Autonomous Robotics

Control Theory (Part II)

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Learning Outcomes

Today's goals:

- Review common control laws: bang-bang control, P, PD, PID control
- Analyze control laws mathematically to see how their properties arise.
- Discuss Braitenberg vehicles.

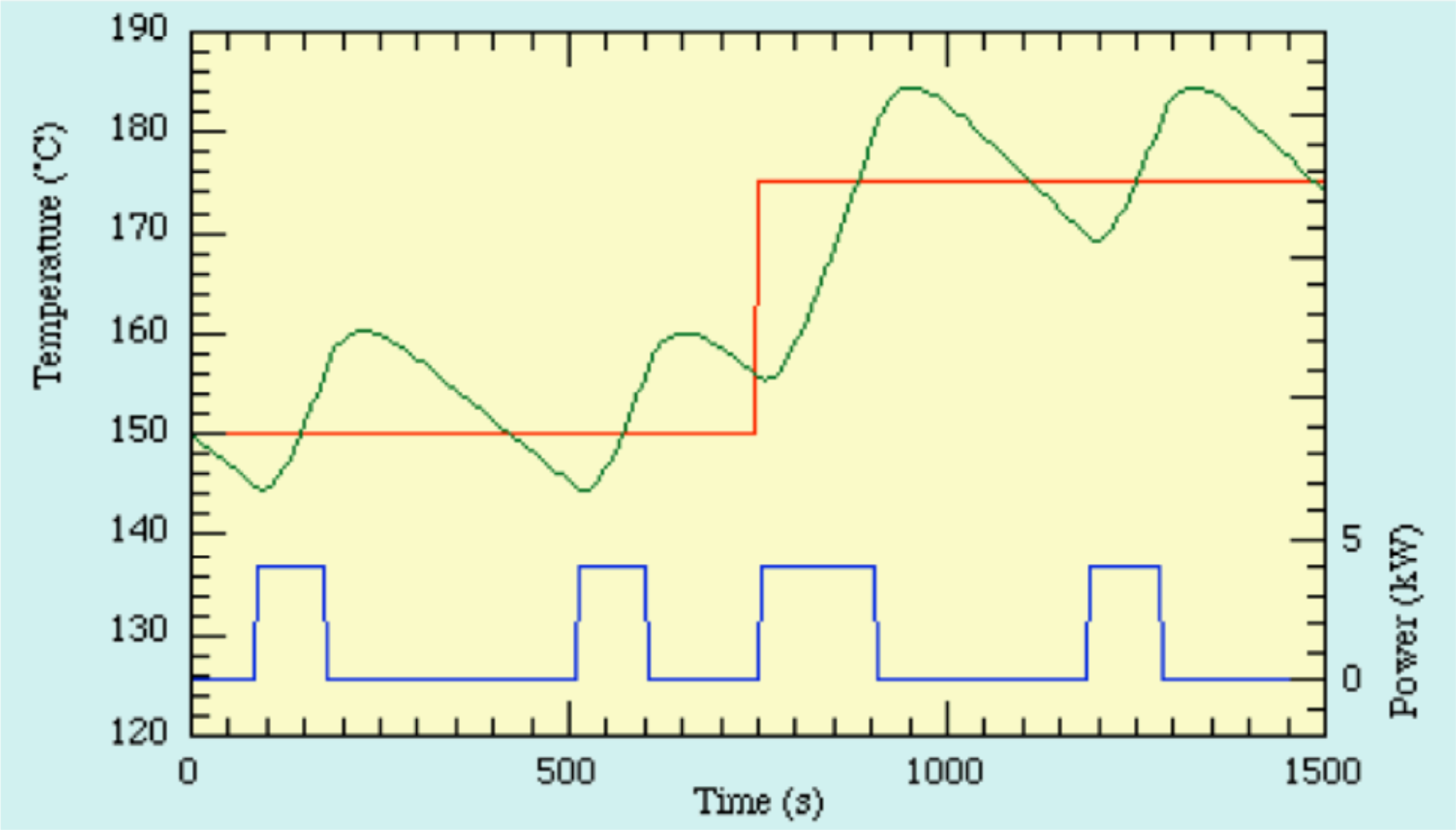
Control Objective

- Goal: bring the robot's state, x, to a desired state, $x_{\texttt{set}}$.
- Use error, $e = x - x_{\texttt{set}}$, to measure how close the robot is to achieving this.
- Assumptions:
  - The state x is observable.
  - Increasing u will increase x.
  - Simplification: everything is 1-dimensional

Bang-Bang Control

- Simplest control law: toggle between choosing one of two values for u.

$$\begin{array}{llll} \text{if} & e < -\epsilon & \text{then} & u := on \\ \text{if} & e > +\epsilon & \text{then} & u := off \end{array}$$



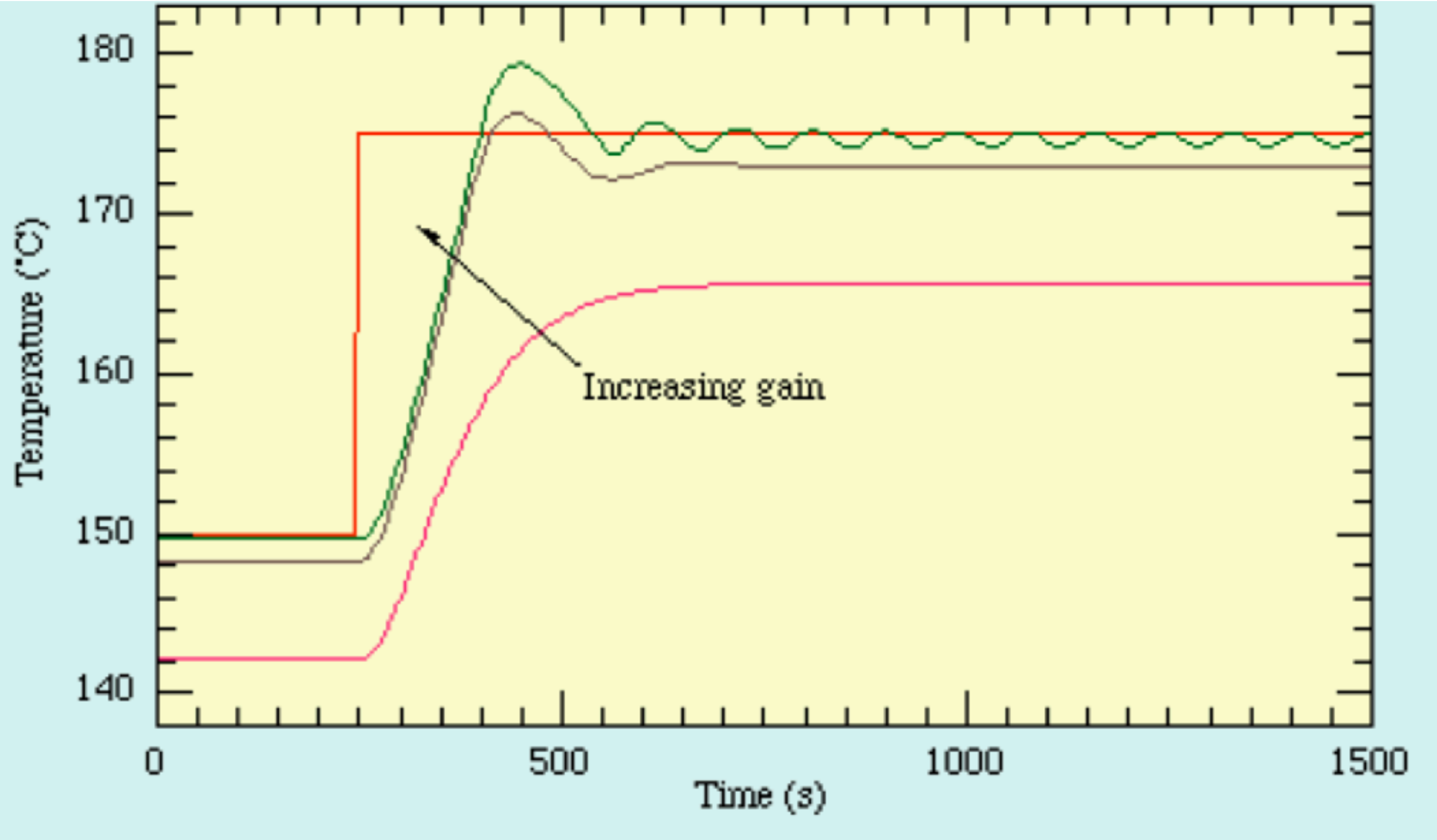
Red: Set point (desired temperature)
Blue: control value (Power)
Green: State (temperature) resulting from bang-bang control.

On: 4 kW
Off: 0 kW
ϵ: 5 degrees Celcius

$$e = x - x_{\texttt{set}}$$

P-Control

$$u = -k_P * e + u_b$$



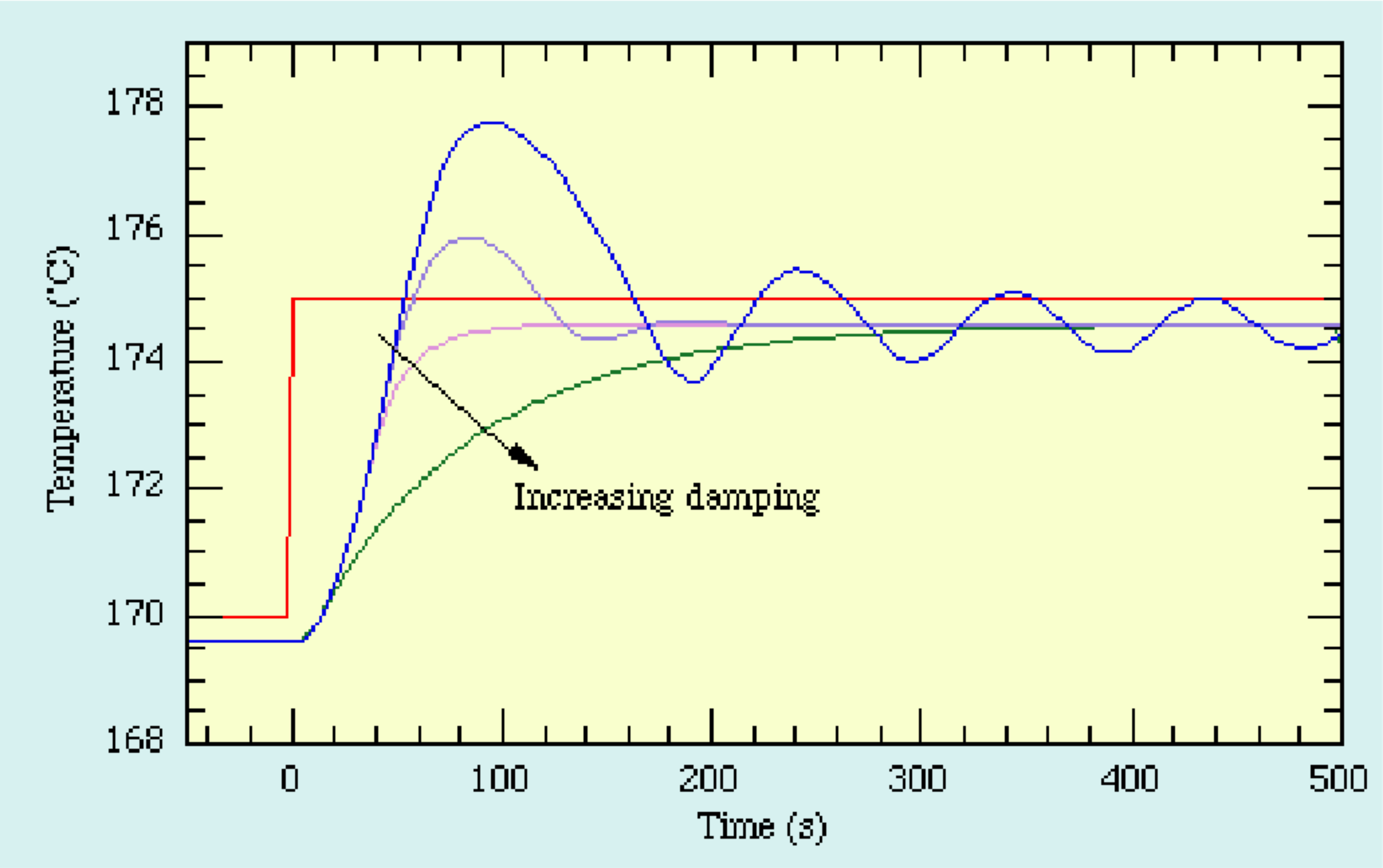
State (temperature) response curves for varying P gain.
Red: set-point (desired temperature)

k_P is the P gain.

$$e = x - x_{\mathtt{set}}$$

PD-Control

$$u_t = -k_P e - k_D \dot{e}$$



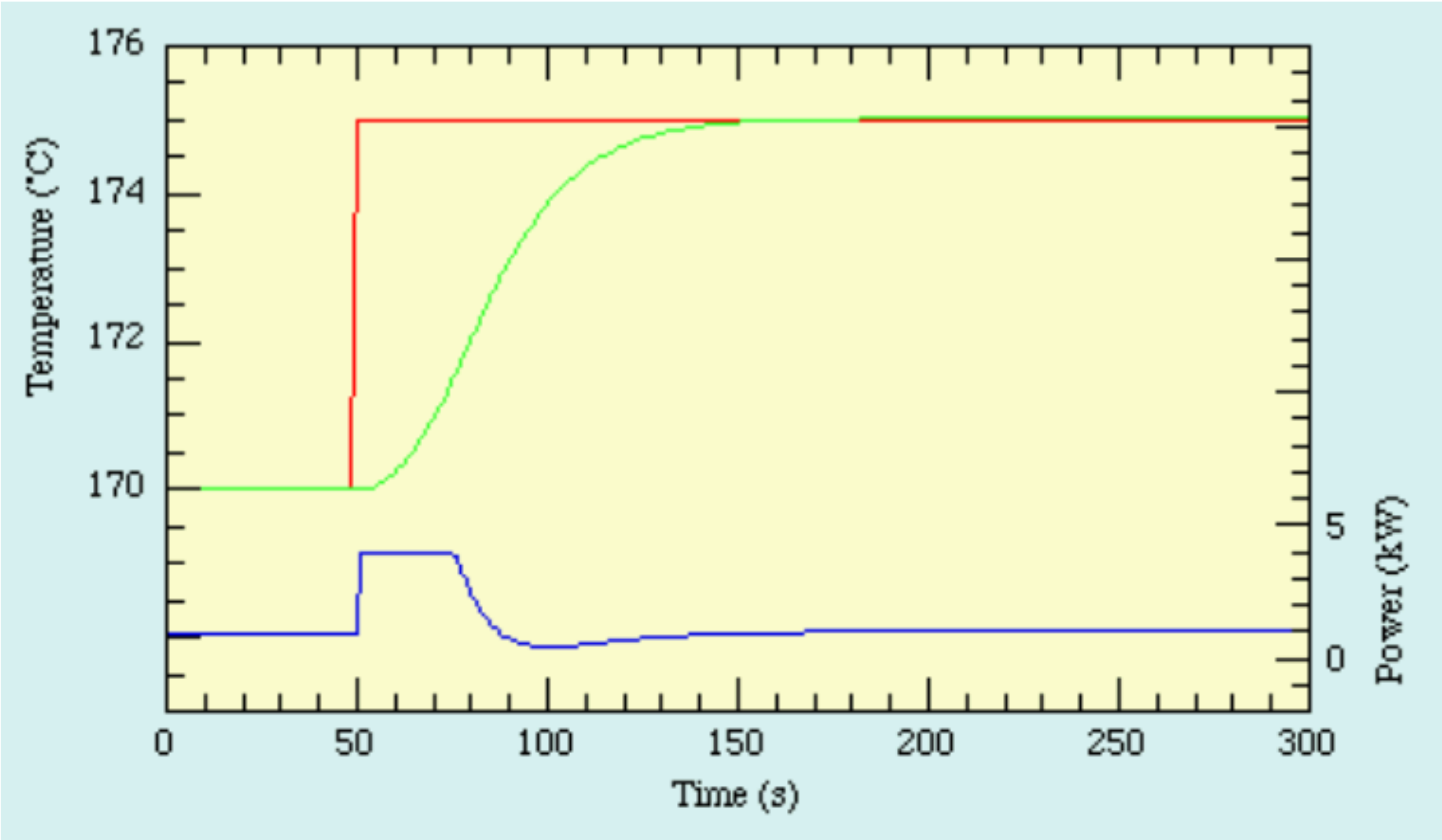
k_D is the D gain (damping).

State (temperature) response curves for varying D gain while the P gain is held constant.
Red: set-point (desired temperature)

$$e = x - x_{\text{set}}$$

PID-Control

$$u_t = -k_P e(t) - k_I \int_{i=0}^{t} e(i)di - k_D \dot{e}$$



Red: set-point (desired temperature)
Green: State response curve
Blue: PID control output (kW)

$$e = x - x_{\texttt{set}}$$

Example Analysis

- When system dynamics, $F(x, u)$, are known, we can mathematically analyze behavior of system under a control law and use results to set parameters.

$$\dot{x} = ax + bu \qquad u = -k_P e + u_b \qquad e = x - x_{\mathtt{set}}$$

$$= ax + b(-k_P(x - x_{\mathtt{set}}) + u_b)$$

$$= -\alpha x + \beta$$

Constants

$$\alpha = k_P b - a$$

$$\beta = (k_P x_{\mathtt{set}} + u_b) b$$

Example Analysis

- $\dot{x}$ and x are implicitly functions of time, t.
- Solve the differential equation to get a function, $x(t)$, that does not depend on $\dot{x}(t)$.

$$\dot{x}(t) = -\alpha x(t) + \beta$$

*See blackboard or control reading *

$$x(t) = Ce^{-\alpha t} + \frac{\beta}{\alpha}$$

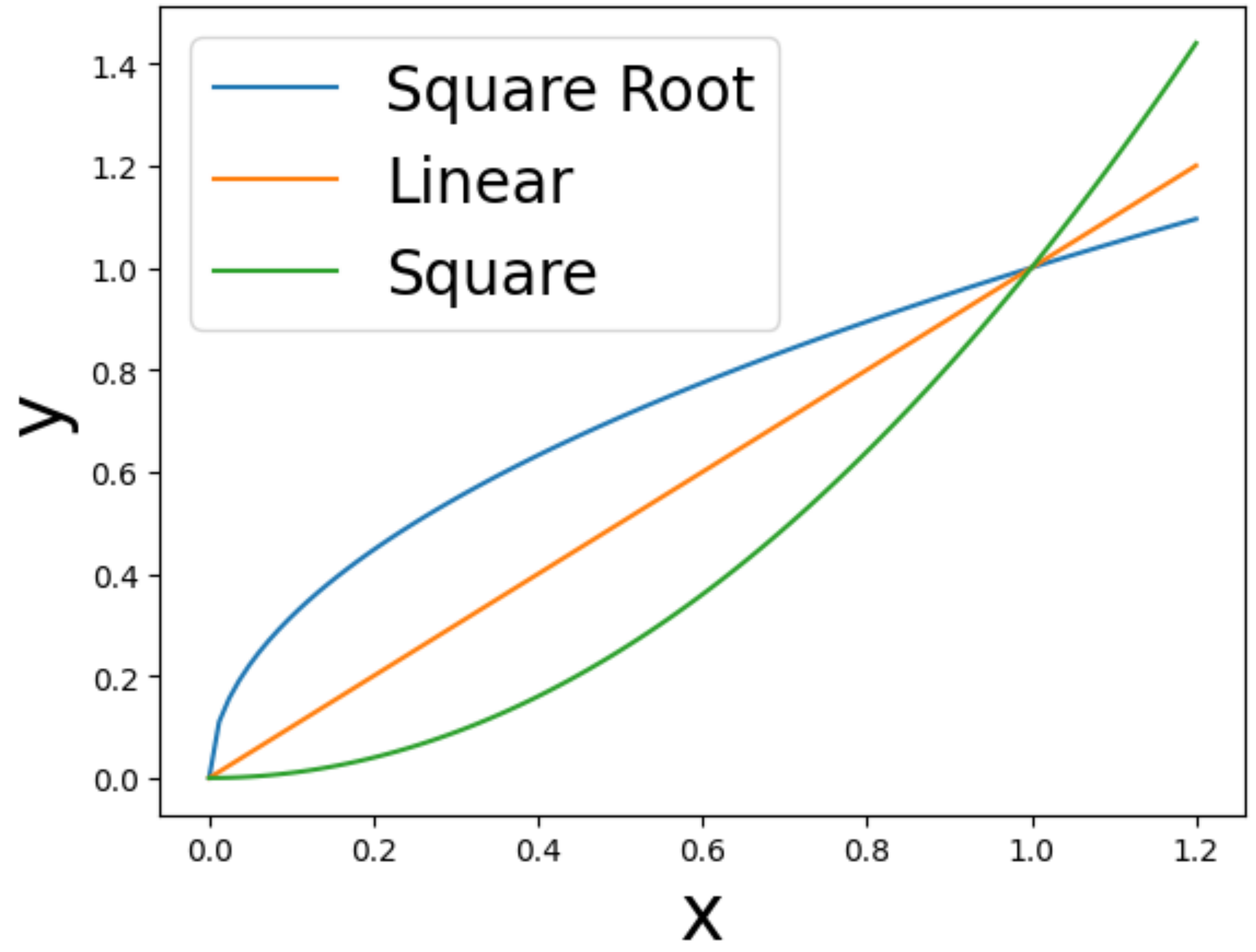
Some constant

Stability: $x(\infty) = \texttt{const}$

Correct convergence: $x(\infty) = x_{\texttt{set}}$

Non-Linear P-Control

- In P-control, control is a linear function of error.
- Generalization: control is a non-decreasing function of error.
  - If the error increases, the control will not decrease.
- Advantages:
  - Can reach set-point (quiescence) in finite time.
  - More suitable for non-linear systems.
- Disadvantages: increased difficulty in tuning.



Application of Non-Linear P-Control

- The stopping controller provides an example of how non-linear P-control can enable a finite stopping time while the P-controller cannot.

- Linear:

$$\dot{x} = -k_P x \qquad x(t) = e^{-k_P t} \qquad t_s = \infty$$

- Non-Linear:

$$\dot{x} = -k_P \sqrt{x} \qquad x(t) = (1 - \frac{kt}{2})^2 \qquad t_s = \frac{2}{k}$$

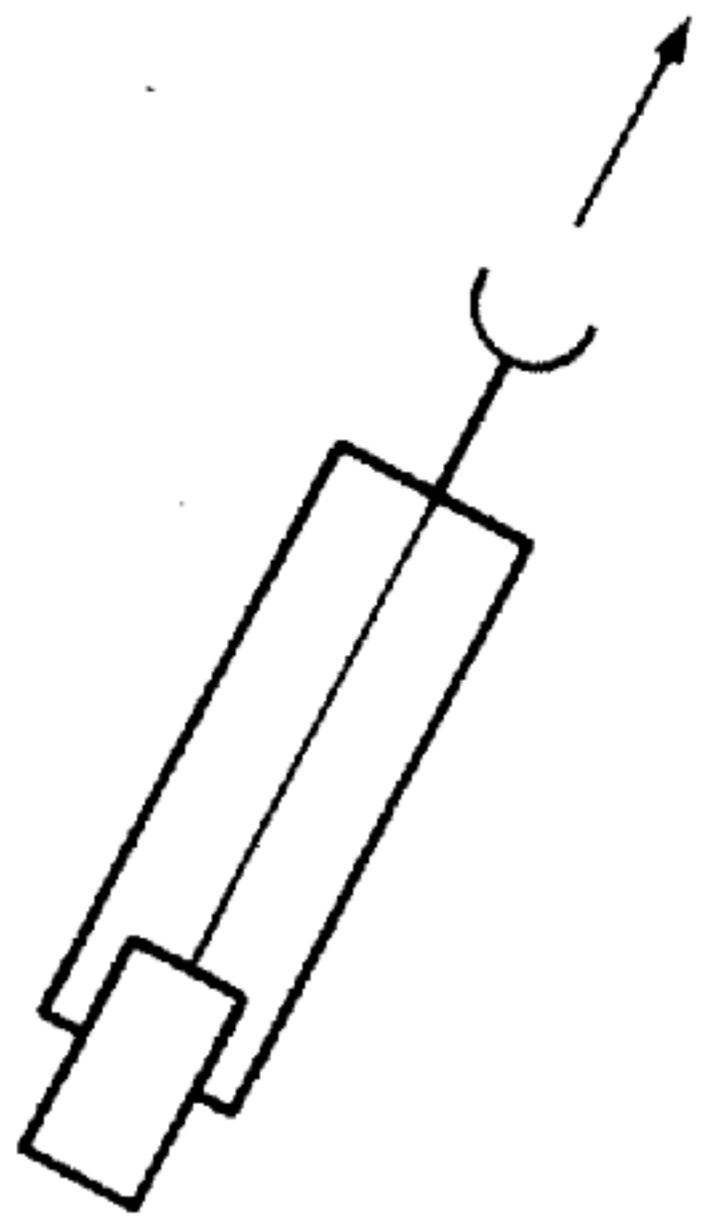
Tuning Strategy for PID Control

- PID controllers are complex to tune because they have three degrees of freedom: k_P, k_I, k_D.
- Initialize all gains to zero. Start by tuning k_P so that x quickly approaches $x_{\texttt{set}}$ without excessive overshoot.
- Next, tune k_I to reduce steady-state offset.
- Finally, tune k_D to minimize damping.

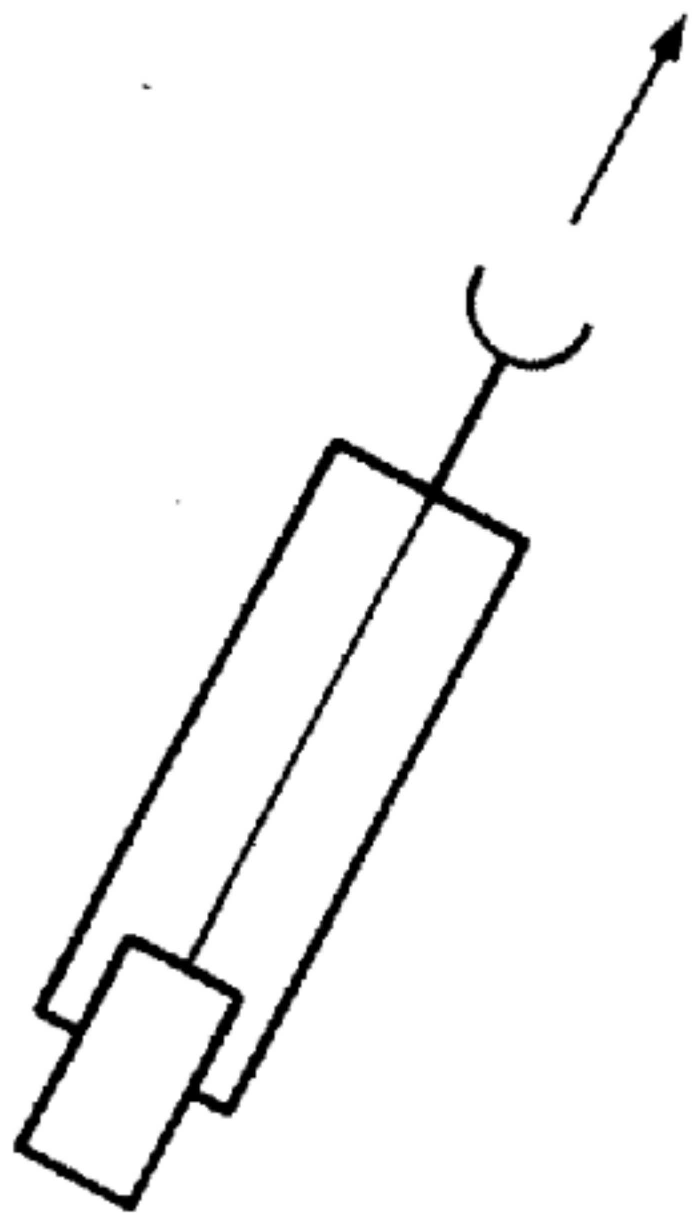
Tuning Example

Braitenberg Vehicles

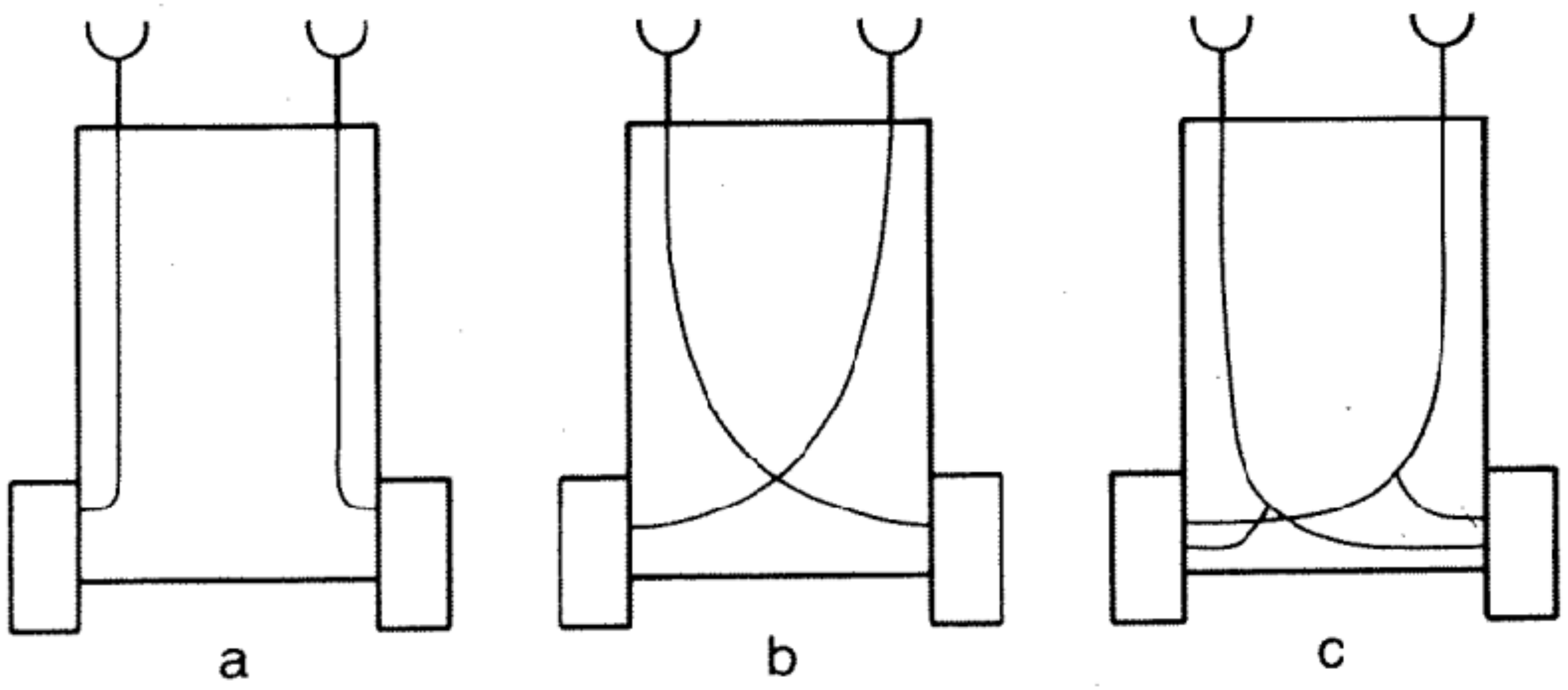
- Reactions?
- Where does the complexity of behavior arise from?
- Do you agree with the author’s descriptions of vehicle behavior?



Braitenberg: Vehicles 1 and 2

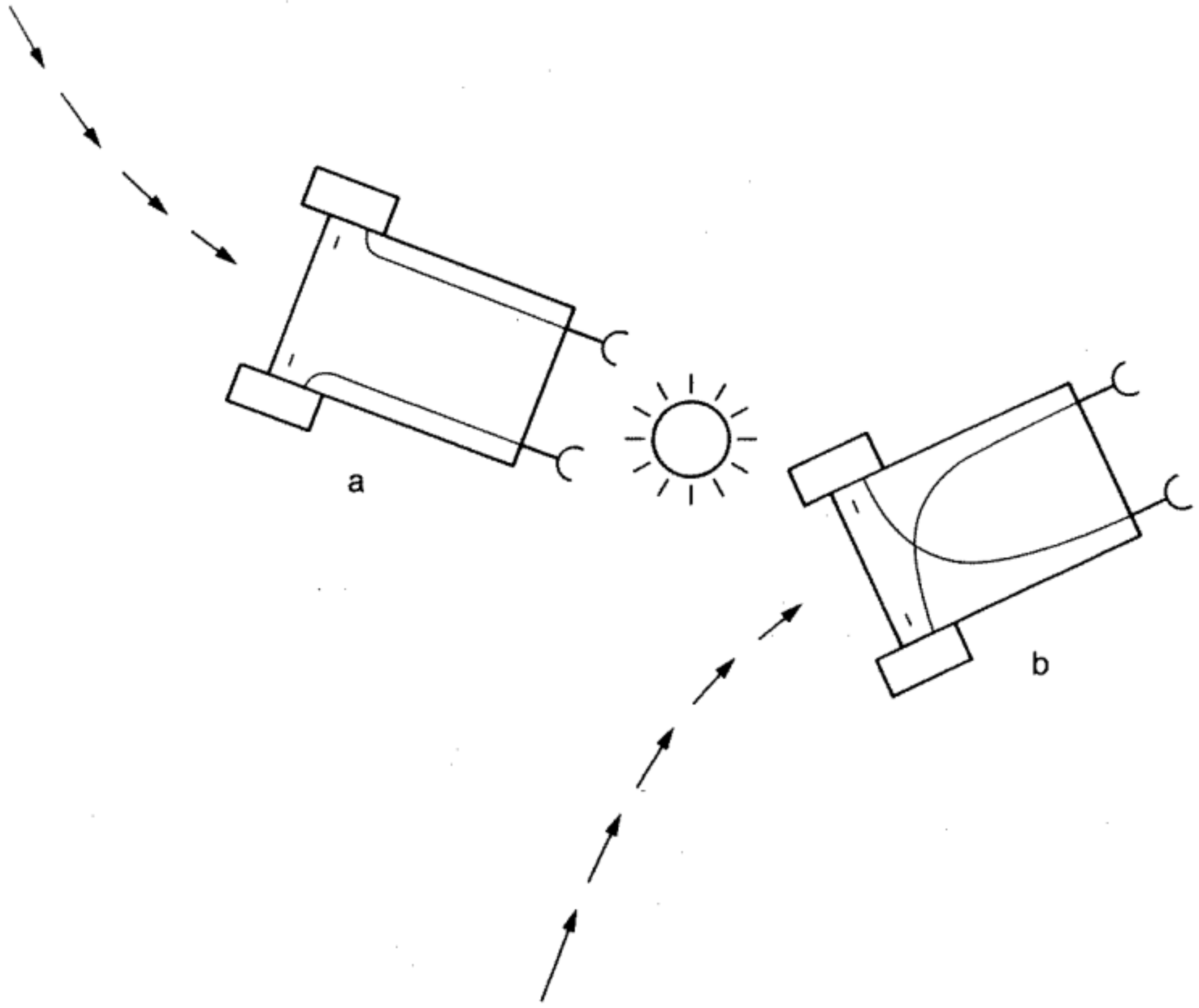


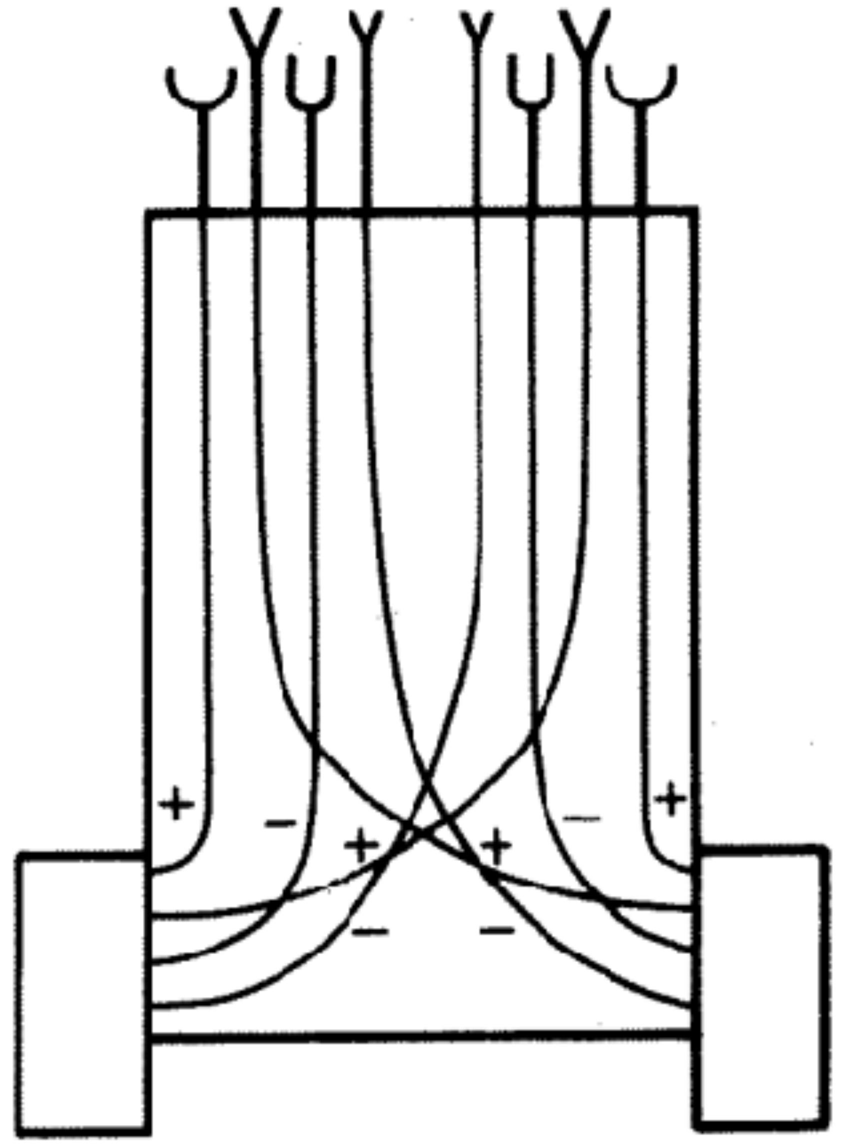
Vehicle 1



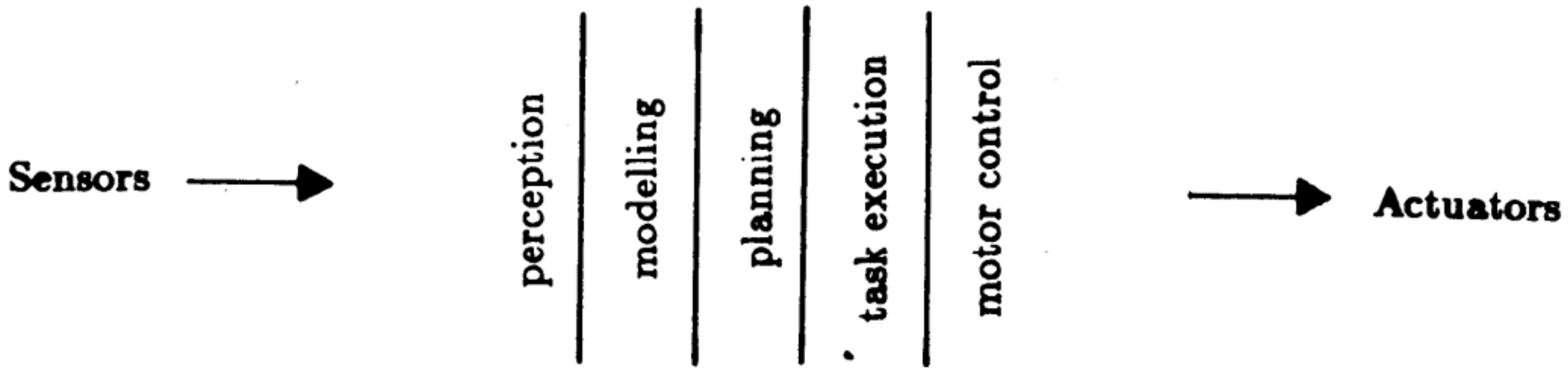
Vehicle 2

Braitenberg: Vehicle 3



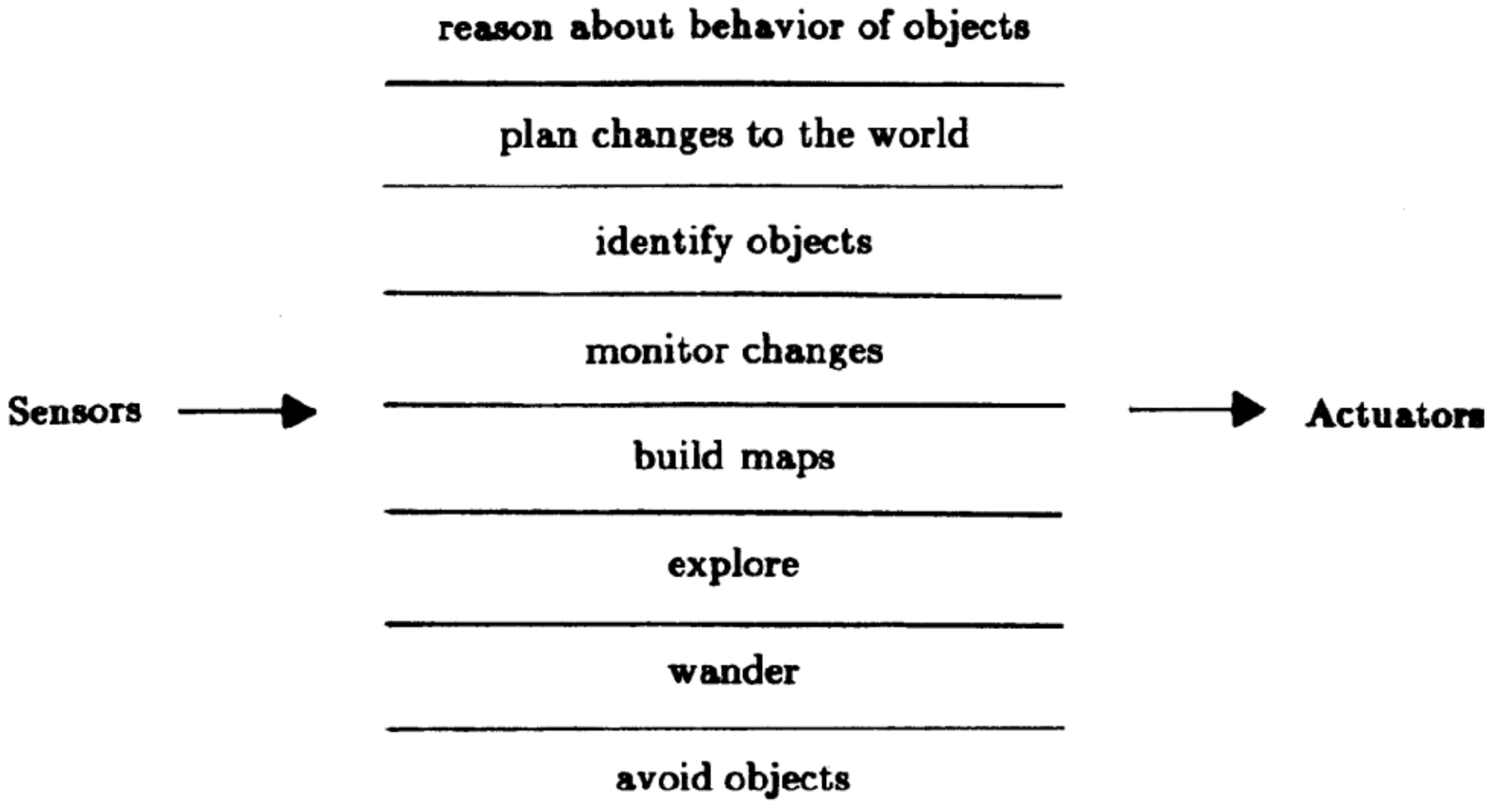


Behavior Architectures



Sense-think-act

Simple, layered behaviors can give rise to complex capabilities.



Subsumption

https://people.csail.mit.edu/brooks/papers/AIM-864.pdf

Summary

- Reviewed basic control laws: bang-bang, proportional or P, PI, PD, and PID.
- Analyzed P-controllers to reveal properties.
- Discussed controller tuning.
- Discussed Braitenberg readings

Action Items

- Complete the background survey: https://forms.gle/d8hmnQGWQc9SMVcN6

- Begin the first programming assignment on control.

- Read on Bayes filter for next week; send a reading response by 12 pm on Monday.