Autonomous Robotics

Simultaneous Localization and Mapping

Josiah Hanna
University of Wisconsin — Madison

Announcements

- Homework #2 due Thursday at 9:30am.
    - Questions?
- Reading assignment for next week (Advanced SLAM) has been posted.
- Office hours start at 11:30 today.

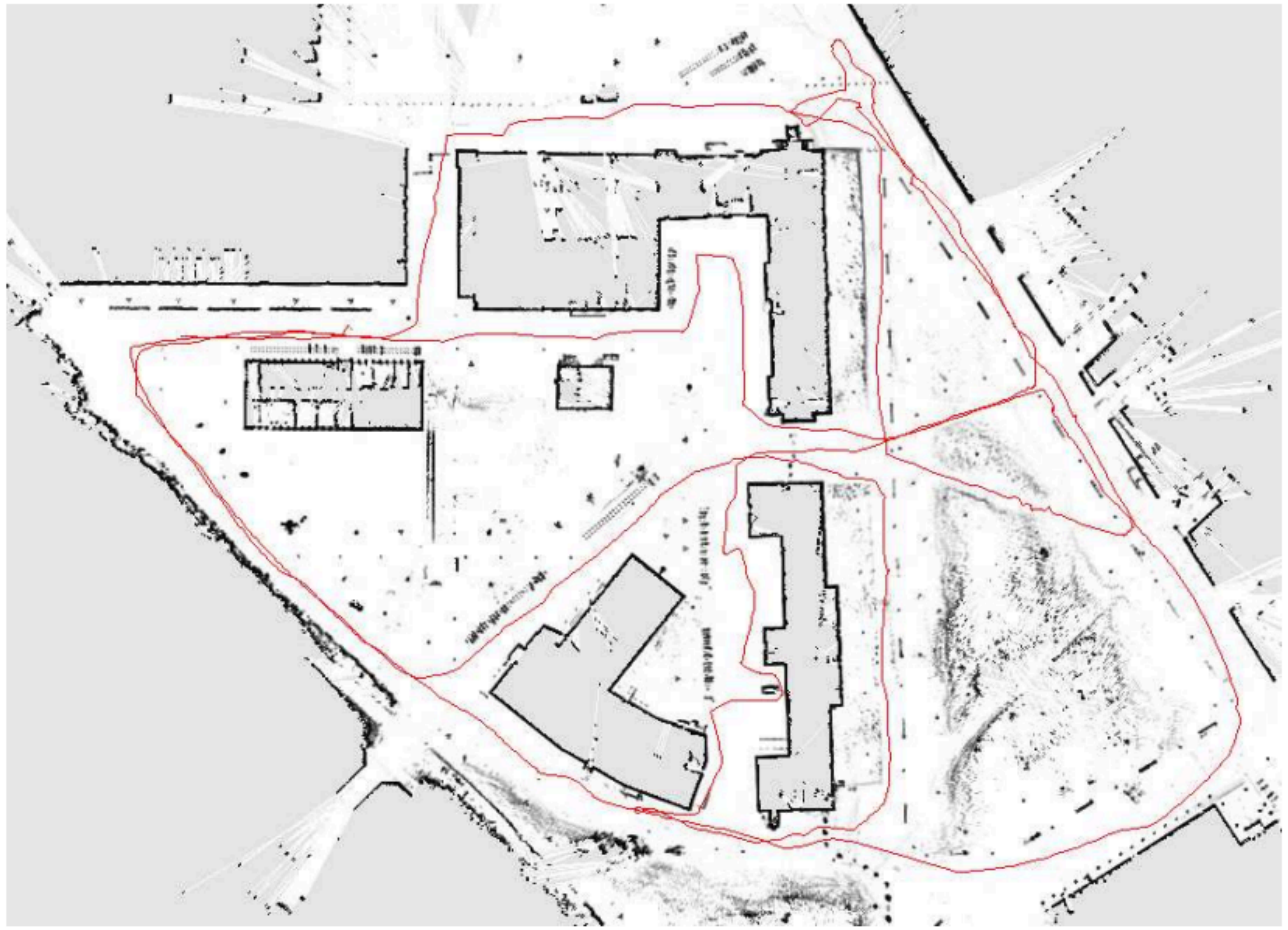
Learning Outcomes

After today's lecture, you will:

- Be able to define the SLAM problem.
- Be able to define key SLAM concepts such as data association and loop closure.
- Understand the correlation structure of a SLAM problem.
- Be able to give the steps of EKF-SLAM.

Review

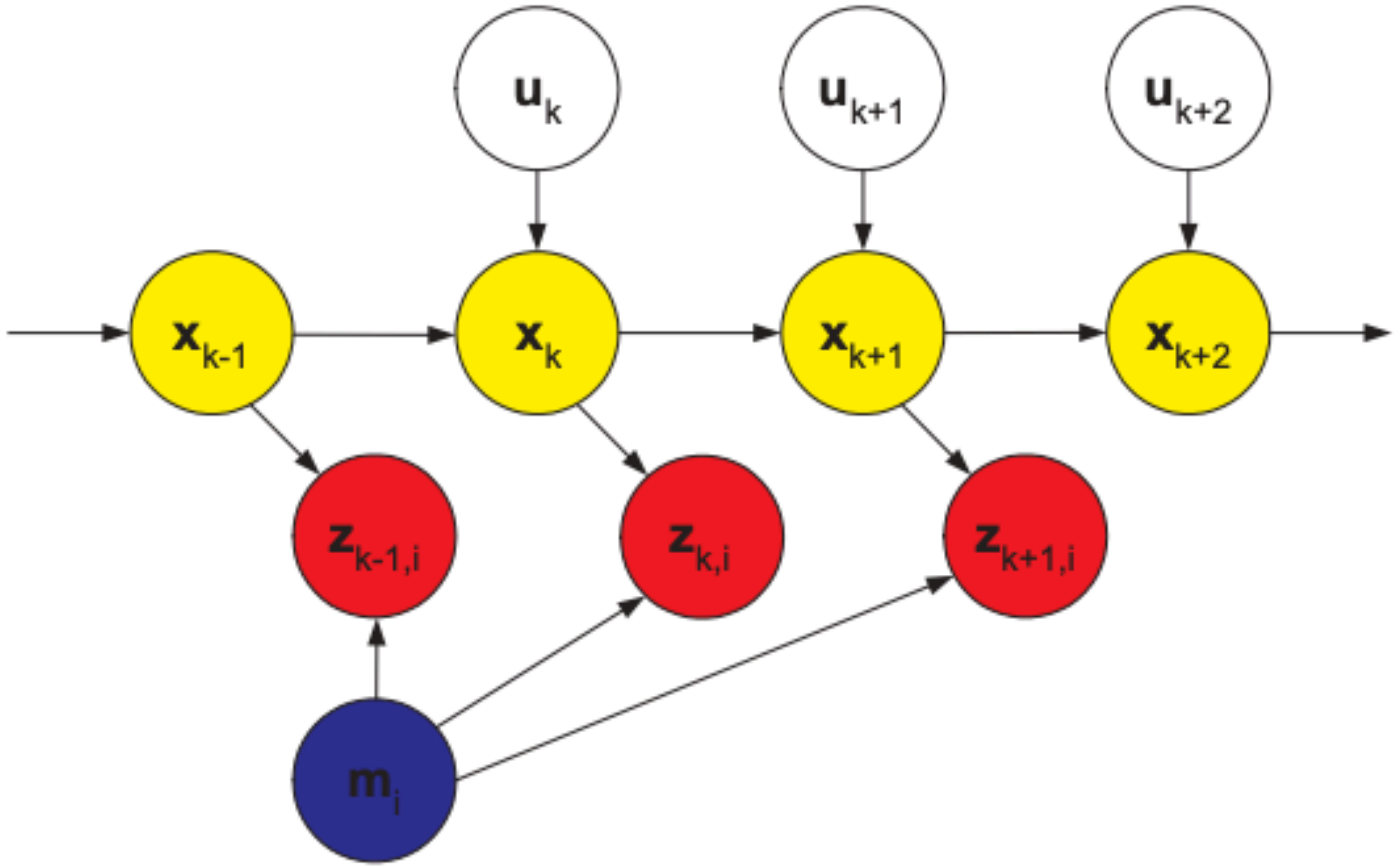
- Recall localization
  - Estimate $p(x_t \mid z_{1:t}, u_{1:t}, m)$
  - Example: localization with known landmarks.
- Recall mapping
  - Estimate $p(m \mid x_{1:t}, z_{1:t})$
  - Example: robot is outdoors and has a GPS sensor.



http://www2.informatik.uni-freiburg.de/~stachnis/pdf/grisetti07tro.pdf

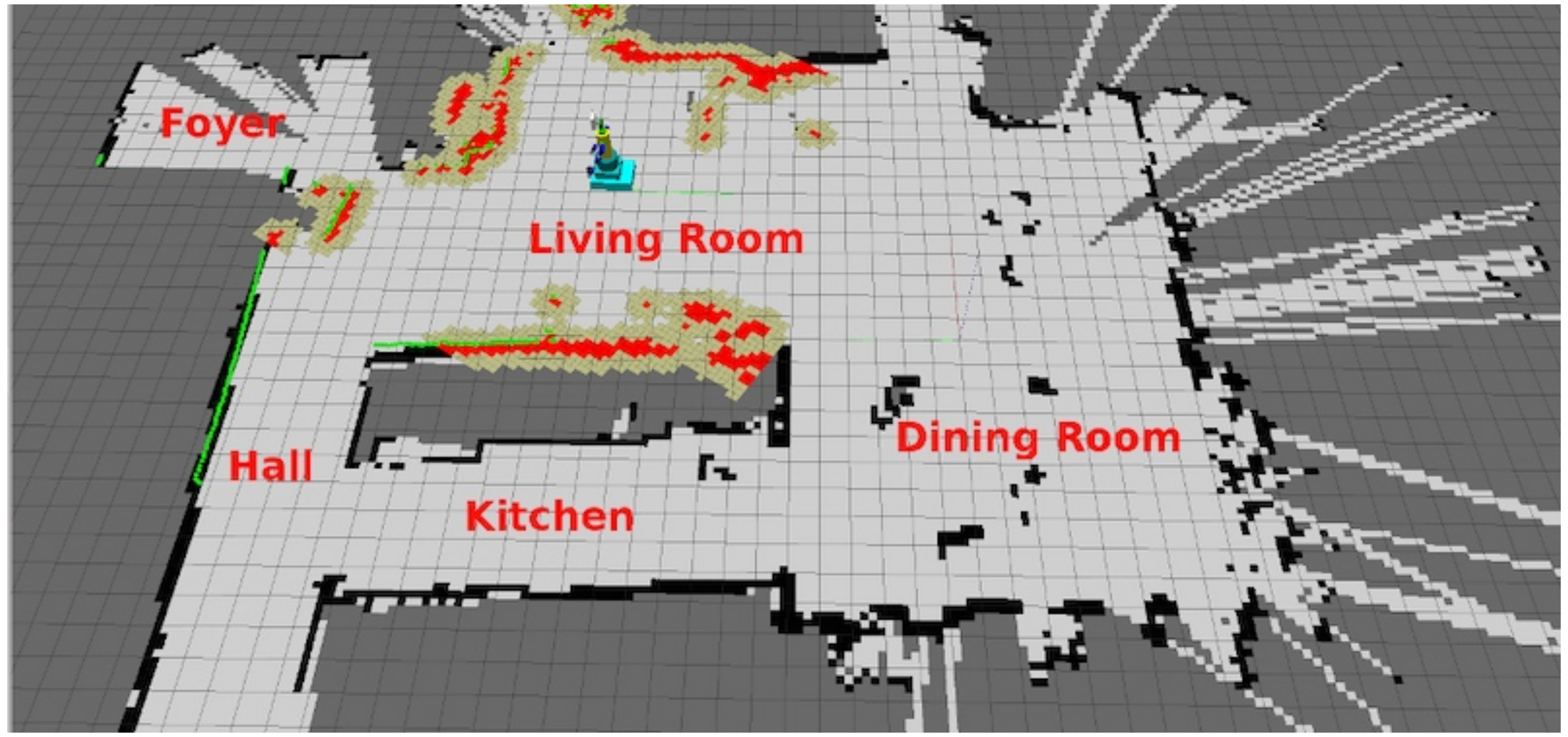
SLAM

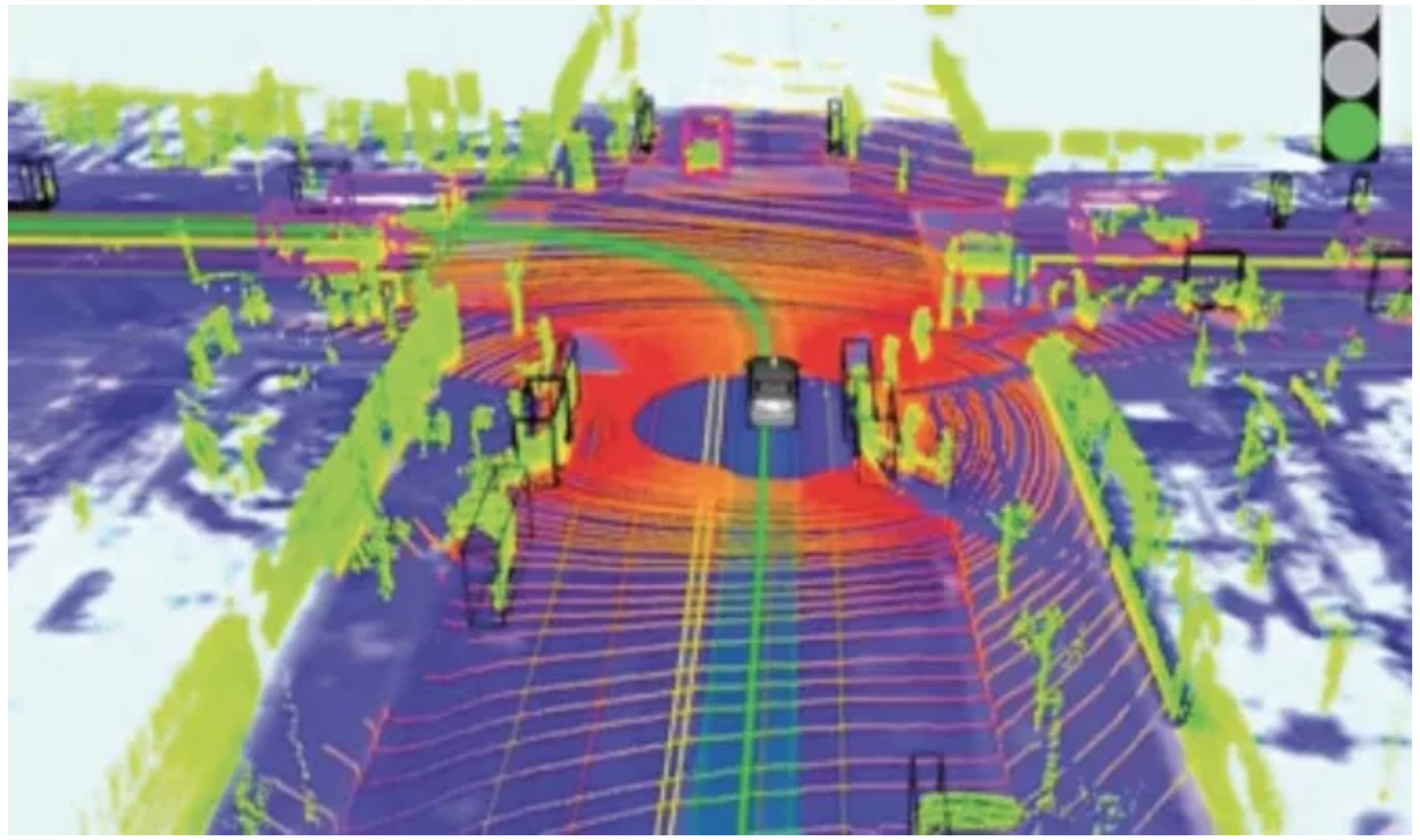
- Localize and map at the same time.
- Formally, estimate $p(x_t, m \mid z_{1:t}, u_{1:t}, x_0)$
- Let's just consider $p(x_t, m \mid z_t)$
  - Not factorable into $p(x_t \mid z_t)p(m \mid z_t)$. Why?
    - Observations depend on both m and x_t.
- Assume we have a motion and observation model:
  - $p(x_t \mid x_{t-1}, u_t)$ and $g(z_t \mid x_t, m)$.



http://www2.informatik.uni-freiburg.de/~stachnis/pdf/grisetti07tro.pdf

Applications

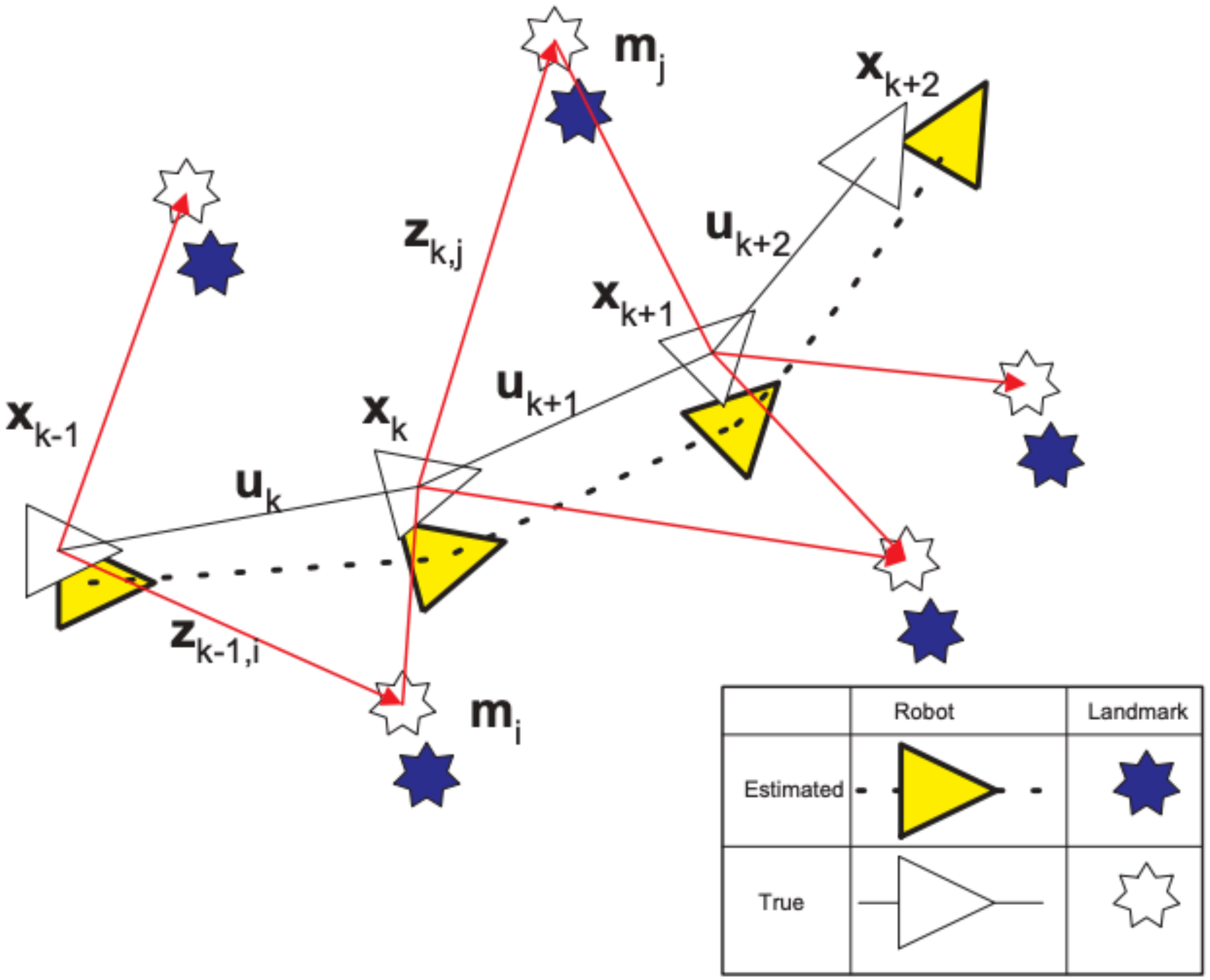


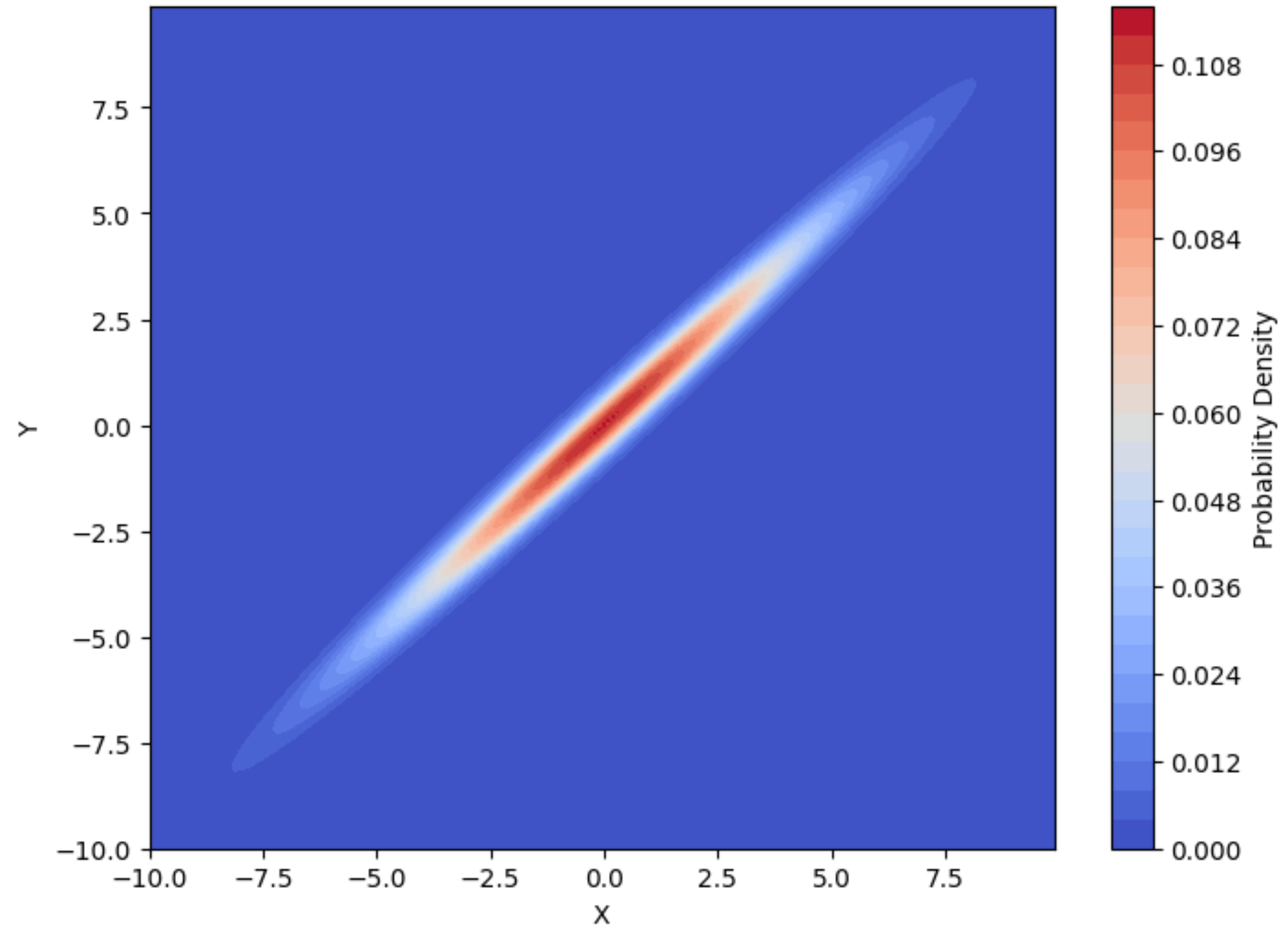


Location Representation

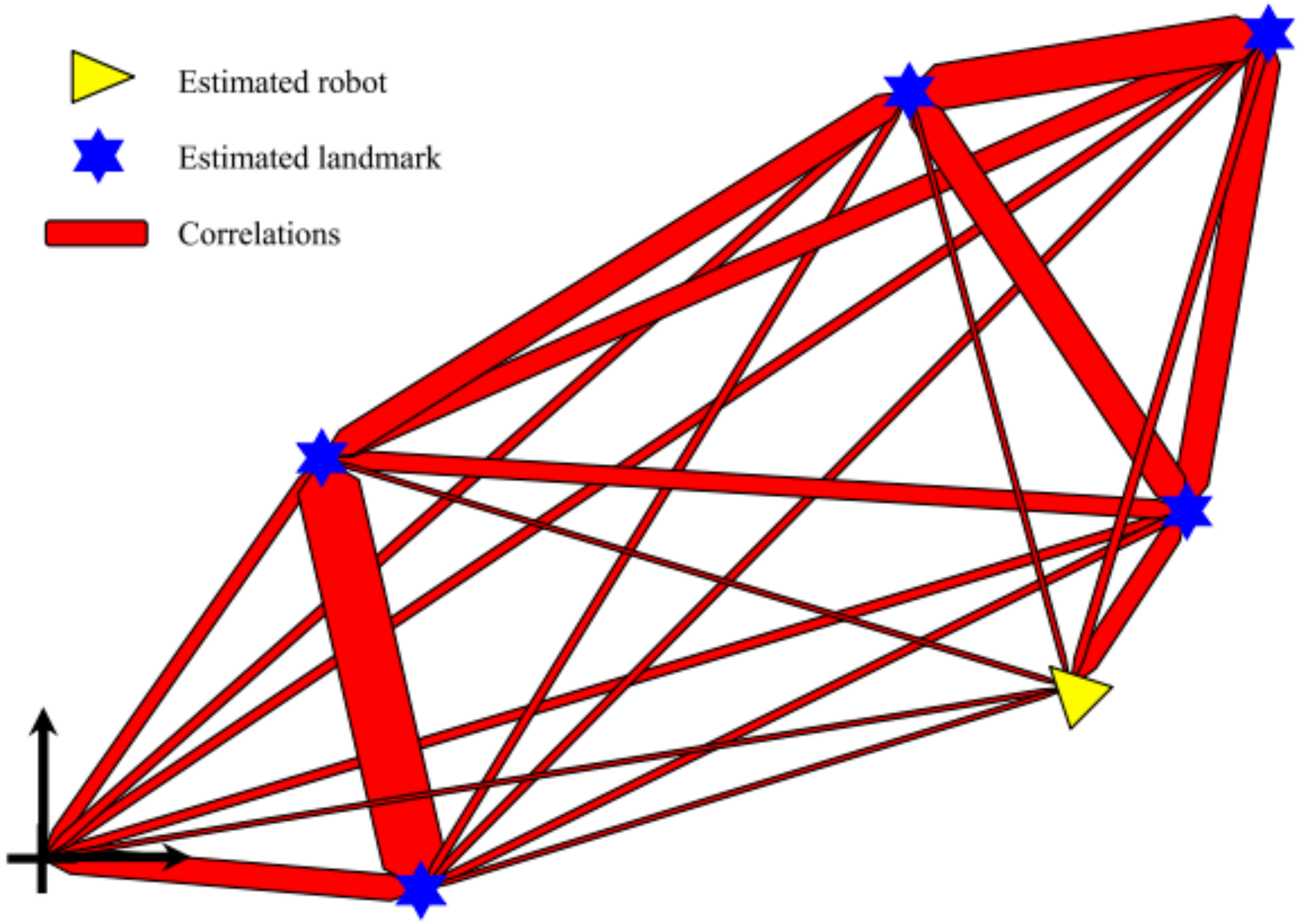
- What coordinate system should we use for the robot's pose (x_t)?
    - I.e., what did the reading refer to as "absolute location?"
    - One idea: Set the robot's initial pose to be the origin or other arbitrary point.
    - Or: We might know the initial pose of the robot in some other coordinate system (e.g., the robot knows that it begins at some world coordinates).
- Localization and mapping are then done in this coordinate system.

SLAM Structure





SLAM Structure



EKF SLAM with Landmarks

- Key idea: make landmarks part of the state and then run an extended Kalman filter.
- Map representation: a set of landmarks with unknown locations.
  - Let m_x^i, m_y^i be the coordinates of the ith landmark and $m = (m_x^1, m_y^1, \ldots, m_x^k, m_y^k)$ be the vector of all landmark coordinates.
- Define z_t^i as the observation of the ith landmark at time t.
- Assume $p(z_t^i \mid x_t, m_x^i, m_y^i) = \mathcal{N}(h(x_t, m_x^i, m_y^i), R)$.
- Initialize belief $\texttt{bel}(x_0, m) = \mathcal{N}([x_0, m]; \mu_0, \Sigma_0)$
- In practice, incrementally add landmarks as found.

$$\mu_0 = \begin{bmatrix} x \\ y \\ \theta \\ m_x^1 \\ m_y^1 \\ \cdots \\ m_x^k \\ m_y^k \end{bmatrix}$$

EKF-SLAM

$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

EKF SLAM with Landmarks

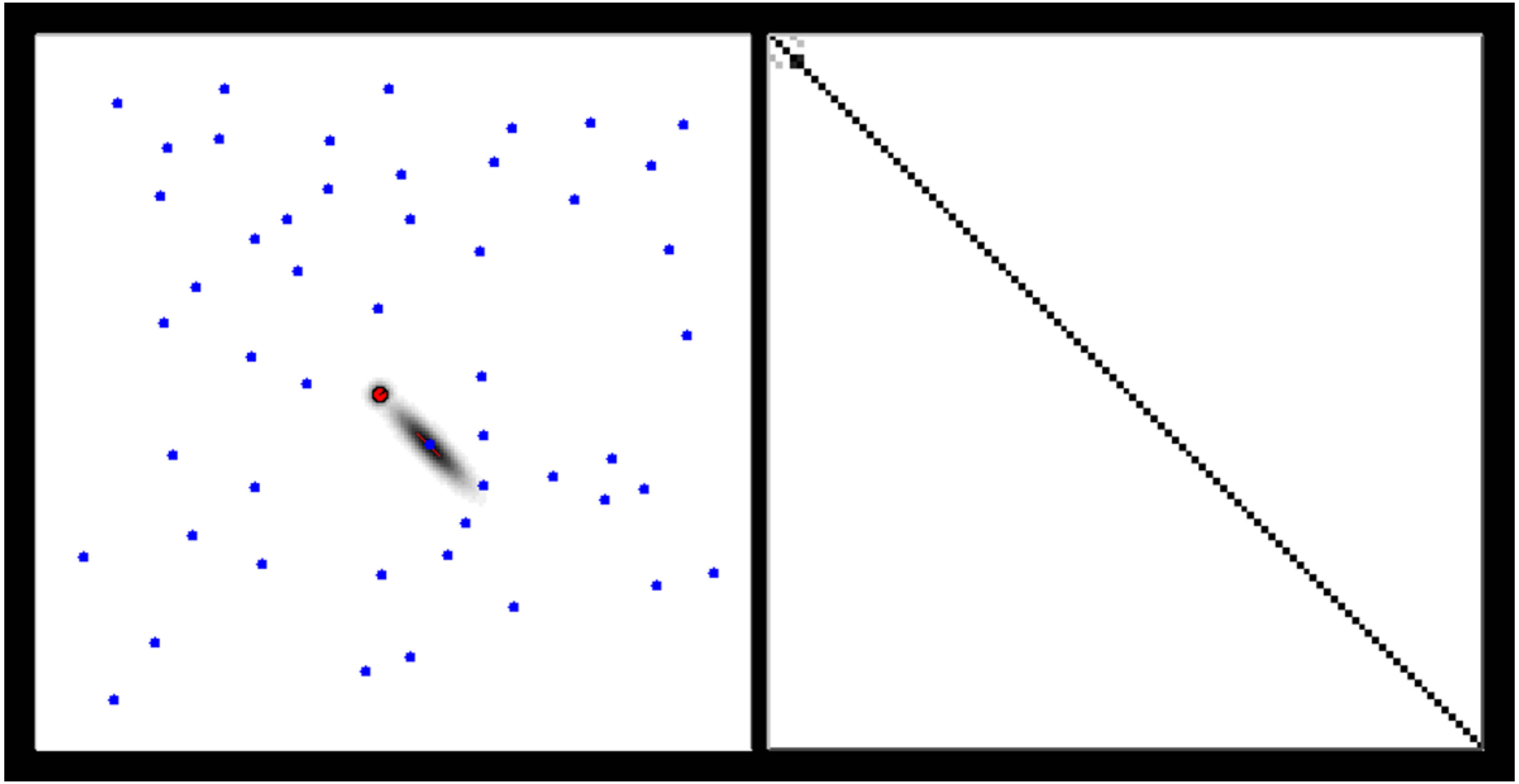
- Covariance matrix Σ_t captures correlation between landmarks.
  - Improves estimate landmark estimates in μ_t even for landmarks that weren't observed at time t.
- Prediction step: only changes μ_t for pose components; increases uncertainty for all components.
  - $\bar{\mu}_t \leftarrow f(\mu_{t-1}, u_t)$, $\overline{\Sigma}_t \leftarrow G_t^\top \Sigma G_t + Q$
- Update step: run for each landmark observation z_t^i:
  - $\bar{\mu}_t, \overline{\Sigma}_t \leftarrow$ update step with z_t^i.
  - Can decrease uncertainty for all landmarks.

EKF Update Step

$$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$$

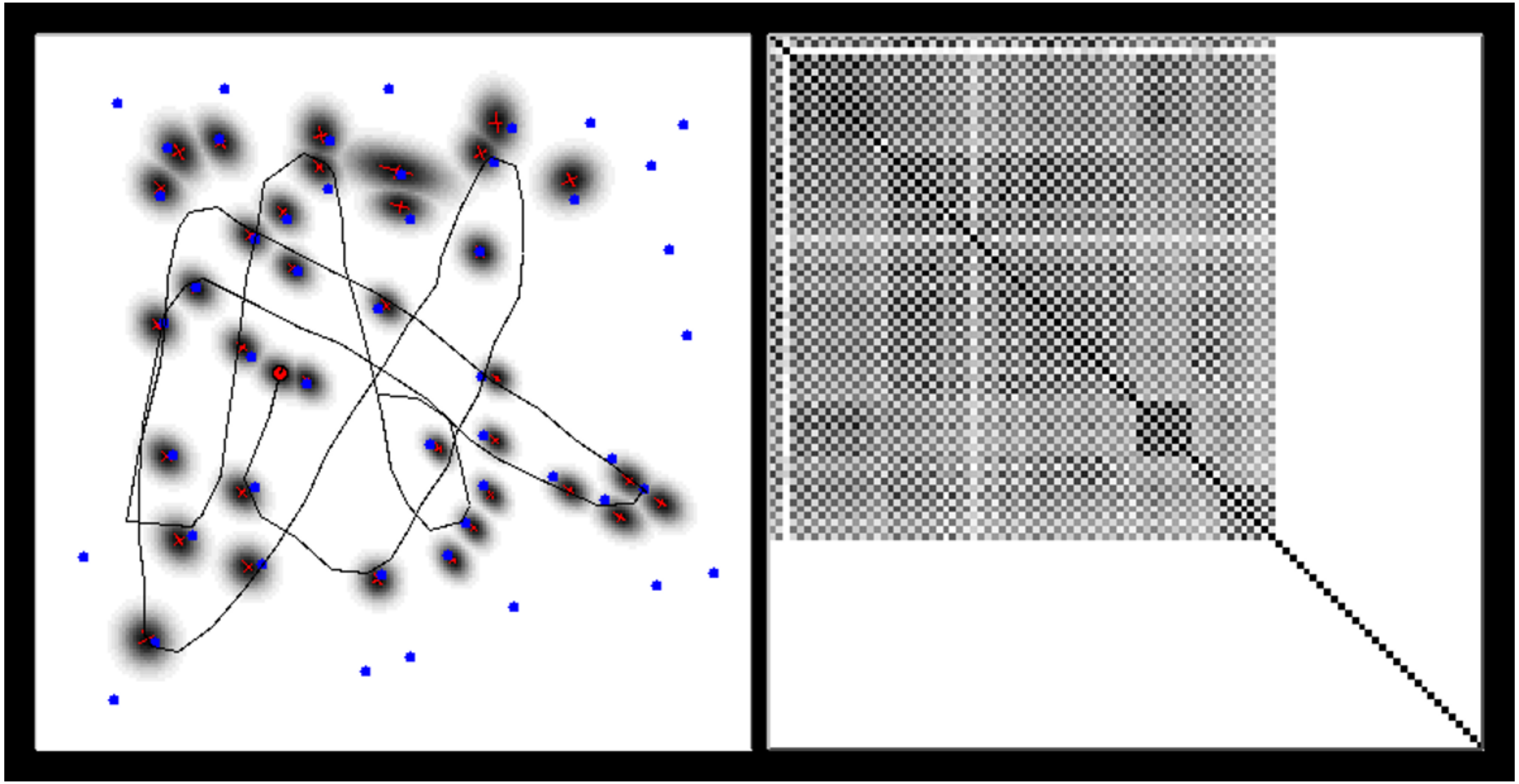
$$\Sigma_t = (I - K_t H_t)\overline{\Sigma}_t$$

EKF-SLAM



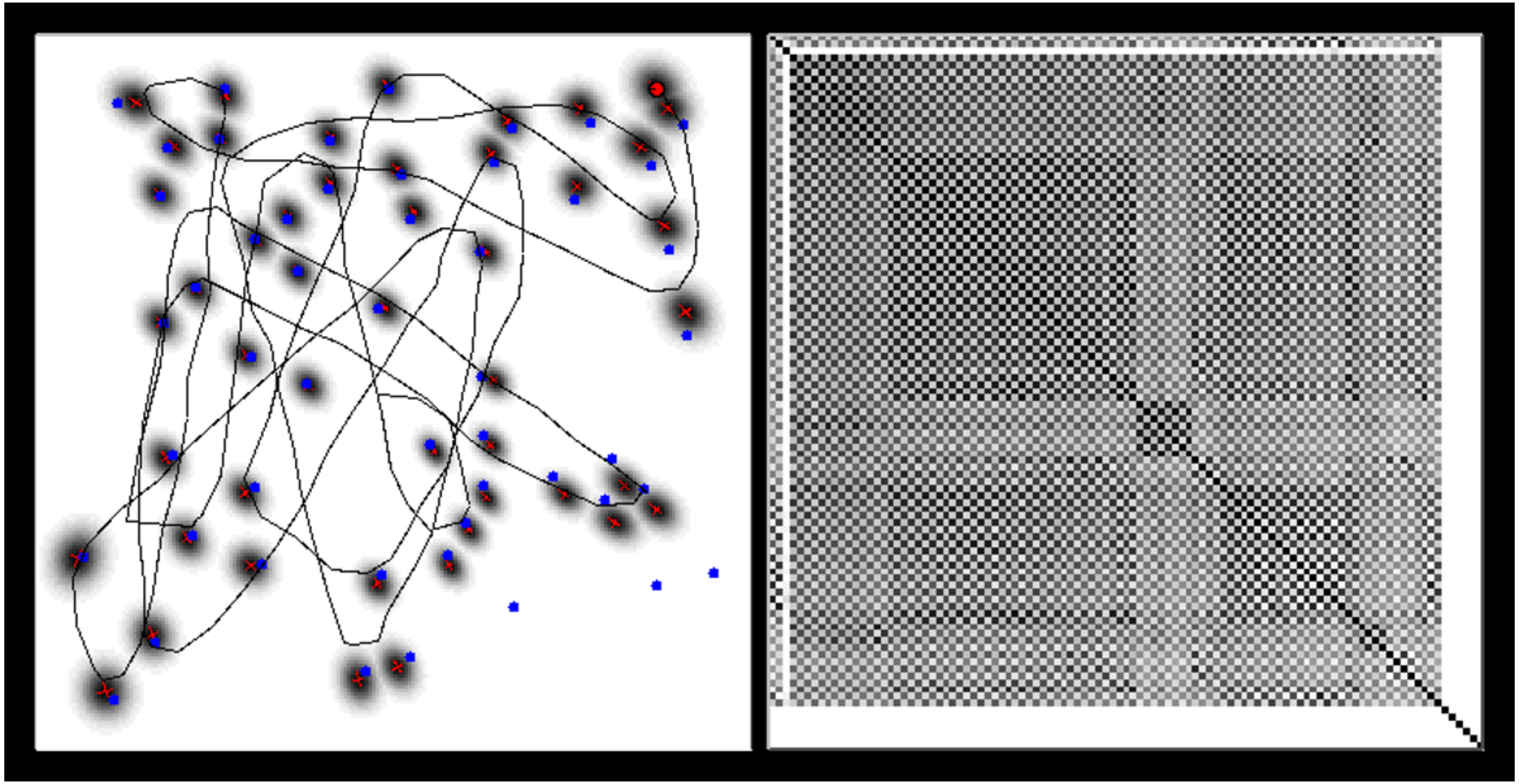
Map **Covariance Matrix**

EKF-SLAM



Map **Covariance Matrix**

EKF-SLAM

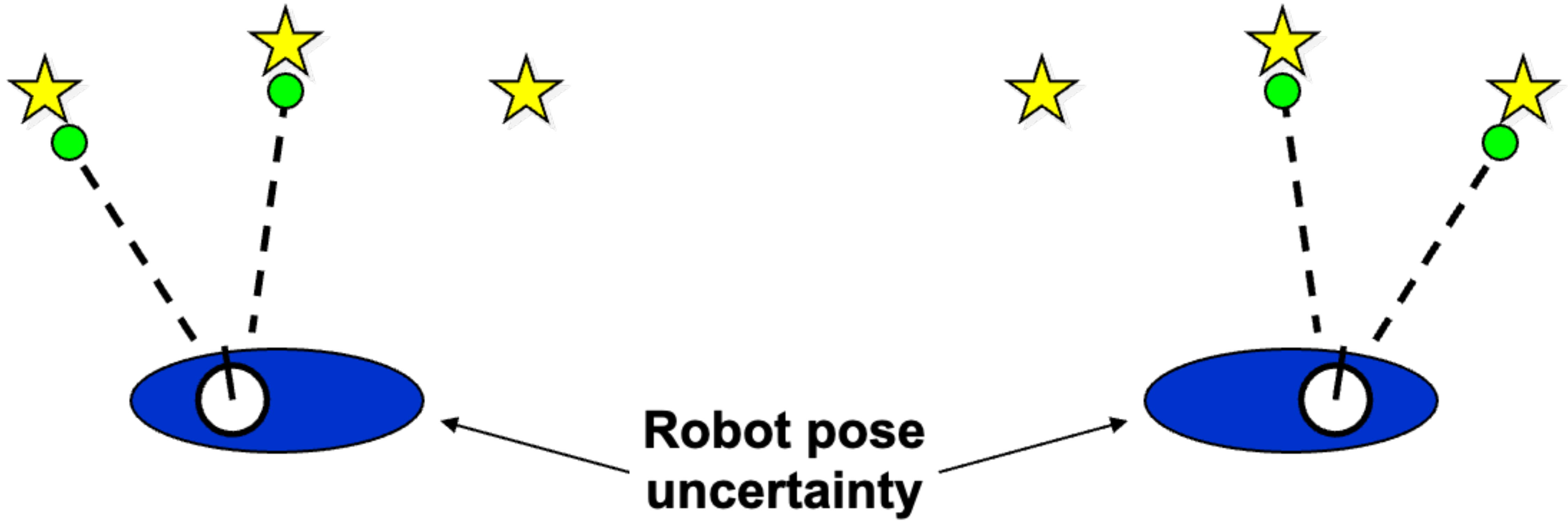


Map

Covariance Matrix

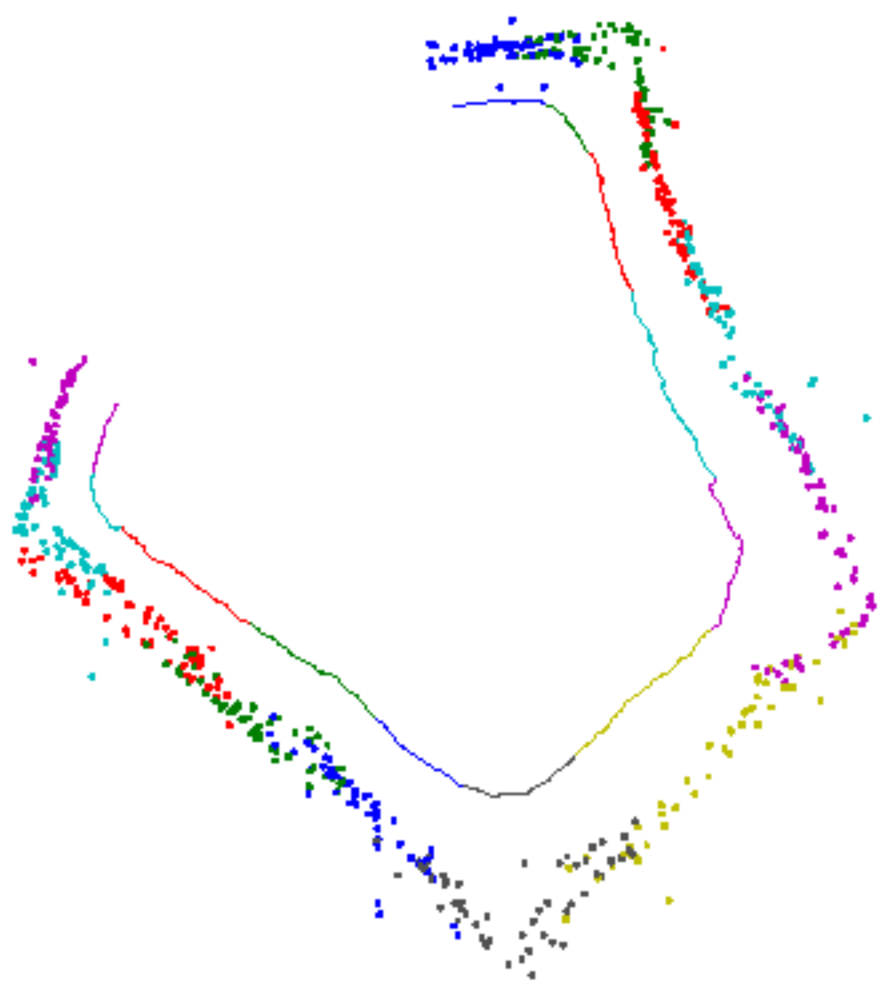
Data Association

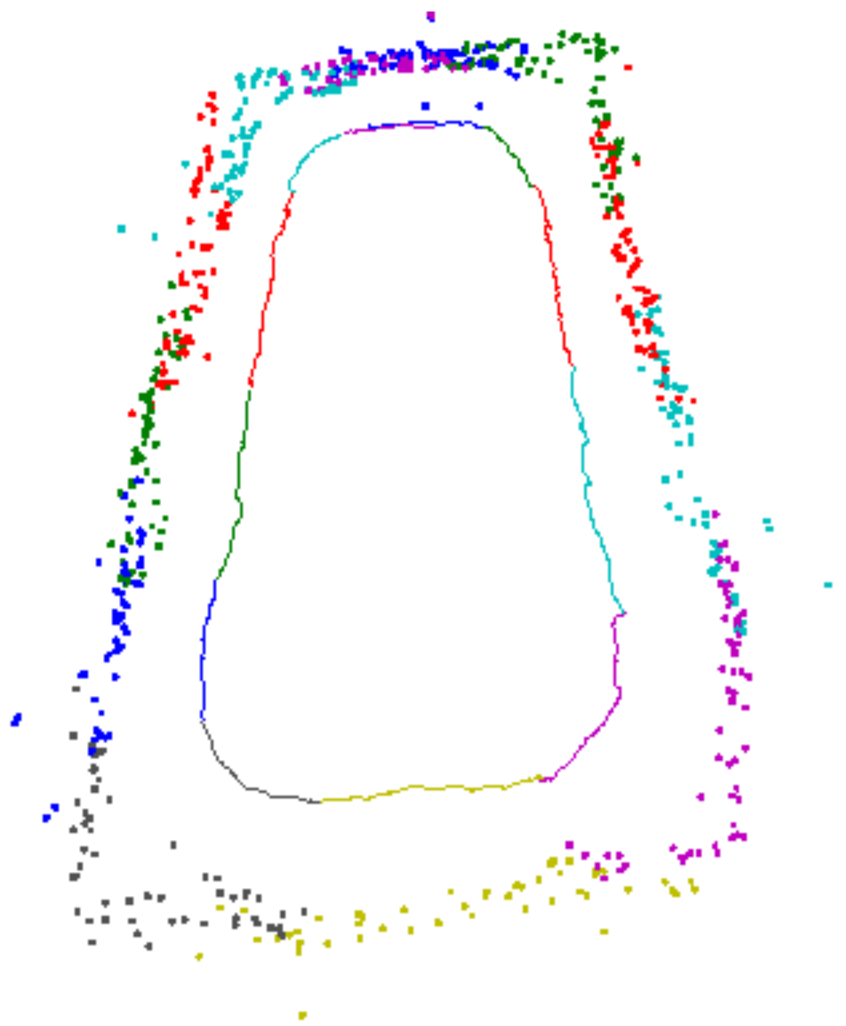
- How to determine which landmark z_t corresponds to?
  - Defined observations based on (noisy) polar coordinates relative to robot. Could be unclear which landmark an observation represents.
- Challenging cases:
  - What if the robot has discovered a new landmark?
  - What if two landmarks are close together?
- Solution:
  - Estimate maximum likelihood correspondence.
  - Choose spatially far apart landmarks for the map.



Loop Closure

- Detect when a previously visited location is being revisited.





Summary

- Introduced the SLAM problem.
- Discussed the correlation structure inherent in the SLAM problem.
- Introduced EKF-SLAM as one approach.

Action Items

- Finish programming assignment #2.
- 2nd SLAM reading for next week; send a reading response by 12 pm on Monday.