CS 540 Introduction to Artificial Intelligence
Reinforcement Learning I

University of Wisconsin-Madison

Spring 2023

Announcements

Homeworks:

- Homework 9 due Thursday April 27
- Homework 10 due Thursday May 4

Course Evaluation:

- Complete by Friday May 5

Class roadmap:

Tuesday, April 25	Reinforcement Learning I
Thursday, April 27	Reinforcement Learning I
Tuesday, May 2	Advanced Search
Thursday, May 4	Ethics and Trust in AI

Final Exam: May 12 5:05 - 7:05 pm

Outline

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- Introduction to reinforcement learning
  - Basic concepts, mathematical formulation, MDPs, policies.

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  - Basic concepts, mathematical formulation, MDPs, policies.
- Learning policies
  - Q-learning, action-values, exploration vs exploitation.

Back to Our General Model

We have an **agent** **interacting** with the **world**

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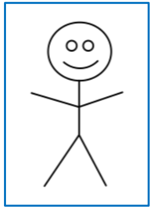

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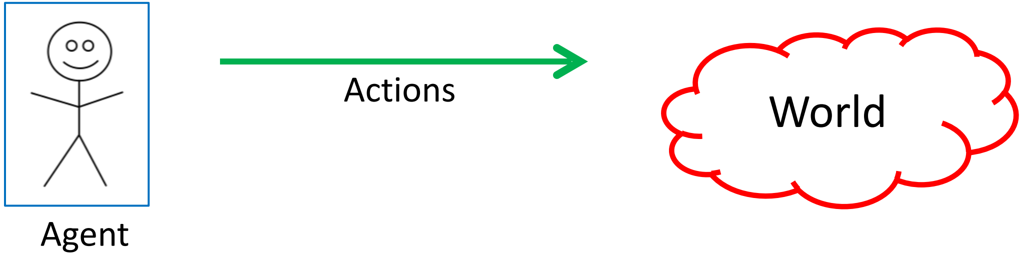

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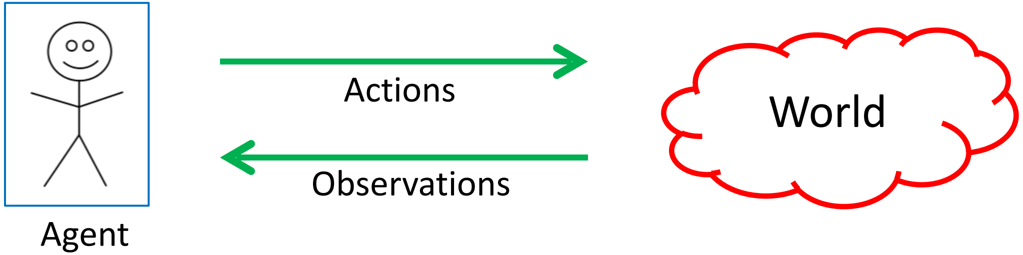

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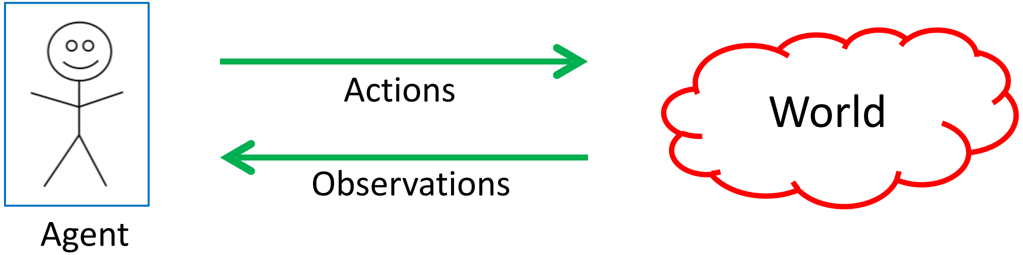

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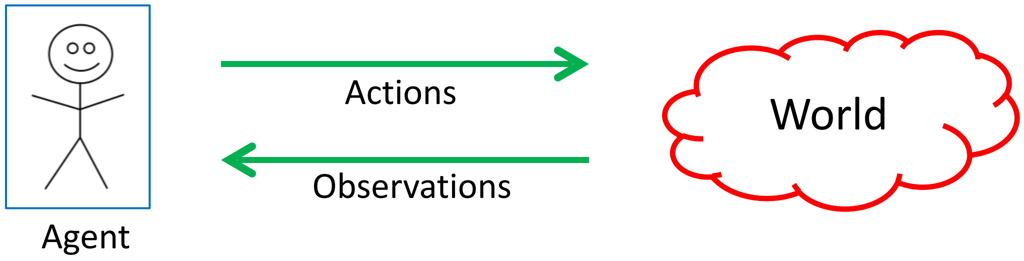

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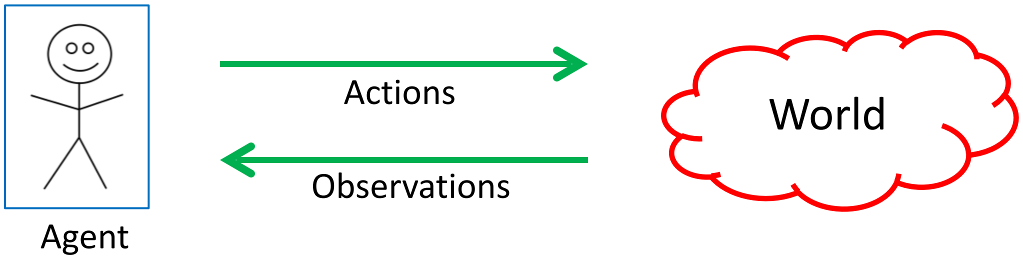

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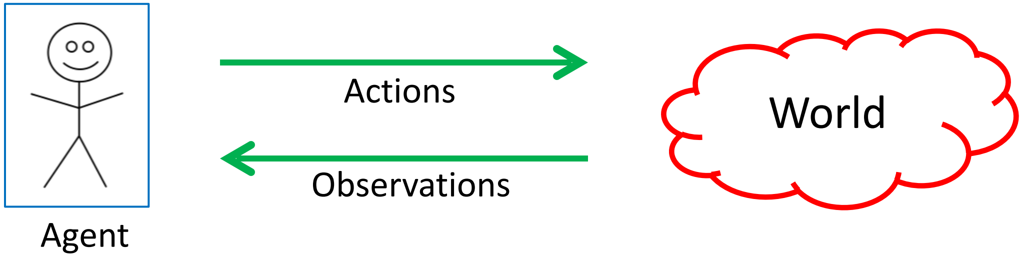

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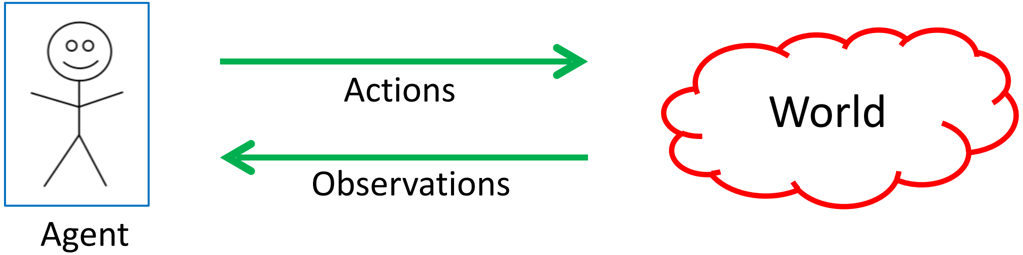

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    - Compare to unsupervised learning and supervised learning

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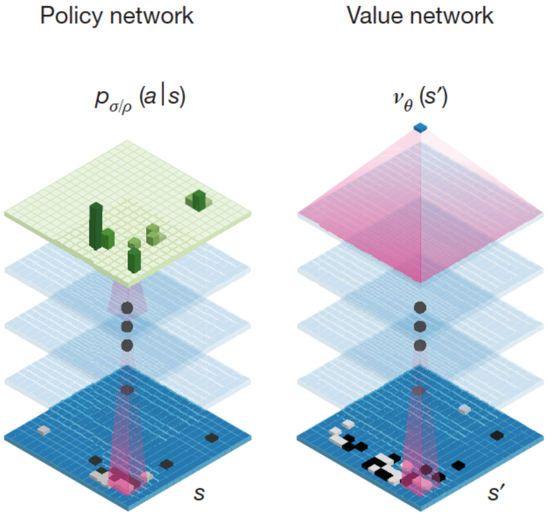

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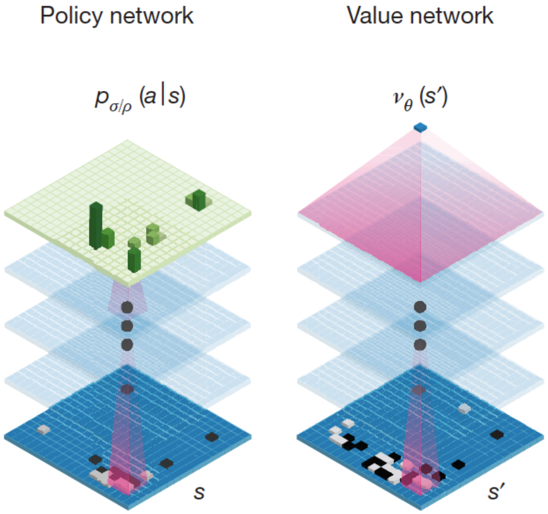
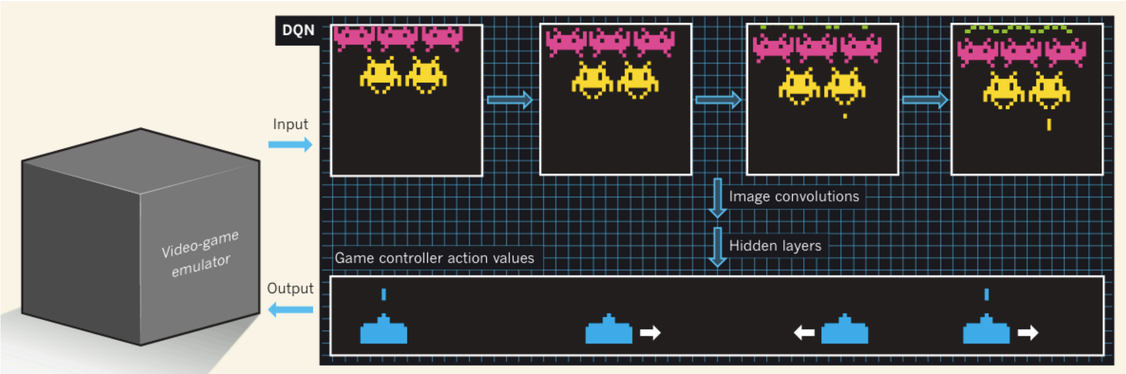


https://deepmind.com/research/alphago/

Examples: Video Game Agents

Pong, Atari



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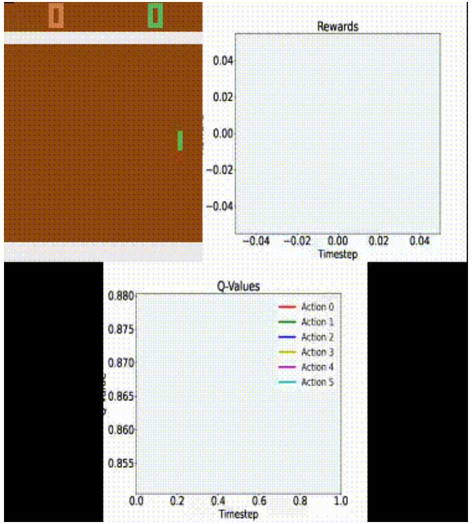

A. Nielsen

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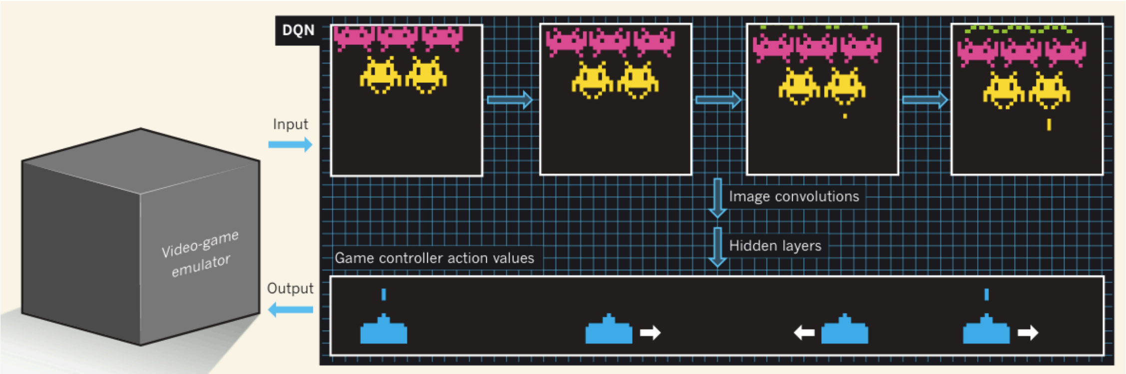

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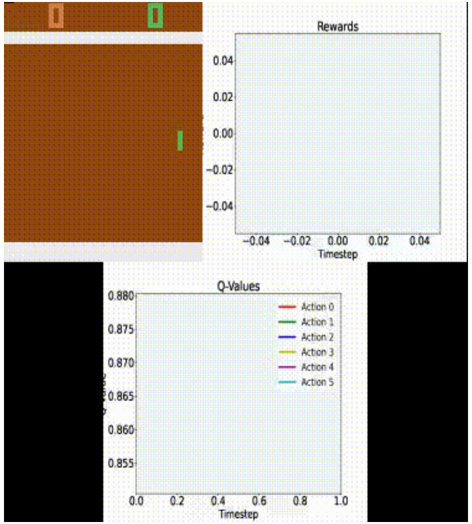

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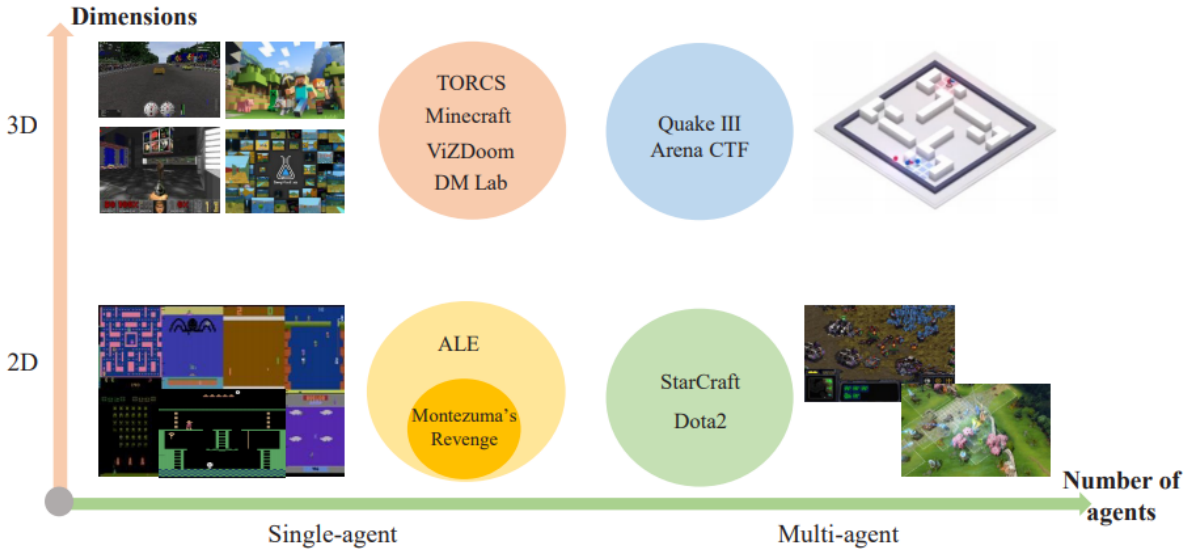

Shao et al, "A Survey of Deep Reinforcement Learning in Video Games"

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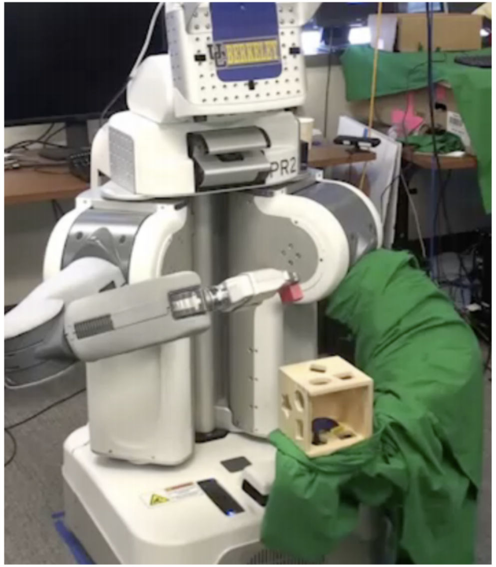

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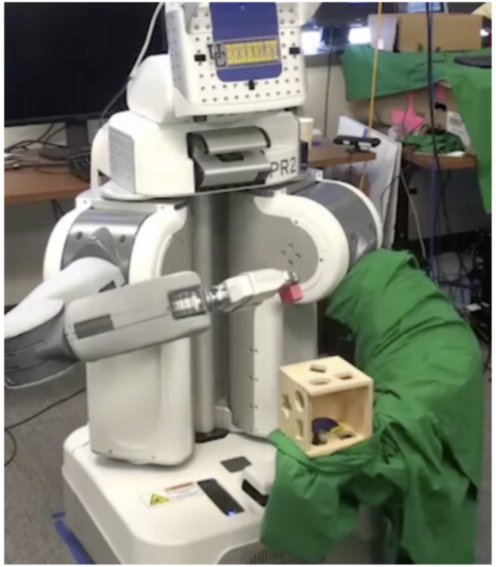

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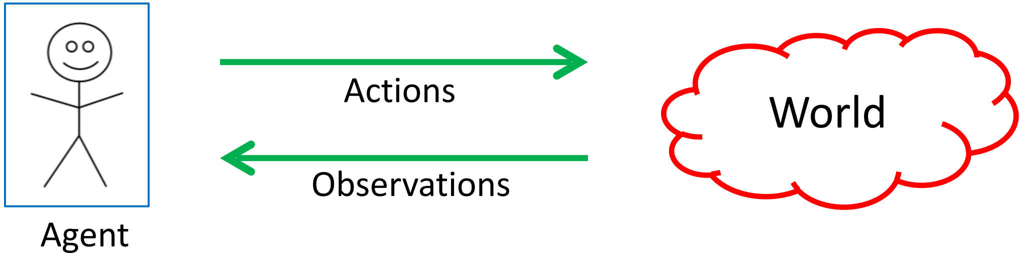

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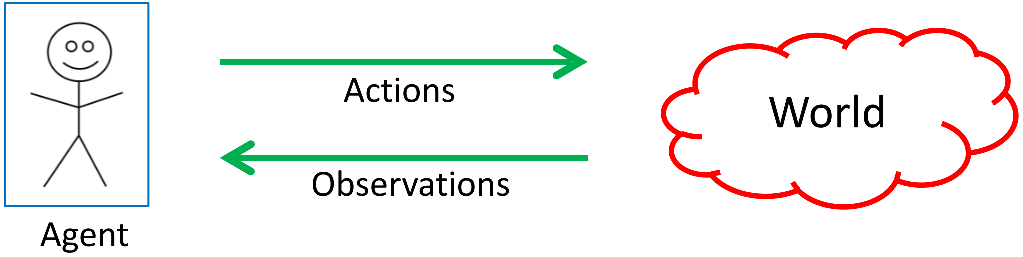

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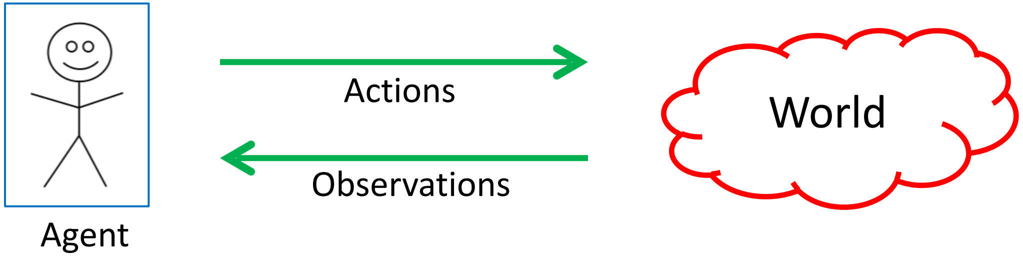

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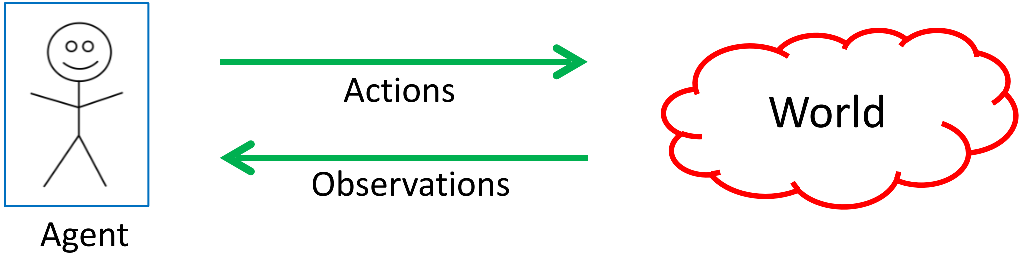

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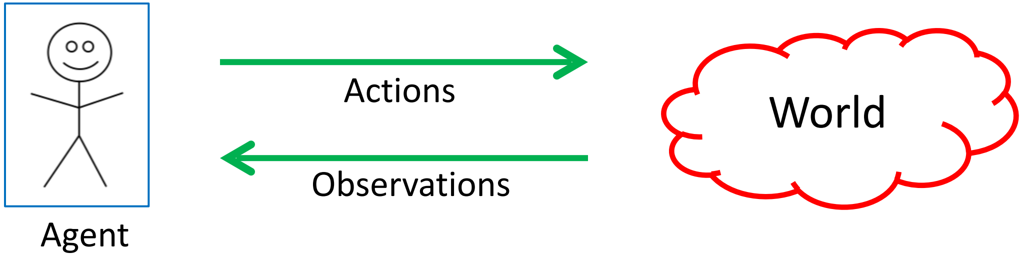

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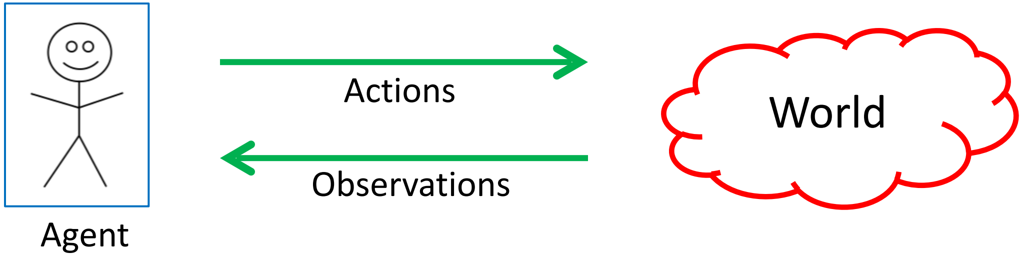

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A “policy”

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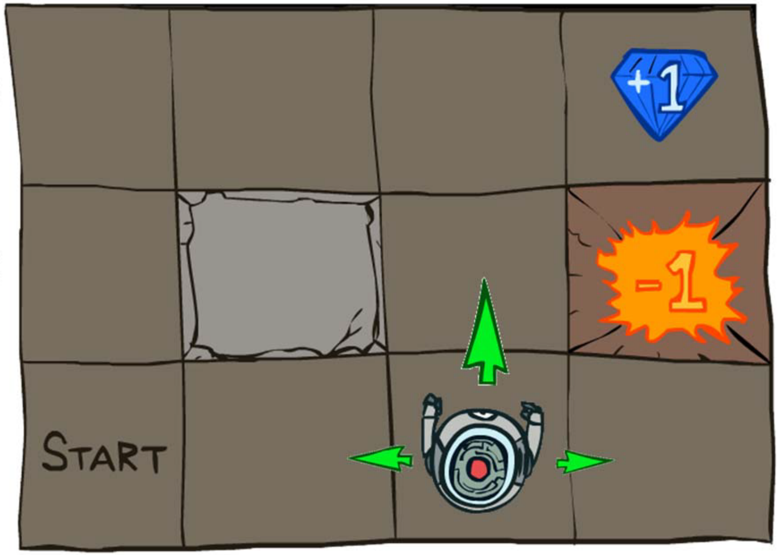

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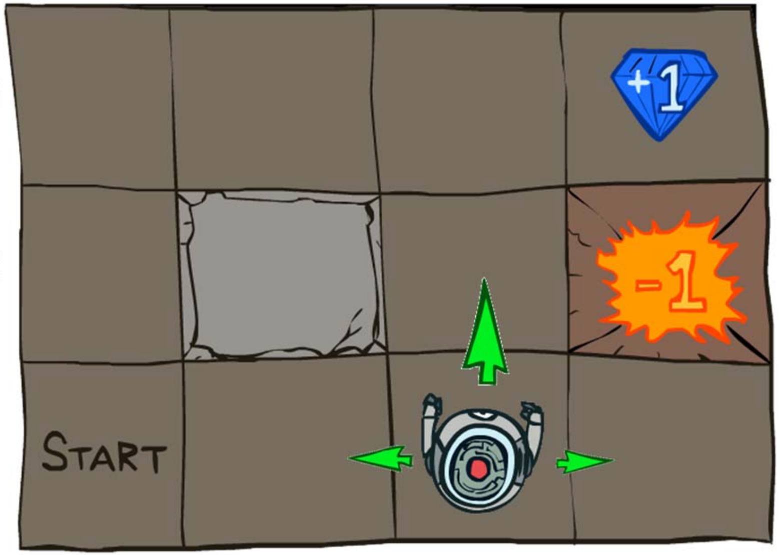

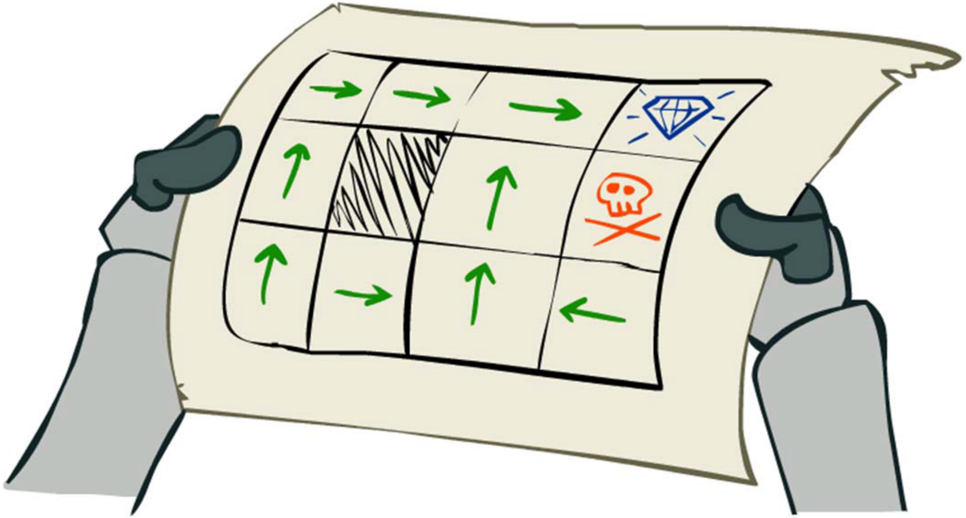

Source: P. Abbeel and D. Klein

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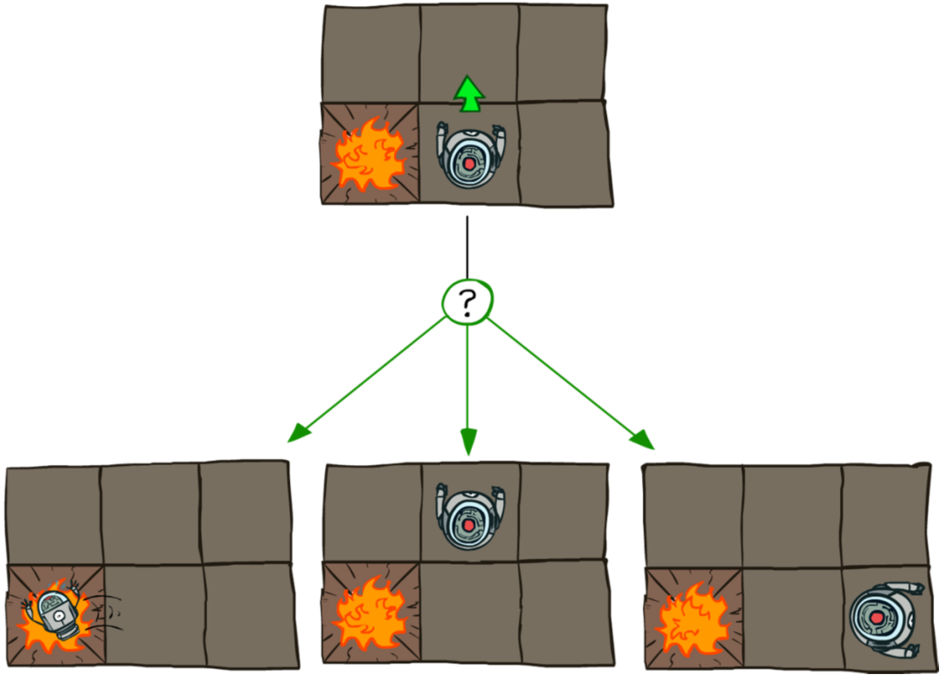

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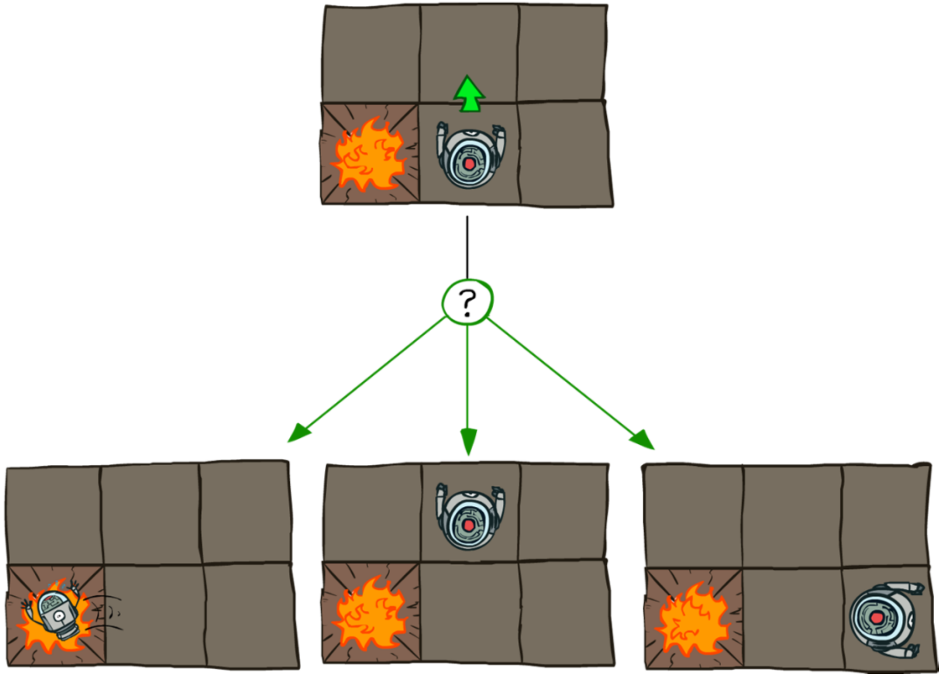

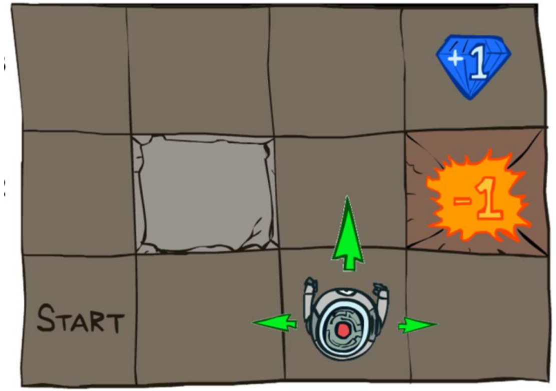

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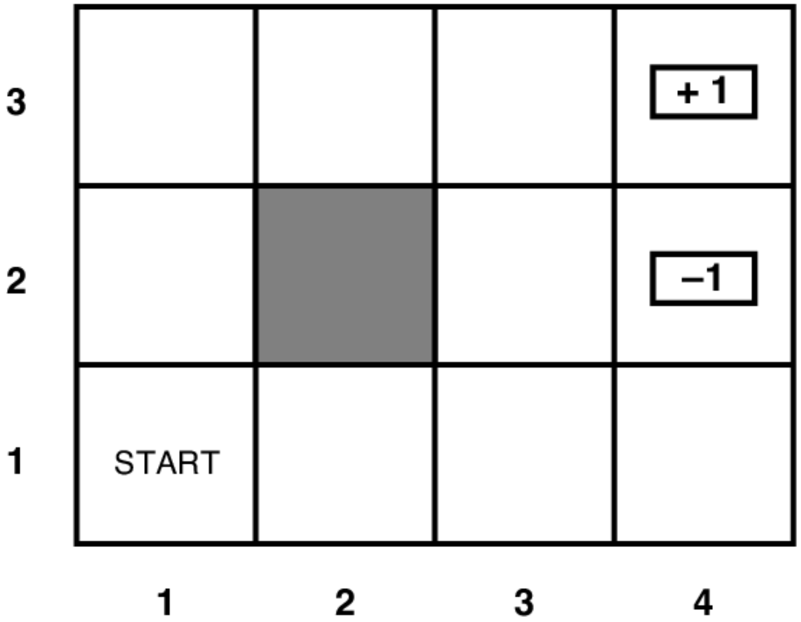

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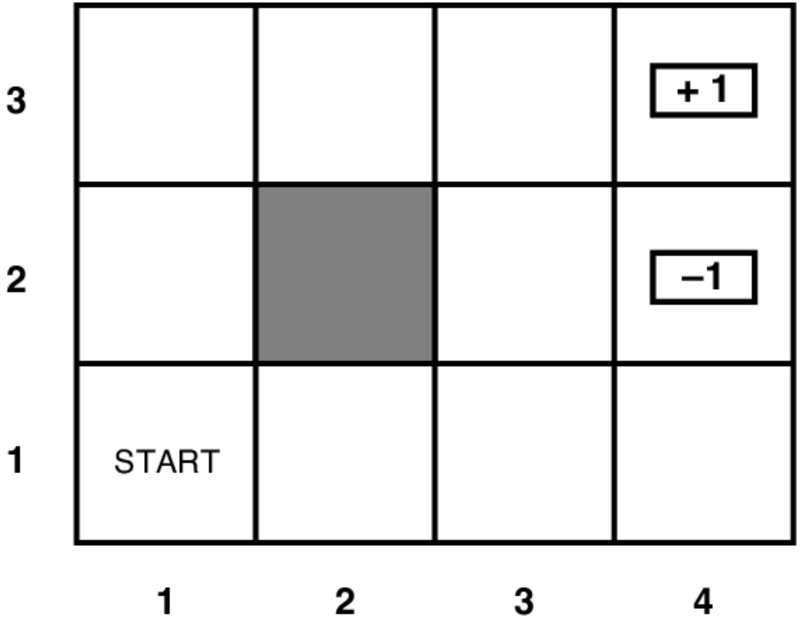

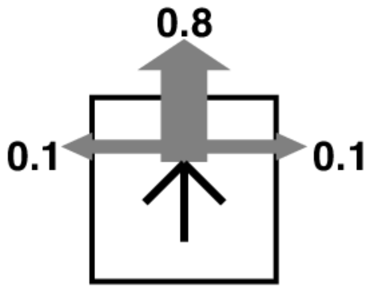

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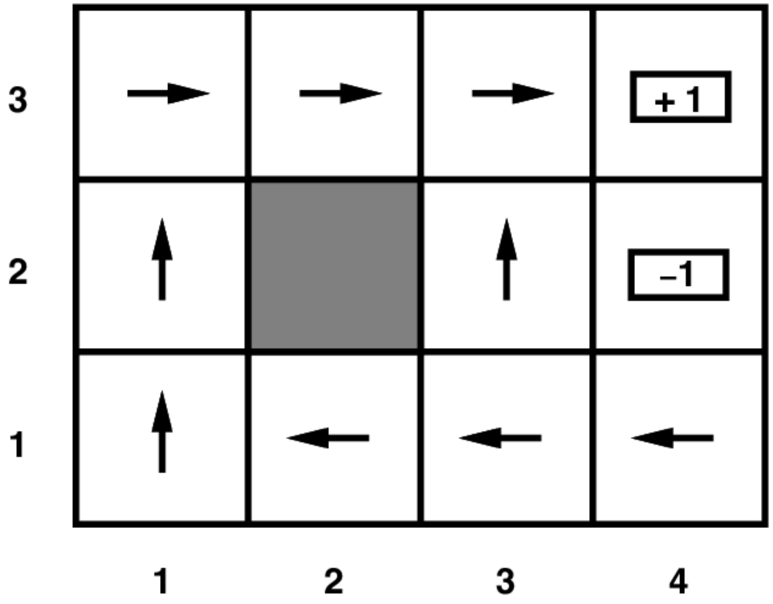

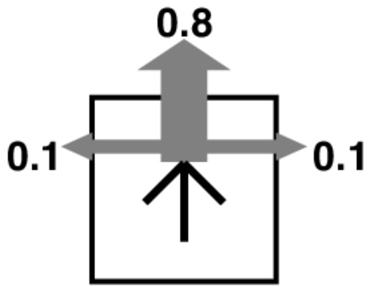

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How do we find the best policy?

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Exploration vs. Exploitation:

- Transition probabilities and reward may be unknown to the learner.
- Should you keep trying actions that led to reward in the past or try new actions that might lead to even more reward?

Break & Quiz

Q 1.1 Which of the following statement about MDP is **not** true?

- A. The reward function must output a scalar value
- B. The policy maps states to actions
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- A. The reward function must output a scalar value (**True: need to be able to compare**)
- B. The policy maps states to actions (**True: a policy tells you what action to take for each state**).
- **C. The probability of next state can depend on current and previous states** (**False: Markov assumption**).
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards (**True: want to maximize rewards overall**).

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- What's the expected utility following an action?

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  - Instead, learn about the utility of actions directly.

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- Equivalent to

$$Q^*(s,a) = r(s) + \gamma \sum_{s'} P(s' \mid s,a) \max_{a'} Q^*(s',a')$$

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Estimate Q*(*s*,*a*) from data $\{(s_t, a_t, r_t, s_{t+1})\}$:

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   1. Except terminal states $Q(s_{terminal},.)=0$
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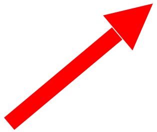

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Input: step size α, exploration probability ϵ

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Converges to Q*(s,a) in limit if all states and actions visited infinitely often.

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$$a = \begin{cases} \text{argmax}_{a \in A} Q(s, a) & \text{uniform}(0,1) > \epsilon \\ \text{random } a \in A & \text{otherwise} \end{cases}$$

Q-Learning Iteration

How do we get $Q(s,a)$?

- Iterative procedure

Idea: combine old value and new estimate of future value.

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Summary

- Reinforcement learning setup
- Mathematical formulation: MDP
- The Q-learning Algorithm

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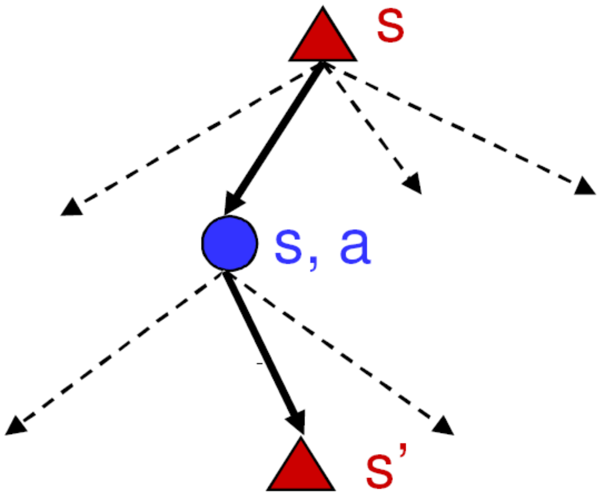

Credit L. Lazbenik

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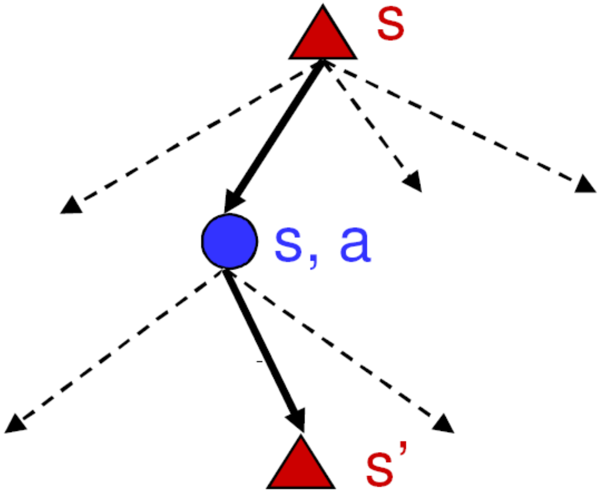

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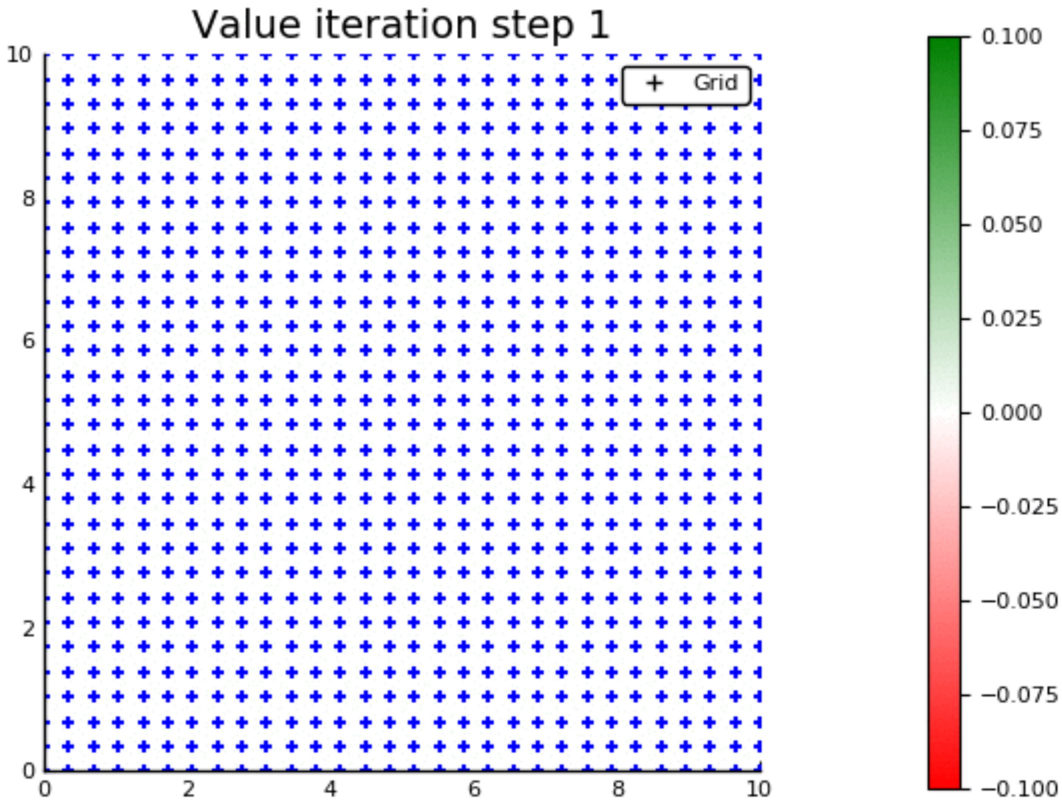

Source: POMDPBGallery Julia Package

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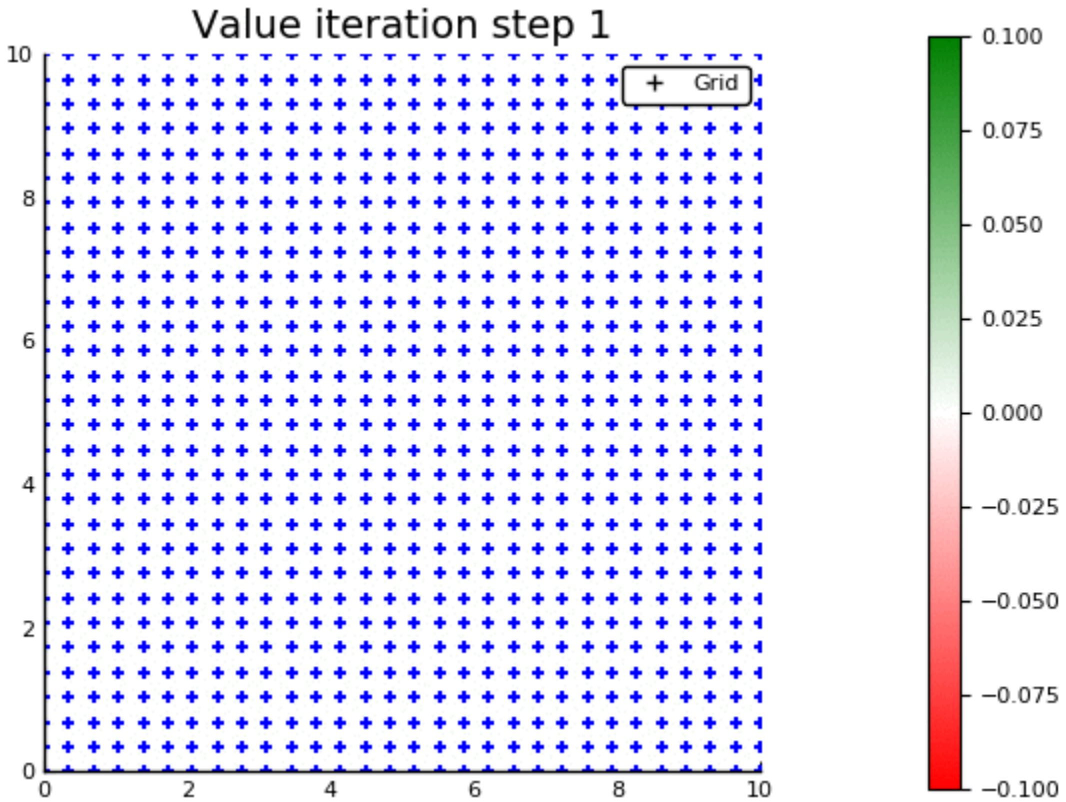

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- A. 0
- B. 1 / (1 -γ)
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