Autonomous Robotics

Particle Filters

Josiah Hanna
University of Wisconsin — Madison

Figure credit: Probabilistic Robotics and Drew Bagnell's course notes.

Announcements

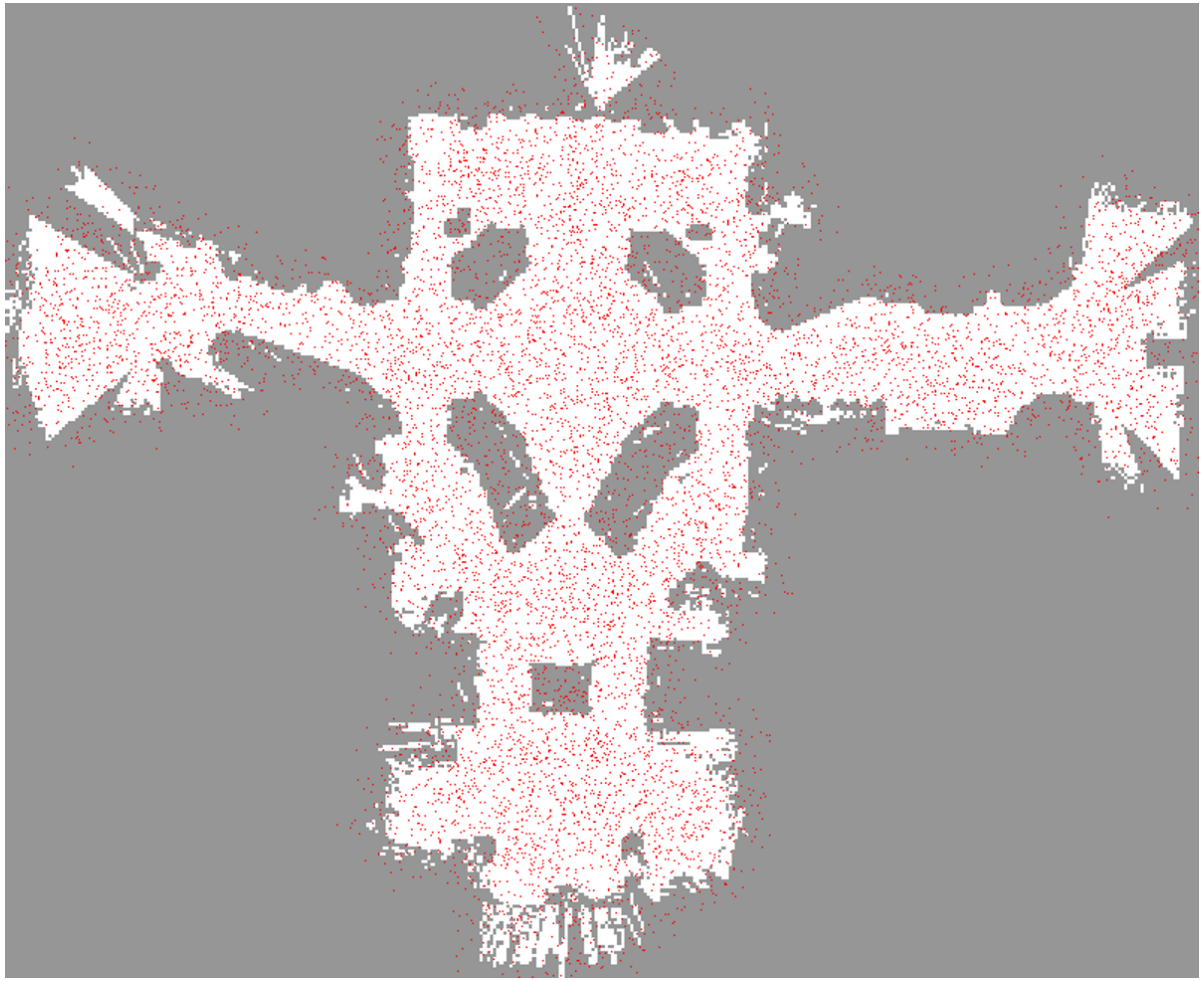
- Programming assignment 2 will be released today. Localization!
- Reading assignment for next week (localization) has been posted.
- Asynchronous lecture (no in-person class) on 2/19.

Learning Outcomes

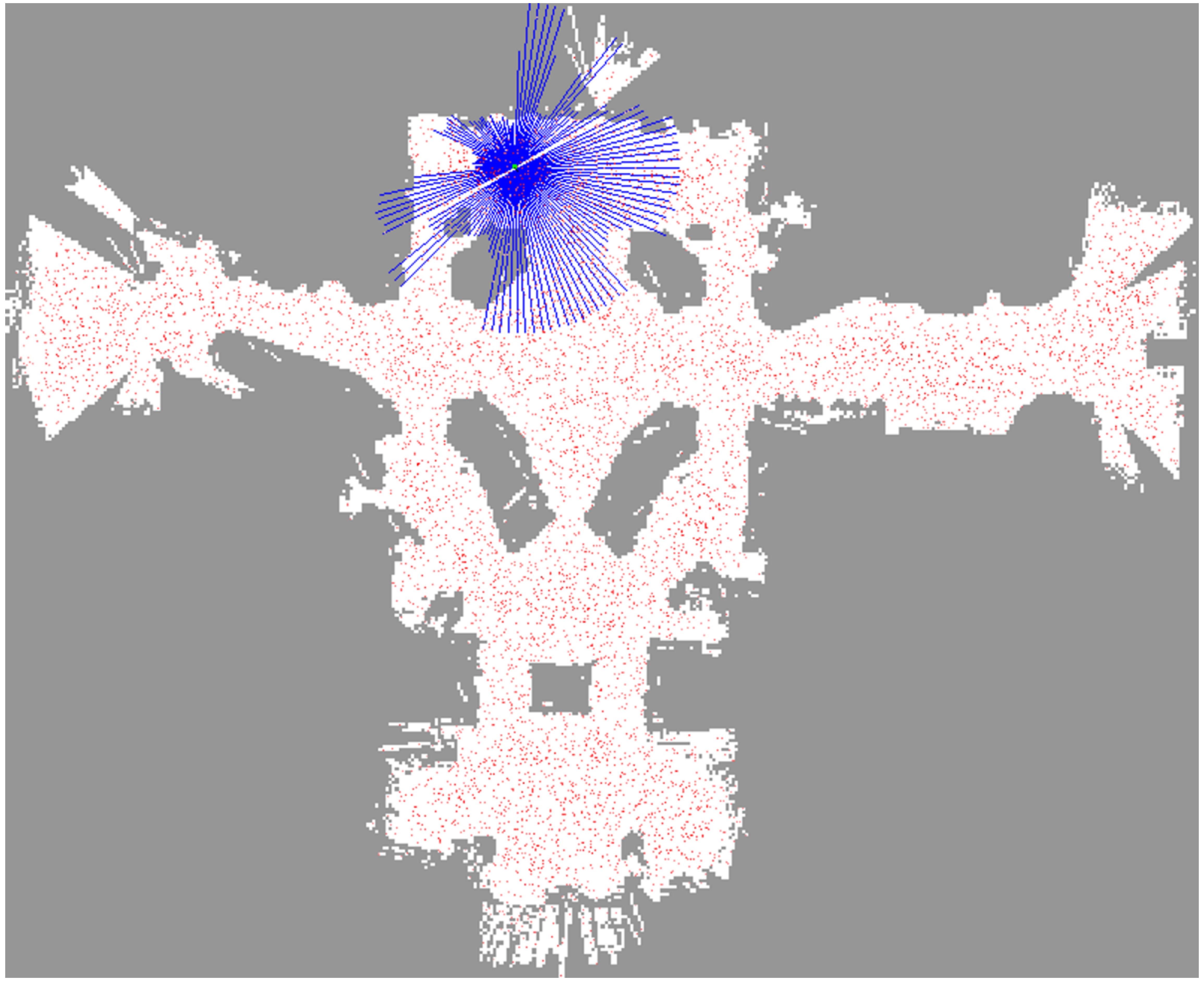
After today's lecture, you will:

- Understand particle approximations of belief distributions.
- Be able to compute a robot's state estimate using a set of weighted particles.
- Understand the weighting and re-sampling schemes used by particle-based methods.

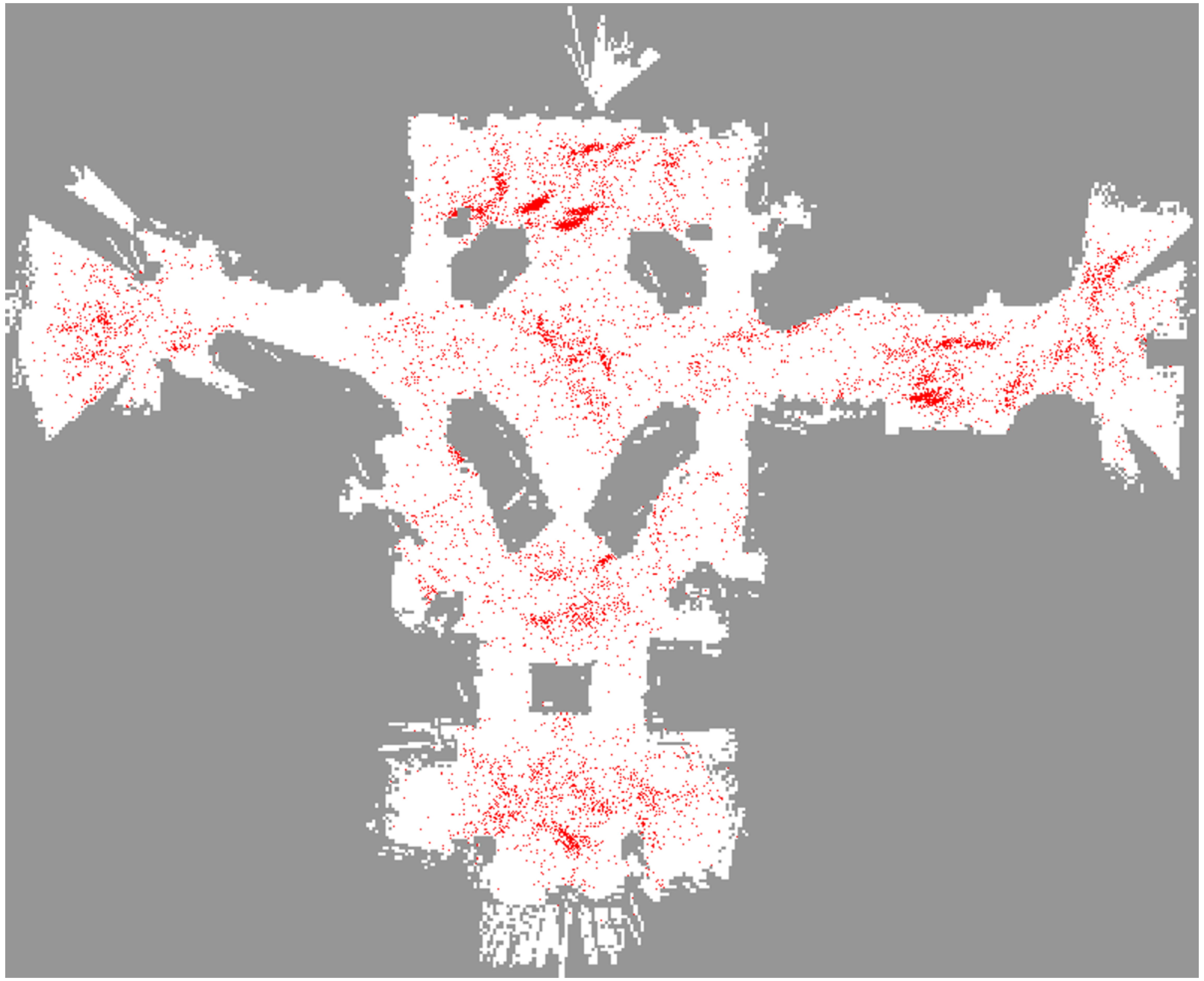
Particle Filter Applications



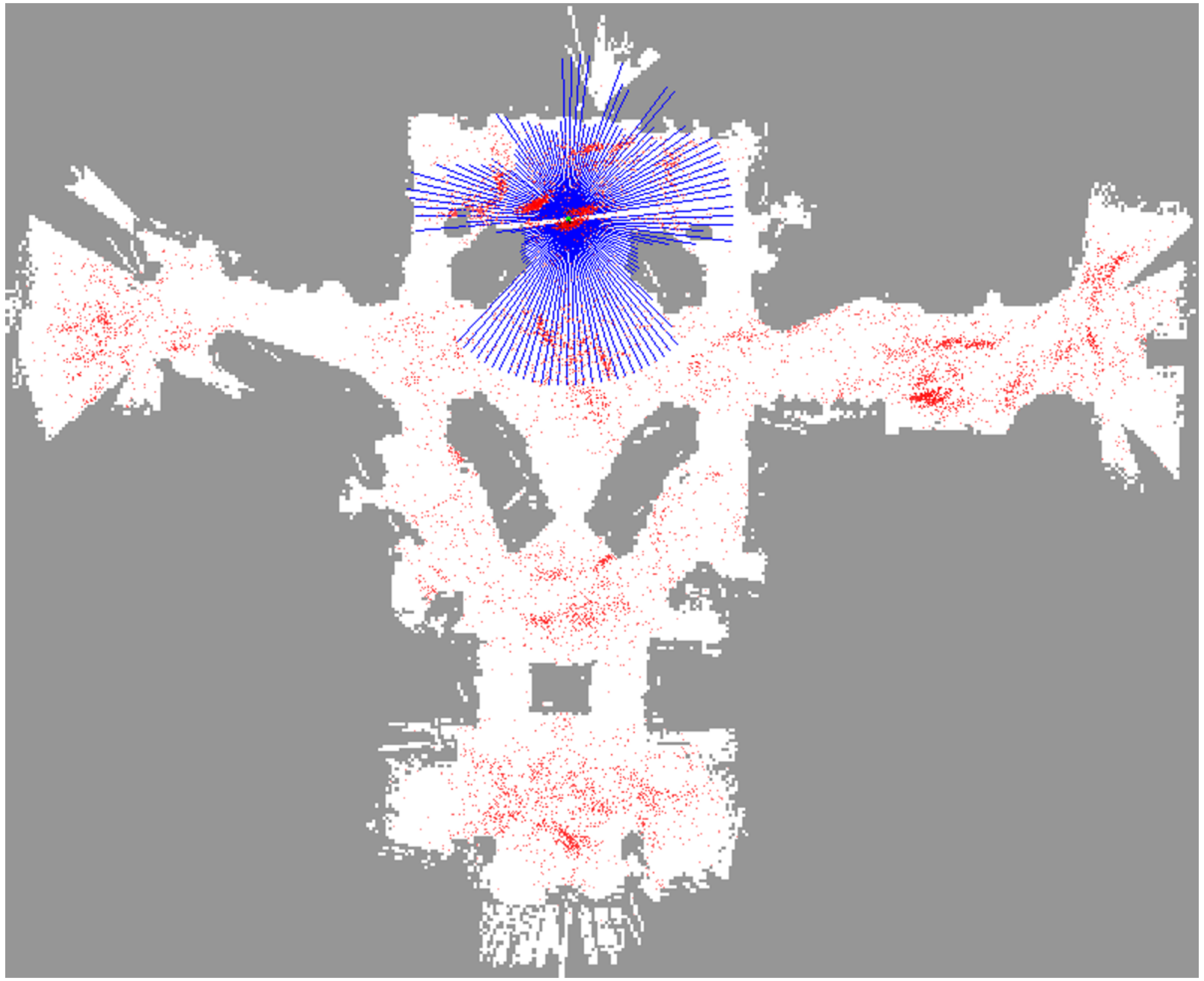
Particle Filter Applications



Particle Filter Applications



Particle Filter Applications



Particle Filter Applications

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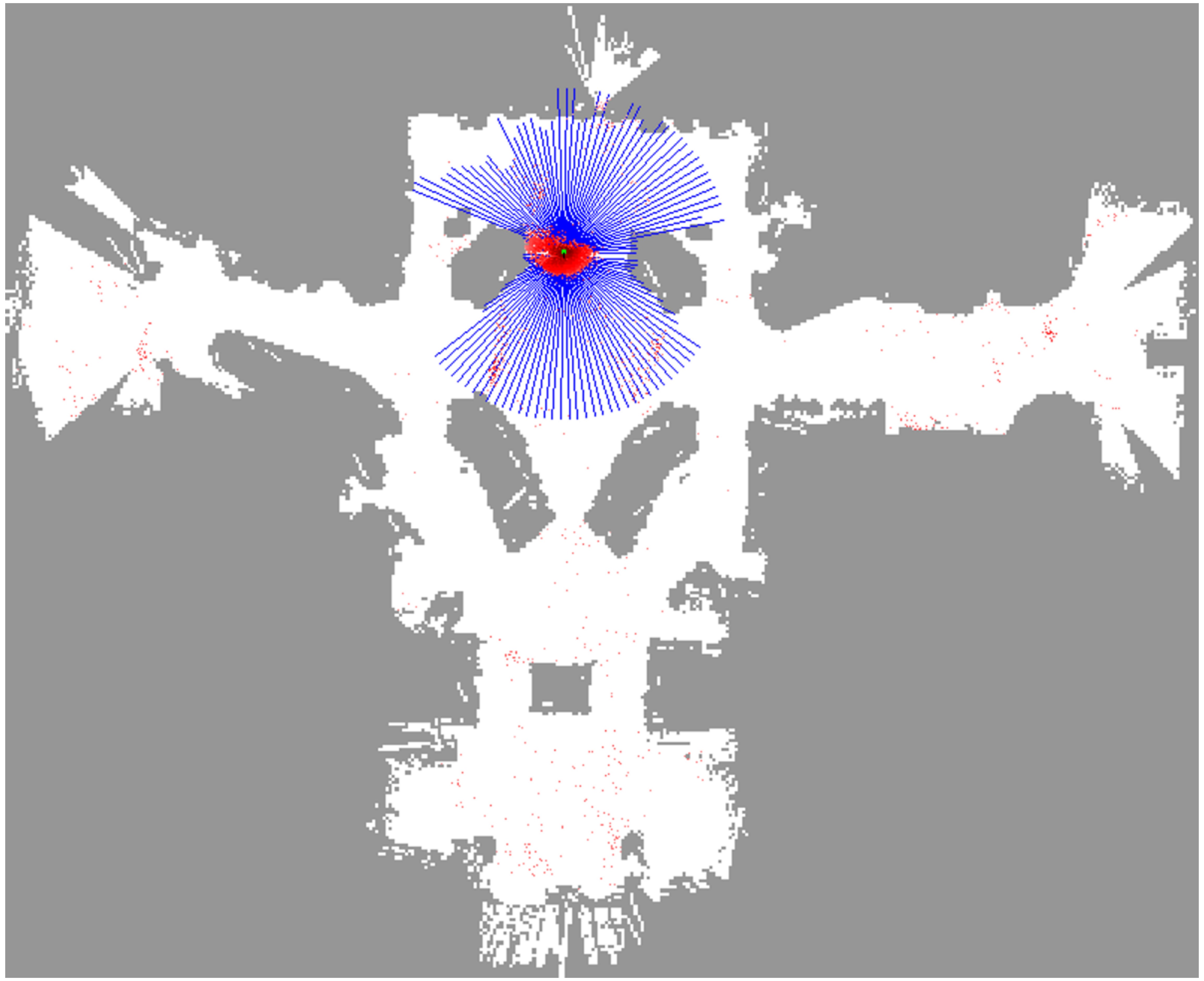
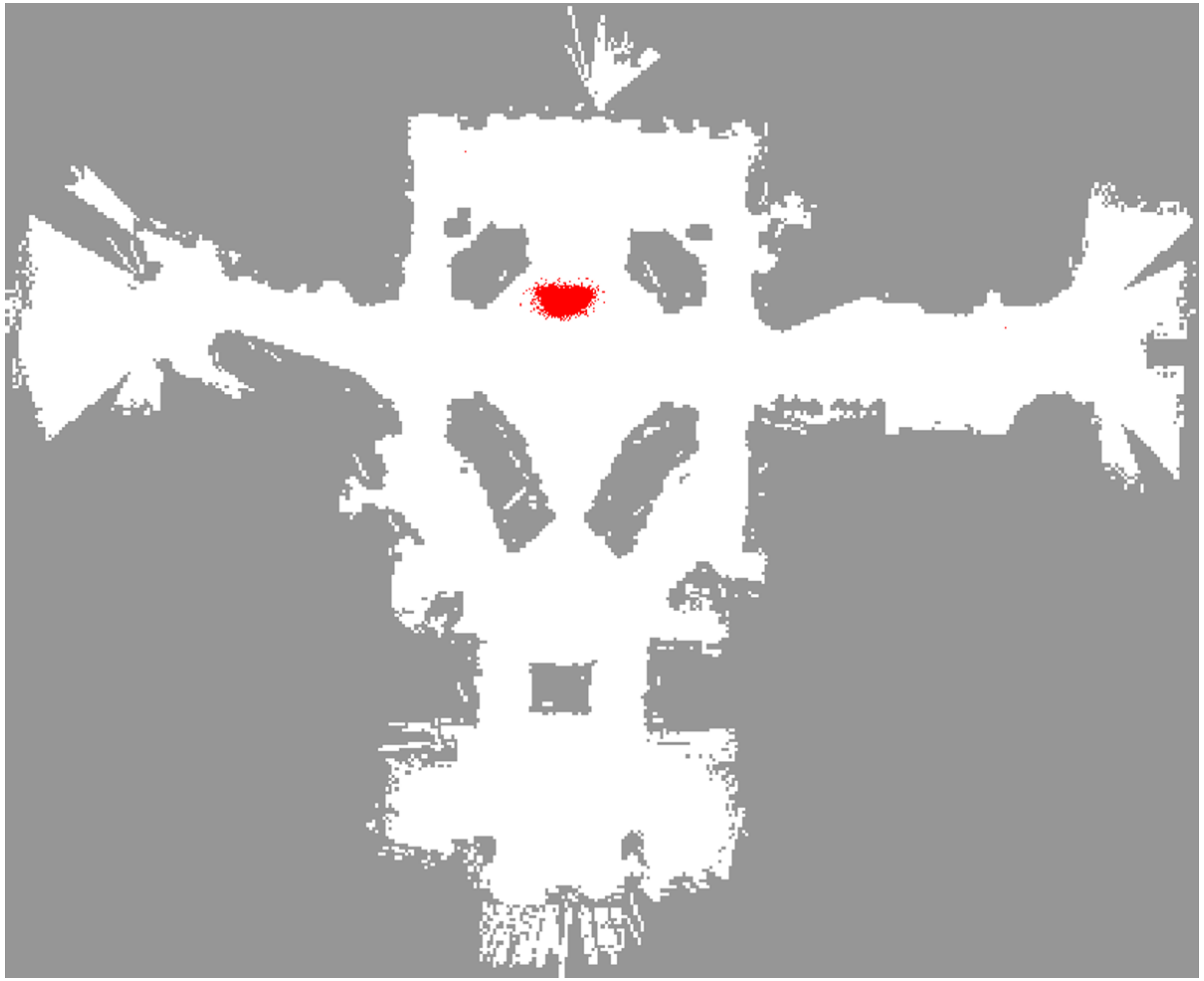


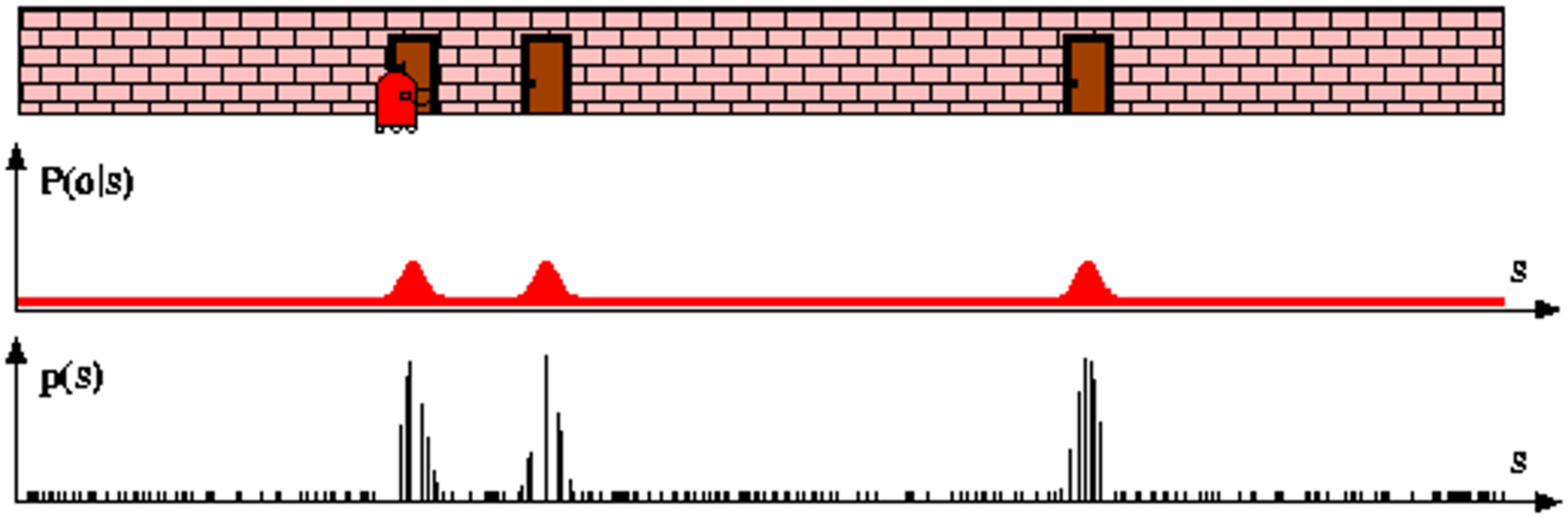
Particle Filter Applications



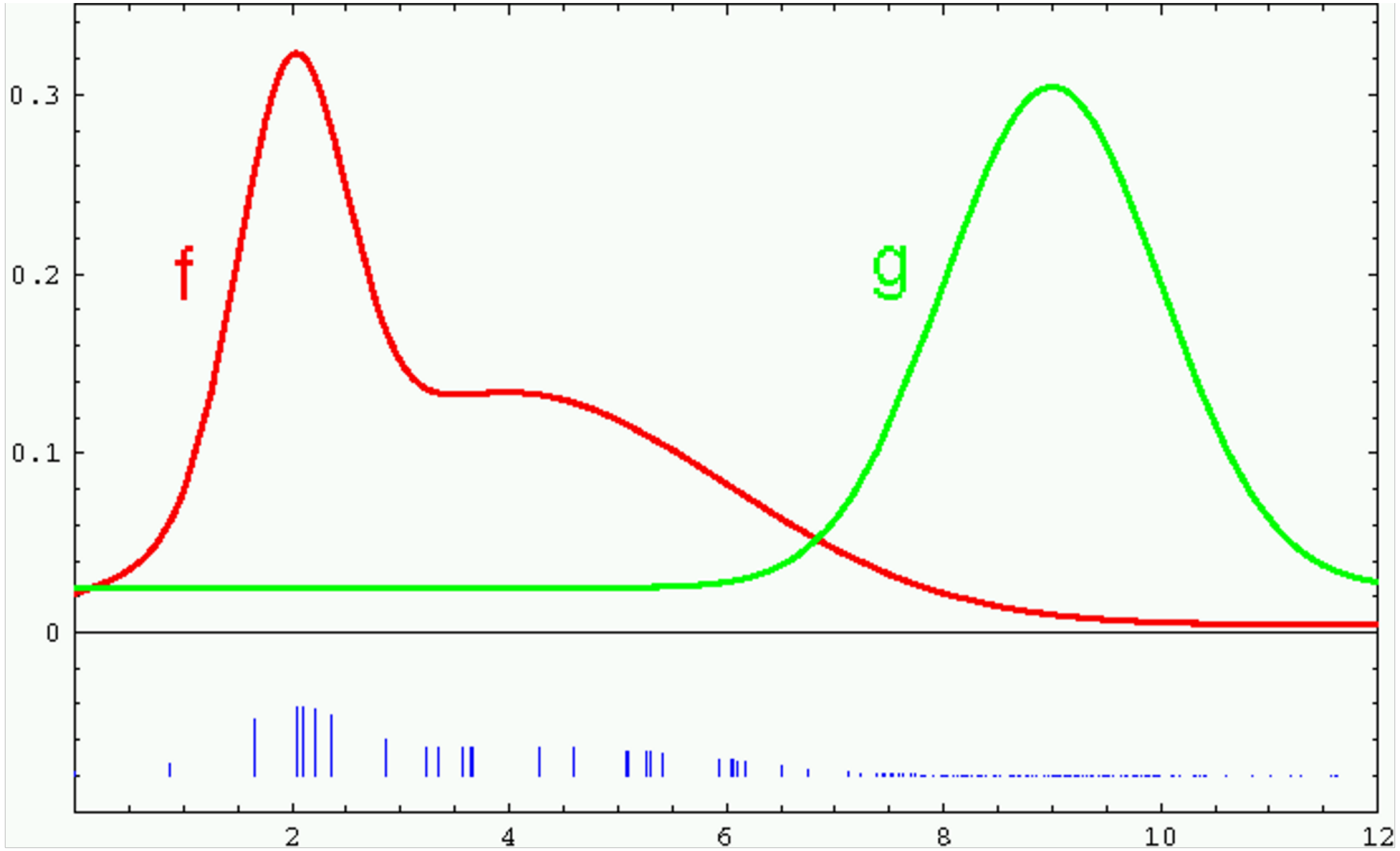
Particle-based Beliefs

- Difficult to exactly represent the robot's belief in continuous or large state spaces.
  - Bayes filter is usually intractable.
  - Kalman filter and EKF restrict to Gaussian beliefs.
- New idea: the robot represents its belief with a set of N particles.
  - Each particle represents a possible robot state.
  - Particles are explicitly or implicitly weighted based on the likelihood the robot is in a particular state.
  - Known as a **non-parametric** belief representation.

Illustration of Particle-Based Beliefs



Underlying Idea: Importance Sampling



Weight samples: $w = f/g$

Draw samples from g; re-weight according to f/g.
Weighted samples approximate f.

Normalized Importance Sampling

- Belief is represented by a set of particles, $\{(x_t^i, w_i)\}$.
- Robot takes action u_t and then observes z_t.
- For each particle:
  - $x_{t+1}^i \sim p(\cdot \mid x_t^i, u_t)$
  - $w_i \leftarrow w_i * p(z_t \mid x_{t+1}^i)$
- Normalize weights so that $\sum_{i=1}^{N} w_i = 1$.

Intuition: Each particle represents a path through the state space and weights represent the plausibility of the path.

Problem: Most paths become unlikely very quickly.

Effective sample size:

$$\frac{1}{\sum_{i=1}^{N} (w_i)^2}$$

Computing the Mean State

- In state estimation, you often want summary statistics:
  - What state is the robot expected to be in?
  - How spread out is the robot's belief.
- Expected value of robot's state is a weighted average:

$$\mu_t = \sum_{i=1}^{N} w_i x_t^i \qquad \texttt{bel}(x_t) = \sum_{i=1}^{N} w_i \cdot \mathbf{1}\{x_t^i = x_t\}$$

Particle Filters

- Belief is represented by a set of particles, $\{(x_t^i, w_i)\}$.
- Robot takes action u_t and then observes z_t. Set $w_i \leftarrow \frac{1}{N}$.
- For each particle:
    - $x_{t+1}^i \sim p(\cdot \mid x_t^i, u_t)$
    - $w_i \leftarrow w_i * p(z_t \mid x_{t+1}^i)$
- Normalize weights so that $\sum_{i=1}^{N} w_i = 1$.
- Sample N new particles (with replacement) to form a new particle set.

Intuition: Survival of the fittest. More likely particles are more likely to be resampled (possibly multiple times).

Particle Filters

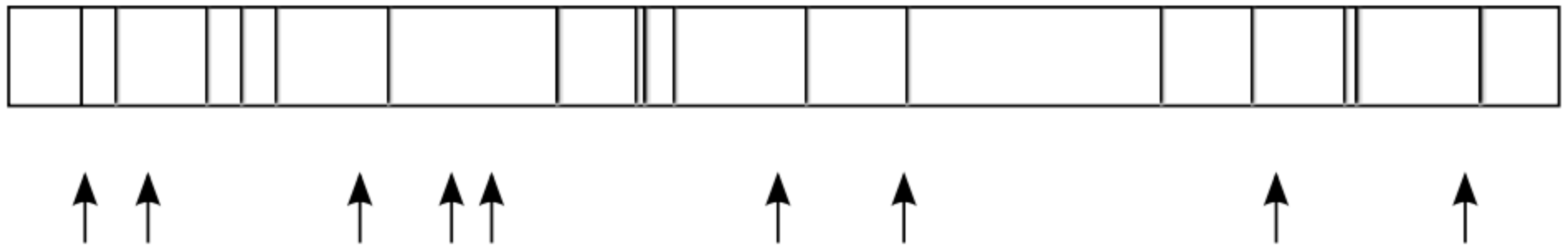


Figure 1: Resampling

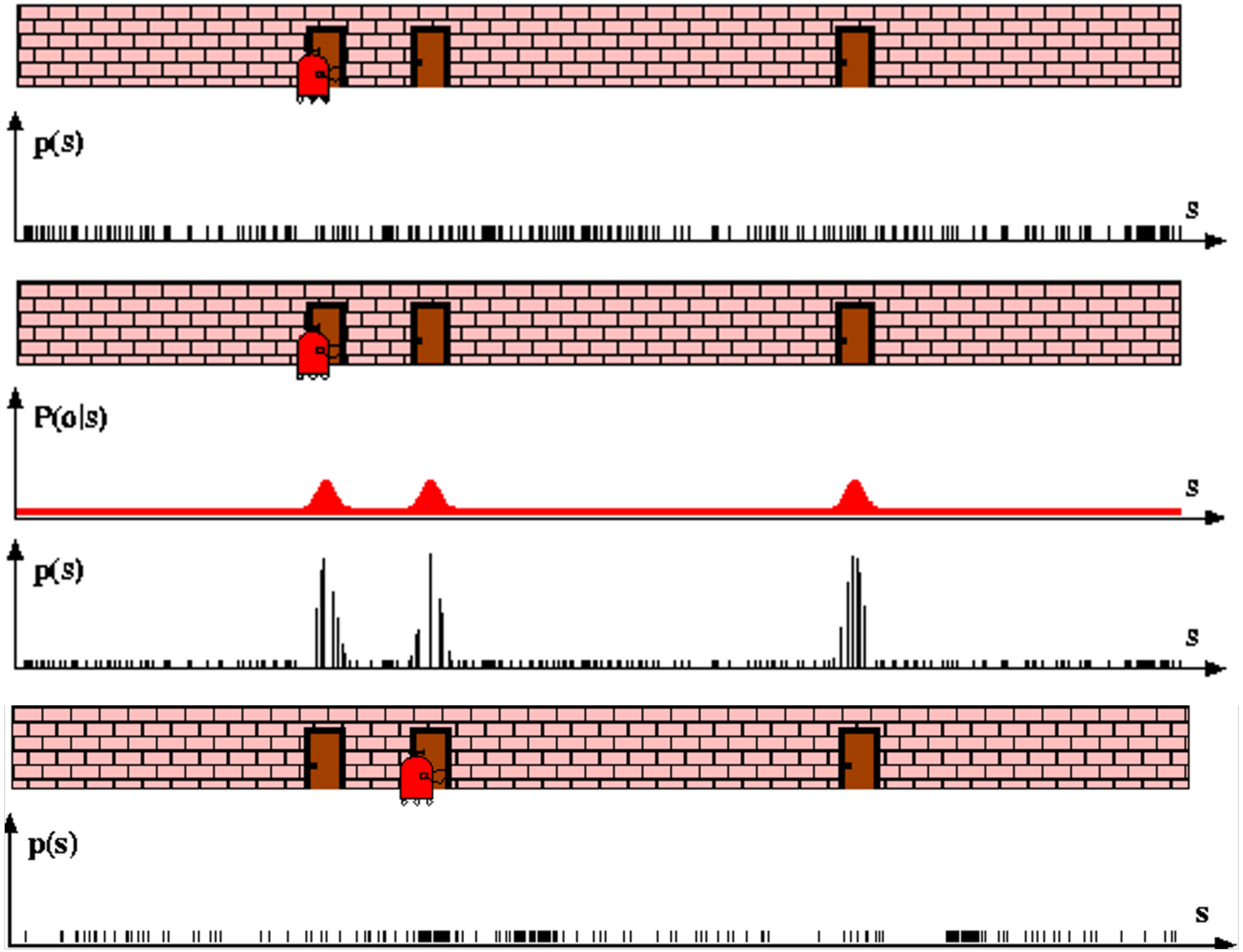
- Survival of the fittest particles.

- Randomly select particles according to their weights.

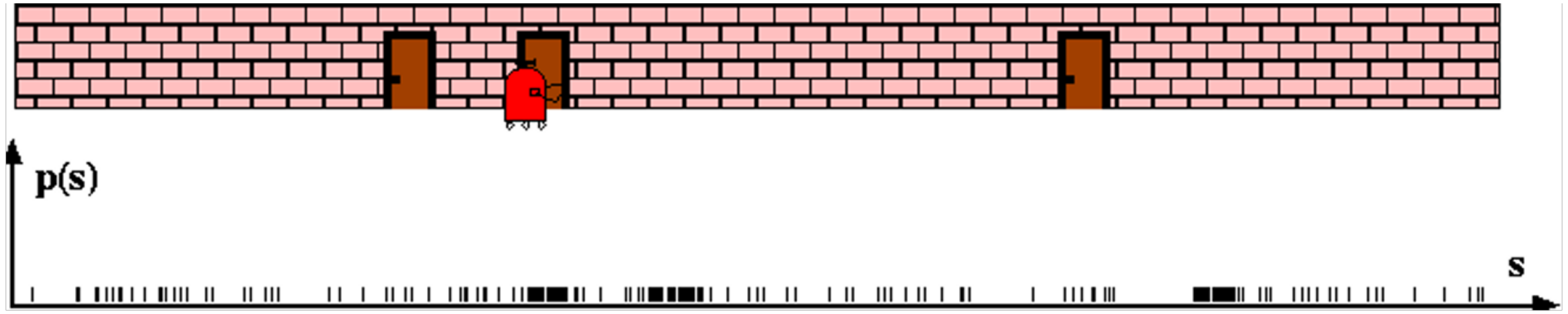
- Reset weights to $\frac{1}{N}$ after each iteration. Why?

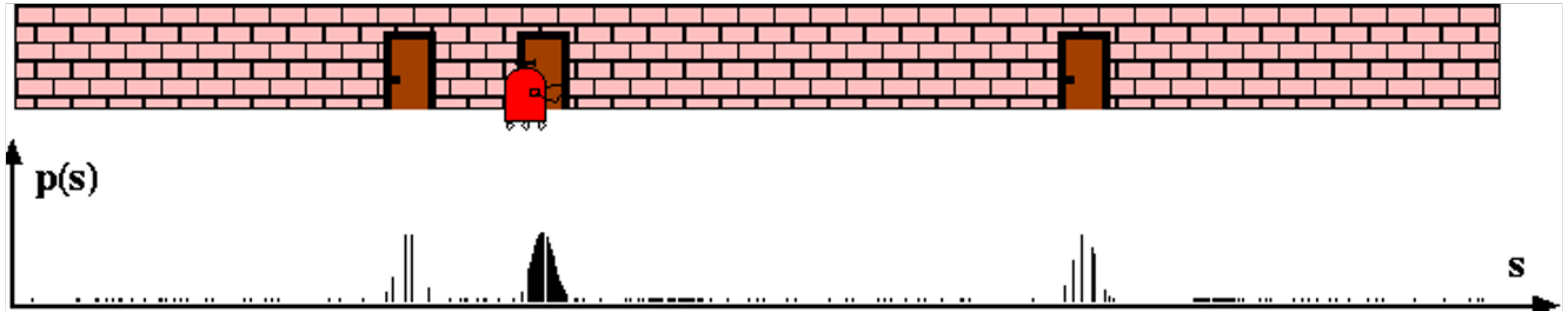
$$\mu_t = \sum_{i=1}^{N} w_i x_t^i$$

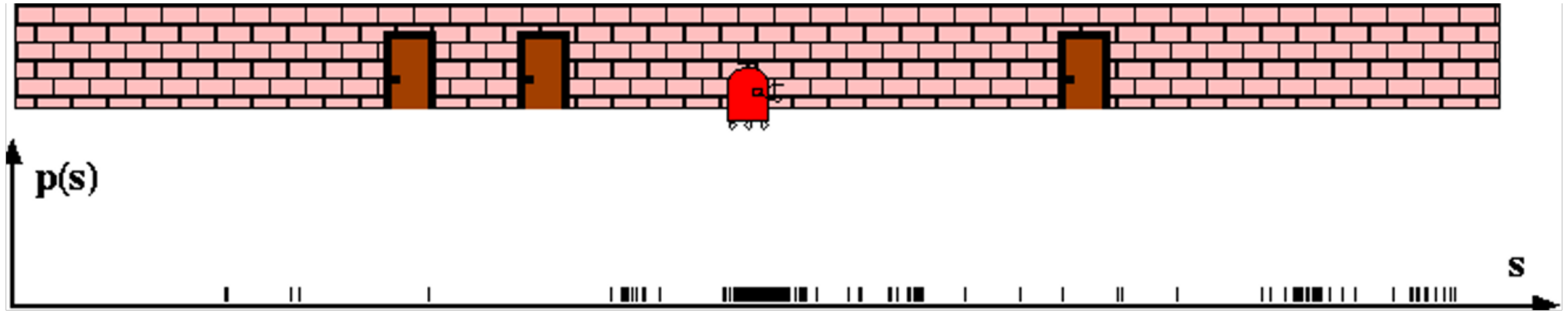
Particle Filter Illustration



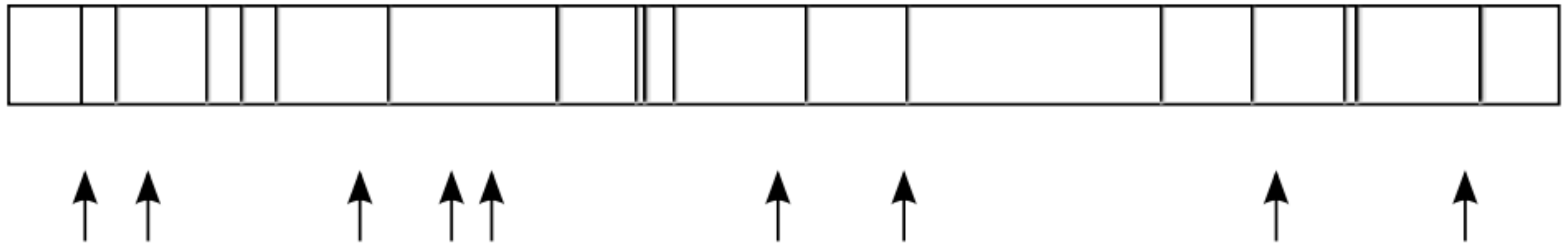
Particle Filter Illustration







Low Variance Resampler



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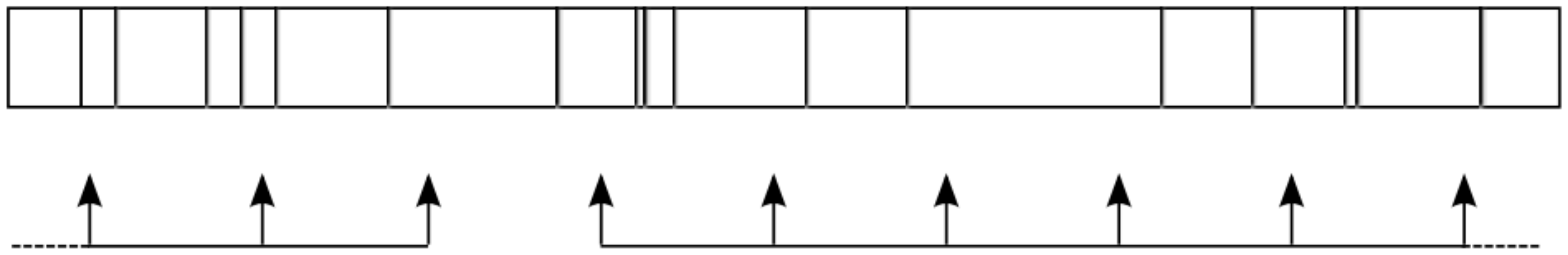
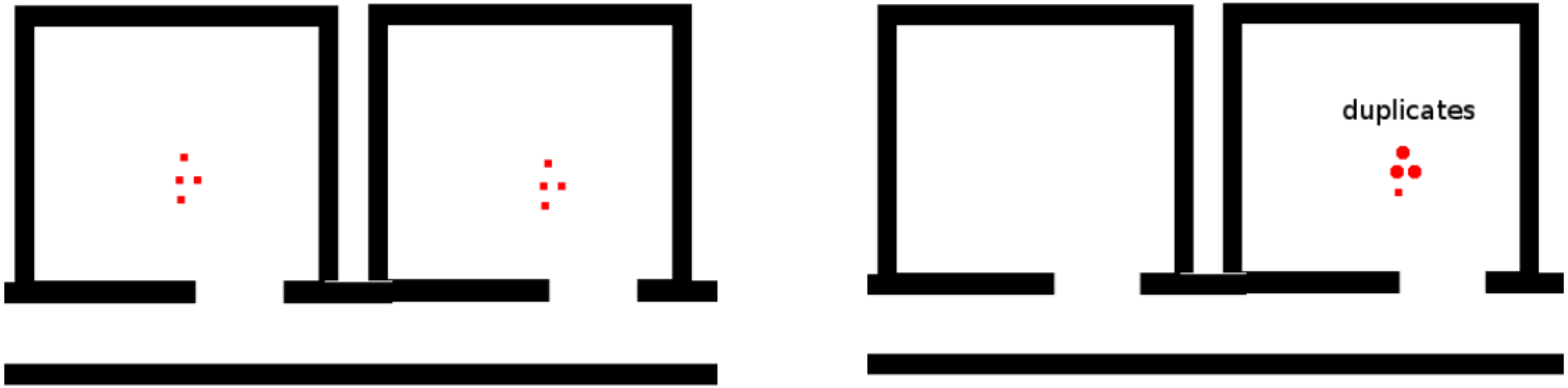


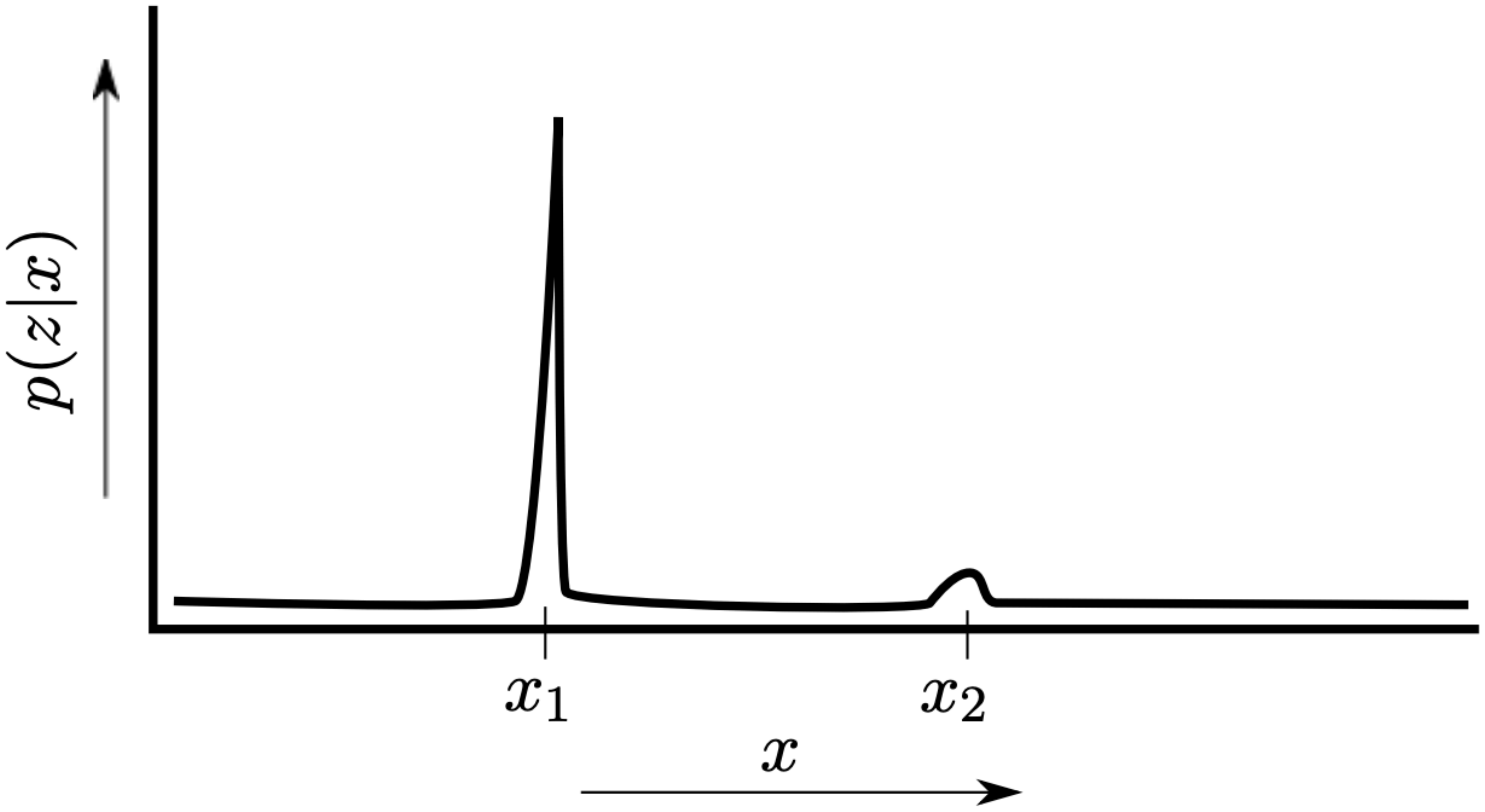
Advantages / Disadvantages

- Particle filters:
  - Can be used for continuous state spaces.
  - Can approximate any belief distribution (compare to Kalman/EKF).
  - Scale with computation
- But…
  - Only approximate belief.
  - Limited for high-dimensional state-spaces.

Loss of Diversity



Good observation models are bad?



Particle Filter Demo

- https://amrl.cs.utexas.edu/interactive-particle-filters/

Credit: Joydeep Biswas

Summary

- Saw examples of particle-based belief representations.
- Discussed differences between NIS and particle filters.
- Discussed pitfalls and remedies for particle filters.

Action Items

- Work on programming assignment #2.
- Localization reading for next week; send a reading response by 12 pm on Monday.