1 Furst Sax Sipser

In this lecture we will describe the Furst Sax Sipser Theorem which states that there is an oracle A s.t. $(\Sigma_d^P)^A \neq PSPACE^A$. Since we already know that $(\Sigma_d^P)^A \subset PSPACE^A$, what we need to show is that there is a language that is in recognizable by a $PSPACE^A$ machine that is not recognizable by a $(\Sigma_d^P)^A$ machine.
We will be considering an enumeration of all p($\cdot$) time, d alternating oracle turning machines $\{M_1, M_2, \cdots\}$. The particular language we will consider will be defined by $L^A = \{1^n | A^{=n} is odd\}$ where each element of A is written as a binary number. Obviously this language will be recognizable by a $PSPACE^A$ machine.
What we will now need to do is show that there is an oracle set A such that L^A can't be recognized by a $(\Sigma_d^P)^A$ machine.
Our stratagy will be to consider each d alternating turing machine in order and incrementally remove each from consdiration.
Also we will look at the choices for what is in A as random on the sequence $\epsilon, 0, 1, 00, 01, 10, 11, \cdots$.
From this we can look at each choice of A as a binary decimal such as $0.b_0b_1b_2b_3b_4\cdots$ where $b_0 = 1$ if $\epsilon \in A$, $b_1 = 1$ if $0 \in A$ $b_2 = 1$ if $1 \in A$ and so on.
For example, if A=$\{\epsilon, 1, 10, \}$, then A can be viewed as 0.101001. Thus this will give use a mapping from all the possible sets A to the interval [0,1]. More formally $\{A | A \subset \{0,1\}^*\} \leftrightarrow [0,1]$