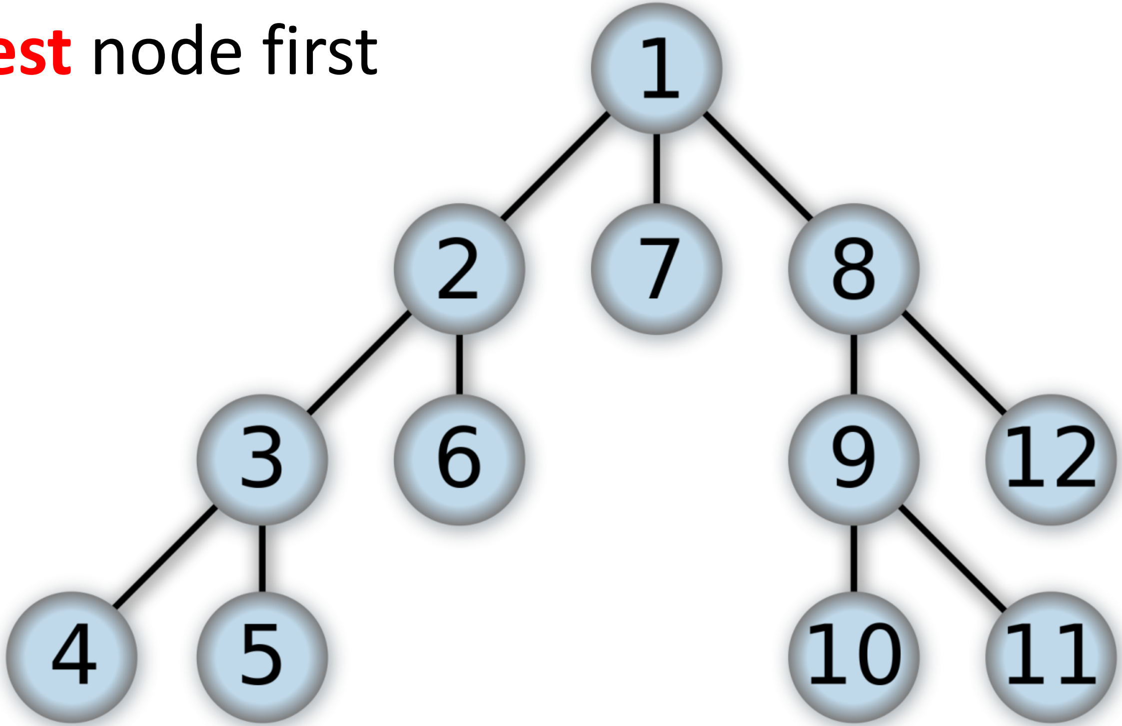

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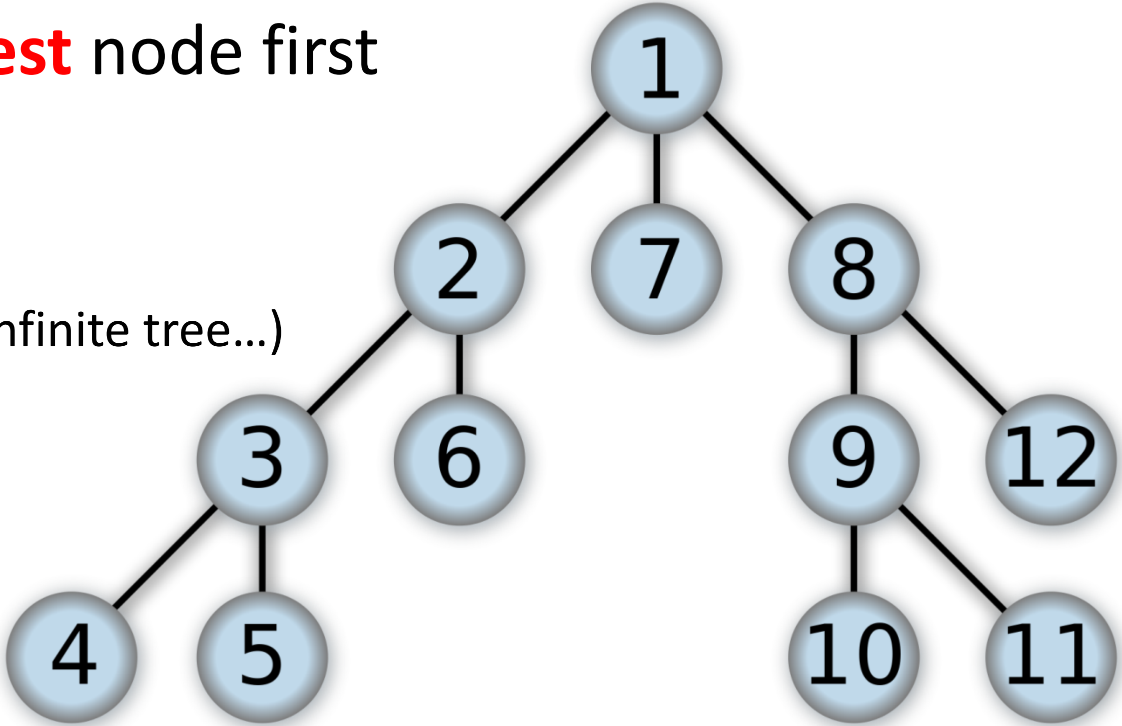

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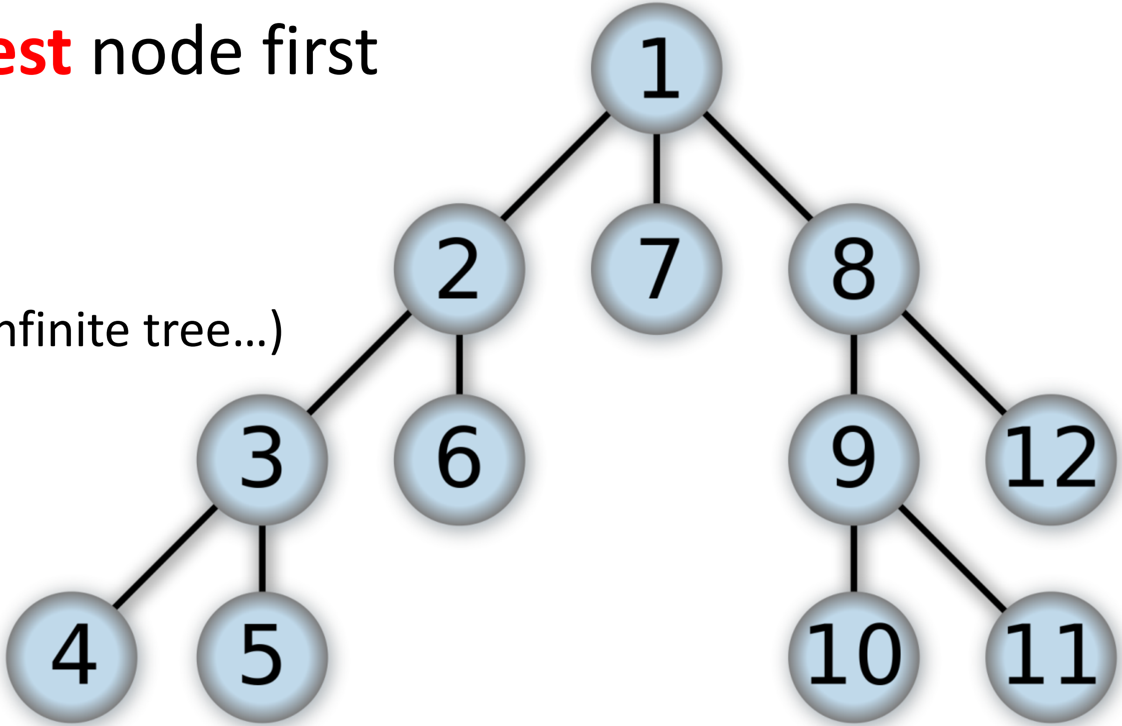

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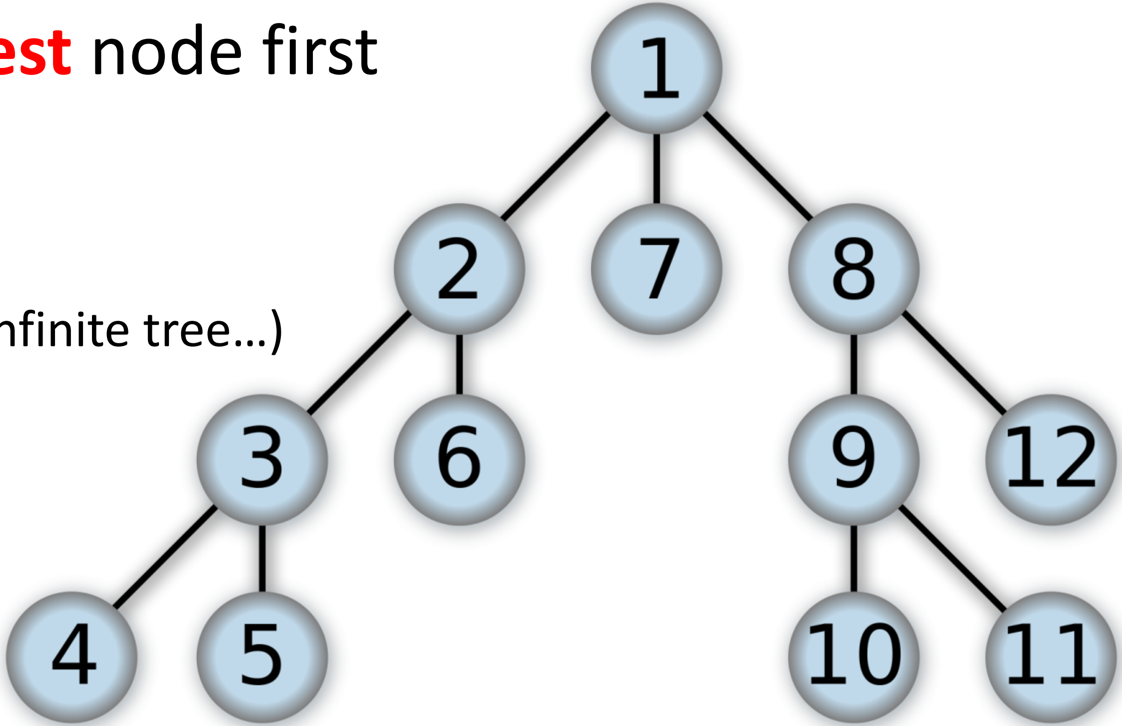

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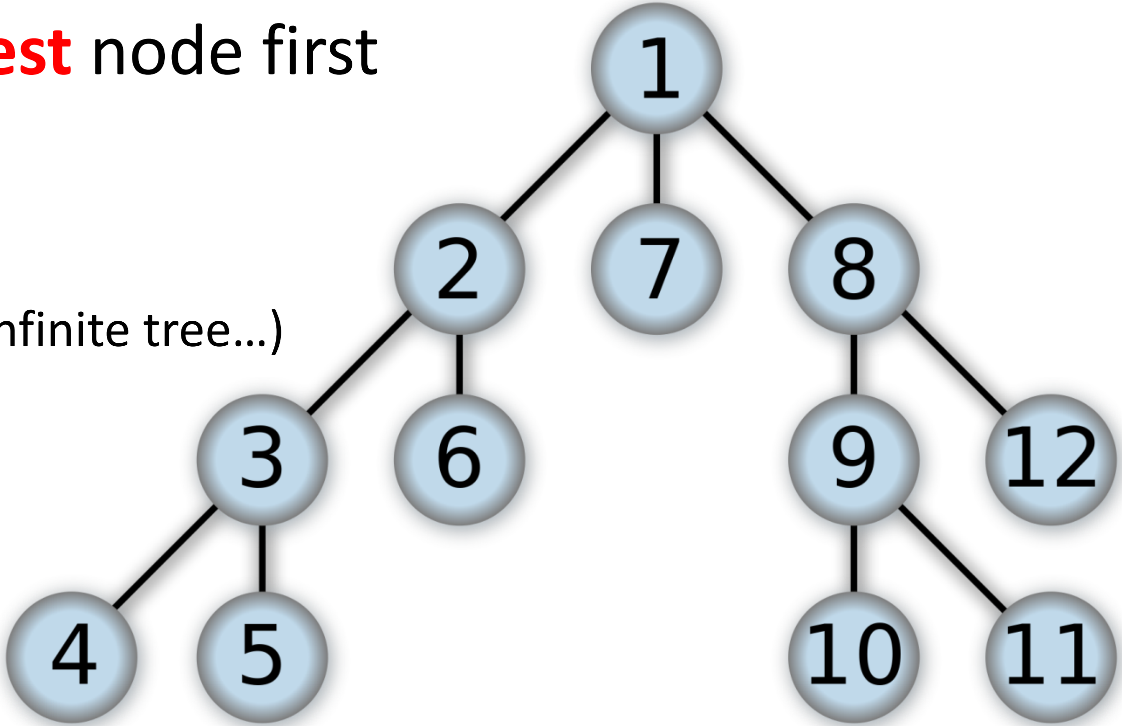

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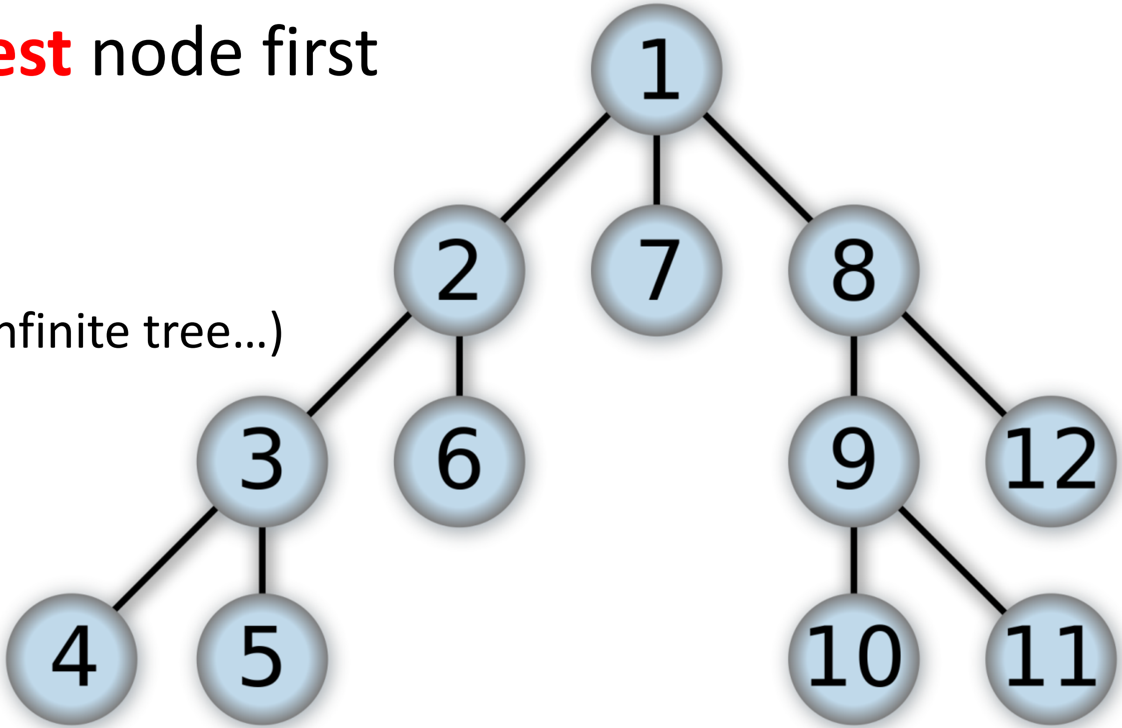

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Iterative Deepening DFS

Repeated limited DFS

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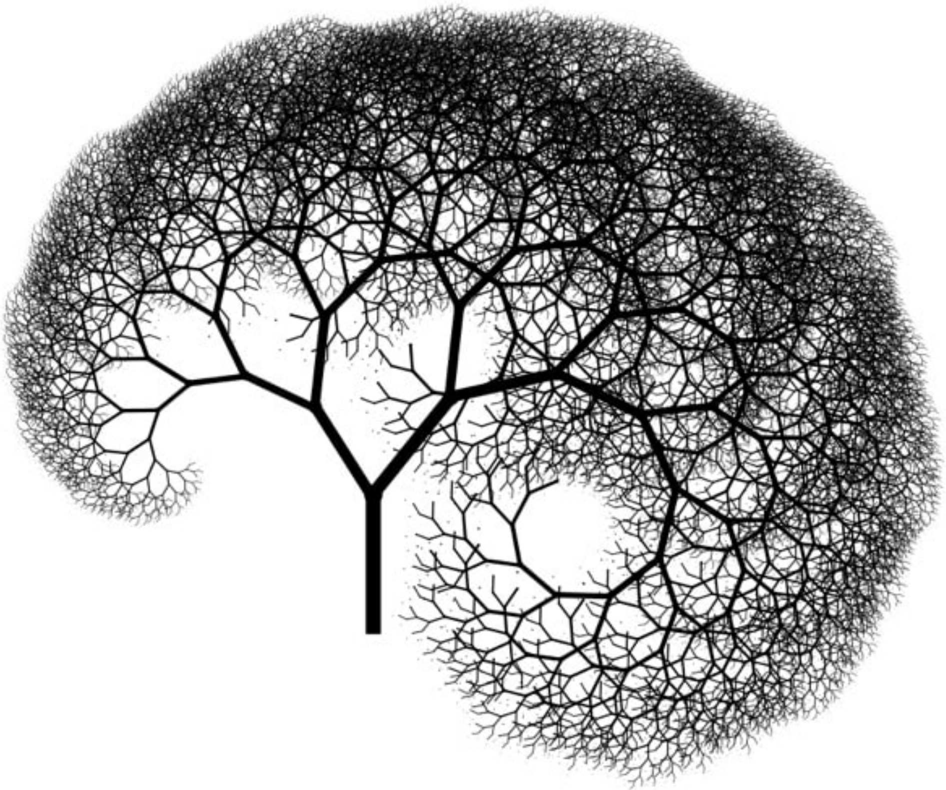

Fractalsaco

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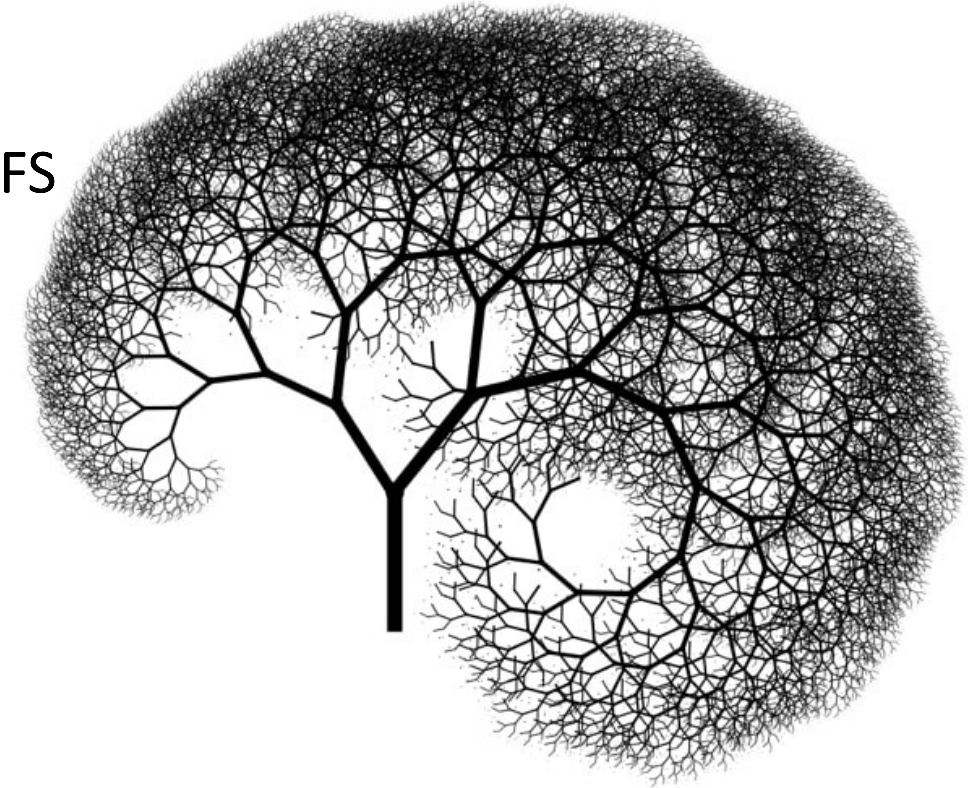
- Search like BFS, fringe like DFS



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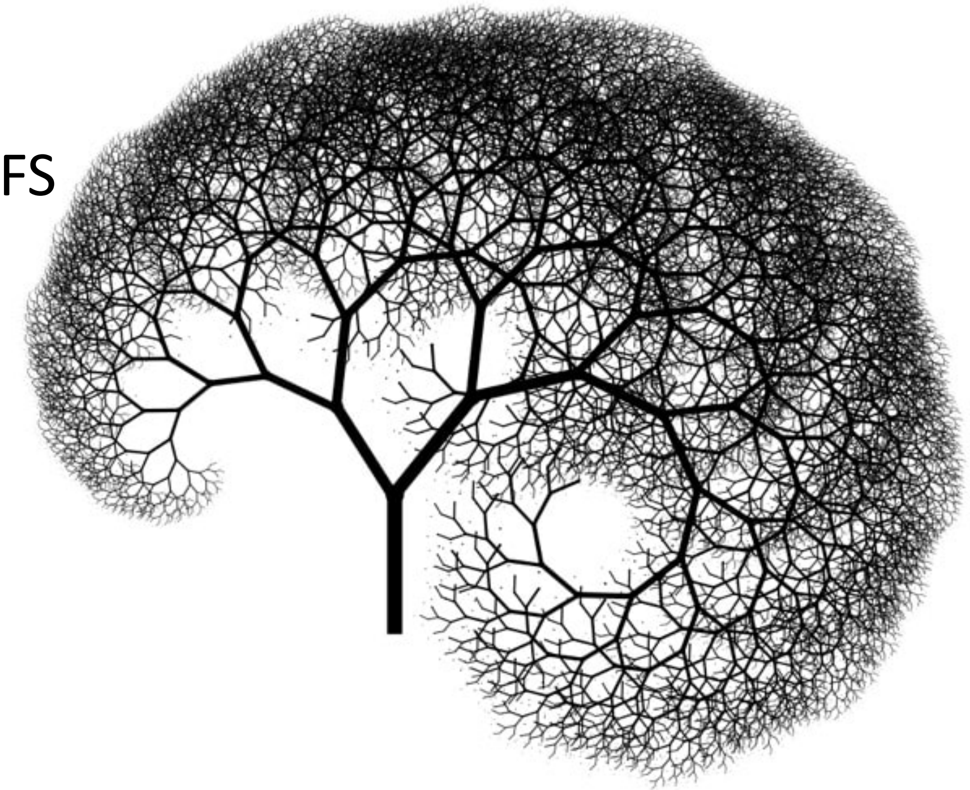

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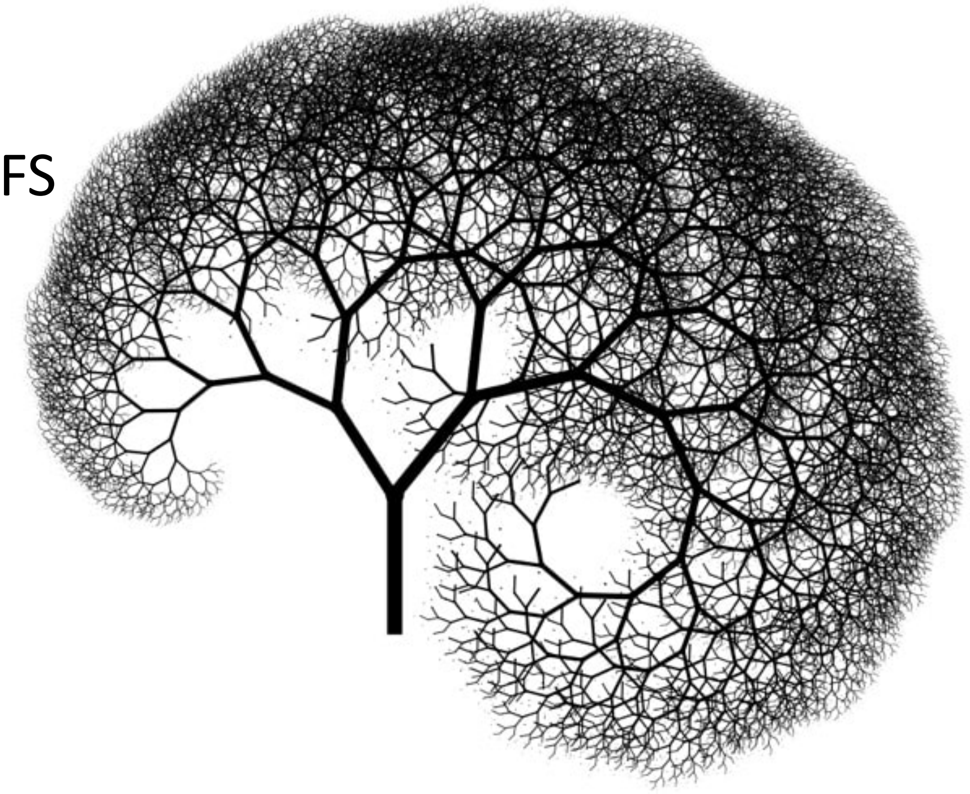

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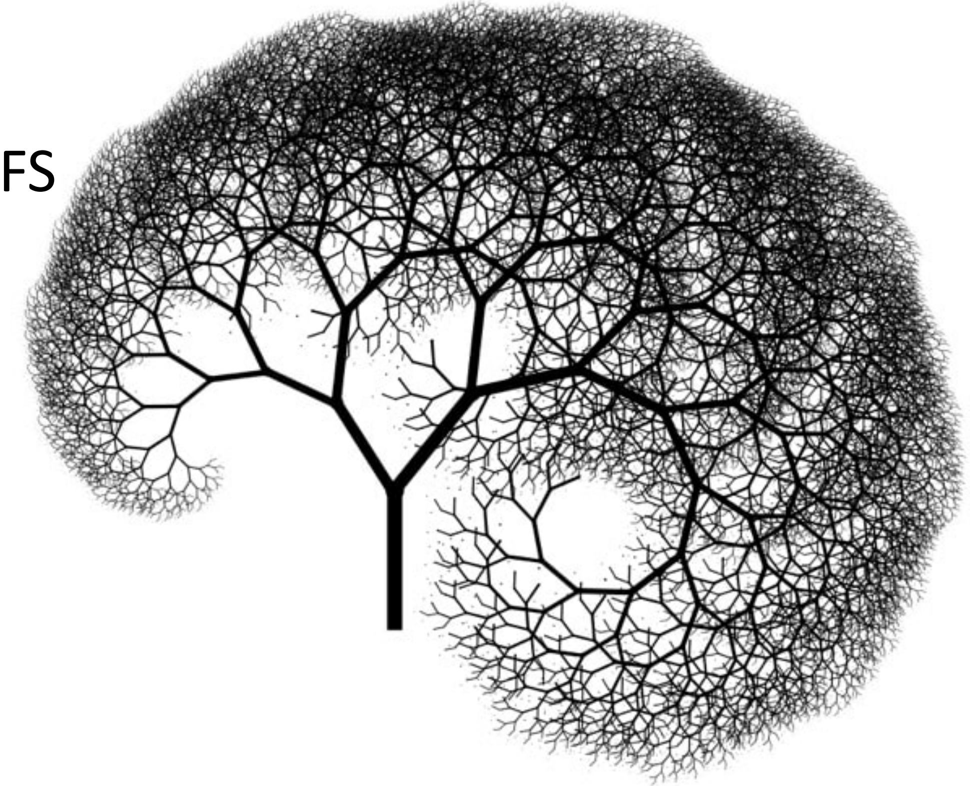

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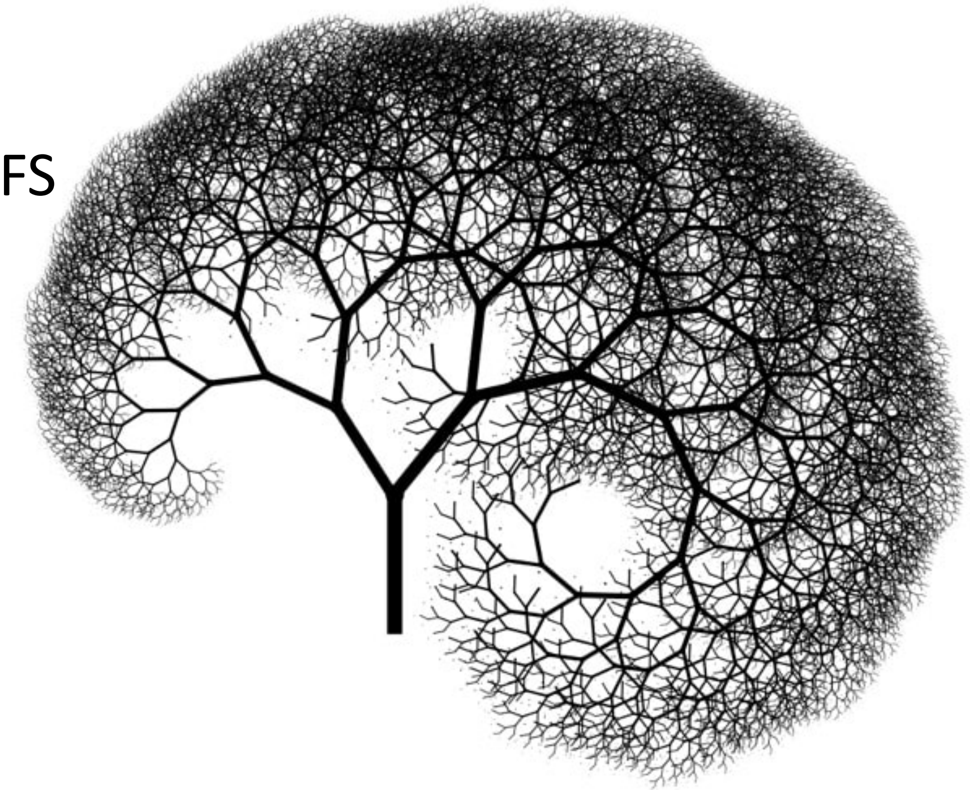

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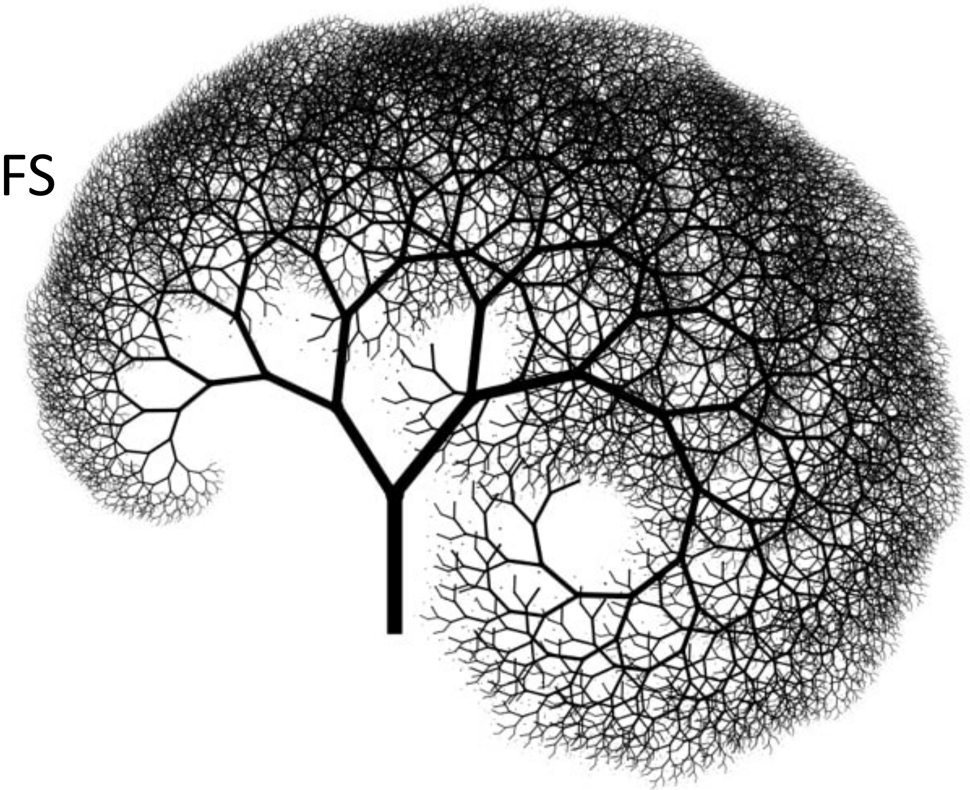

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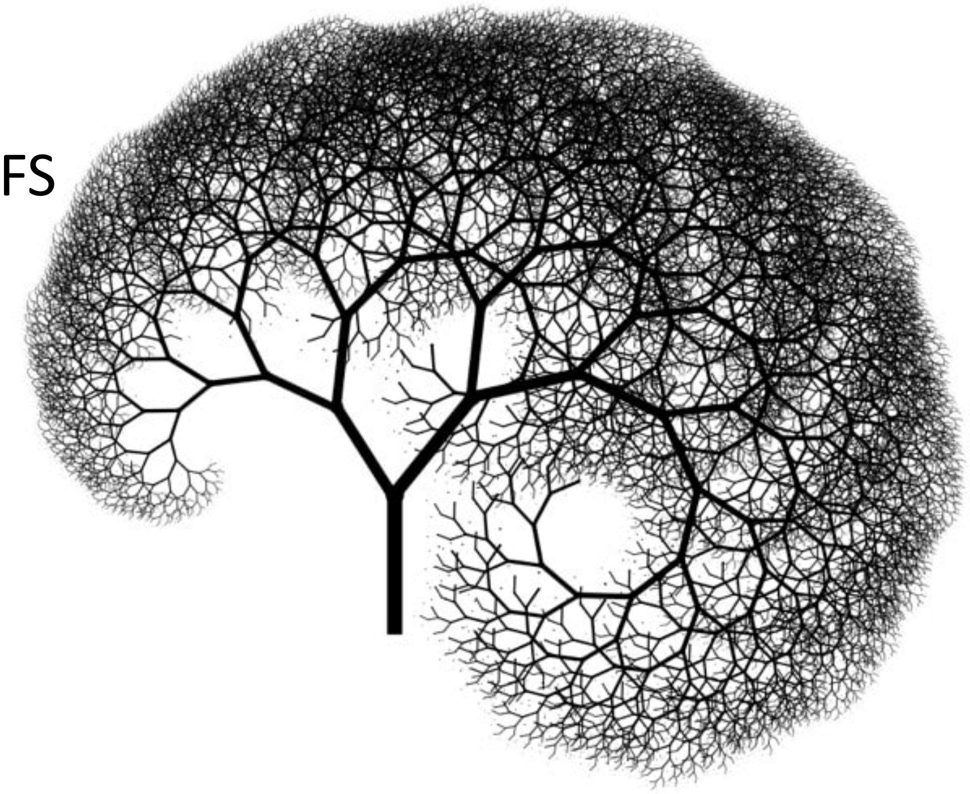
A good option!



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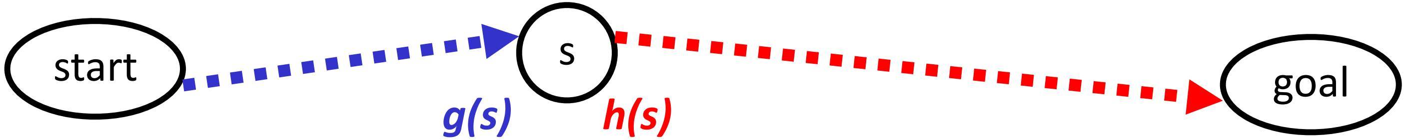

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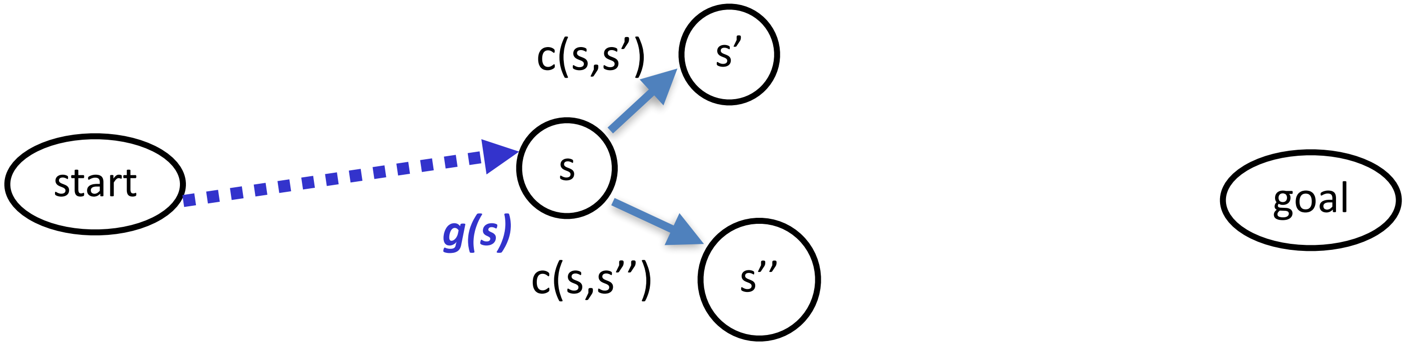

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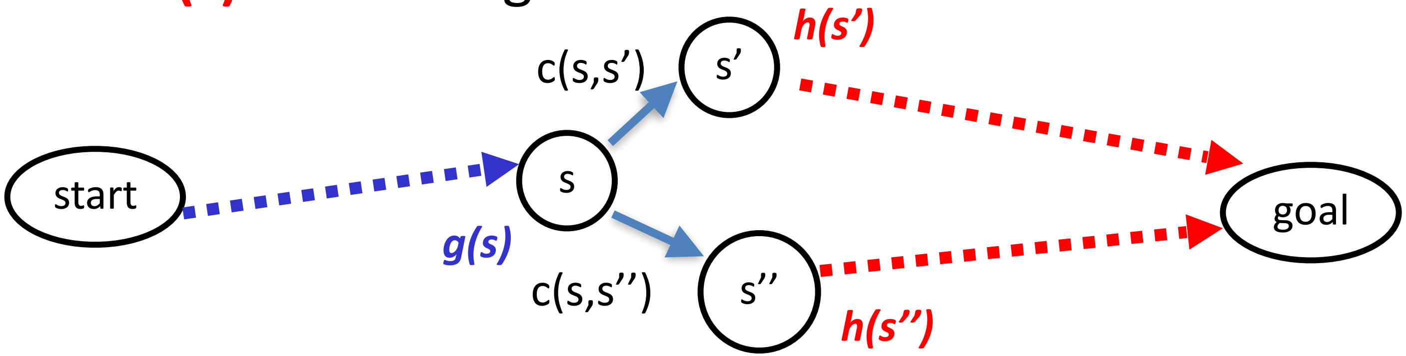

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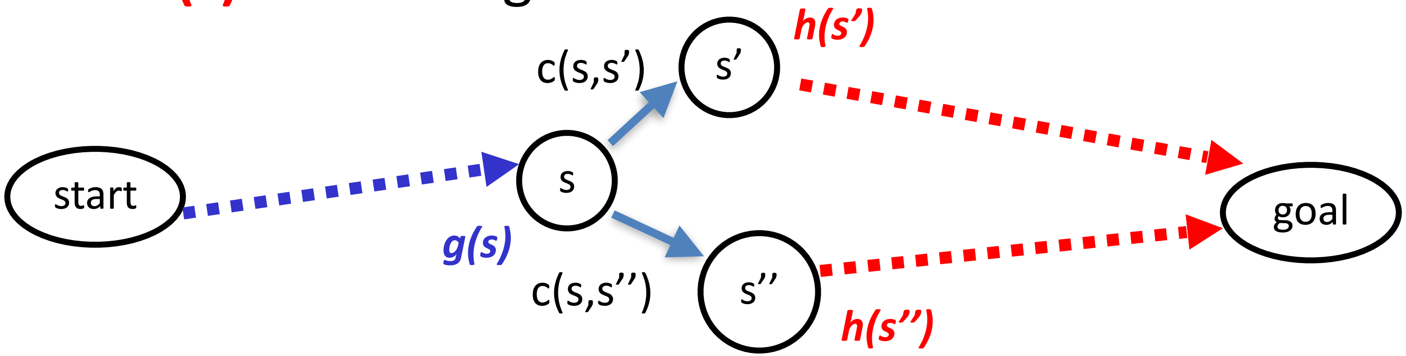

- Goal: **speed up search.**

Using the Heuristic

Recall uniform-cost search

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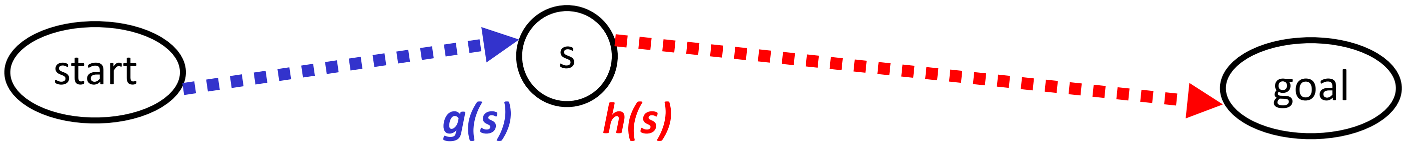

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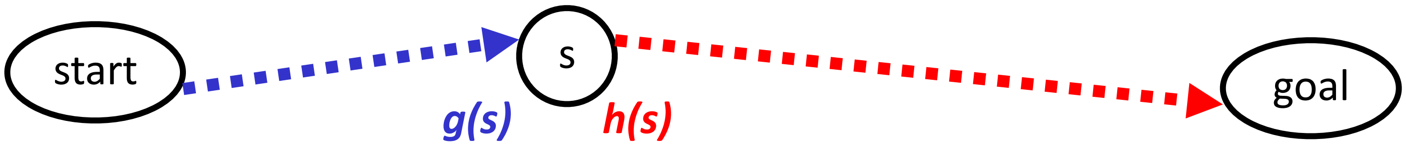

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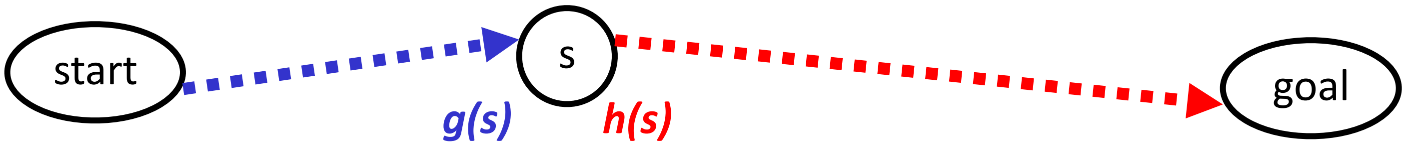

- Now let's use the heuristic ("second-half-cost")
  - Several possible approaches: let's see what works

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One approach: just use ***h*(*s*)** alone

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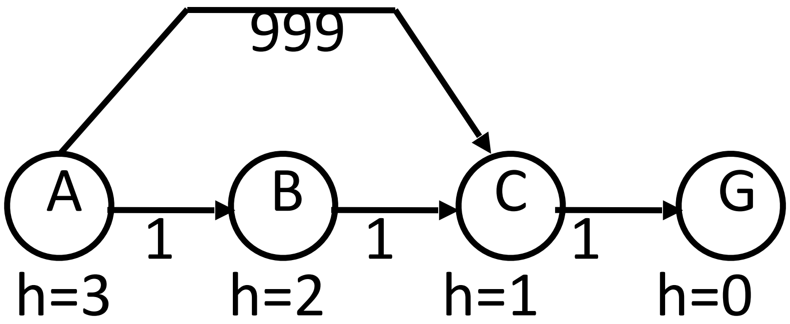

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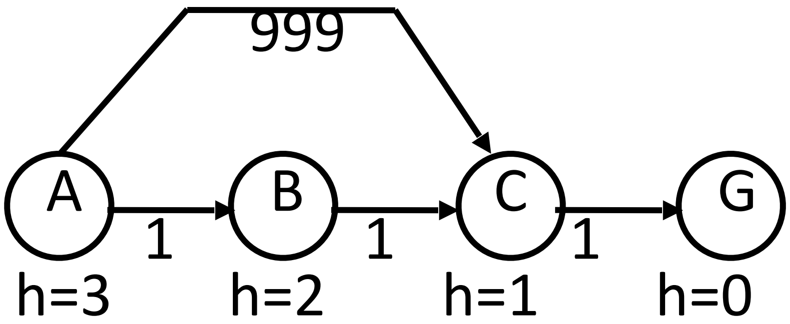

- Not optimal! **Get** A → C → G. **Want**: A →B → C → G

Attempt 2: **A Search**

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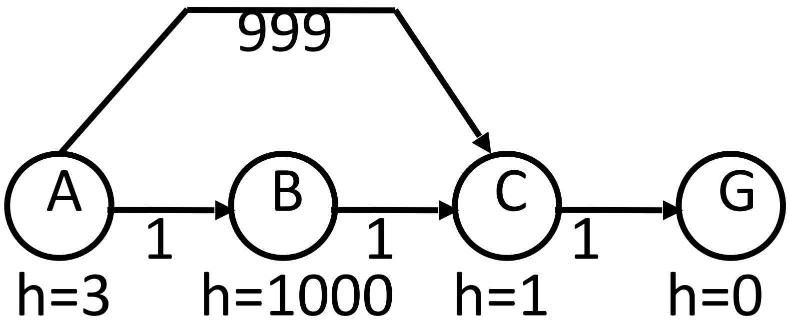

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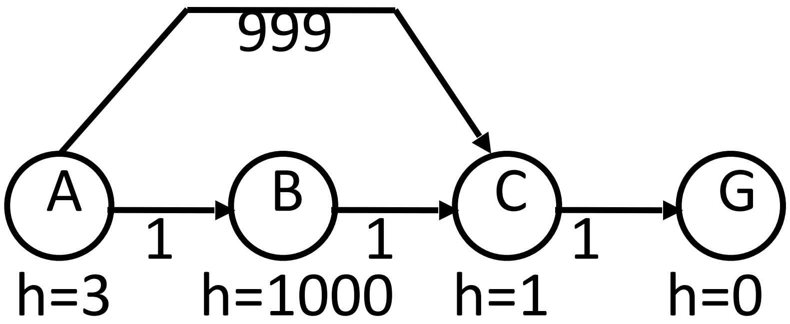

- **Still not optimal!** (Does work for former example).

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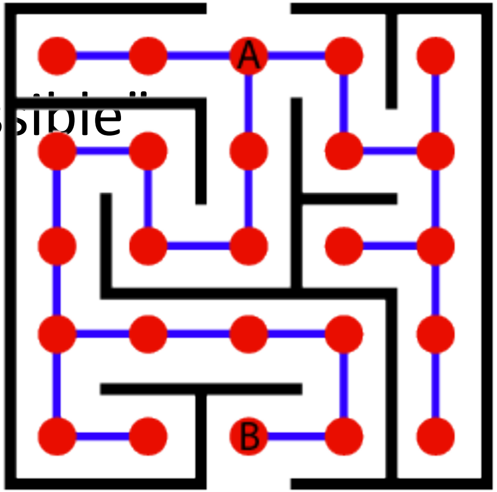

V. Batoćanin

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Origins: robots and planning

Animation: finding a path around obstacle

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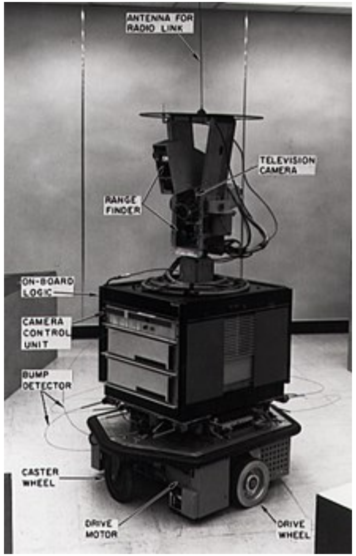

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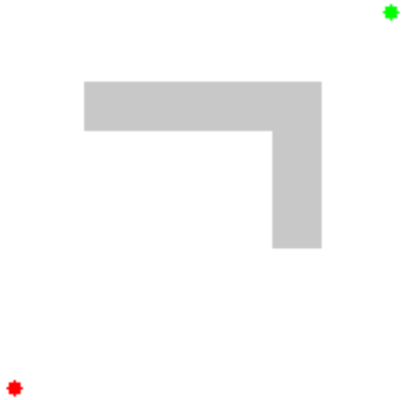

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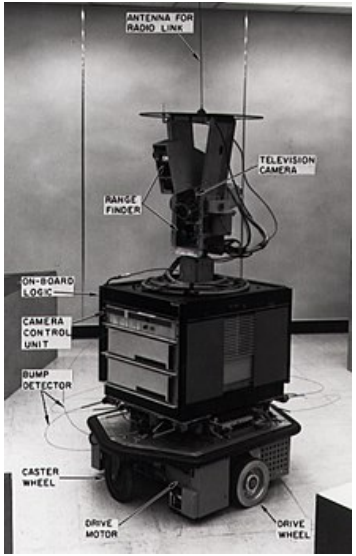

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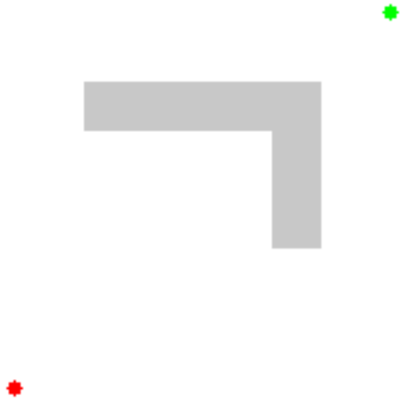

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- One useful approach: **relax constraints**
  - ***h*(*s*)** = number of tiles in wrong position
    - allows tiles to fly to destination in a single step

Break & Quiz

Q 1.1: Consider finding the fastest driving route from one US city to another. Measure cost as the number of hours driven when driving at the speed limit. Let ***h*(*s*)** be the number of hours needed to ride a bike from city s to your destination. ***h*(*s*)** is

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- A. An admissible heuristic No: riding your bike takes longer.
- **B. Not an admissible heuristic**

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Q 1.2: Which of the following are admissible heuristics?

(i) $h(s) = h^*(s)$

(ii) $h(s) = \max(2, h^*(s))$

(iii) $h(s) = \min(2, h^*(s))$

(iv) $h(s) = h^*(s)-2$

(v) $h(s) = \mathrm{sqrt}(h^*(s))$

- A. All of the above
- B. (i), (iii), (iv)
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Q 1.2: Which of the following are admissible heuristics?

(i) $h(s) = h^*(s)$

(ii) $h(s) = \max(2, h^*(s))$ No: $h(s)$ might be too big

(iii) $h(s) = \min(2, h^*(s))$

(iv) $h(s) = h^*(s)-2$ No: $h(s)$ might be negative

(v) $h(s) = \text{sqrt}(h^*(s))$ No: if $h^*(s) < 1$ then $h(s)$ *is bigger*

- A. All of the above
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Heuristic Function Tradeoffs

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- **Idea**: we want to be as close to h^* as possible
  - But not over! **Must under-estimate true cost.**
- **Tradeoff**: being very close might require a very complex heuristic, expensive computation
  - Might be better off with cheaper heuristic & expand more nodes.

A* Termination

When should A* **stop**?

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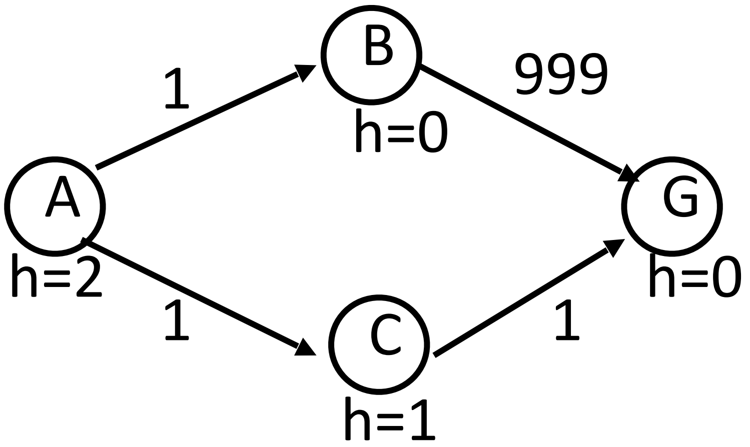

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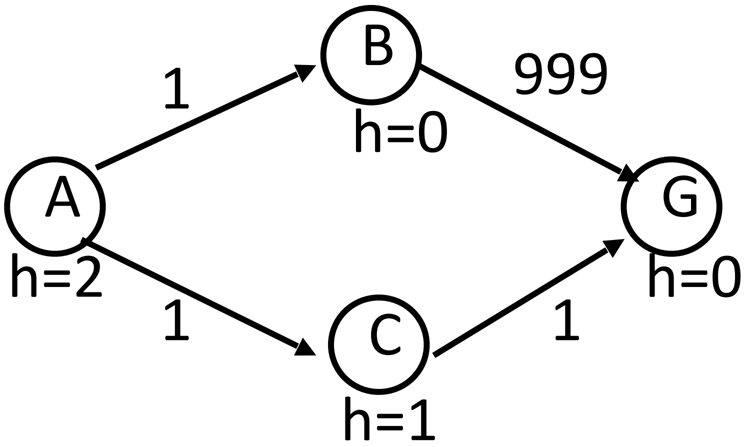

- ***h*** is admissible, but note that we get A →B → G (**cost 1000**)!

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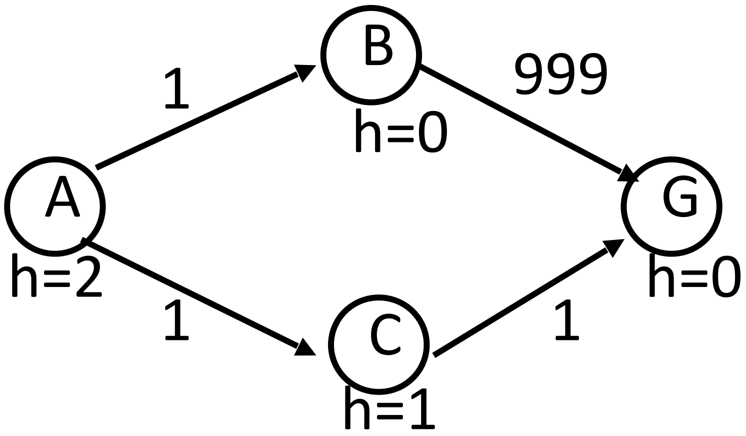

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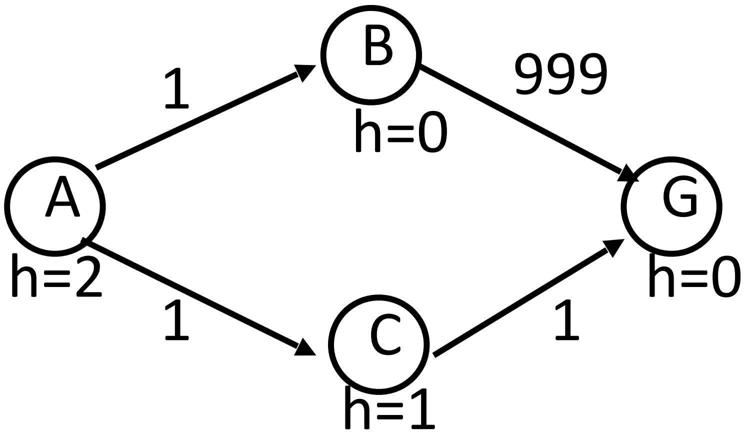

- Note: taking ***h*** =0 reduces to uniform cost search rule.

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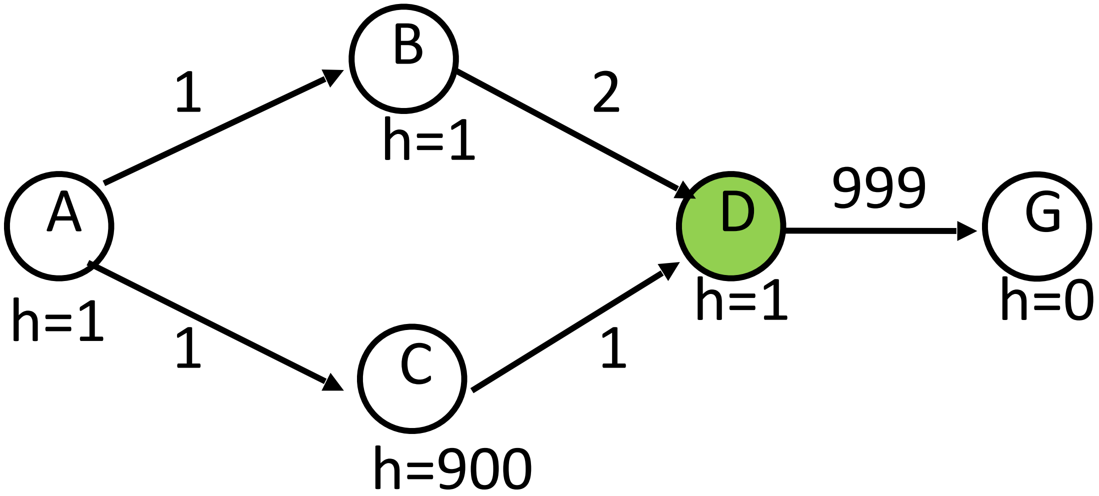

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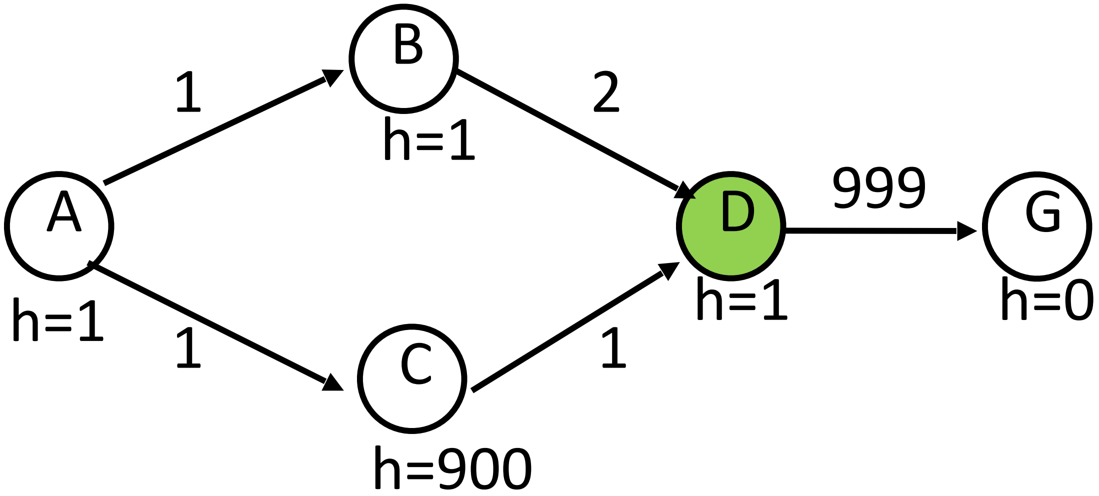

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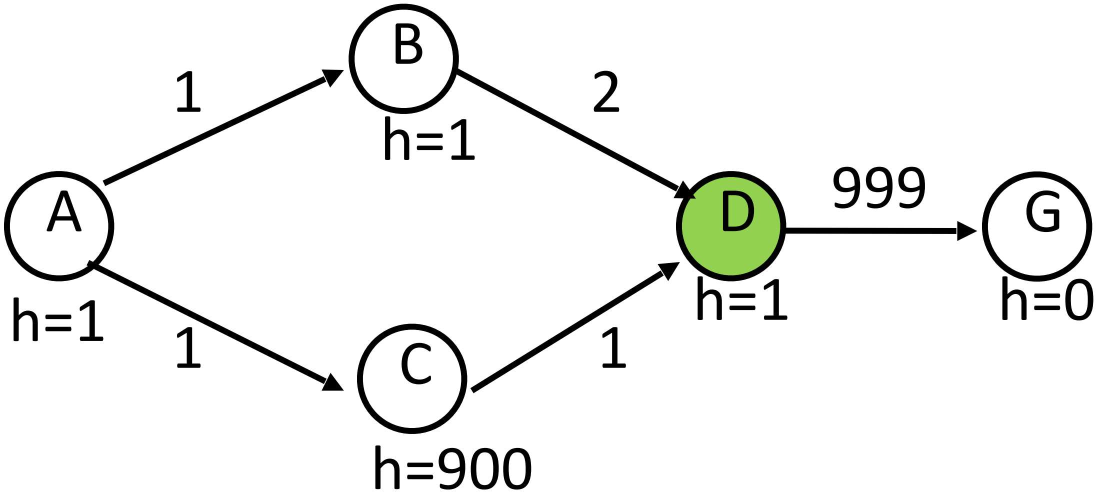

- Put D back into priority queue, smaller **g**+**h.**
- **Note:** uninformed search methods will not revisit expanded states.

A* Full Algorithm

1. Put the start state S on the priority queue. We call the priority queue OPEN
2. If OPEN is empty, exit with failure
3. Remove from OPEN and place on CLOSED a node n for which f(n) is minimum (note that f(n)=g(n)+h(n))
4. If n is a goal node, exit (recover path by tracing back pointers from n to S)
5. Expand n, generating all successors and attach to pointers back to n. For each successor n' of n
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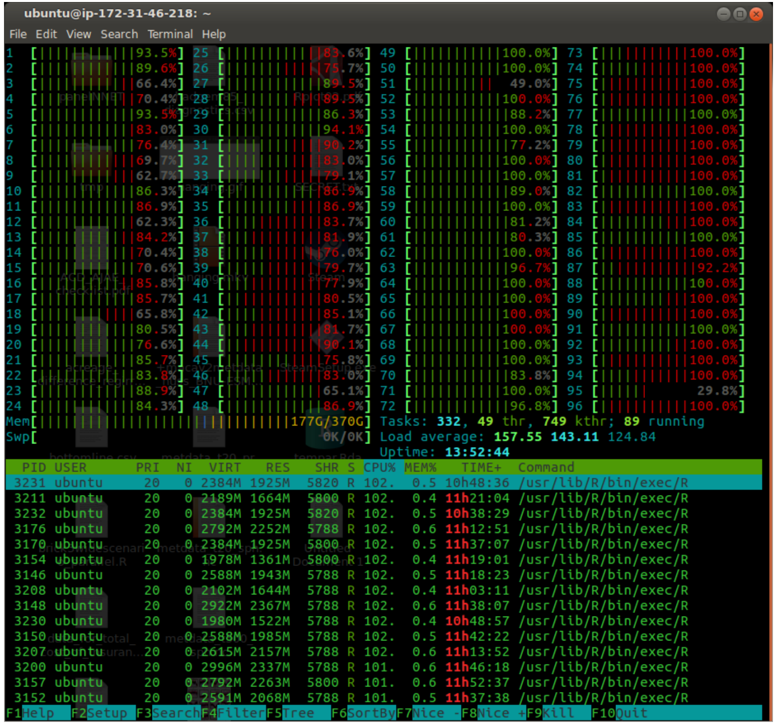

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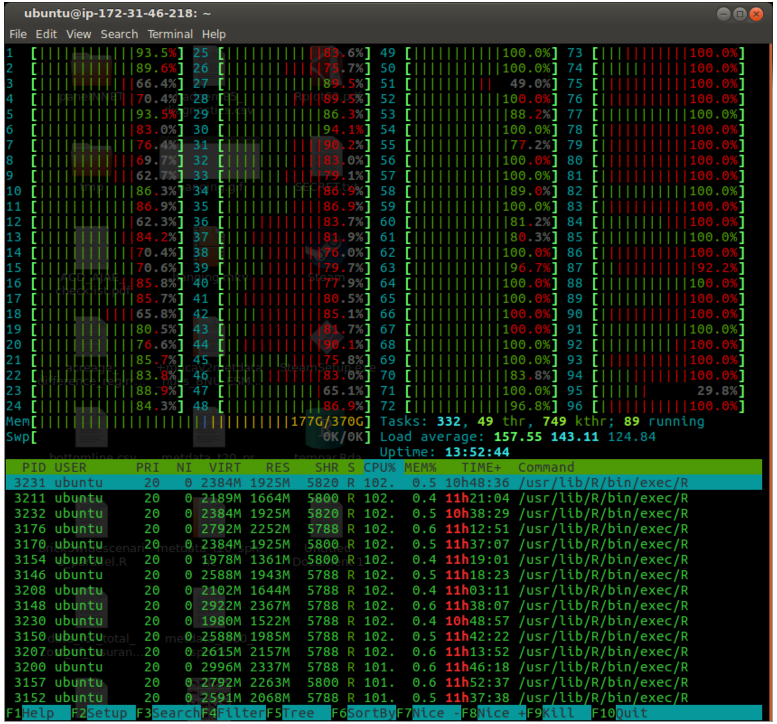
- Next, we will consider some alternatives to deal with this.



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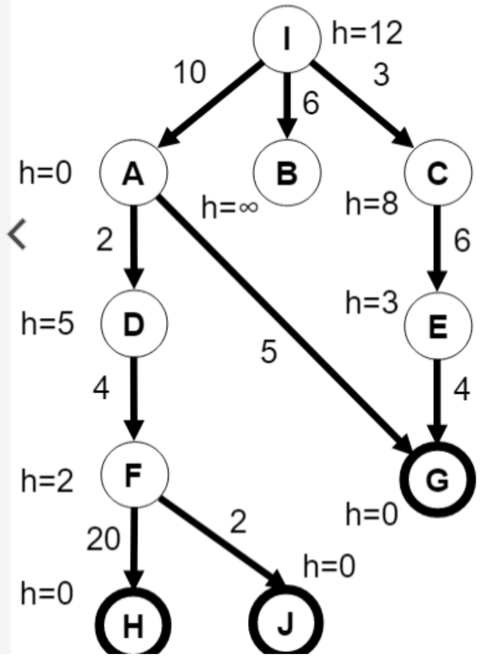

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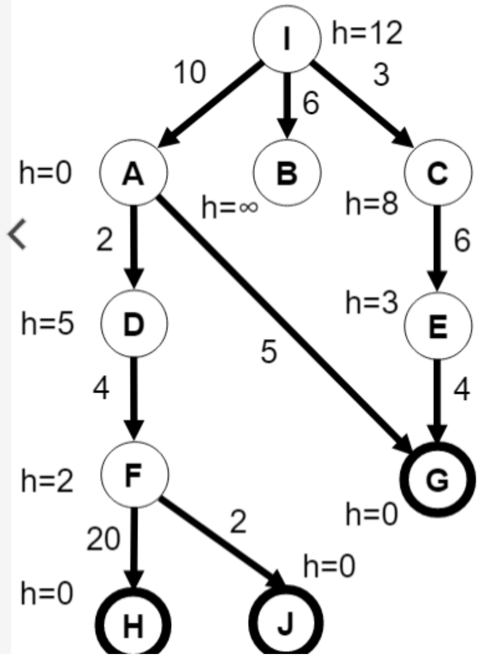

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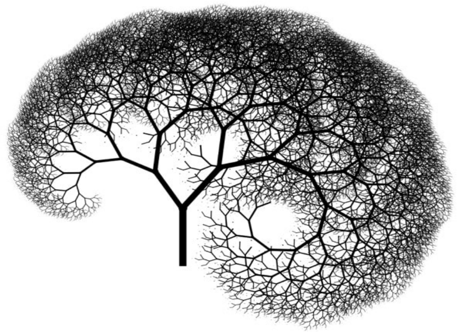

Fractalsaco

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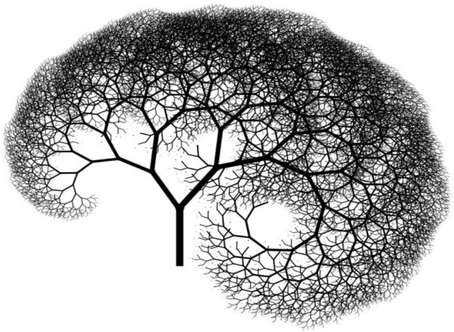

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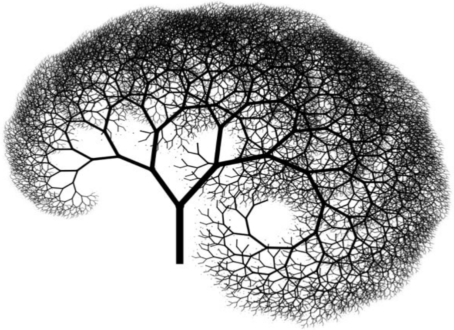

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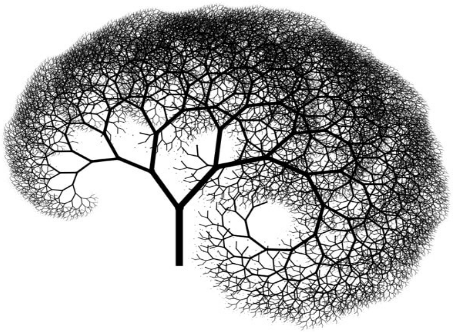
- Complete + optimal, might be costly time-wise
  - Revisit many nodes
- Lower memory use than A*



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- Worst case: restarted for each state

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General approach (beyond A* too)

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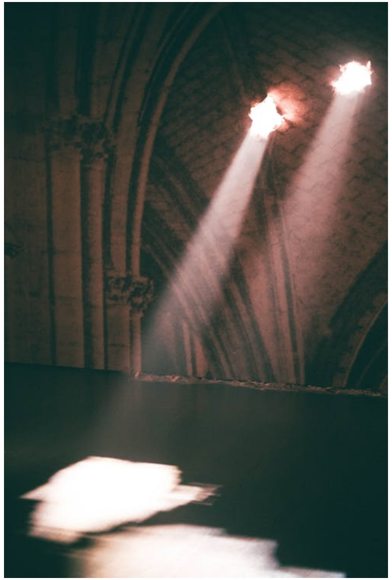

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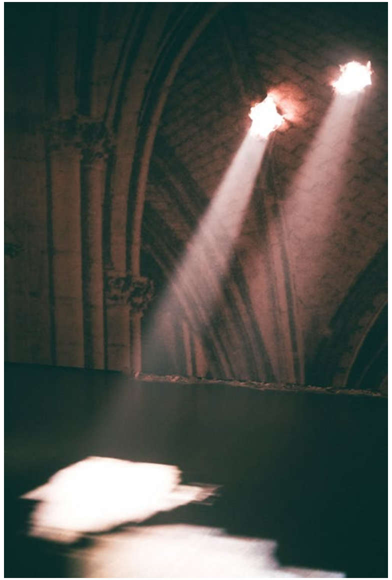

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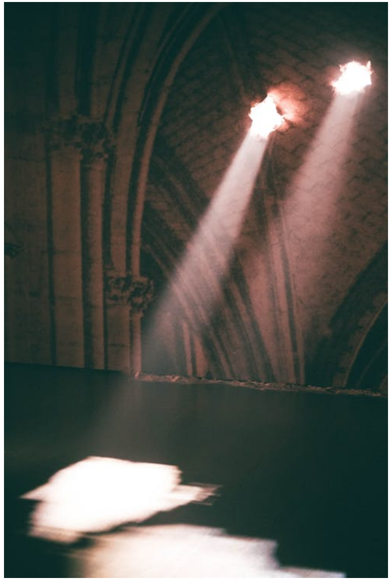

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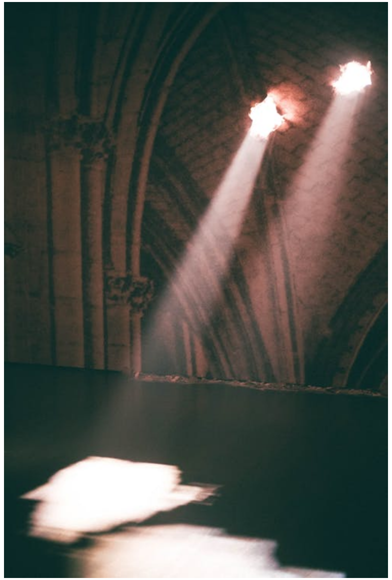

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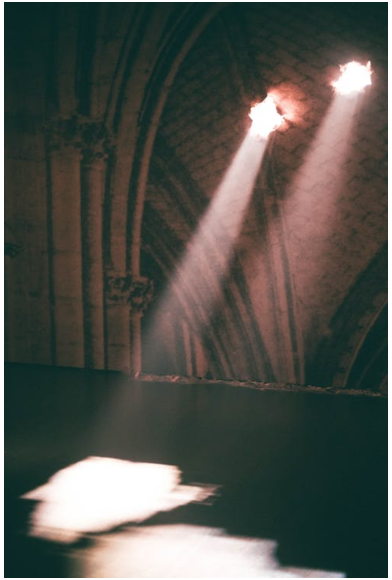
- Priority queue with nodes that **are at most ε worse** than best node.



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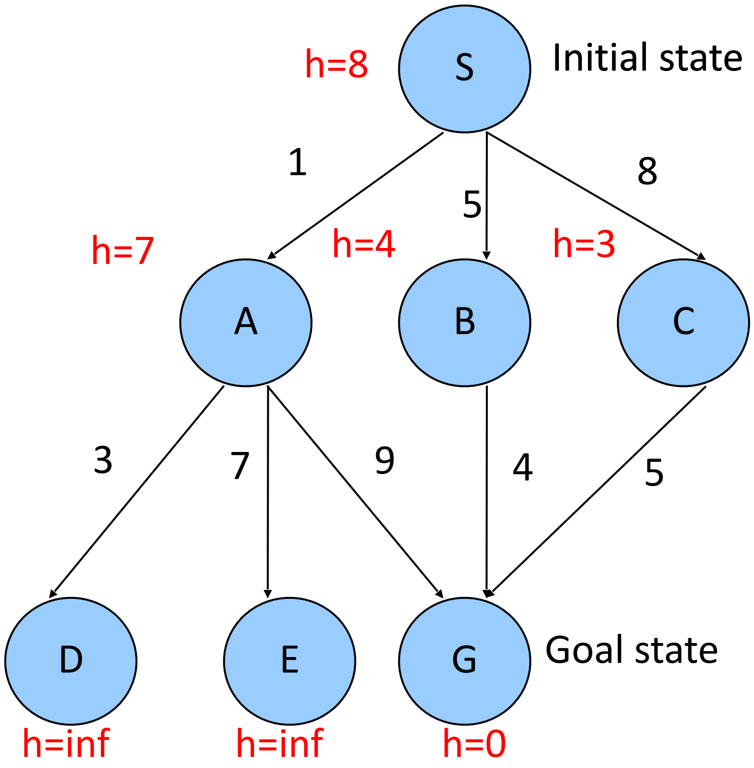

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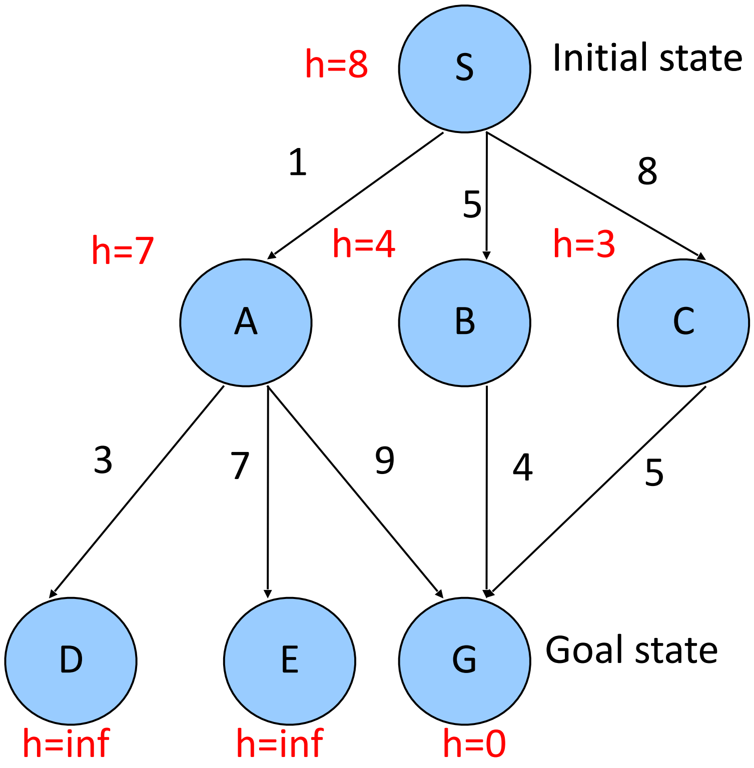
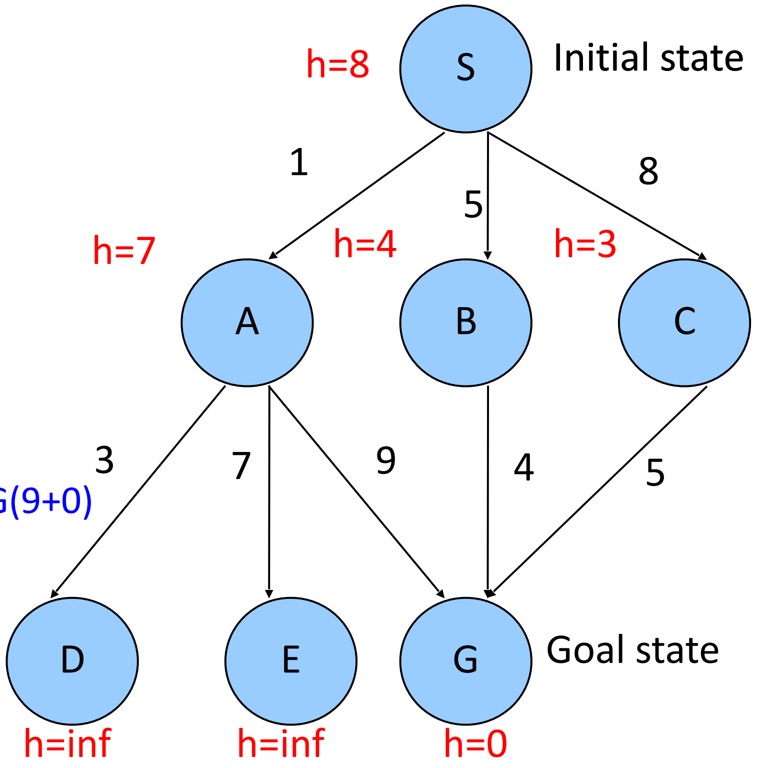


Recap and Examples

Example for A*:

OPEN	CLOSED
S(0+8)	-
A(1+7) B(5+4) C(8+3)	S(0+8)
B(5+4) C(8+3) D(4+inf) E(8+inf) G(10+0)	S(0+8) A(1+7)
C(8+3) D(4+inf) E(8+inf) G(9+0)	S(0+8) A(1+7) B(5+4)
C(8+3) D(4+inf) E(8+inf)	S(0+8) A(1+7) B(5+4) G(9+0)

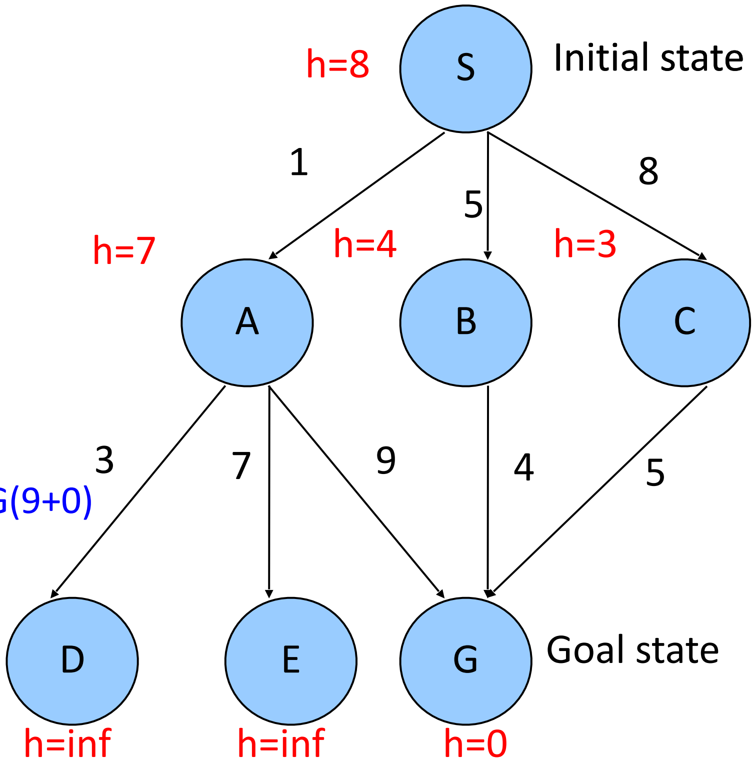


Recap and Examples

Example for A*:

OPEN	CLOSED
S(0+8)	-
A(1+7) B(5+4) C(8+3)	S(0+8)
B(5+4) C(8+3) D(4+inf) E(8+inf) G(10+0)	S(0+8) A(1+7)
C(8+3) D(4+inf) E(8+inf) G(9+0)	S(0+8) A(1+7) B(5+4)
C(8+3) D(4+inf) E(8+inf)	S(0+8) A(1+7) B(5+4) G(9+0)

G → B → S



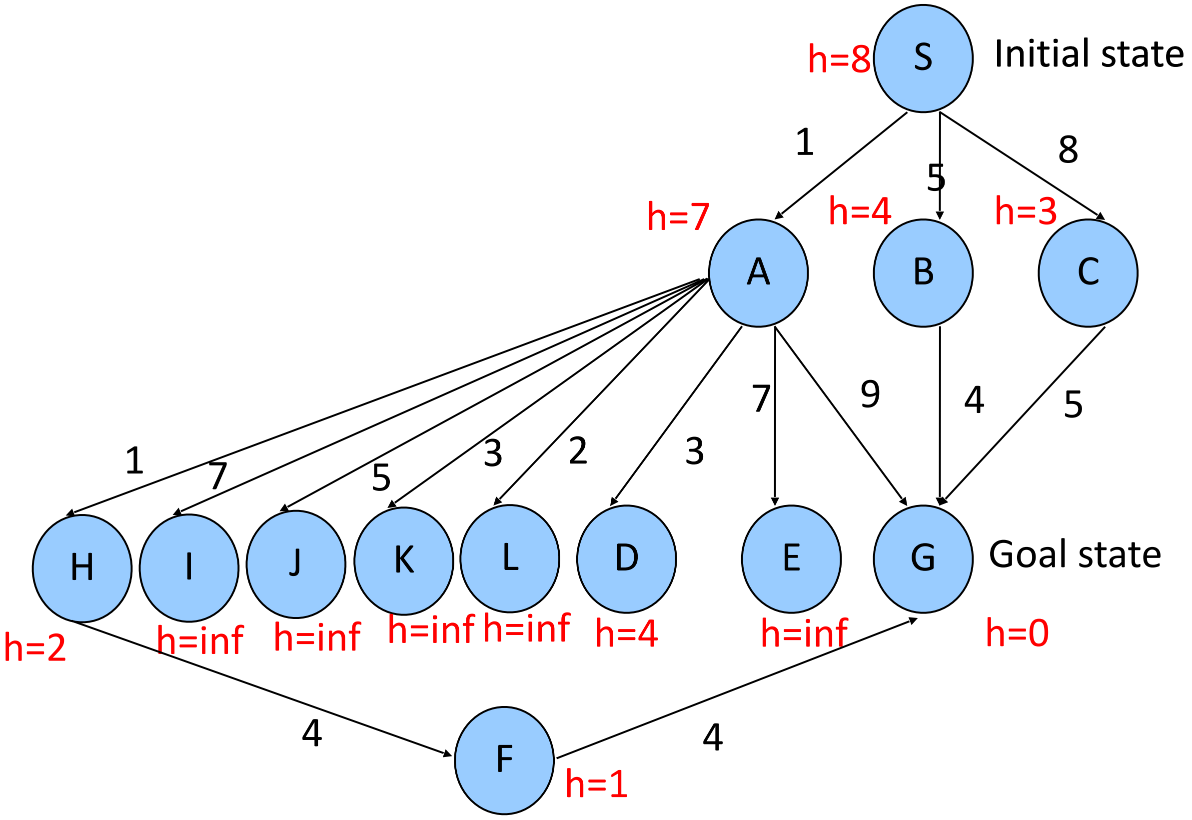
Recap and Examples

Recap and Examples

Example for IDA*:

Recap and Examples

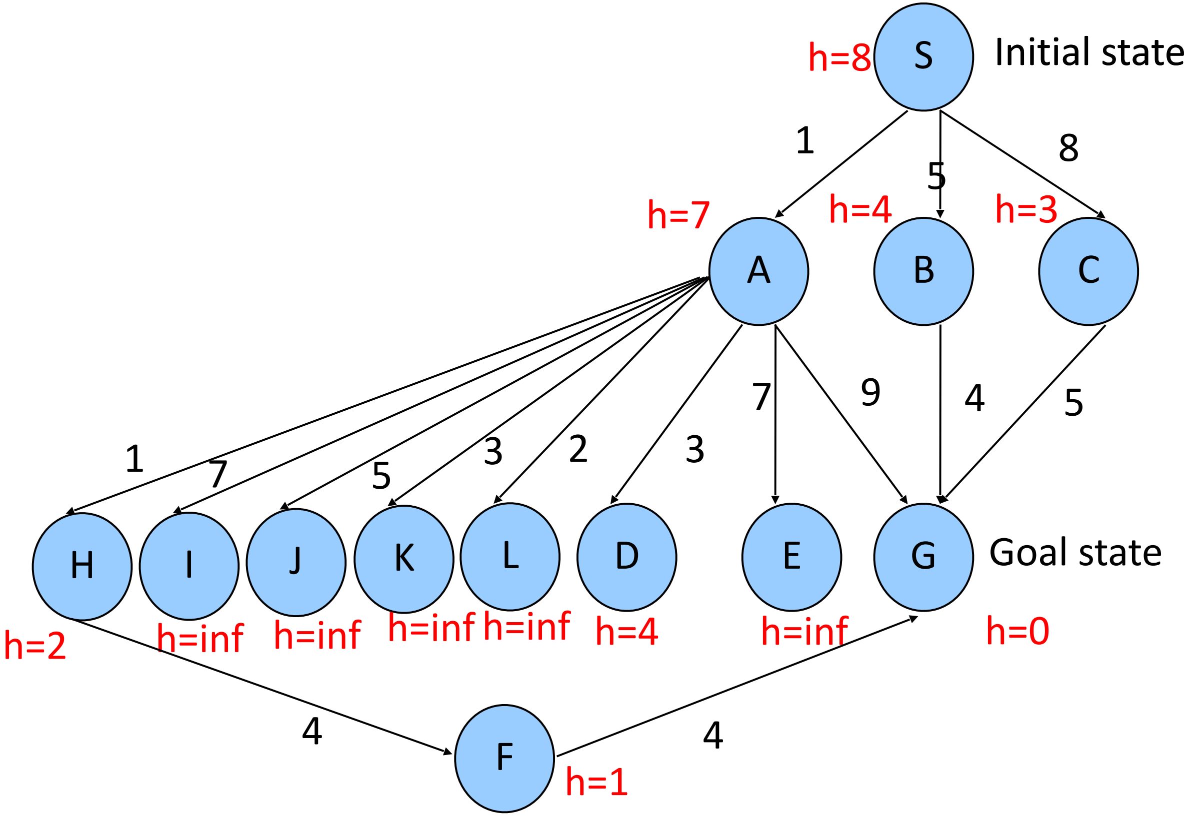
Example for IDA*:



Recap and Examples

Example for IDA*:

Threshold = 8

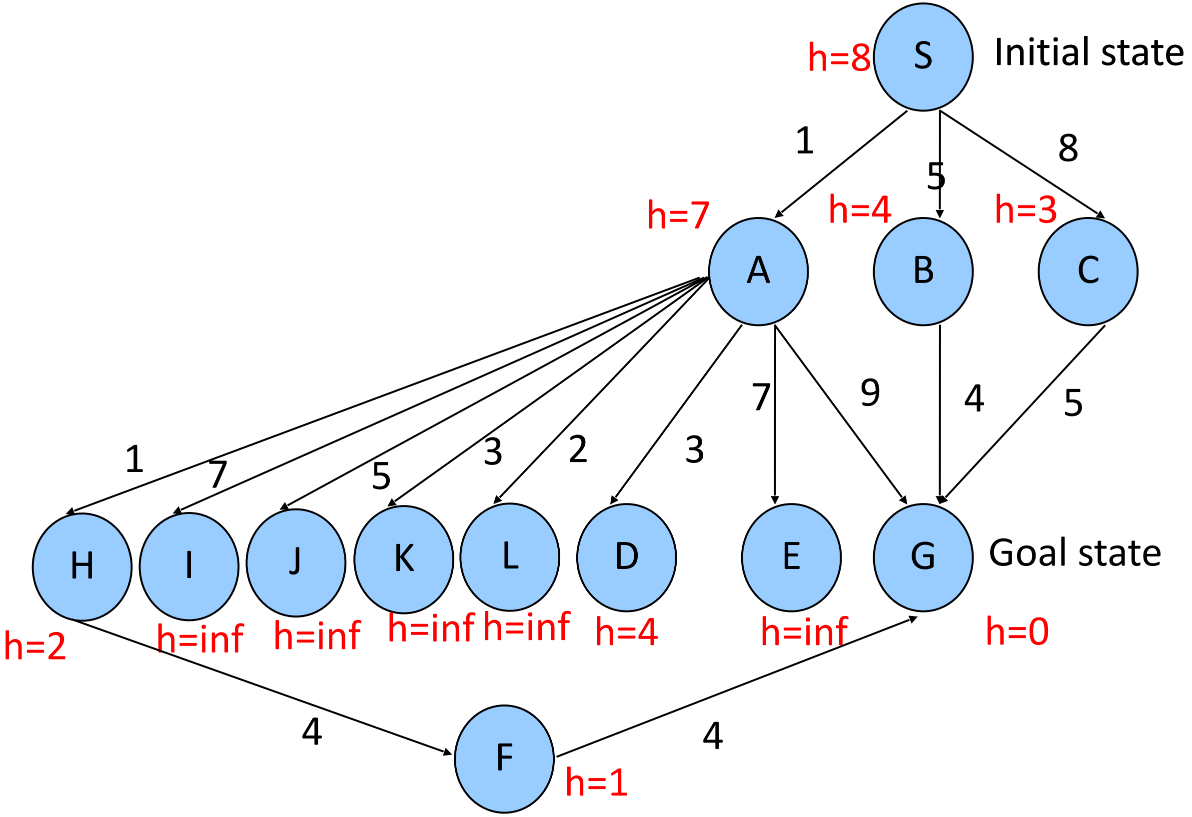


Recap and Examples

Example for IDA*:

Threshold = 8

PATH PREFIX	OPEN
-	S(0+8)
S	A(1+7)
S A	H(2+2) D(4+4)
S A H	D(4+4) F(6+1)
S A H F	D(4+4)
S A D	



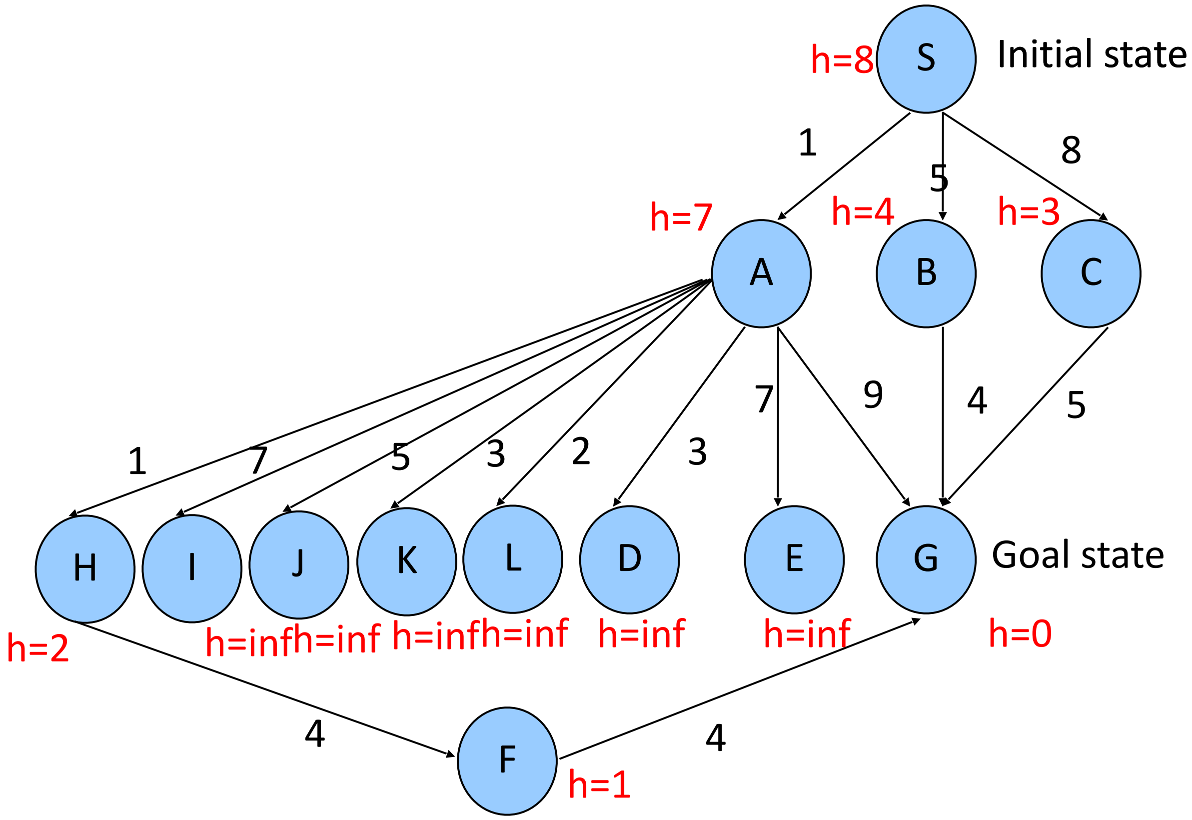
Recap and Examples

Recap and Examples

Example for IDA*:

Recap and Examples

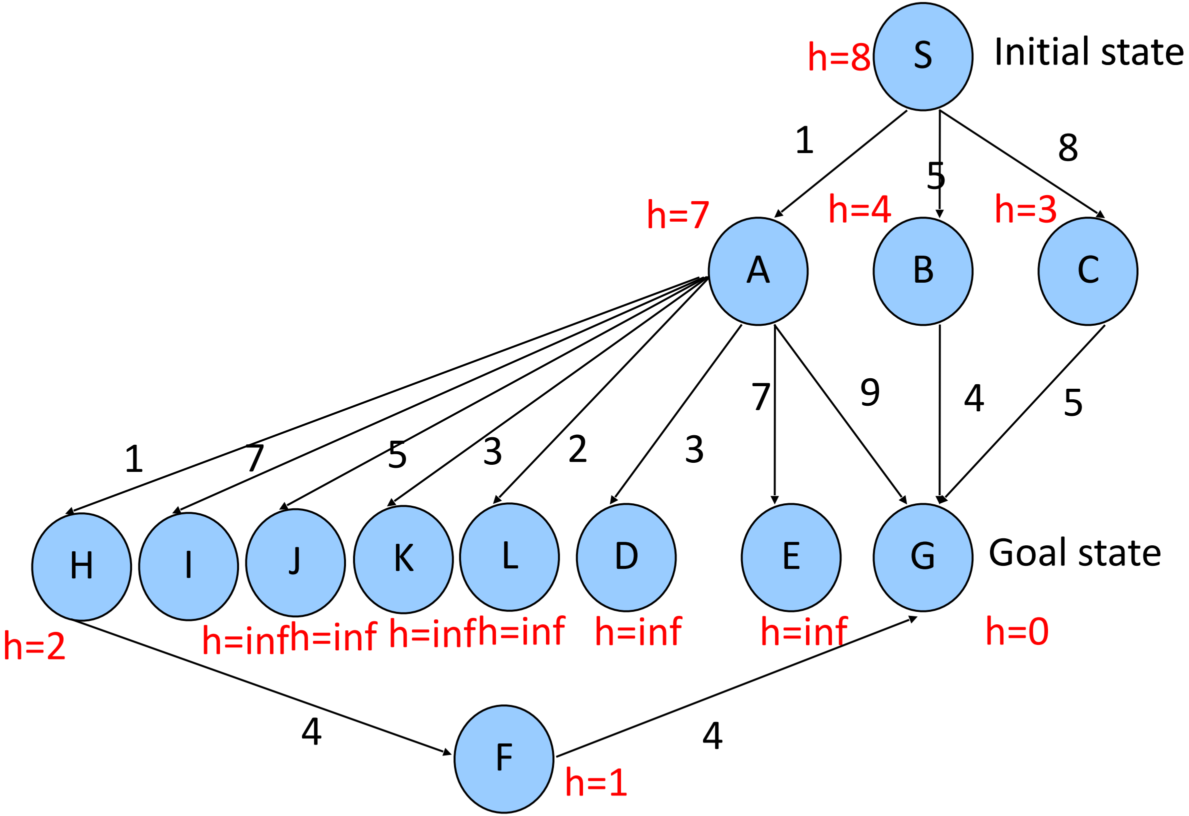
Example for IDA*:



Recap and Examples

Example for IDA*:

Threshold = 9

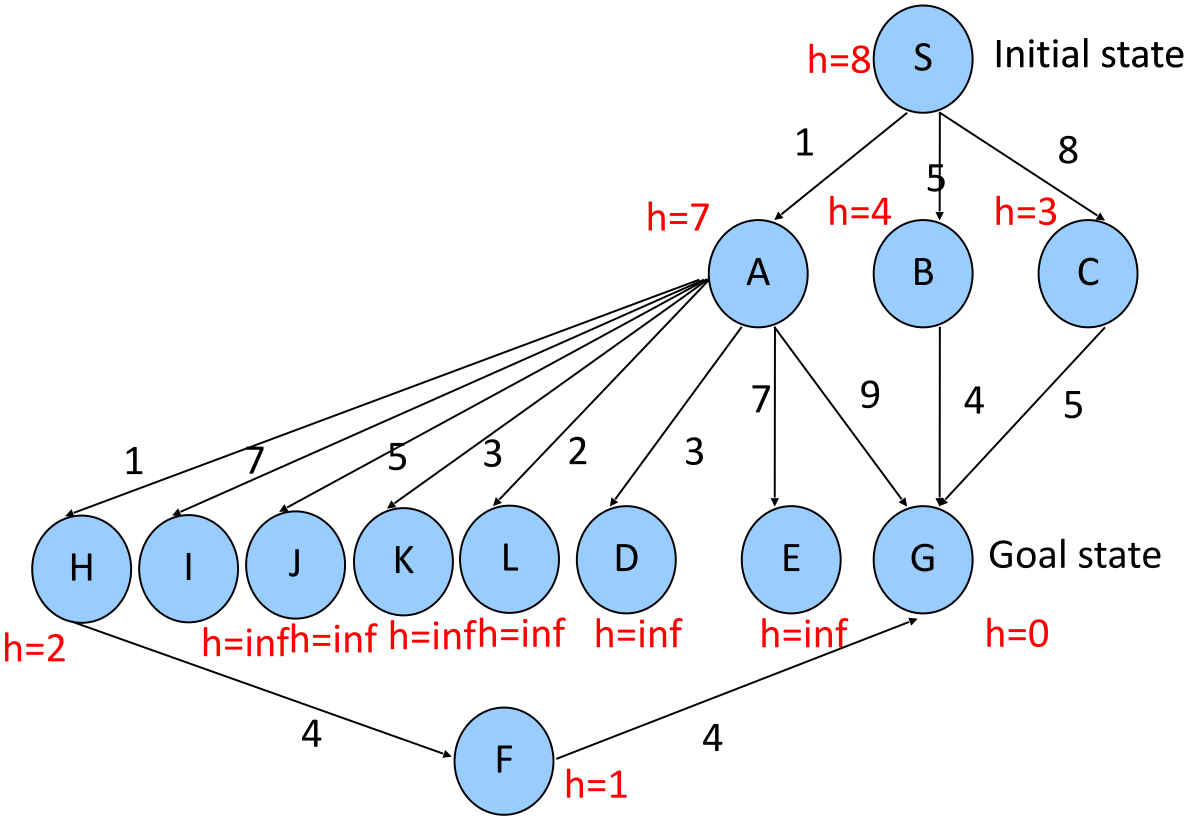


Recap and Examples

Example for IDA*:

Threshold = 9

PREFIX	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
S A	B(5+4) H(2+2) D(4+4)
S A H	B(5+4) D(4+4) F(6+1)
S A H F	B(5+4) D(4+4)
S A D	B(5+4)
S B	G(9+0)
S B G	



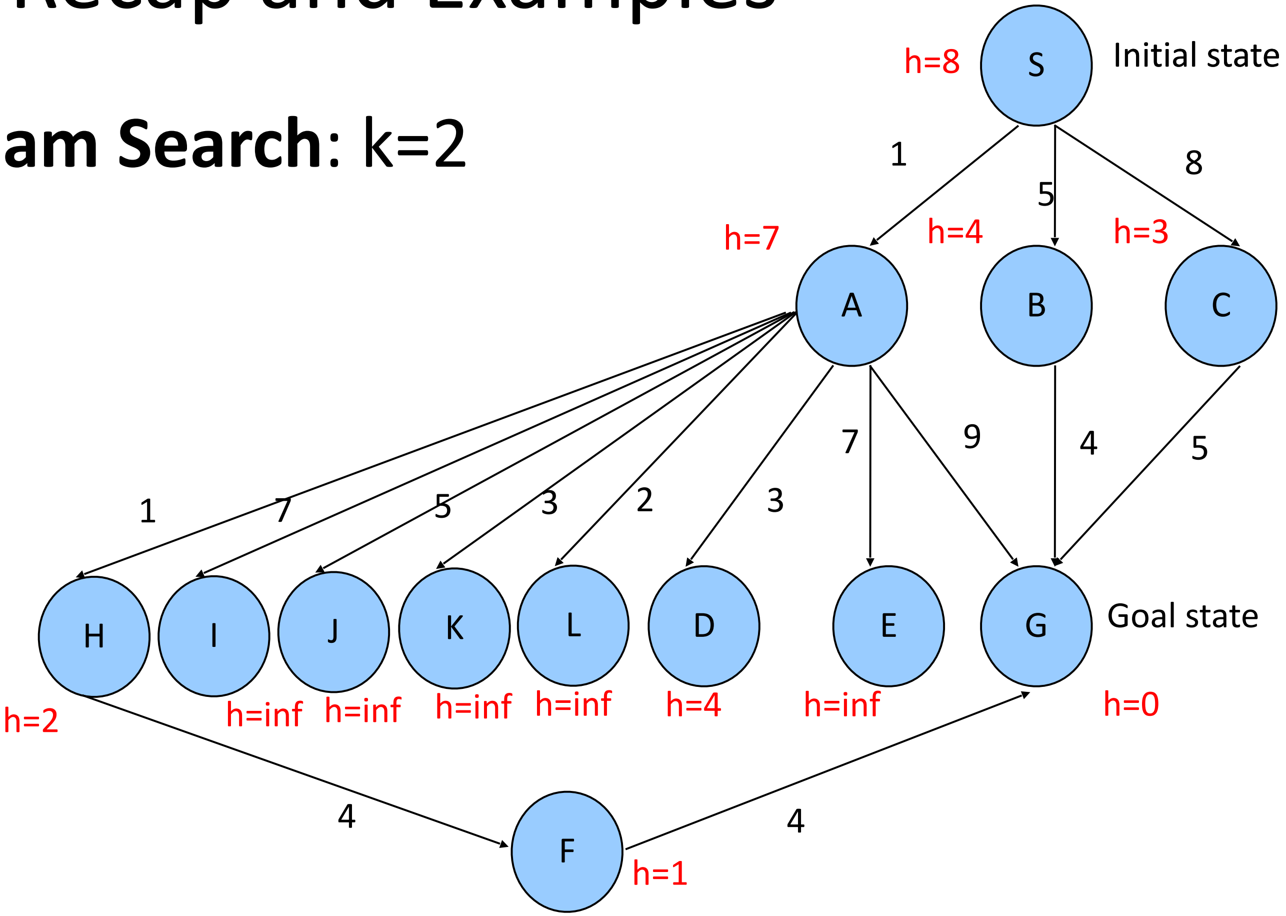
Recap and Examples

Recap and Examples

Example for **Beam Search**: k=2

Recap and Examples

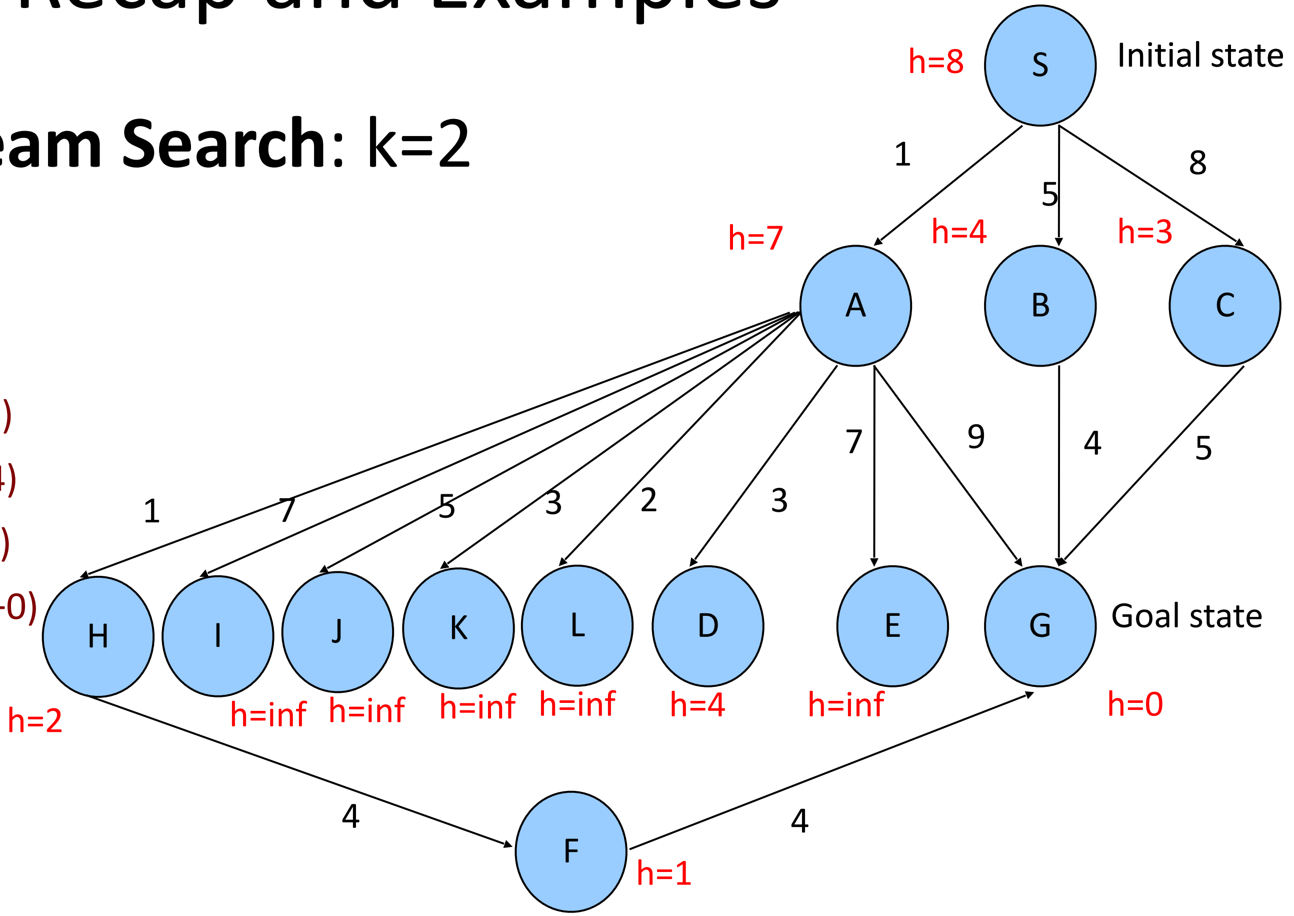
Example for **Beam Search**: k=2



Recap and Examples

Example for **Beam Search**: k=2

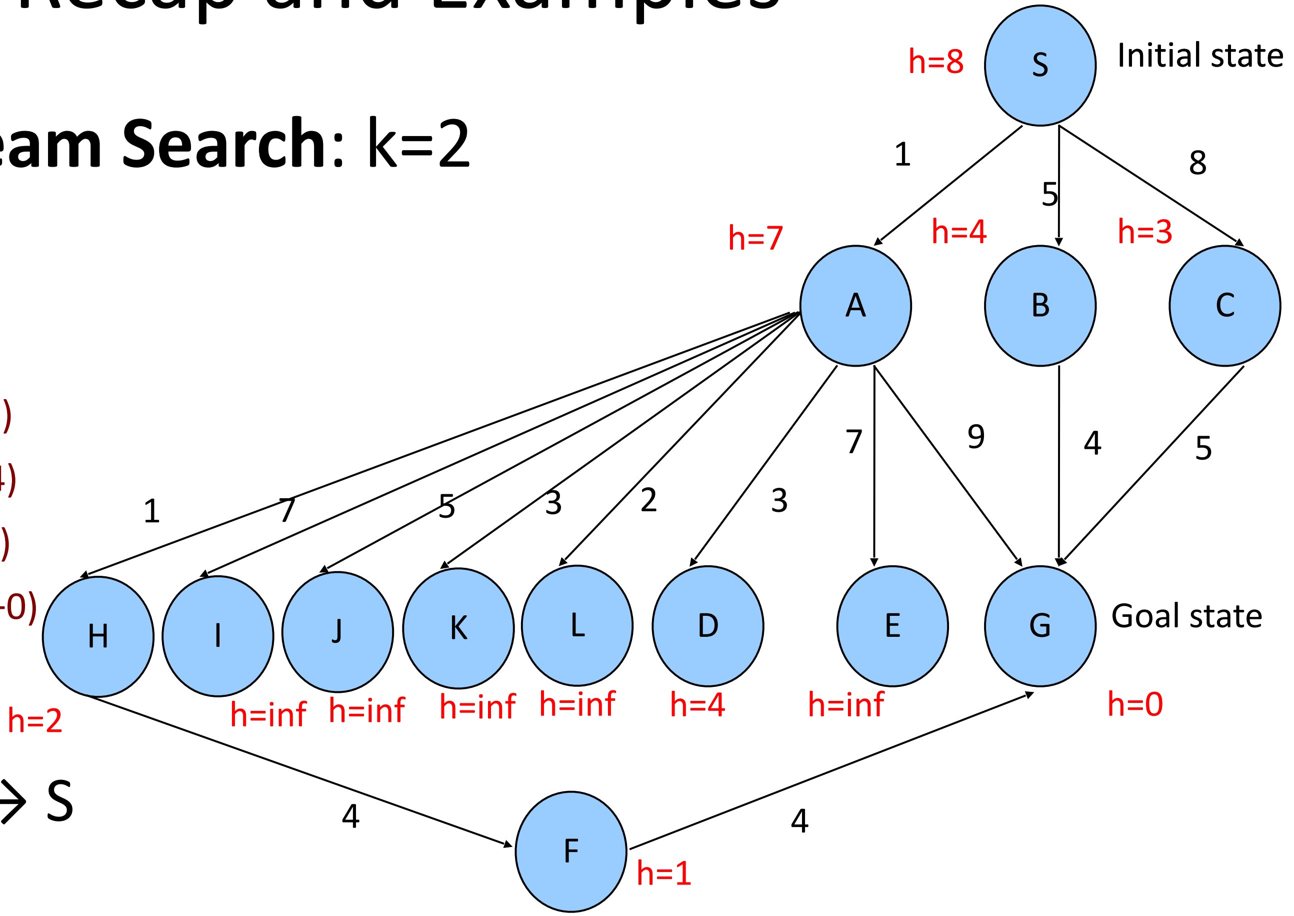
CURRENT	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
A	H(2+2) D(4+4)
H	D(4+4) F(6+1)
F	D(4+4) G(10+0)
D	G(10+0)
G	



Recap and Examples

Example for **Beam Search**: k=2

CURRENT	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
A	H(2+2) D(4+4)
H	D(4+4) F(6+1)
F	D(4+4) G(10+0)
D	G(10+0)
G	



G → F → H → A → S

Not optimal!

Summary

- Informed search: introduce heuristics
  - Not all approaches work: best-first greedy is bad
- A* algorithm
  - Properties of A*, idea of admissible heuristics
- Beyond A*
  - IDA*, beam search. Ways to deal with space requirements.

Acknowledgements: Adapted from materials by Jerry Zhu (University of Wisconsin).