CS 540 Introduction to Artificial Intelligence

Search III: Advanced Search (aka Optimization)

University of Wisconsin-Madison

Spring 2023

Outline

Homeworks:

- Homework 10 due Thursday
- Course evaluation due Friday

Class roadmap:

Tuesday, May 2	Advanced Search
Thursday, May 4	Ethics and Review
Friday, May 12 5:05 - 7:05pm	Final Exam

Advanced Search Overview

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Problem Setting

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Fitness
Population
Cross-over
Mutation

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- Advanced Search & Hill-climbing
  - More difficult problems, basics, local optima, variations

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- Hill Climbing
  - Basic algorithm, local optima
- Genetic Algorithms
  - Basics of evolution, fitness, natural selection

Search vs. Optimization

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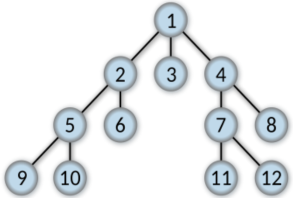

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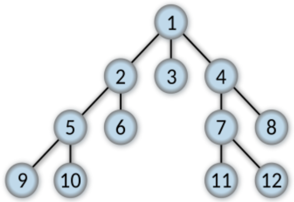

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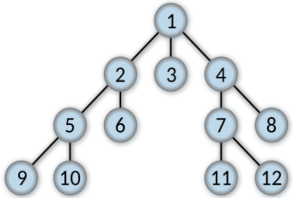

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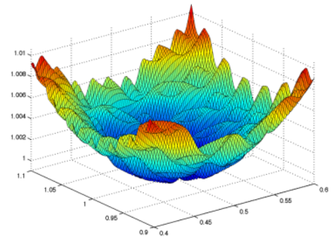

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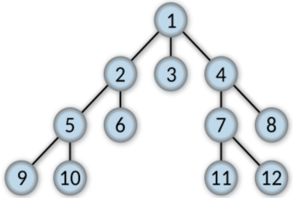

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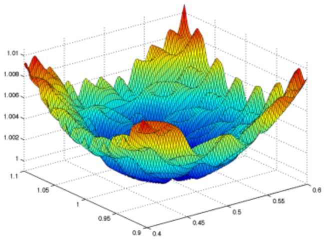

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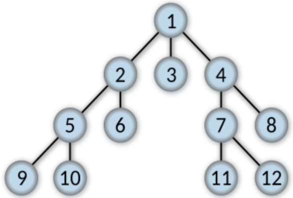

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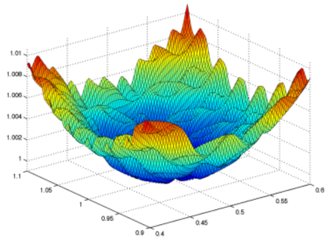

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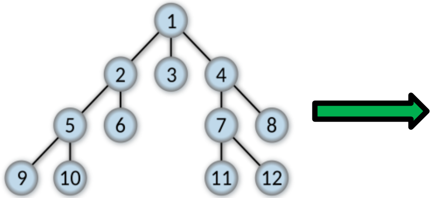

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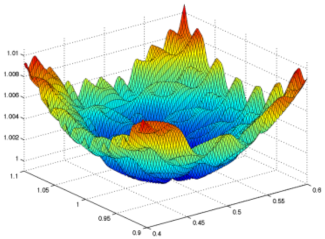

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New setting: optimization

- States *s* have values ***f***(*s*)
- Want: Find *s* with optimal value ***f***(*s*) (i.e, **optimize** over states)
- Challenging settings: **too many states** for previous search approaches, but maybe not a differentiable function for gradient descent.

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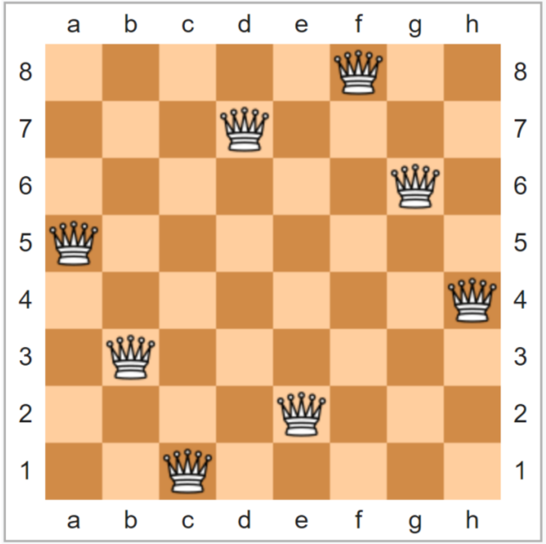

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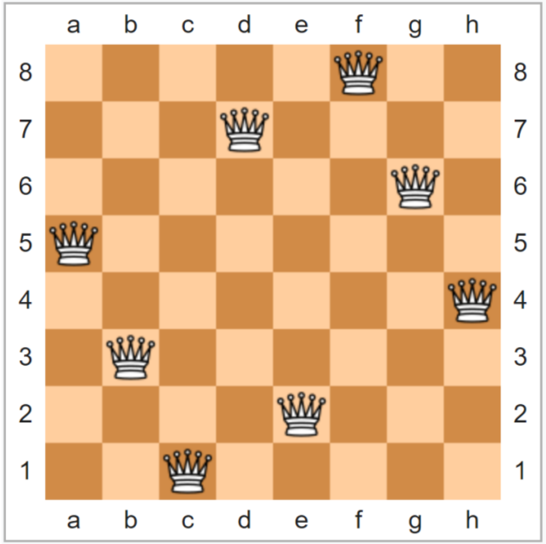

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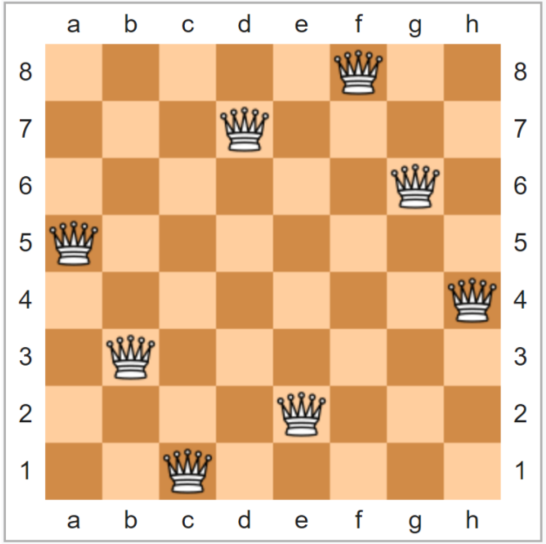

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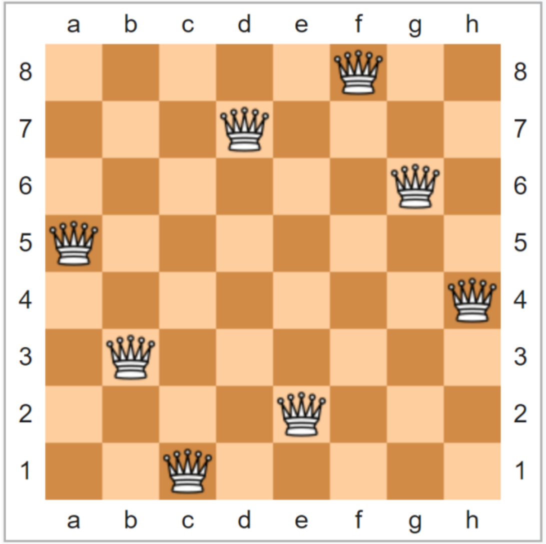
  - ***f***(*s*): # of non-conflicting queens



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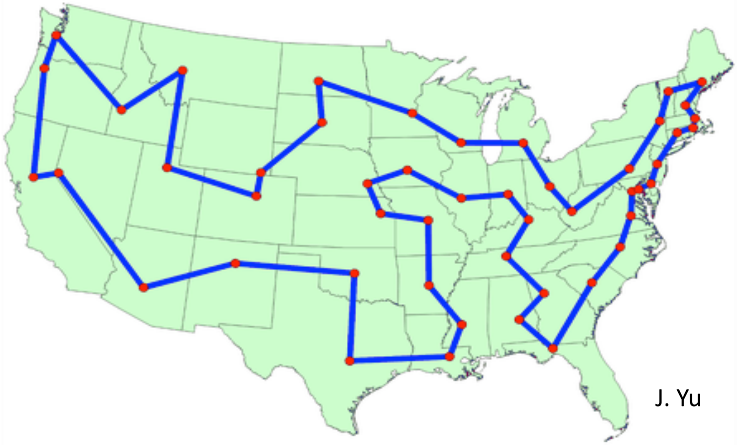

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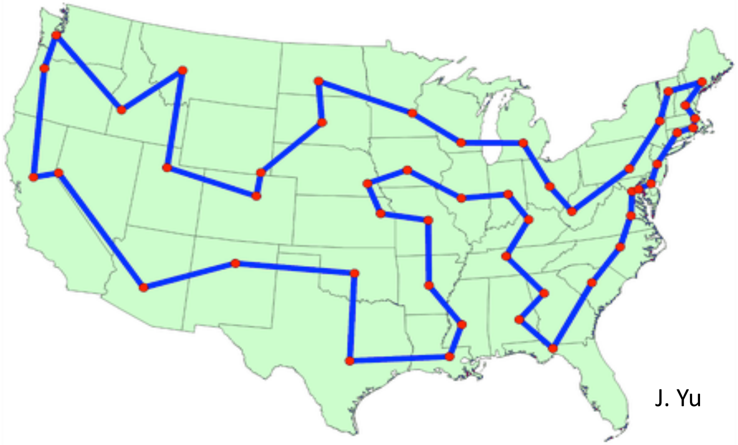

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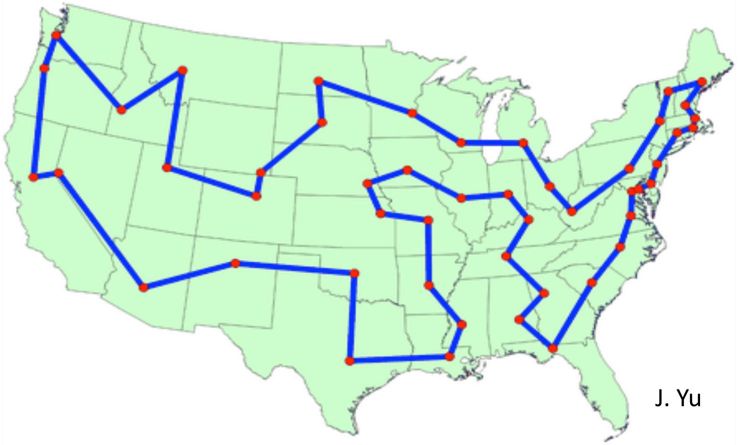

J. Yu

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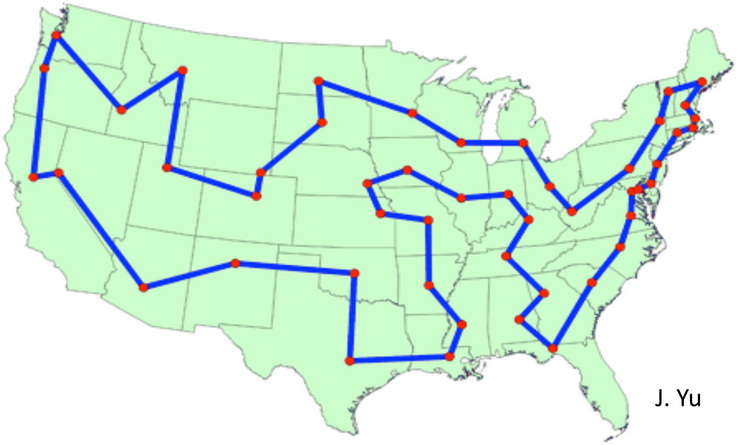

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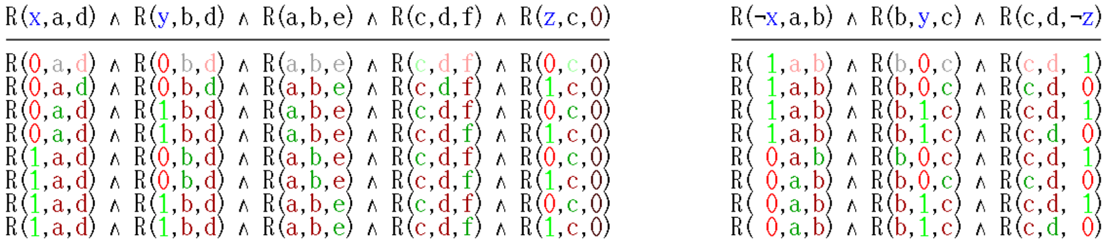

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Hill Climbing

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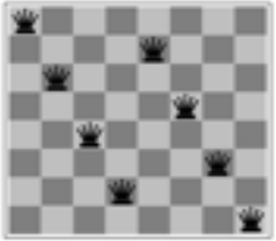
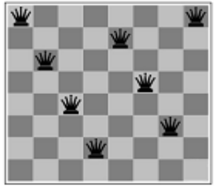
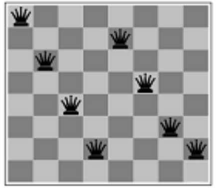
  - As we'll see, needs a careful choice

Defining Neighbors: n Queens

In n Queens, a simple possibility:







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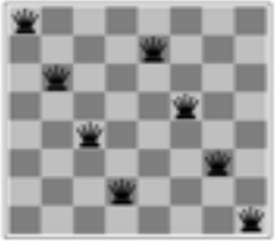

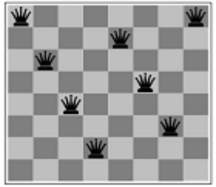

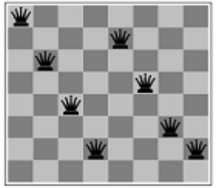

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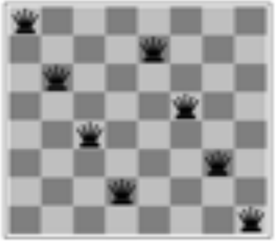

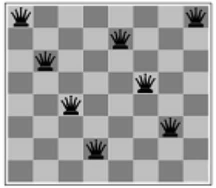

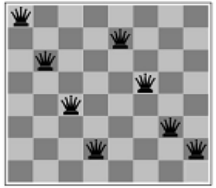

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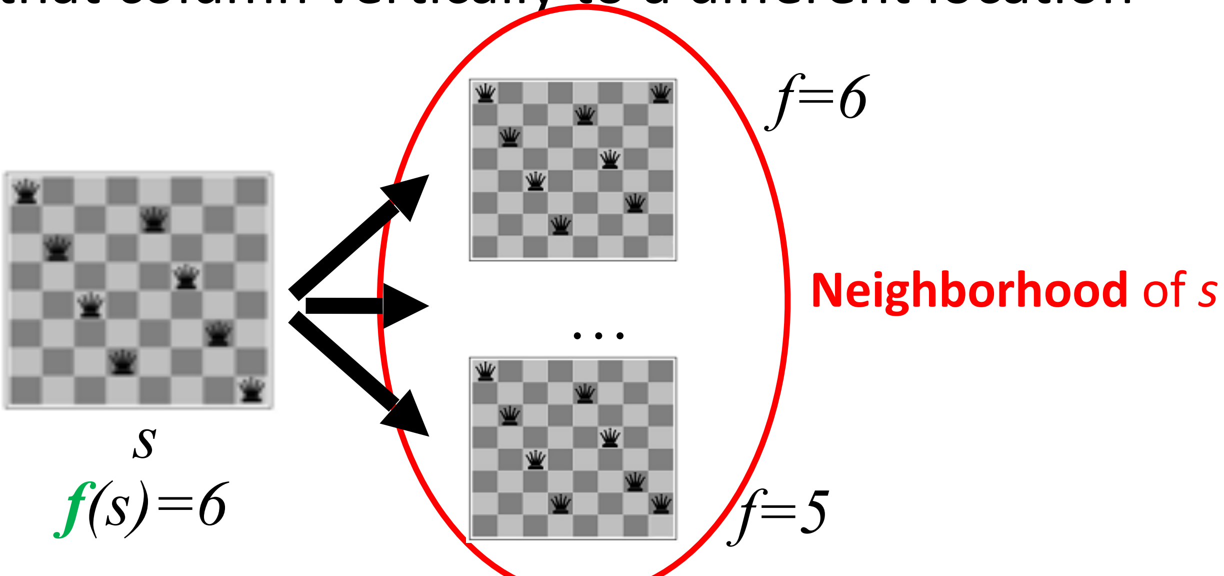

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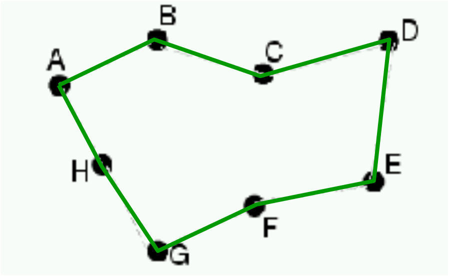

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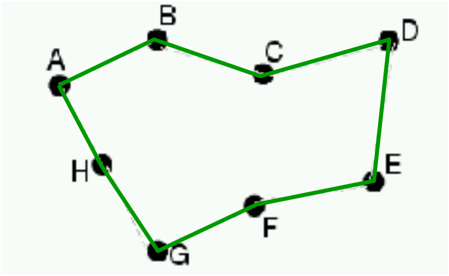

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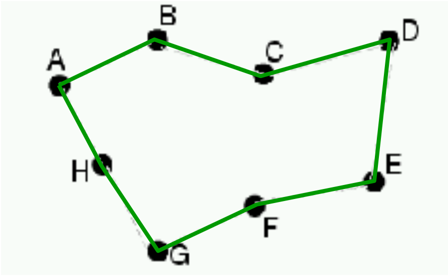

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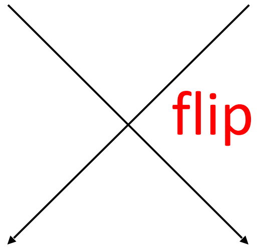

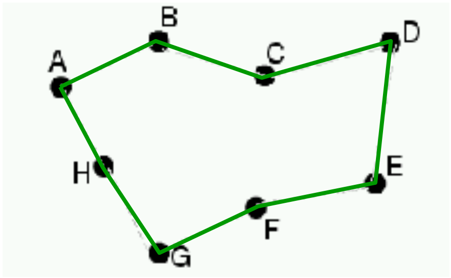

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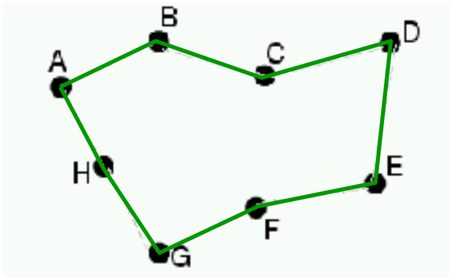

A-B-C-D-E-F-G-H-A

flip

A-E-D-C-B-F-G-H-A

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- **Q**: terminate? When no neighbor has better value

Hill Climbing Algorithm

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Pseudocode:

Hill Climbing Algorithm

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1. Pick initial state *s*
2. Pick *t* in **neighbors**(*s*) with the best ***f***(*t*)
3. if ***f***(*t*) is not better than ***f***(*s*) THEN stop, return *s*
4. *s* ← *t*. goto 2.

Hill Climbing Algorithm

Pseudocode:

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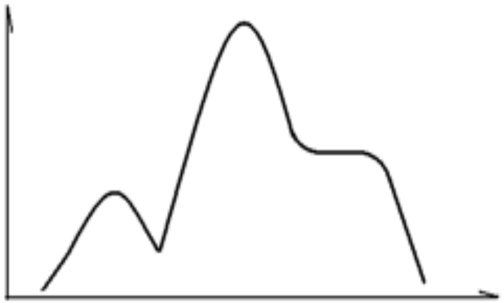
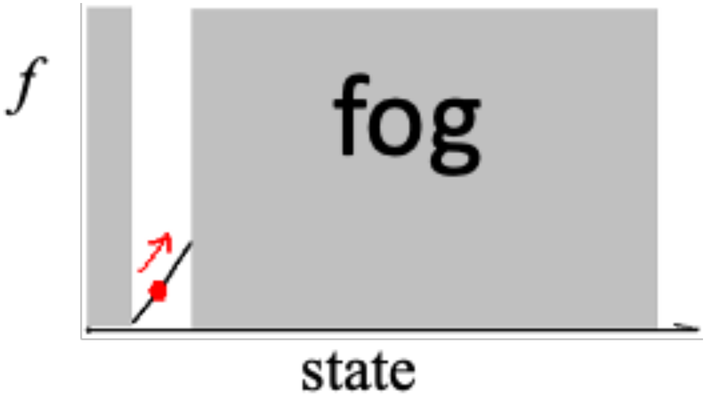
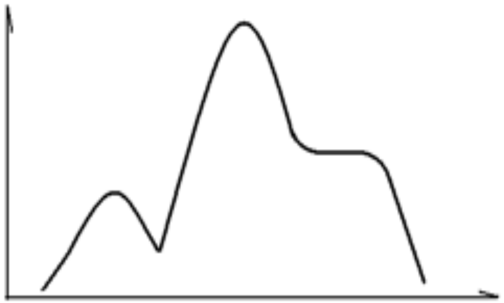
Hill Climbing: Local Optima

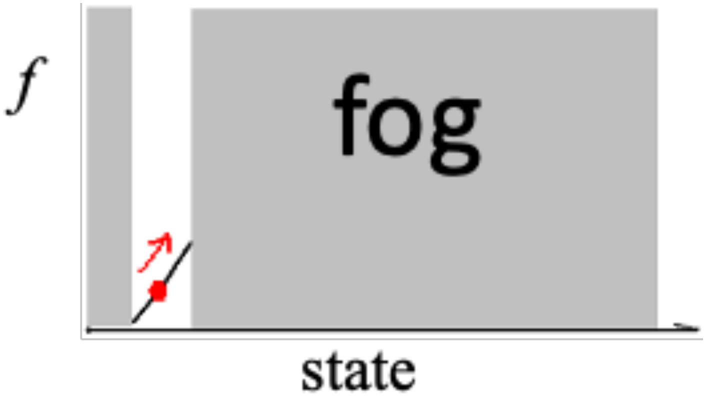




Hill Climbing: Local Optima

Q: Why is it called hill climbing?





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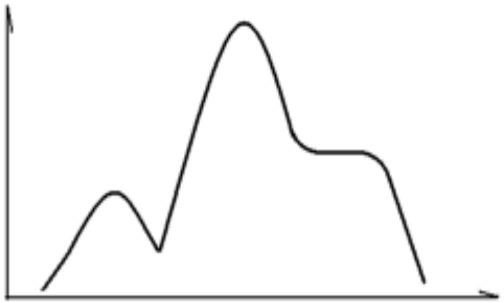

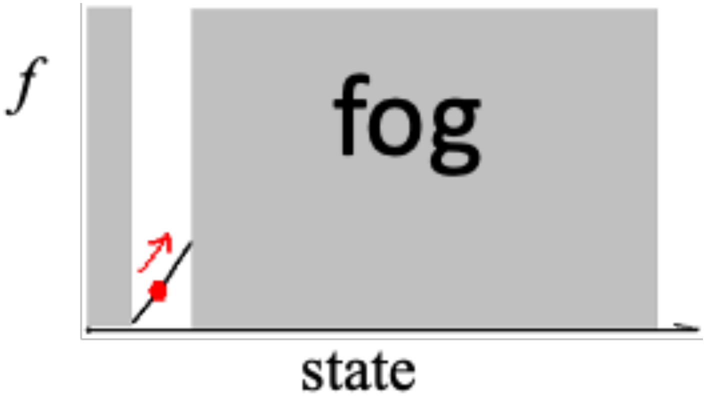

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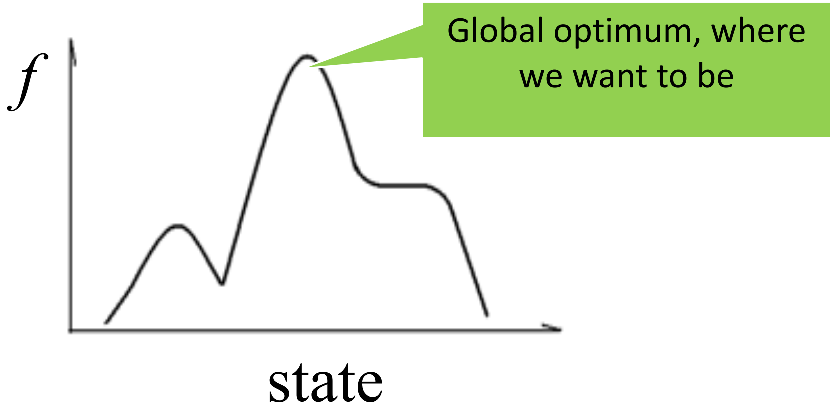

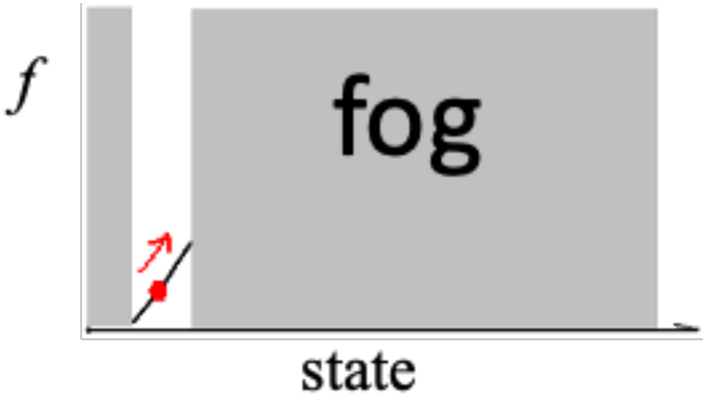

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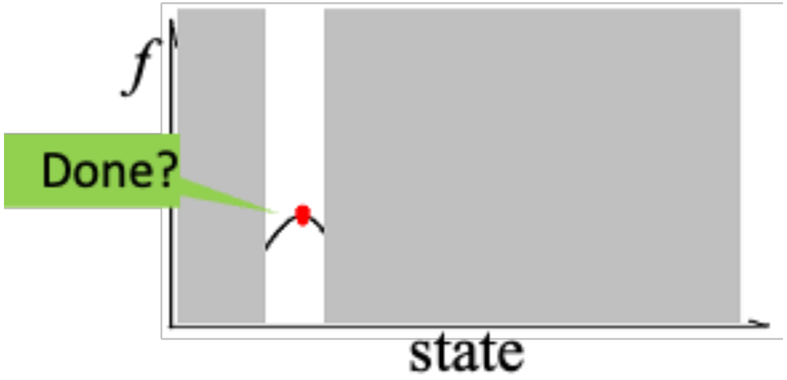
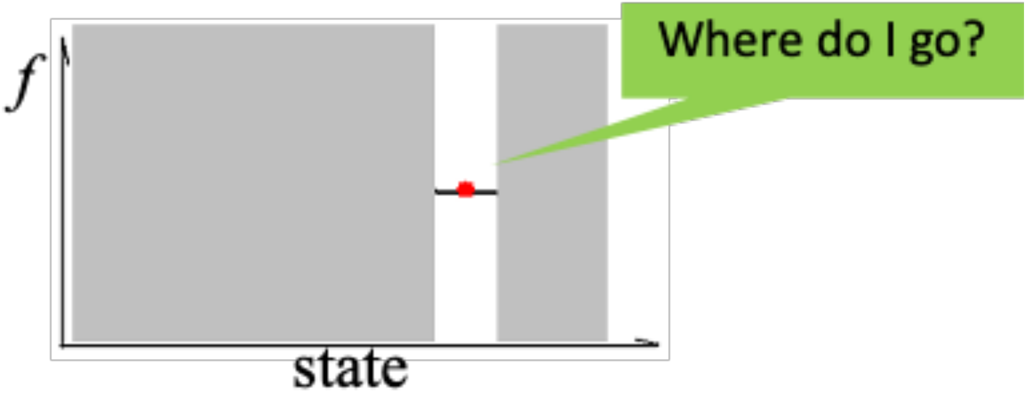
Hill Climbing: Local Optima

Note the **local optima**. How do we handle them?





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- "Stochastic" hill climbing: randomly select between neighbors.
- Probability of selecting a neighbor should be proportional to the value of that neighbor.

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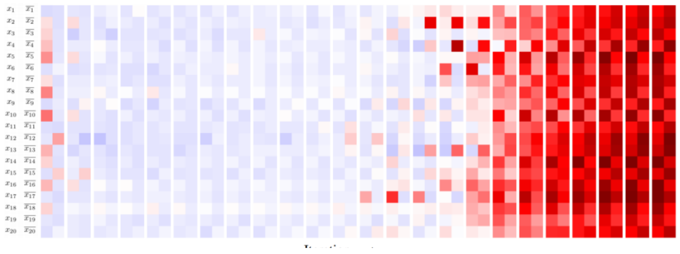

D. Selsam

Break & Quiz

Q 1.1: Hill climbing and stochastic gradient descent are related by

(i) Both head towards optima

(ii) Both require computing a gradient

(iii) Both will find the global optimum for a convex problem (problem where all optima have the same value).

- A. (i)
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- A. (i)
- B. (i), (ii) (No: (ii) is false. Hill-climbing looks at neighbors only.)
- **C. (i), (iii)**
- D. All of the above

Break & Quiz

Q 2.2: Which of the following would be better to solve with hill climbing rather than A* search?

i. Finding the smallest set of vertices in a graph that involve all edges
ii. Finding the fastest way to schedule jobs with varying runtimes on machines with varying processing power
iii. Finding the fastest way through a maze

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- C. (i) and (ii)
- D. (ii) and (iii)

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- A. (i) (No, (ii) better: huge number of states, don't care about path)
- B. (ii) (No, (i) complete graph might have too many edges for A*)
- **C. (i) and (ii)**
- D. (ii) and (iii) (No, (iii) is good for A*: few successors, want path)

Genetic Algorithms

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Optimization approach based on nature

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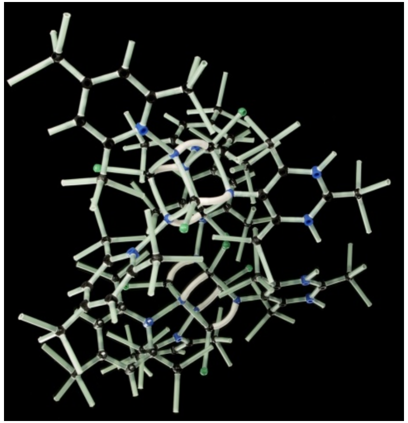

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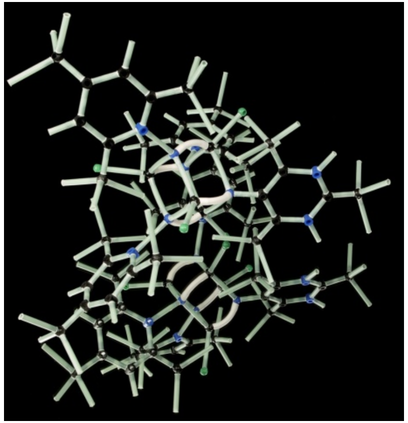

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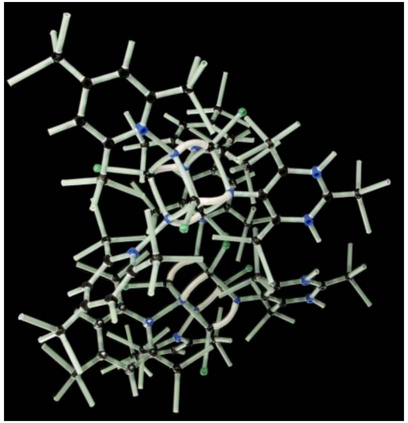

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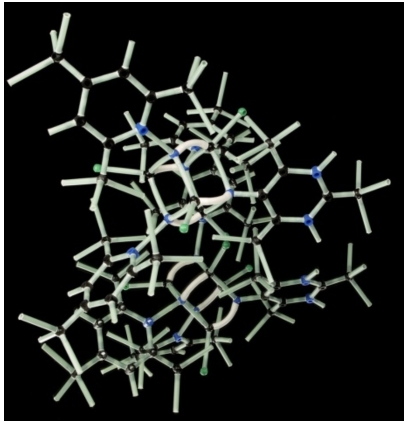

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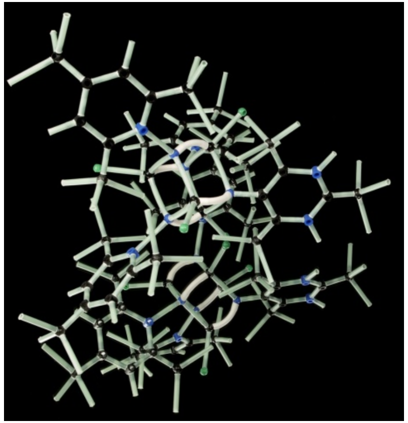
    - Happens at individual level



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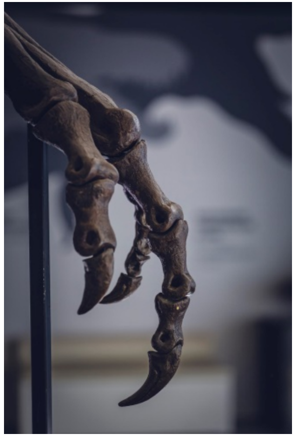

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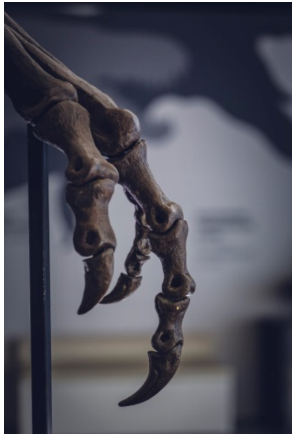

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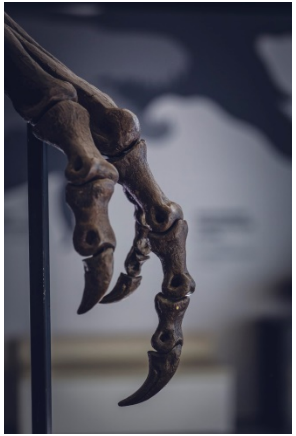

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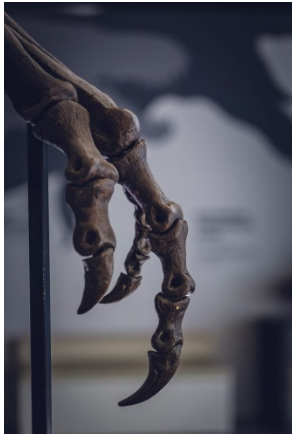

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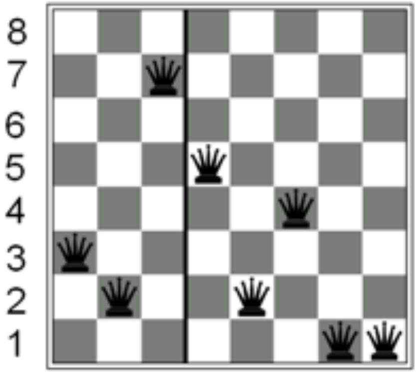
- Value $f(s)$ is now the ‘**fitness**’

Genetic Algorithms Setup I

Keep around a fixed number of states/individuals



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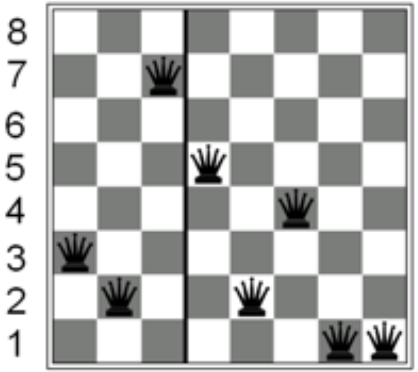

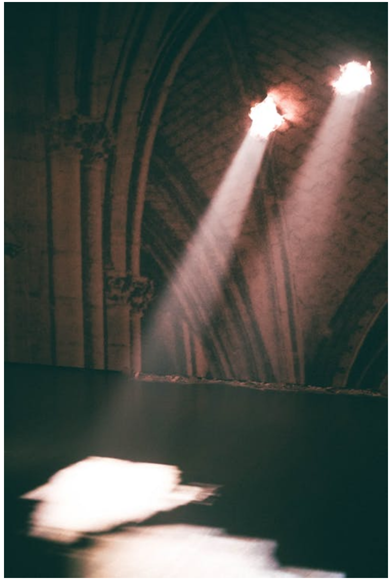

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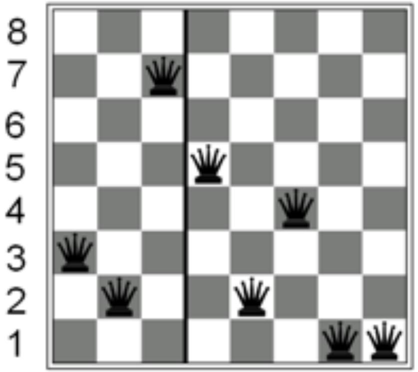

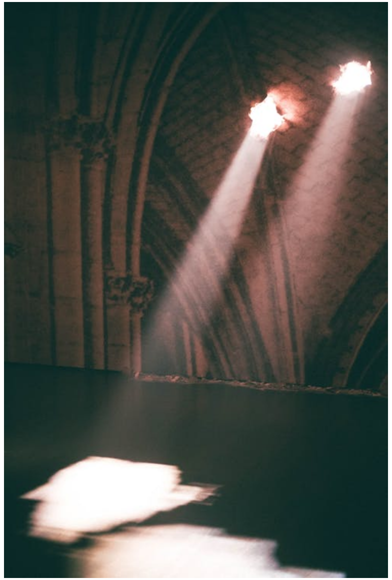

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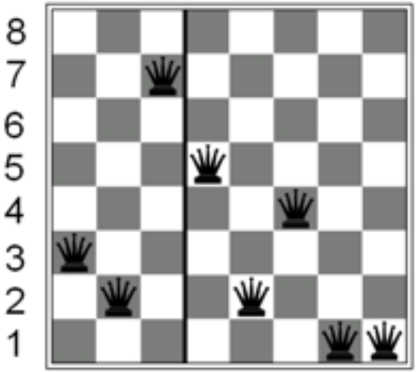

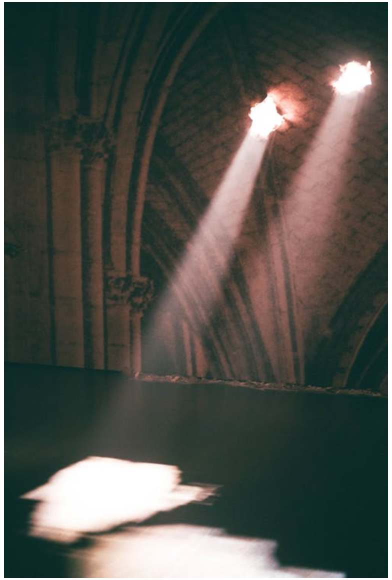

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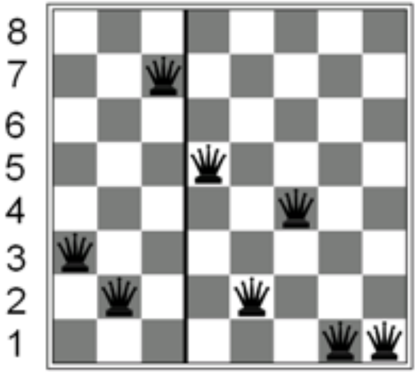
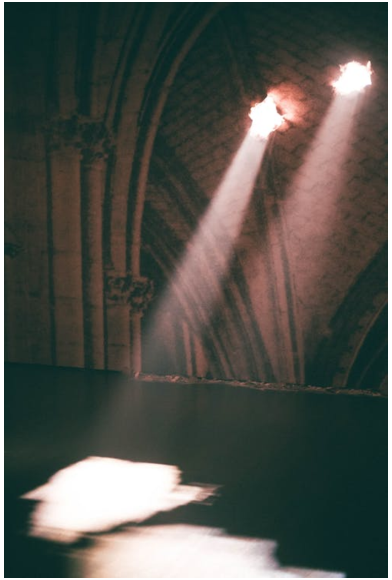
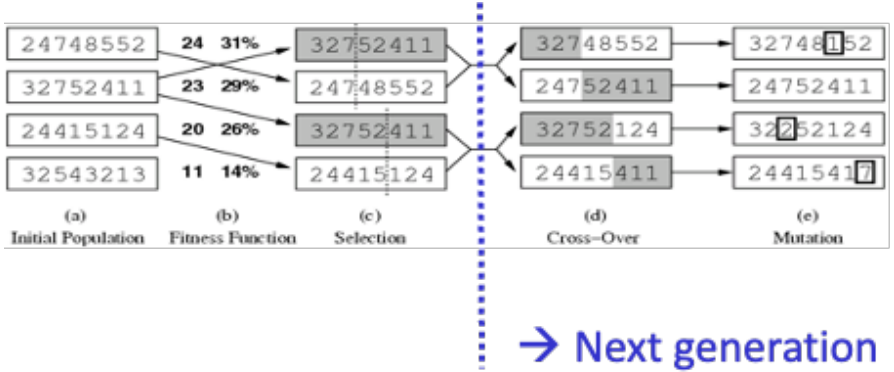


(3 2 7 5 2 4 1 1)



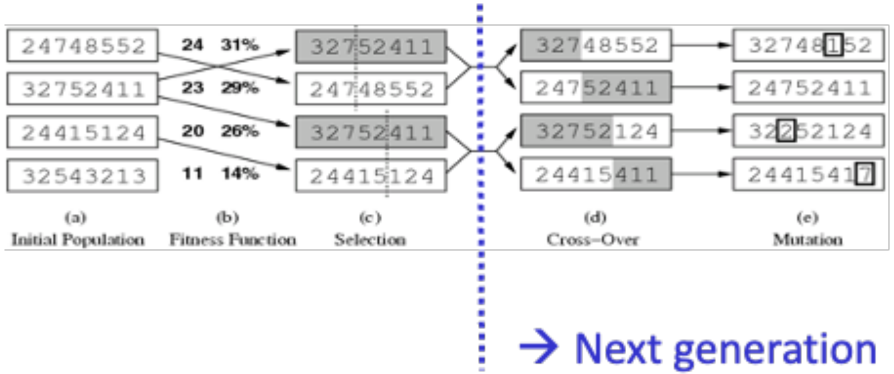
Genetic Algorithms Setup II



Genetic Algorithms Setup II

Goal of genetic algorithms: optimize using principles inspired by mechanism for evolution

- Analogous to **natural selection, cross-over**, and **mutation**



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→ Next generation

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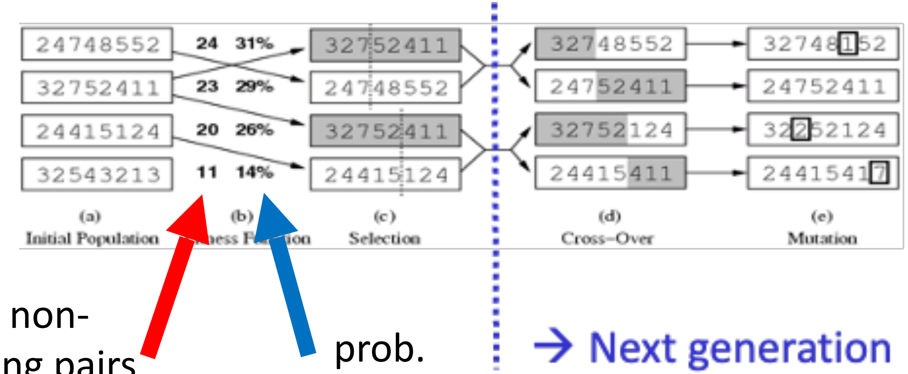

# of non-attacking pairs

prob. reproduction ∝ fitness

→ Next generation

Genetic Algorithms Pseudocode

Just one variant:

1. Let s_1, …, s_N be the current population
2. Let $p_i = f(s_i) / \Sigma_j f(s_j)$ be the reproduction probability
3. for $k = 1$; $k<N$; k+=2
   - parent1 = randomly pick according to p
   - parent2 = randomly pick another
   - randomly select a crossover point, swap strings of parents 1, 2 to generate children $t[k]$, $t[k+1]$
4. for $k = 1$; $k<=N$; k++
   - Randomly mutate each position in $t[k]$ with a small probability (mutation rate)
5. The new generation replaces the old: { s }←{ t }. Repeat

Reproduction: Proportional Selection

Reproduction probability: $p_i = \boldsymbol{f}(s_i) / \Sigma_j \boldsymbol{f}(s_j)$

Reproduction: Proportional Selection

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Individual	Fitness	Prob.
A	5	10%
B	20	40%
C	11	22%
D	8	16%
E	6	12%

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- **Example**: $\Sigma_j f(s_j) = 5+20+11+8+6=50$

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- p_1=5/50=10%

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Let’s run through an example:

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A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

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- Goal: maximize student enrollment

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Courses	Students	Can enroll?
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C D E	5	Yes

Example: Scheduling Courses

Value of a state? Say `MMFTM`

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- Here 4+5=9 students can enroll in desired courses

Example: Scheduling Courses

First step:

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A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

First step:

- Randomly initialize and evaluate states

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

First step:

- Randomly initialize and evaluate states

`MMFTM` = 9

`TTFMM` = 4

`FMTTF` = 19

`MTTTF` = 3

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

First step:

- Randomly initialize and evaluate states

  `MMFTM` = 9

  `TTFMM` = 4

  `FMTTF` = 19

  `MTTTF` = 3

- Calculate reproduction probabilities

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

First step:

- Randomly initialize and evaluate states

MMFTM = 9

TTFMM = 4

FMTTF = 19

MTTTF = 3

MMFTM = 26%

TTFMM = 11%

FMTTF = 54%

MTTTF = 9%

- Calculate reproduction probabilities

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Next steps:

Example: Scheduling Courses

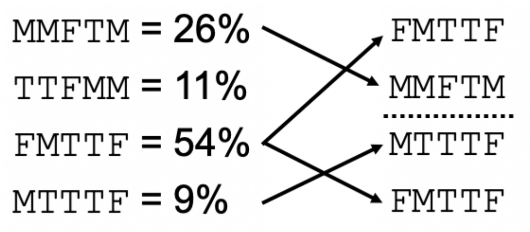
Next steps:

- Select parents using reproduction probabilities

Example: Scheduling Courses

Next steps:

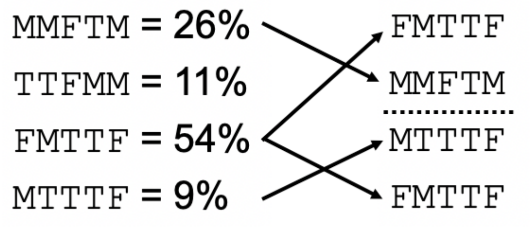
- Select parents using reproduction probabilities



Example: Scheduling Courses

Next steps:

- Select parents using reproduction probabilities
- Perform crossover



Example: Scheduling Courses

Next steps:

- Select parents using reproduction probabilities
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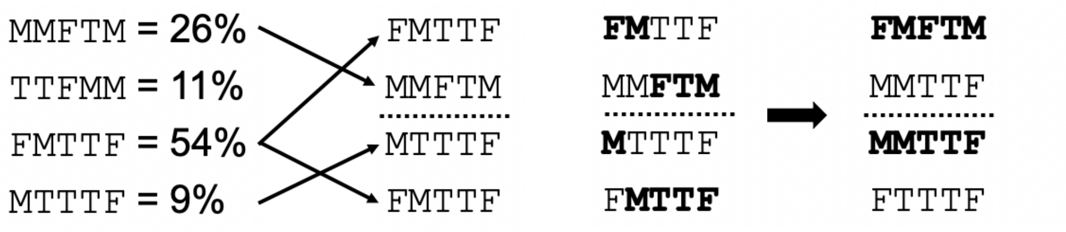
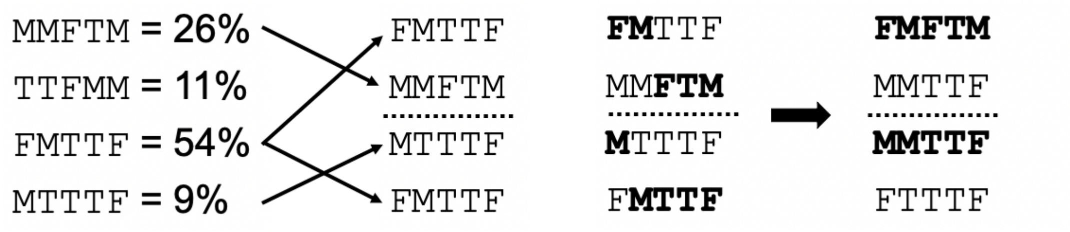


Example: Scheduling Courses

Next steps:

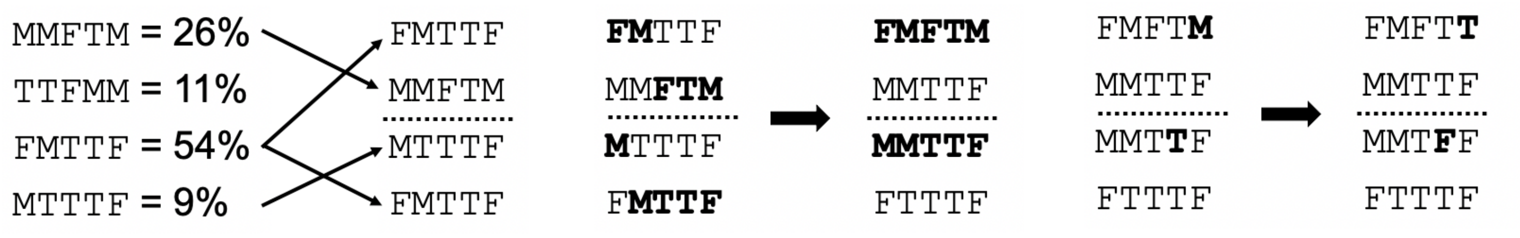
- Select parents using reproduction probabilities
- Perform crossover
- Randomly mutate new children



Example: Scheduling Courses

Next steps:

- Select parents using reproduction probabilities
- Perform crossover
- Randomly mutate new children



Example: Scheduling Courses

Continue:

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Continue:

- Now, get our function values for updated population

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Continue:

- Now, get our function values for updated population

`FMFTT` = 11

`MMTTF` = 13

`MMTFF` = 4

`FTTTF` = 0

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Continue:

- Now, get our function values for updated population
- Calculate reproduction probabilities

`FMFTT` = 11

`MMTTF` = 13

`MMTFF` = 4

`FTTTF` = 0

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Example: Scheduling Courses

Continue:

- Now, get our function values for updated population
- Calculate reproduction probabilities

FMFTT = 11	FMFTT = 39%
MMTTF = 13	MMTTF = 46%
MMTFF = 4	MMTFF = 14%
FTTTF = 0	FTTTF = 0%

Courses	Students
A B C	2
A B D	7
A D E	3
B C D	4
B D E	10
C D E	5

Variations & Concerns

Many **possibilities**:

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- Parents survive to next generation

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- Formulating a good state encoding

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- Lack of diversity: converge too soon

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- Use ranking instead of exact value of ***f***(*s*) for reproduction probabilities (reduce influence of extreme f values)

Some **challenges**

- Formulating a good state encoding
- Lack of diversity: converge too soon
- Must pick a lot of parameters

Summary

- Challenging optimization problems
  - First, try hill climbing. Simplest solution
- Genetic algorithms
  - Biology-inspired optimization routine