CS540 Introduction to Artificial Intelligence

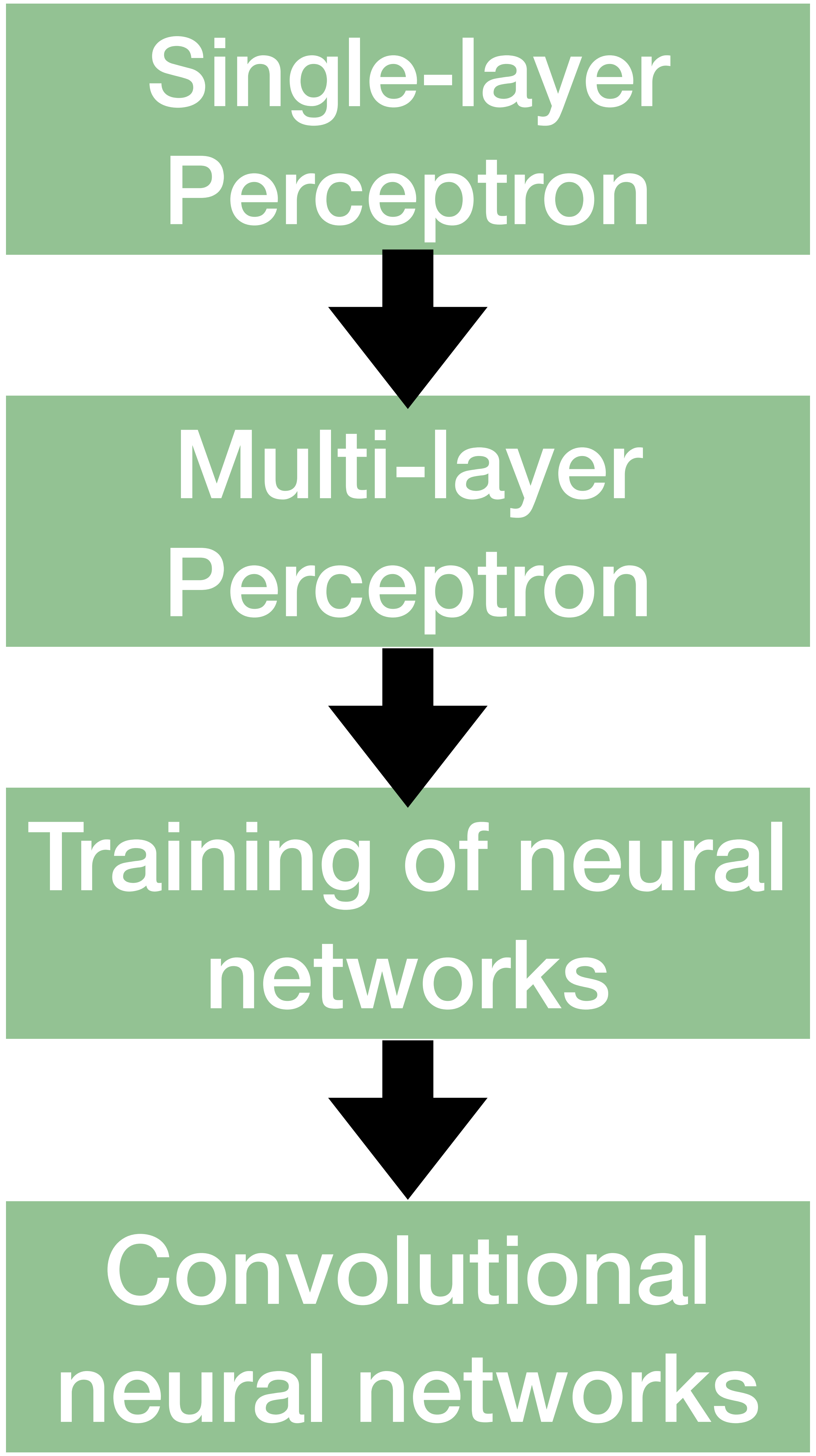
Neural Networks: Review

University of Wisconsin-Madison

Spring 2022

How to classify

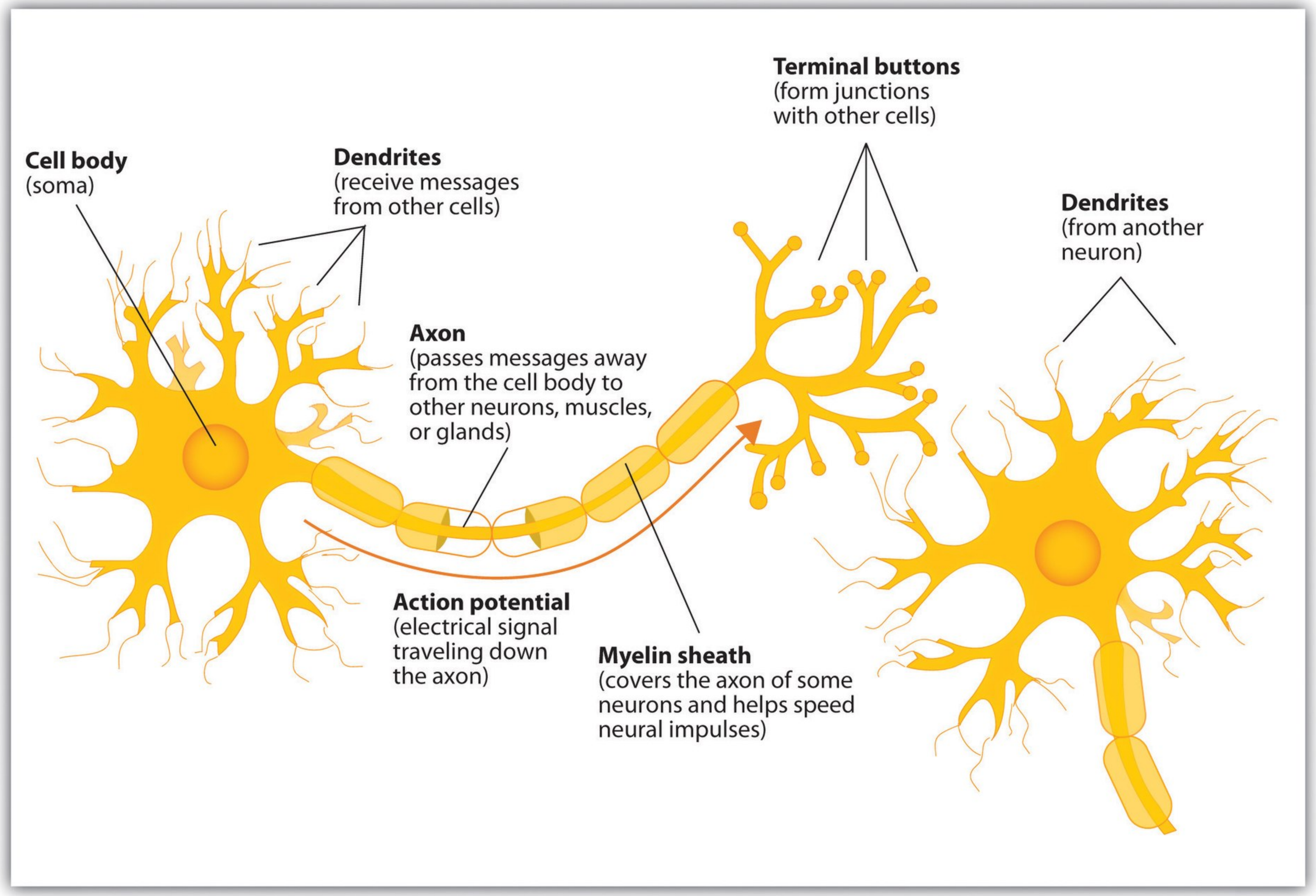
Cats vs. dogs?



Inspiration from neuroscience

- Inspirations from human brains
- Networks of simple and homogenous units (a.k.a **neuron**)

(wikipedia)



Perceptron

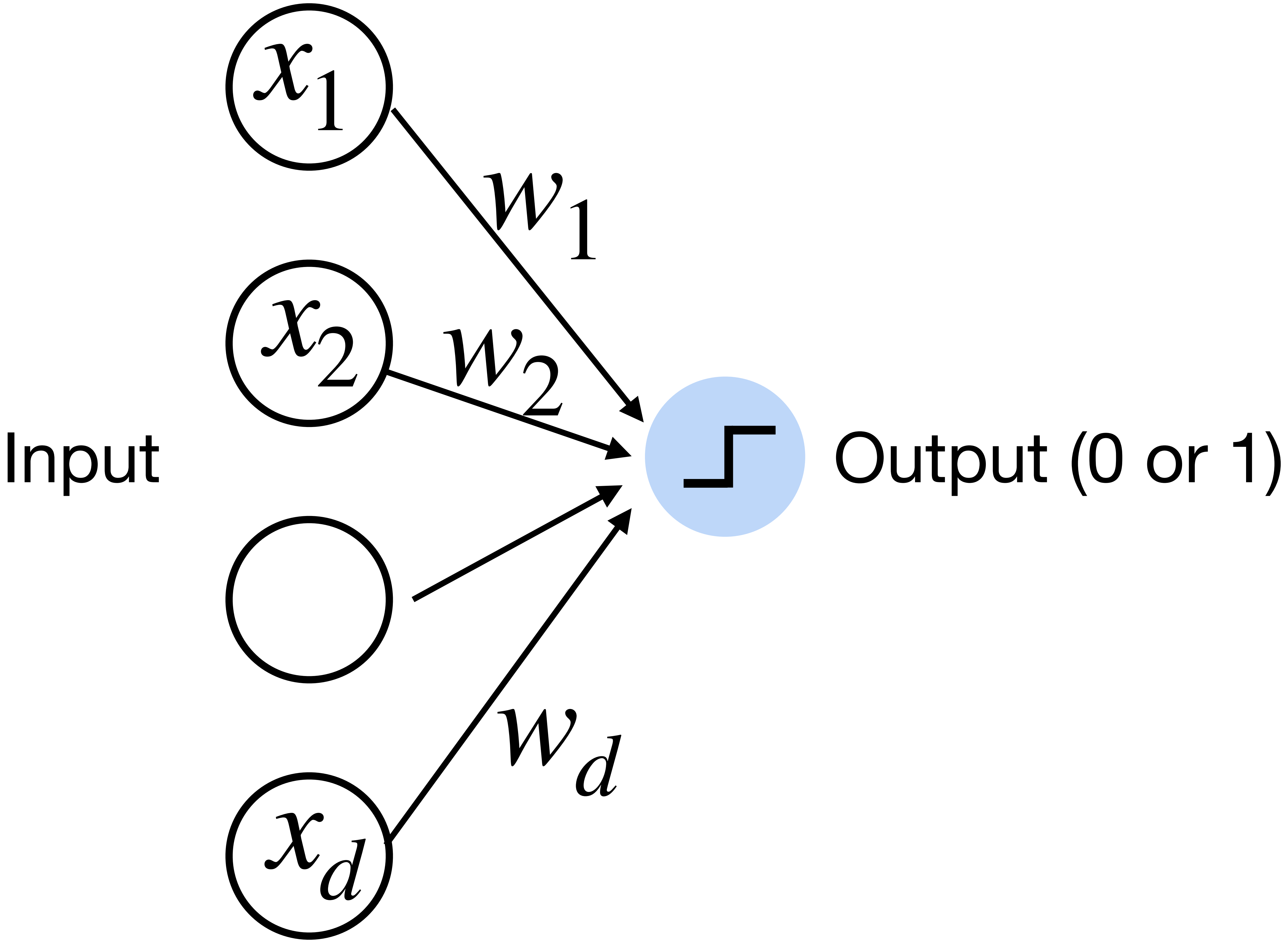
- Given input $\mathbf{x}$, weight $\mathbf{w}$ and bias b, perceptron outputs:

$$o = \sigma\left(\mathbf{w}^\top \mathbf{x} + b\right)$$

$$\sigma(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Activation function

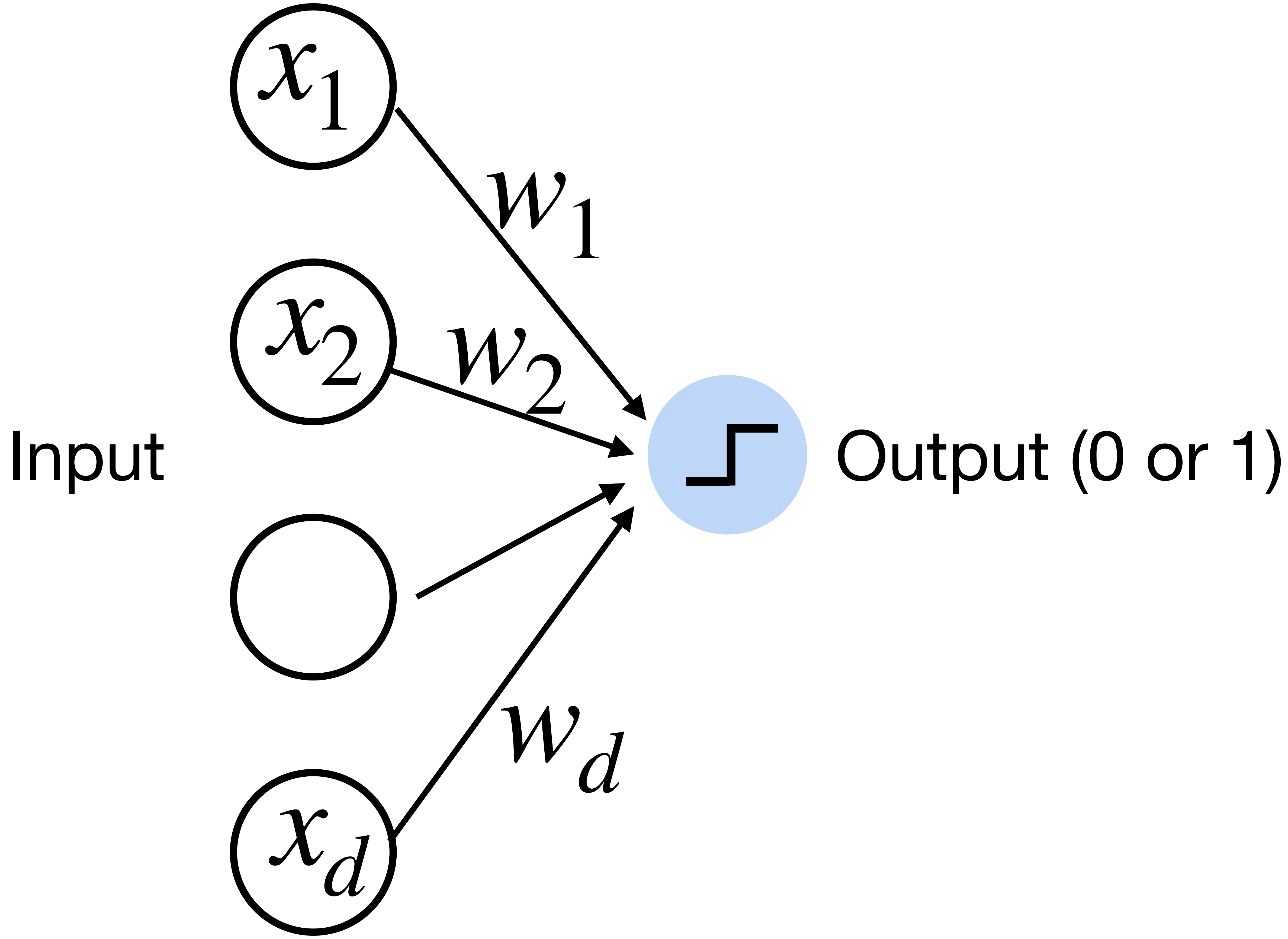
Cats vs. dogs?



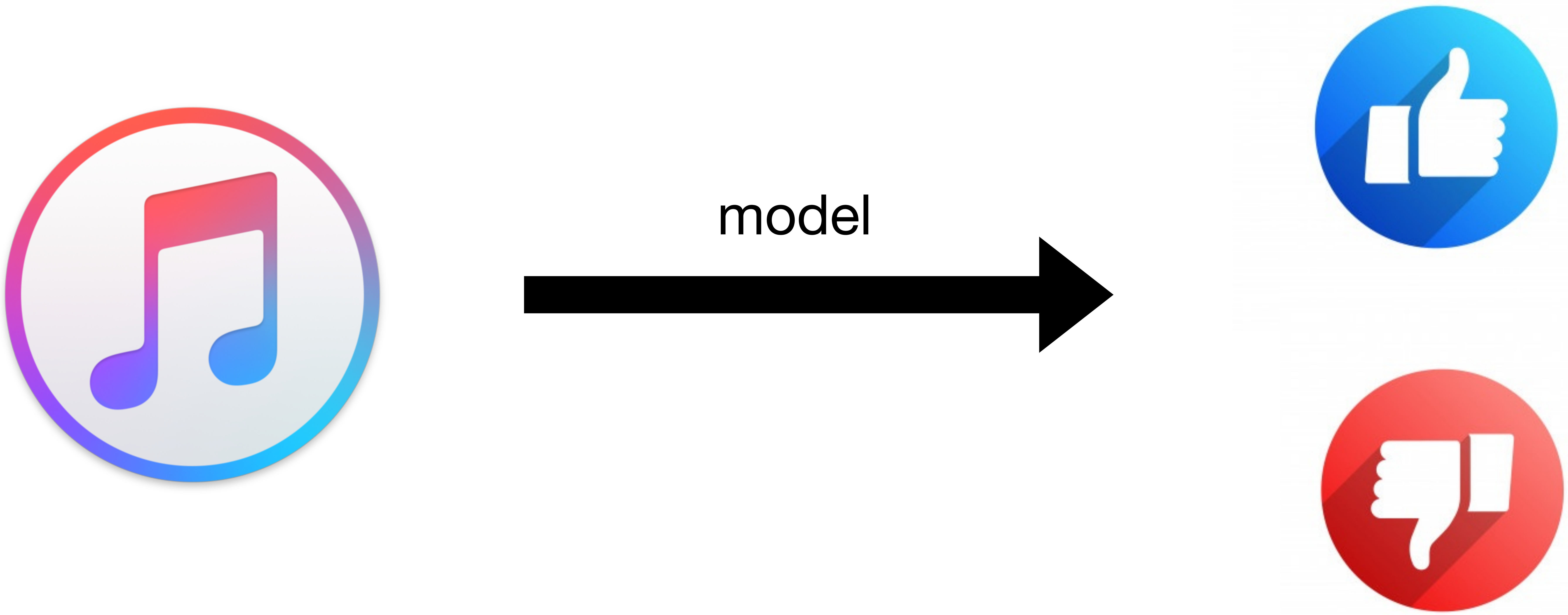
Perceptron

- Goal: learn parameters $\mathbf{w} = \{w_1, w_2, \ldots, w_d\}$ and b to minimize the classification error

Cats vs. dogs?



Example 2: Predict whether a user likes a song or not

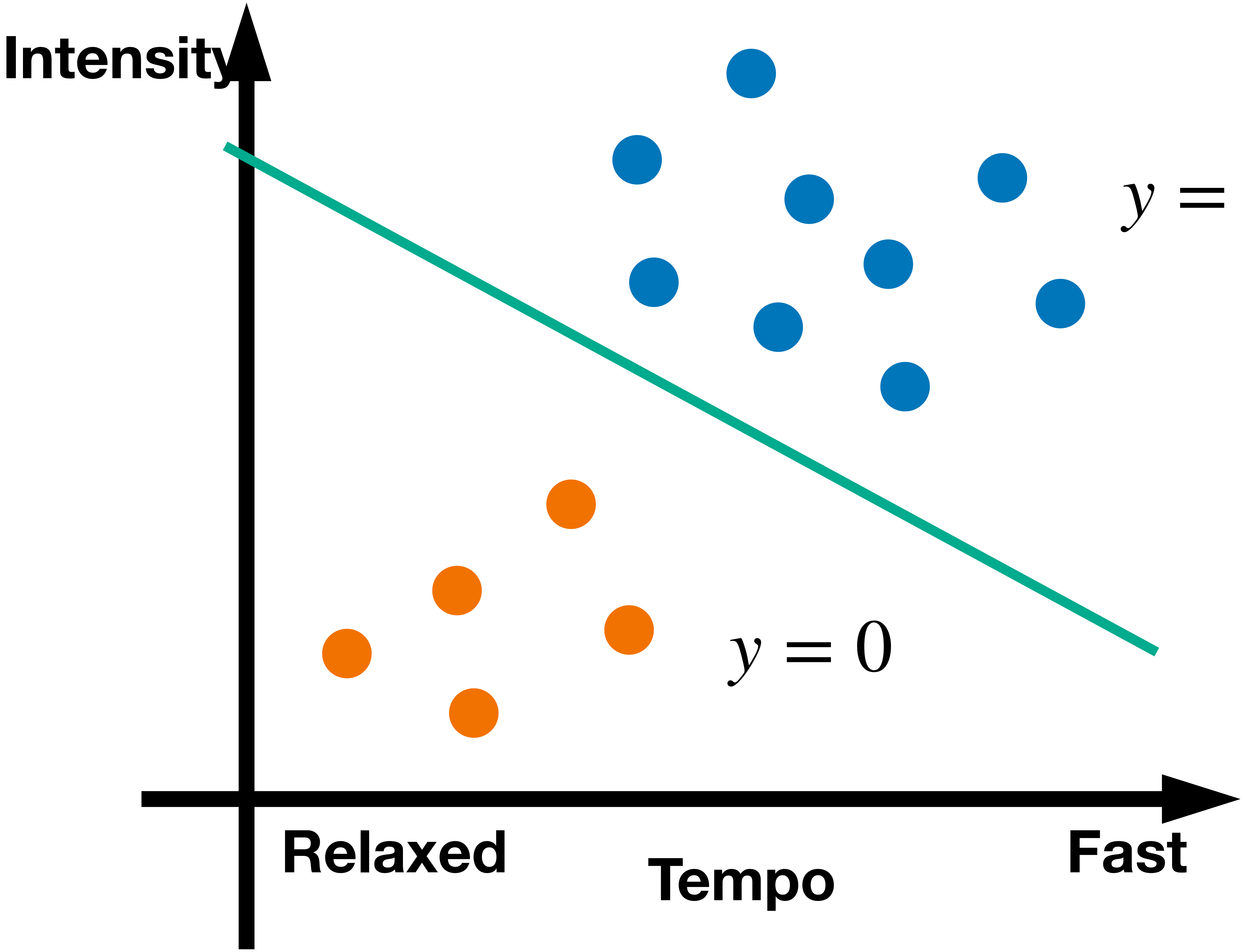


Example 2: Predict whether a user likes a song or not Using Perceptron

User Sharon

DisLike

Like



Learning logic functions using perceptron

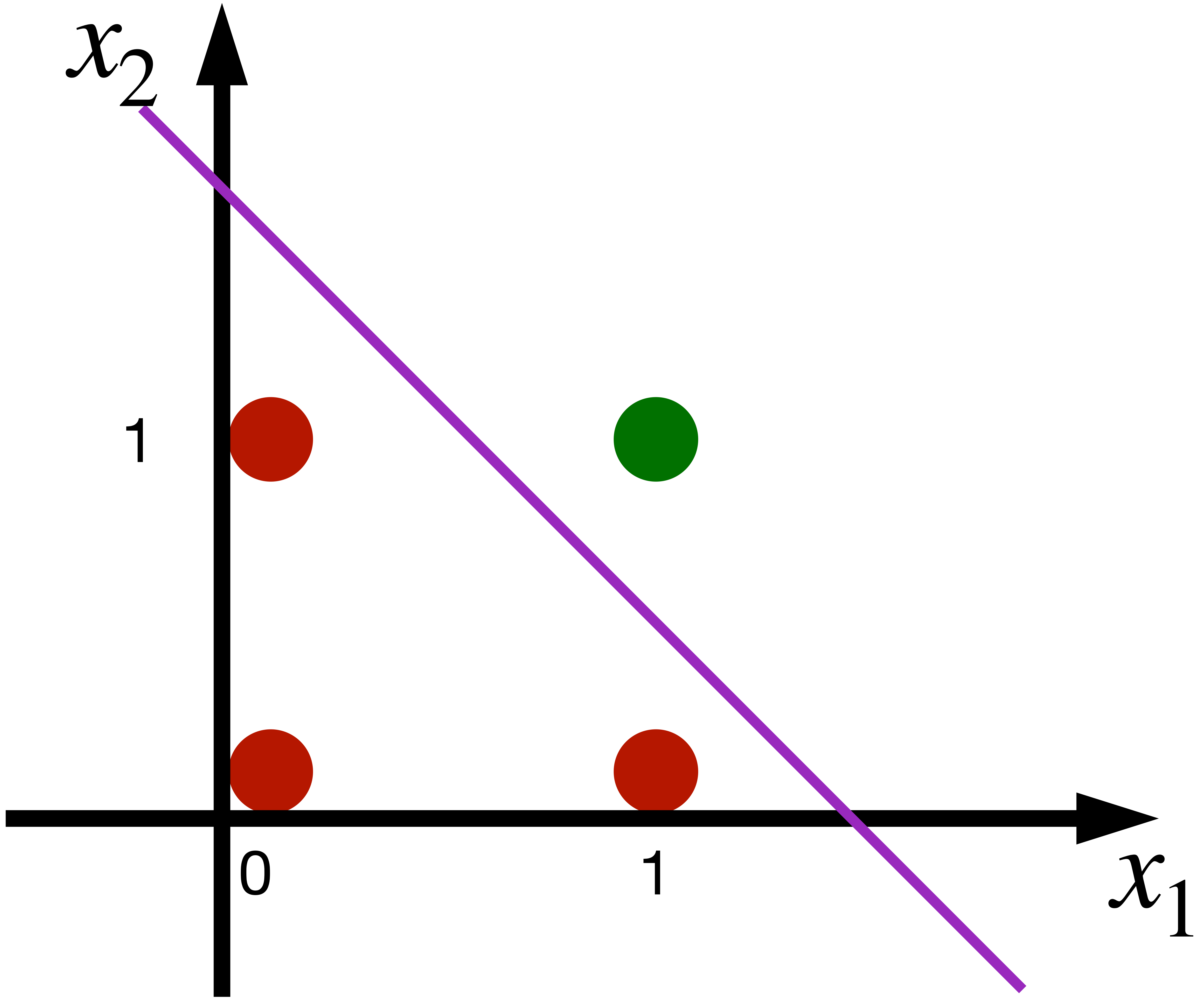
The perceptron can learn an AND function

$x_1 = 1, x_2 = 1, y = 1$

$x_1 = 1, x_2 = 0, y = 0$

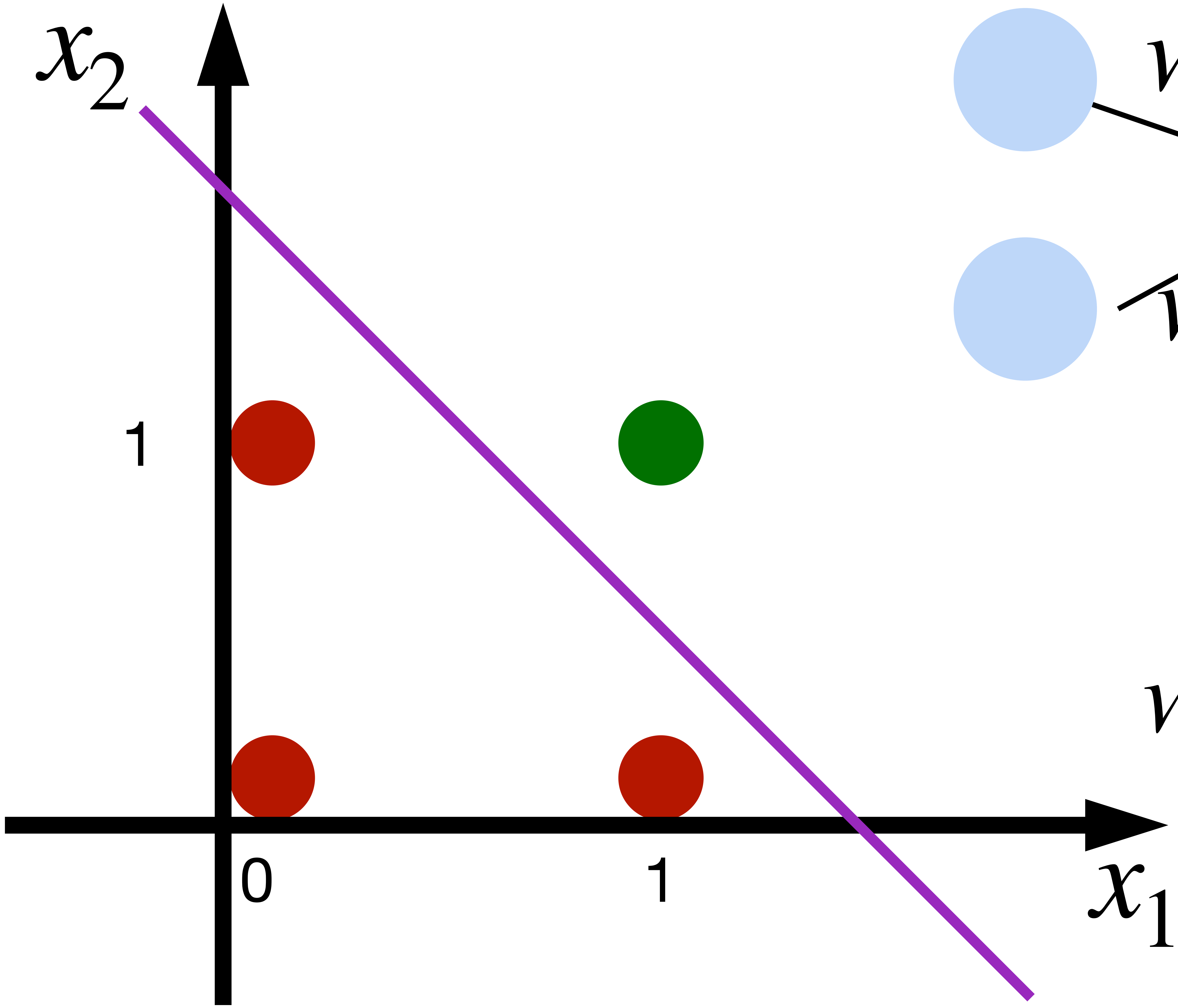
$x_1 = 0, x_2 = 1, y = 0$

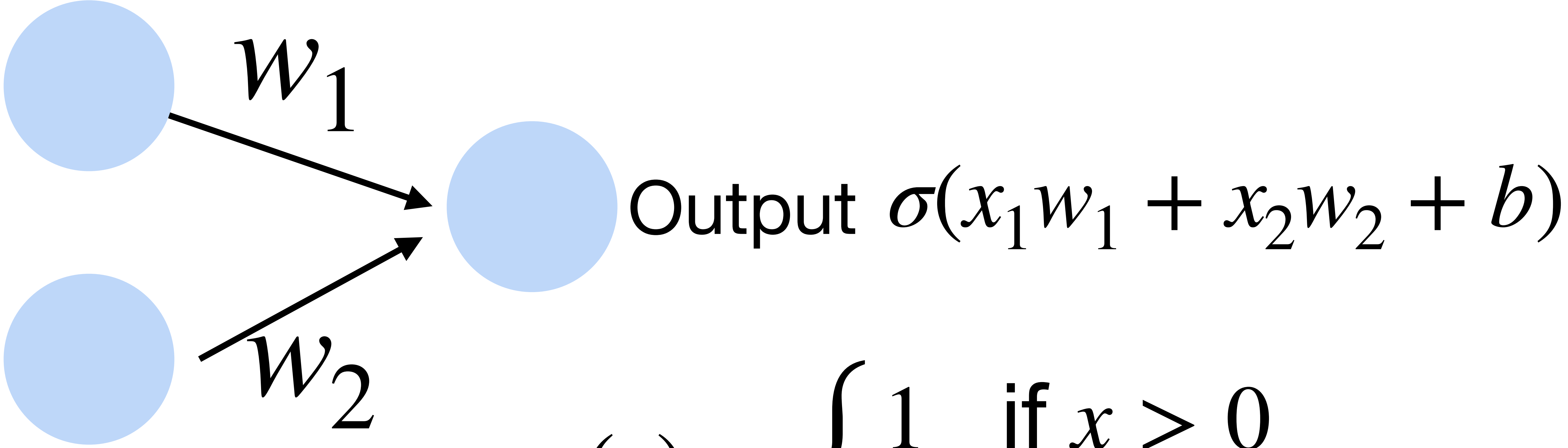
$x_1 = 0, x_2 = 0, y = 0$



Learning logic functions using perceptron

The perceptron can learn an AND function



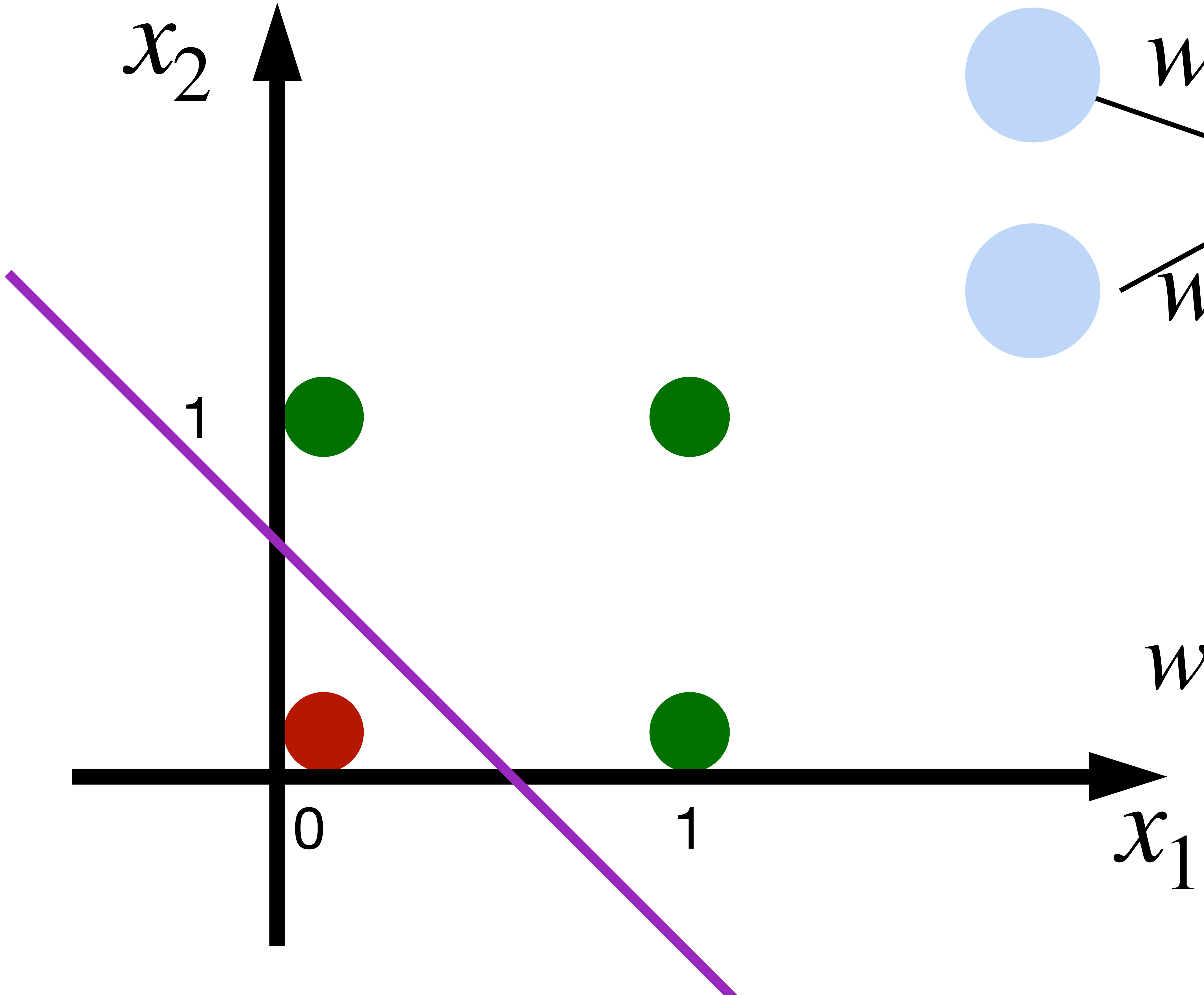


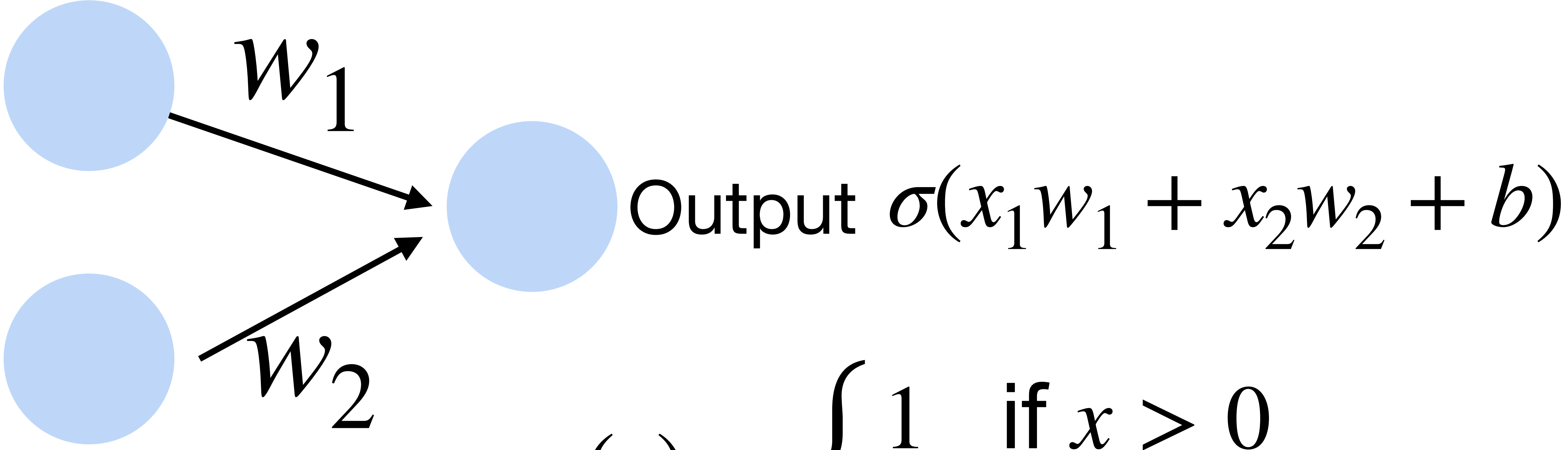
$$\sigma(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$w_1 = 1, w_2 = 1, b = -1.5$$

Learning OR function using perceptron

The perceptron can learn an OR function





$$\sigma(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$w_1 = 1, w_2 = 1, b = -0.5$$

XOR Problem (Minsky & Papert, 1969)

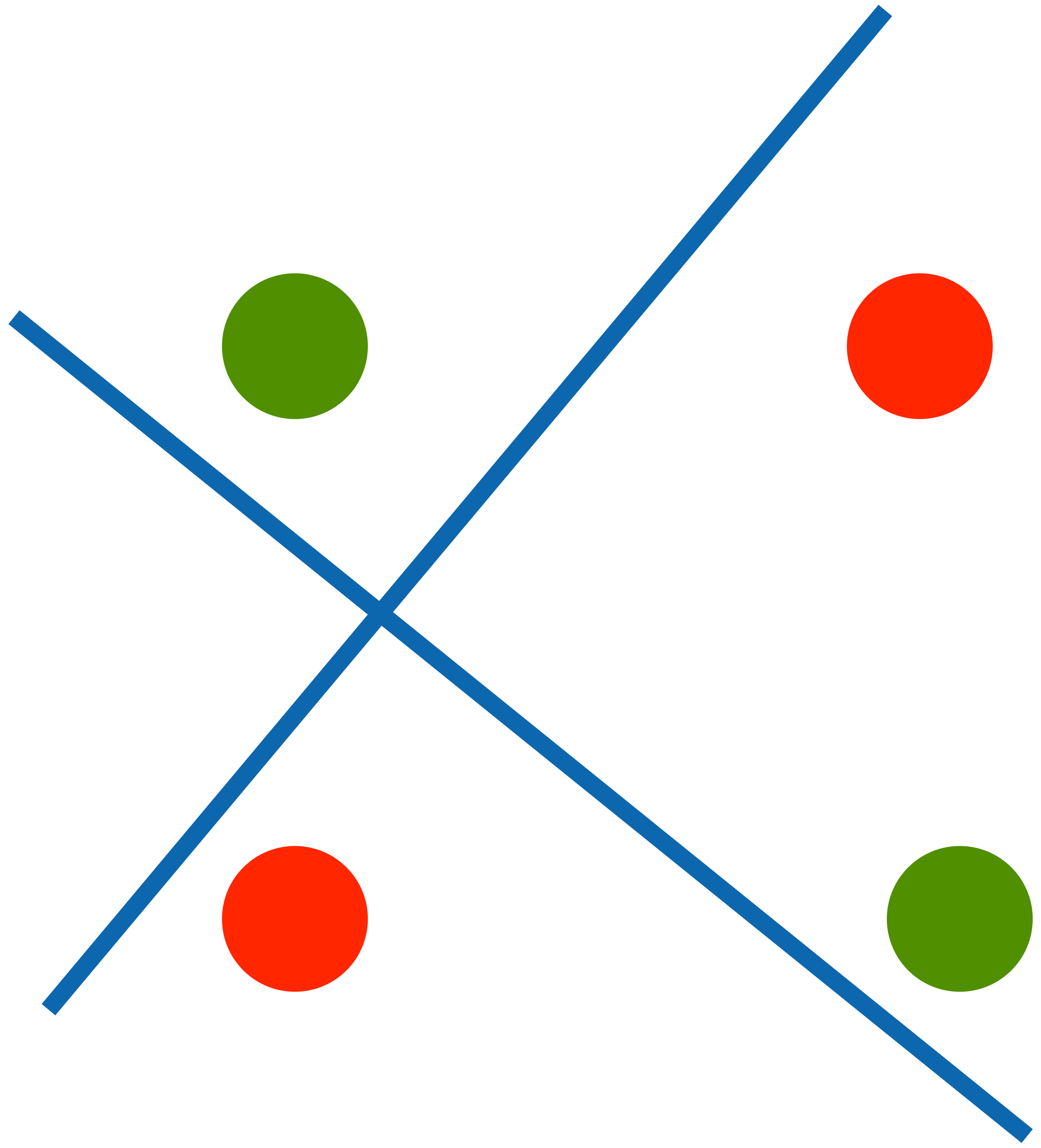
The perceptron cannot learn an XOR function
(neurons can only generate linear separators)

$$x_1 = 1, x_2 = 1, y = 0$$

$$x_1 = 1, x_2 = 0, y = 1$$

$$x_1 = 0, x_2 = 1, y = 1$$

$$x_1 = 0, x_2 = 0, y = 0$$



This contributed to the first AI winter

Quiz break

Which one of the following is NOT true about perceptron?

A. Perceptron only works if the data is linearly separable.

B. Perceptron can learn AND function

C. Perceptron can learn XOR function

D. Perceptron is a supervised learning algorithm

Quiz break

Which one of the following is NOT true about perceptron?

A. Perceptron only works if the data is linearly separable.

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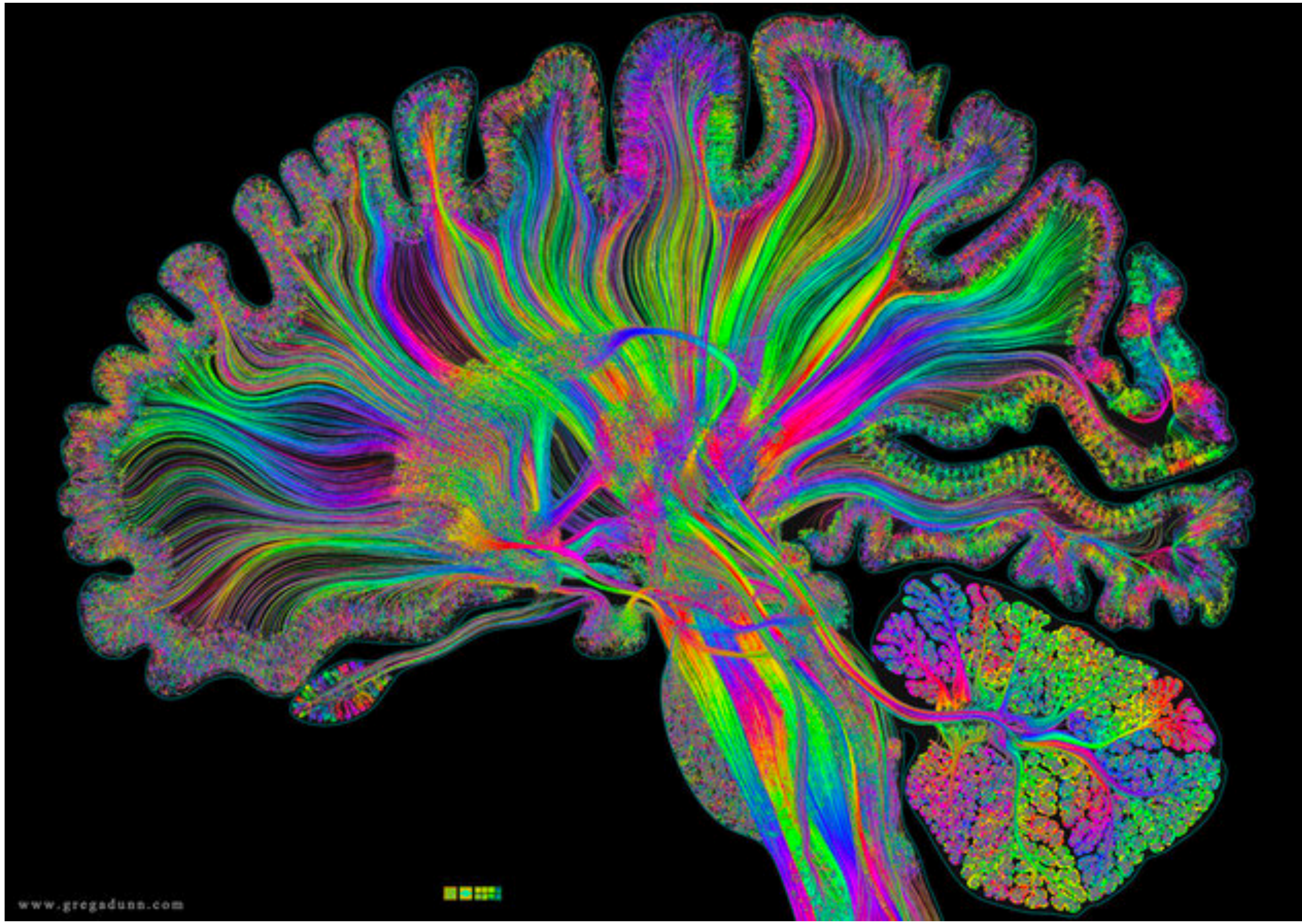
C. Perceptron can learn XOR function

D. Perceptron is a supervised learning algorithm

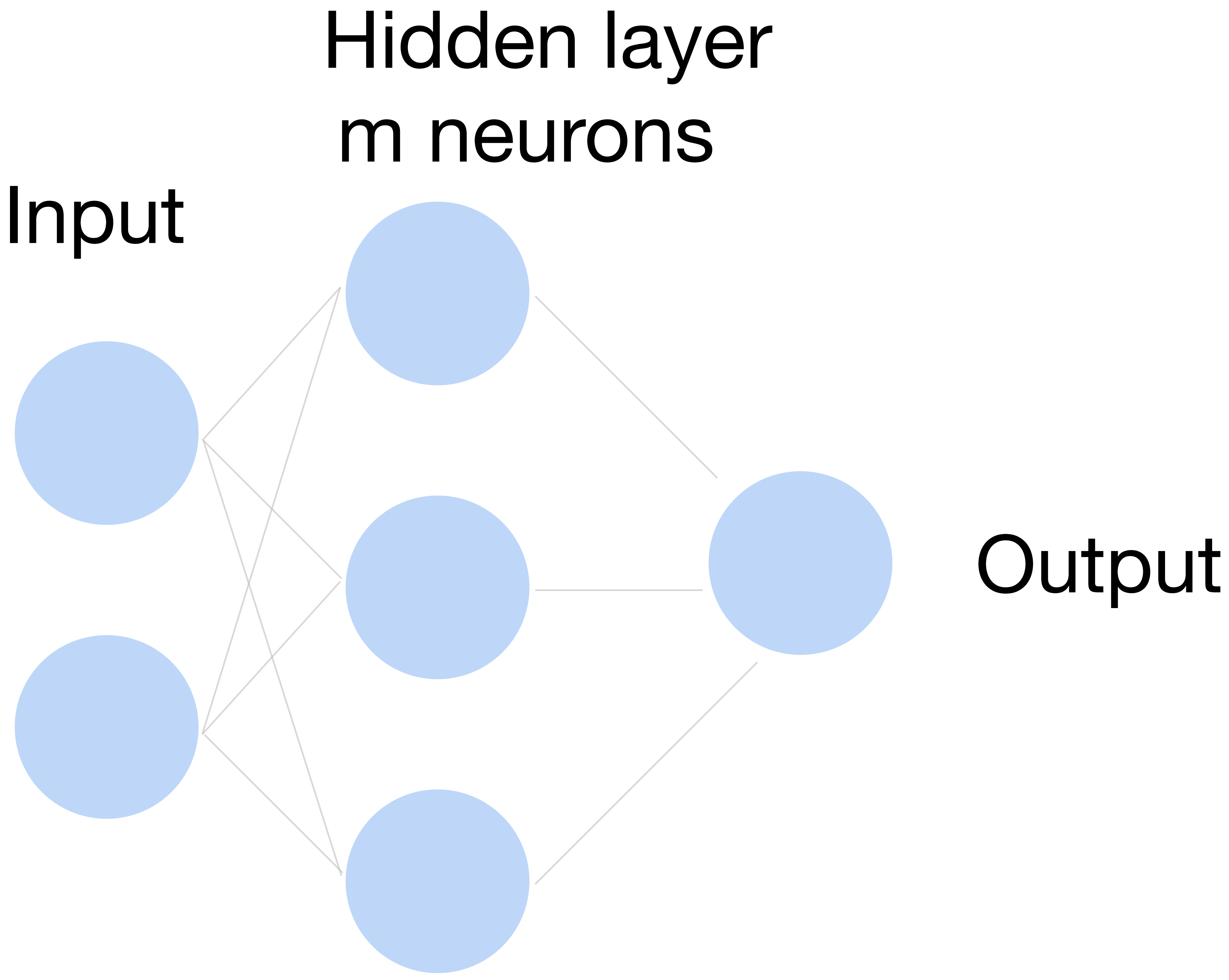
Multilayer Perceptron



Single Hidden Layer

How to classify

Cats vs. dogs?

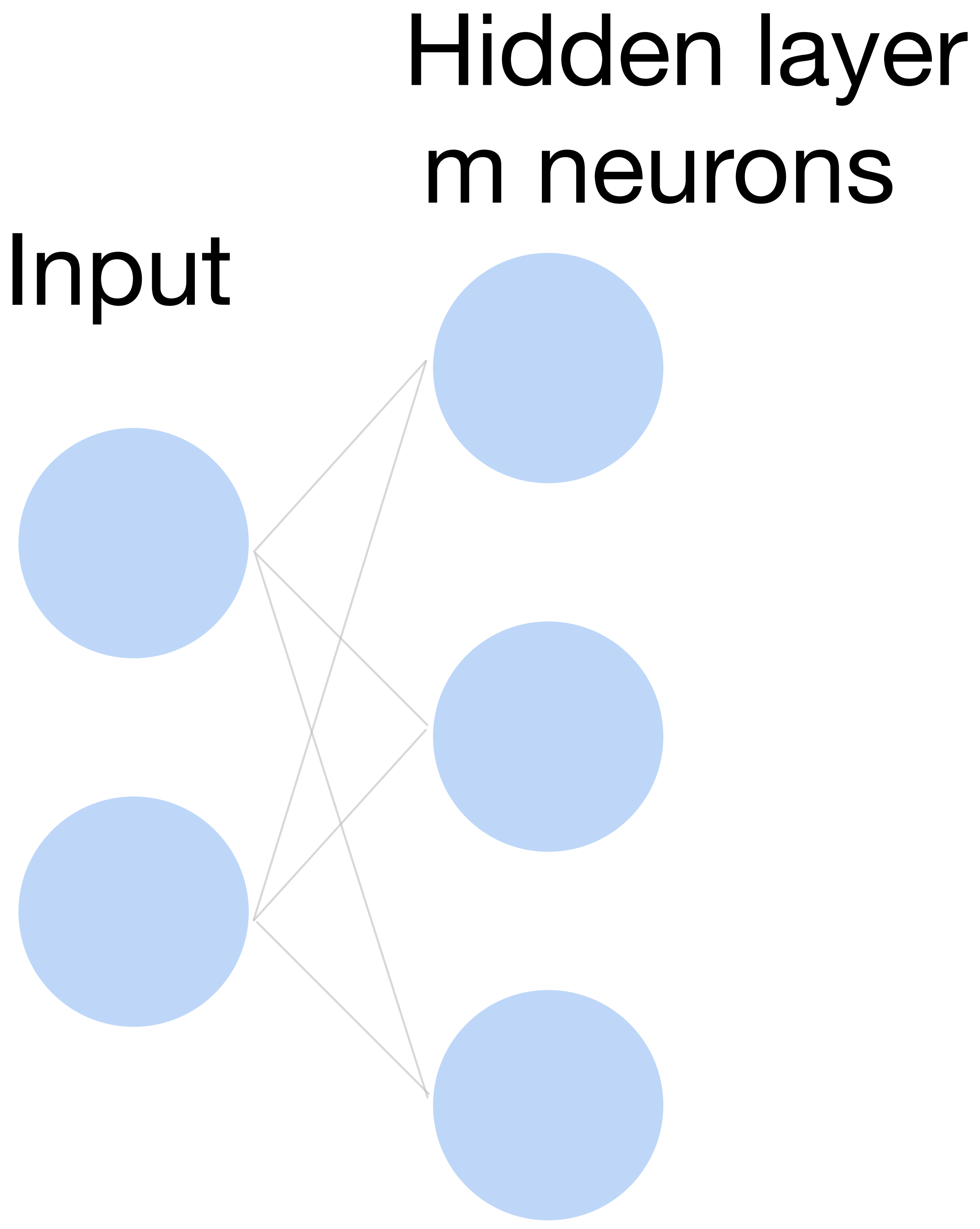


Single Hidden Layer

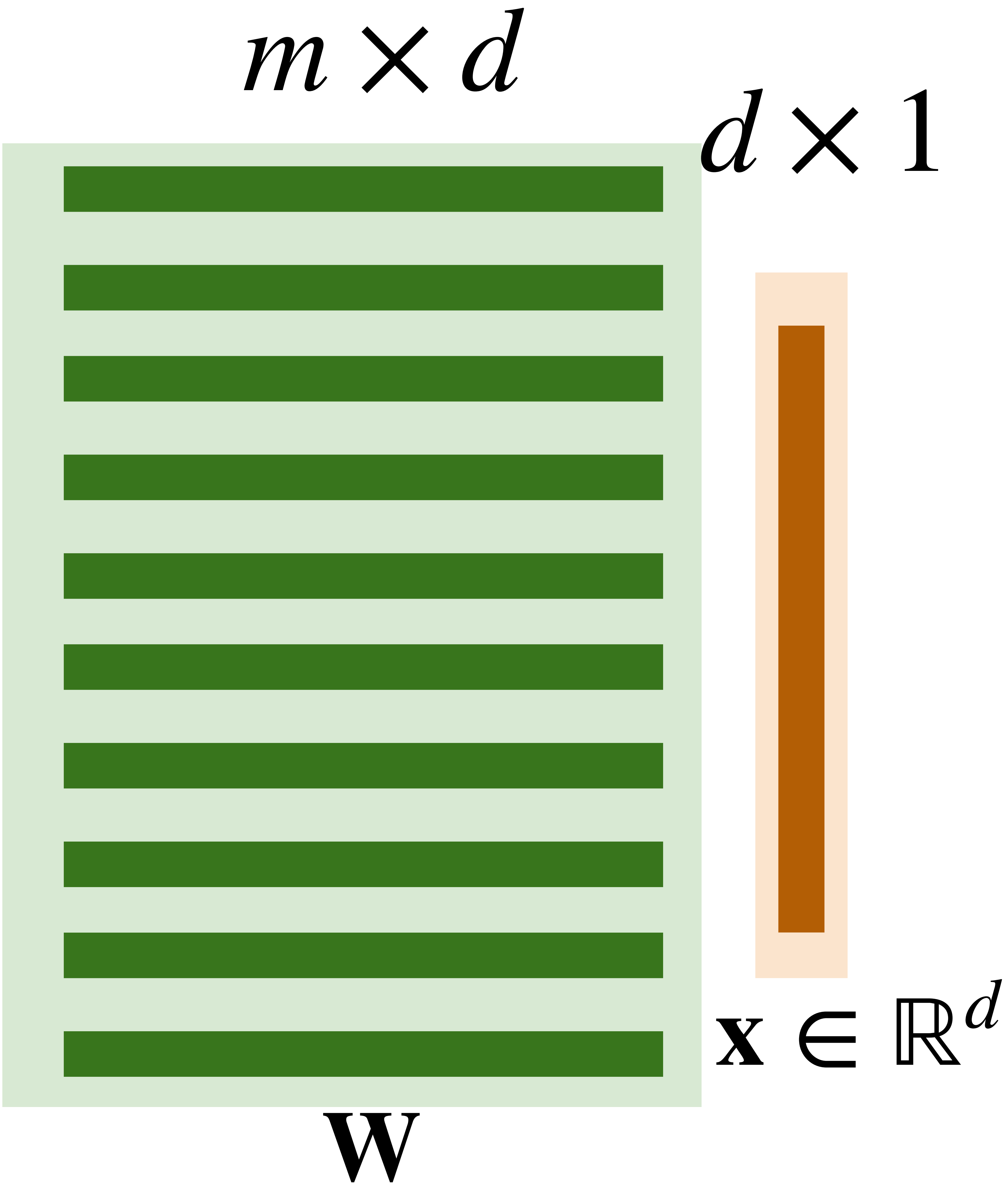
- Input $\mathbf{x} \in \mathbb{R}^d$
- Hidden $\mathbf{W} \in \mathbb{R}^{m \times d}, \mathbf{b} \in \mathbb{R}^m$
- Intermediate output

$$\mathbf{h} = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$$

σ is an element-wise activation function

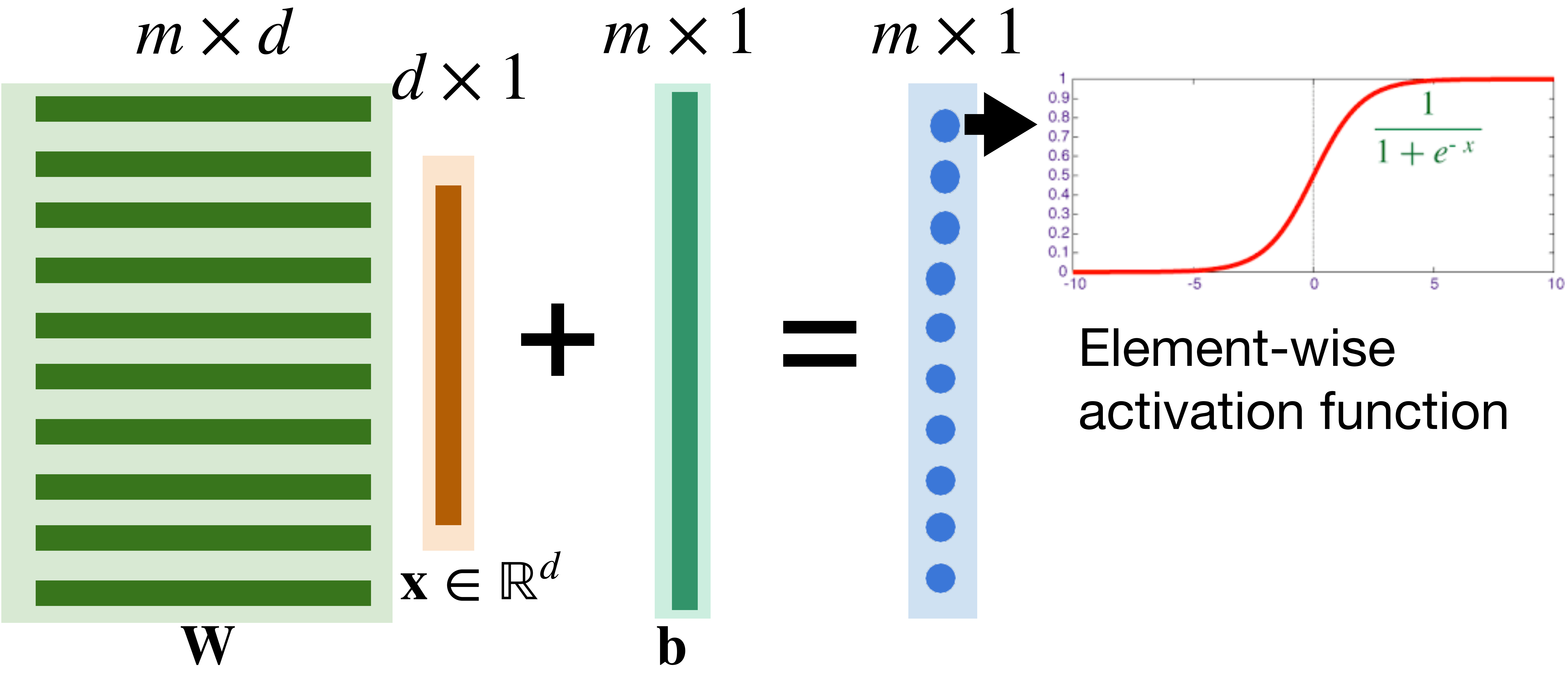


Neural networks with one hidden layer



Neural networks with one hidden layer

Key elements: linear operations + Nonlinear activations

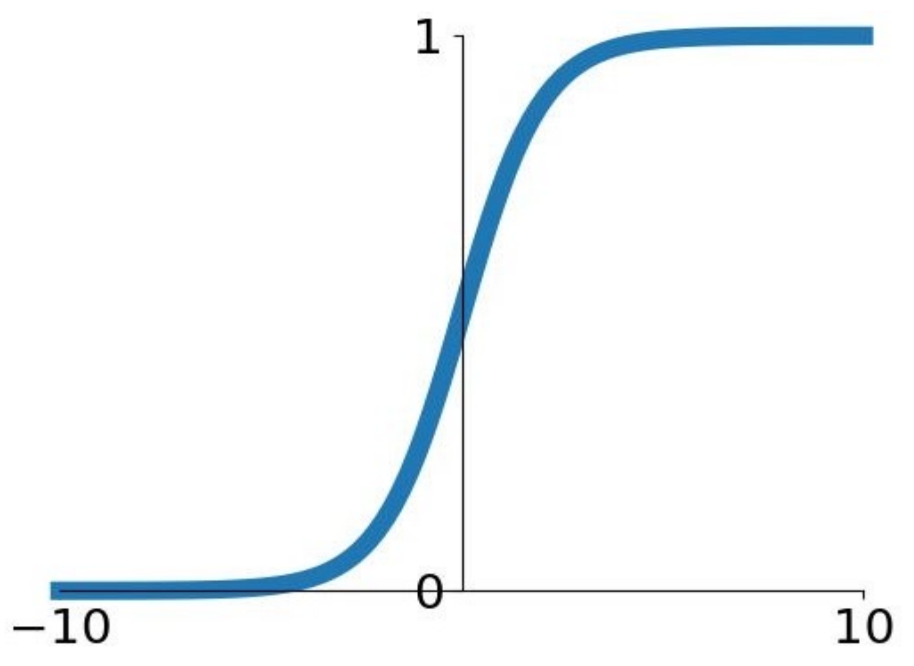


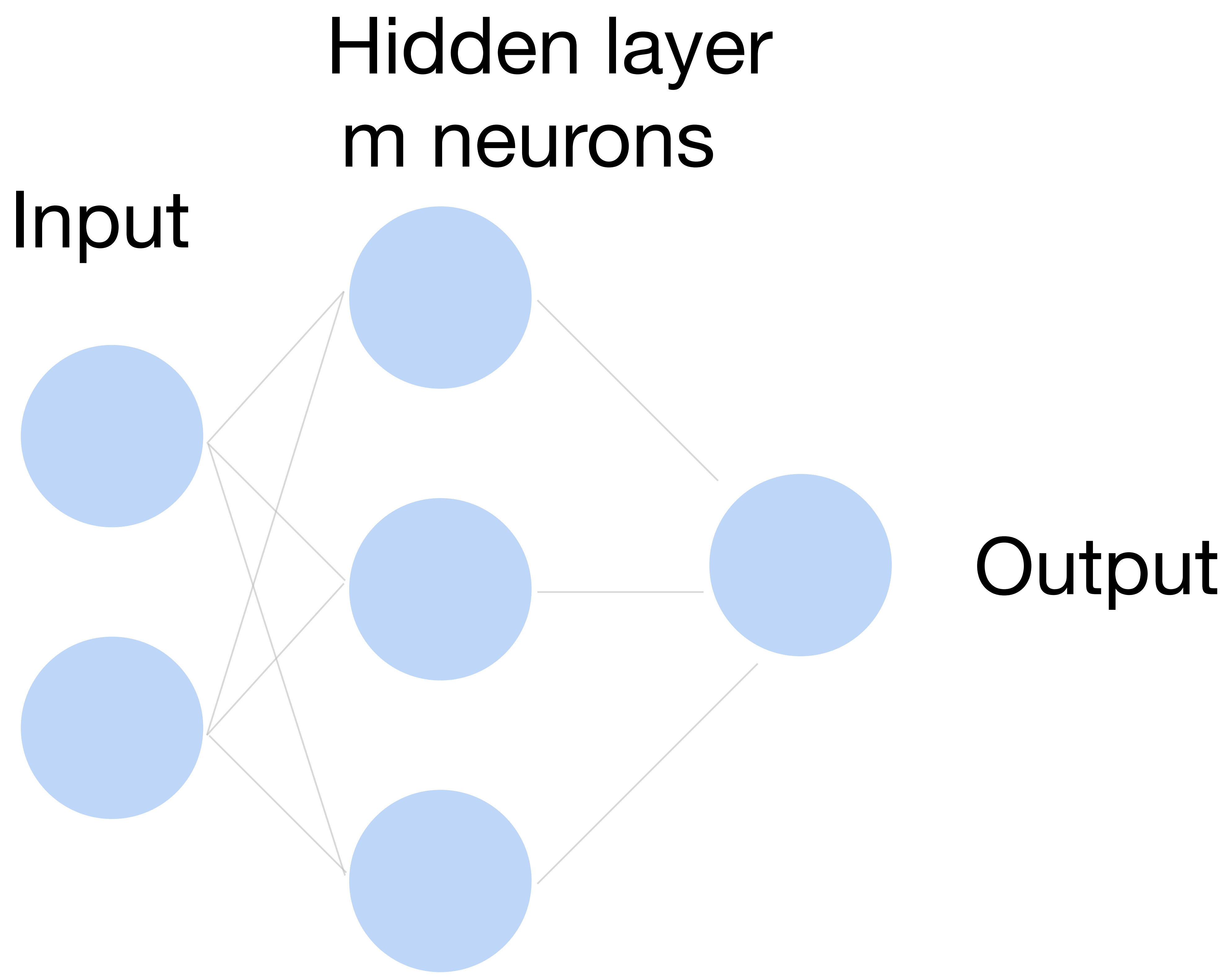
Single Hidden Layer

- Output $f = \mathbf{w}_2^\top \mathbf{h} + b_2$
- Normalize the output into probability using sigmoid

$$p(y = 1 \mid \mathbf{x}) = \frac{1}{1 + e^{-f}}$$

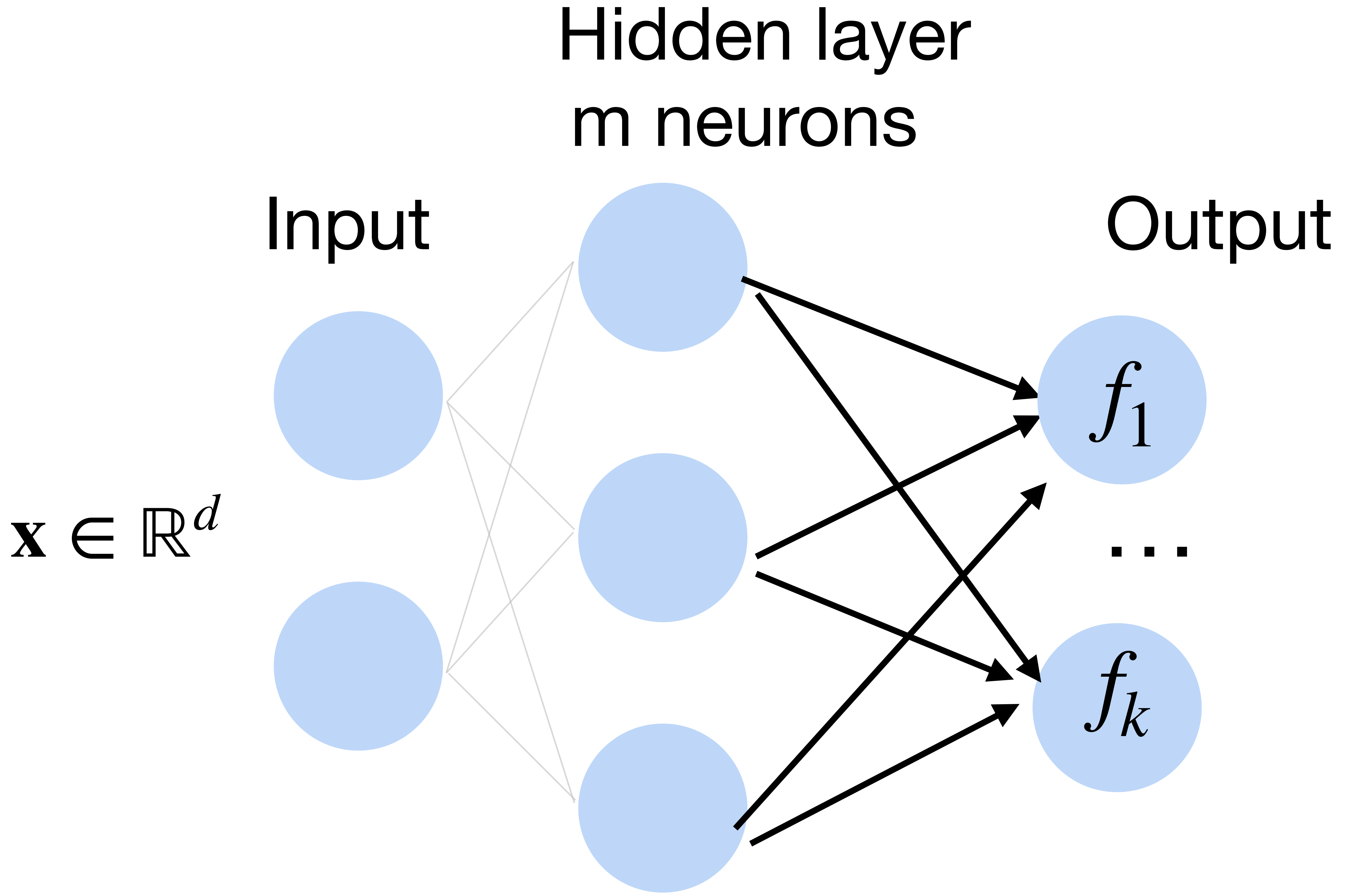
Sigmoid





Multi-class classification

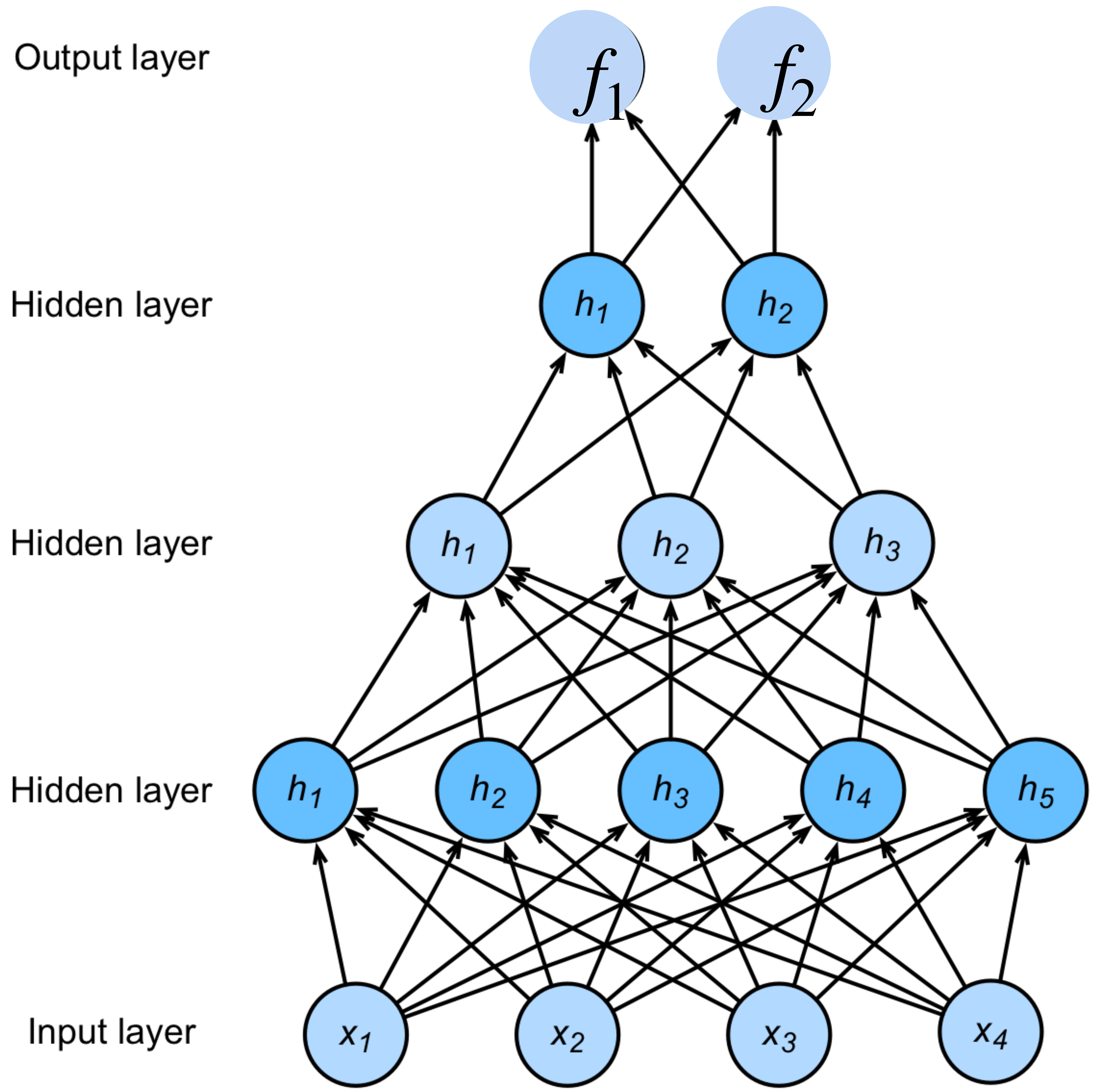
Turns outputs f into k probabilities (sum up to 1 across k classes)



$$p(y \mid \mathbf{x}) = \text{softmax}(\mathbf{f})$$

$$= \frac{\exp f_y(x)}{\sum_i^k \exp f_i(x)}$$

Deep neural networks (DNNs)



$$\mathbf{h}_1 = \sigma(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$
$$\mathbf{h}_2 = \sigma(\mathbf{W}_2\mathbf{h}_1 + \mathbf{b}_2)$$
$$\mathbf{h}_3 = \sigma(\mathbf{W}_3\mathbf{h}_2 + \mathbf{b}_3)$$
$$\mathbf{f} = \mathbf{W}_4\mathbf{h}_3 + \mathbf{b}_4$$
$$\mathbf{y} = \text{softmax}(\mathbf{f})$$

NNs are composition of nonlinear functions

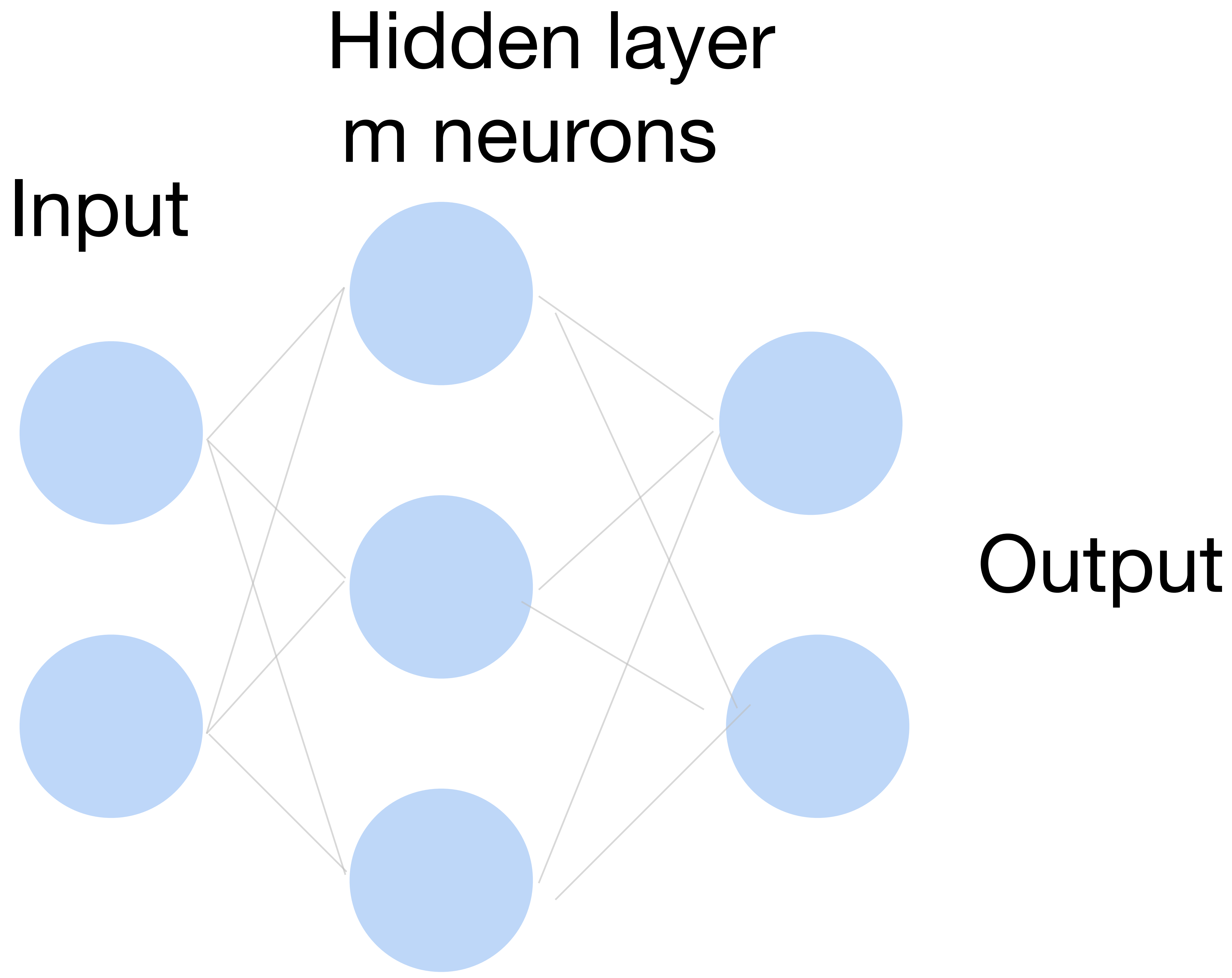
How to train a neural network?

Loss function: $$\frac{1}{|D|}\sum_i \ell(\mathbf{x}_i, y_i)$$

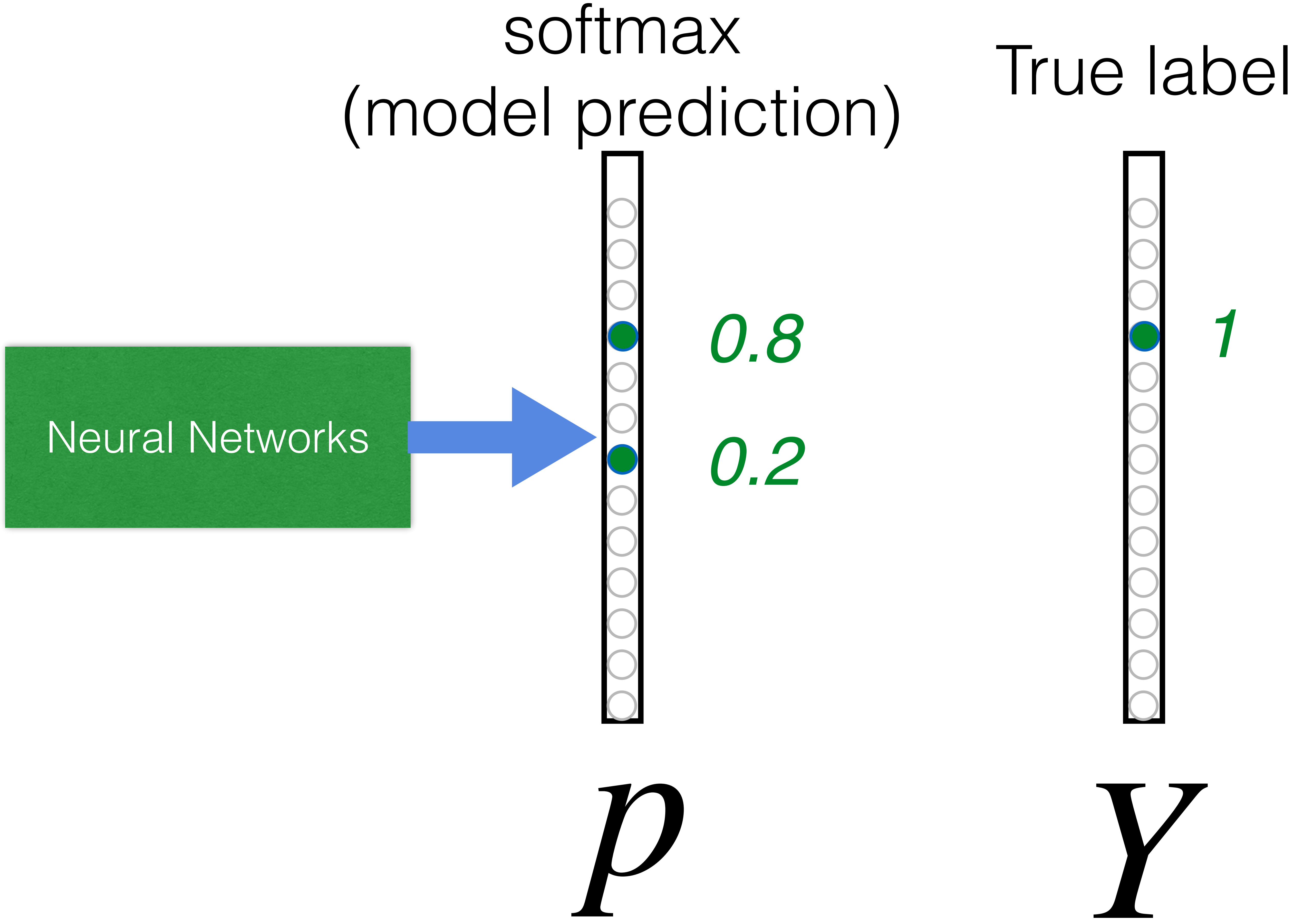
Per-sample loss:

$$\ell(\mathbf{x}, y) = \sum_{j=1}^{K} -y_j \log p_j$$

Also known as cross-entropy loss or softmax loss



Cross-Entropy Loss



$$L_{CE} = \sum_j - y_j \log(p_j)$$

$$= -\log(0.8)$$

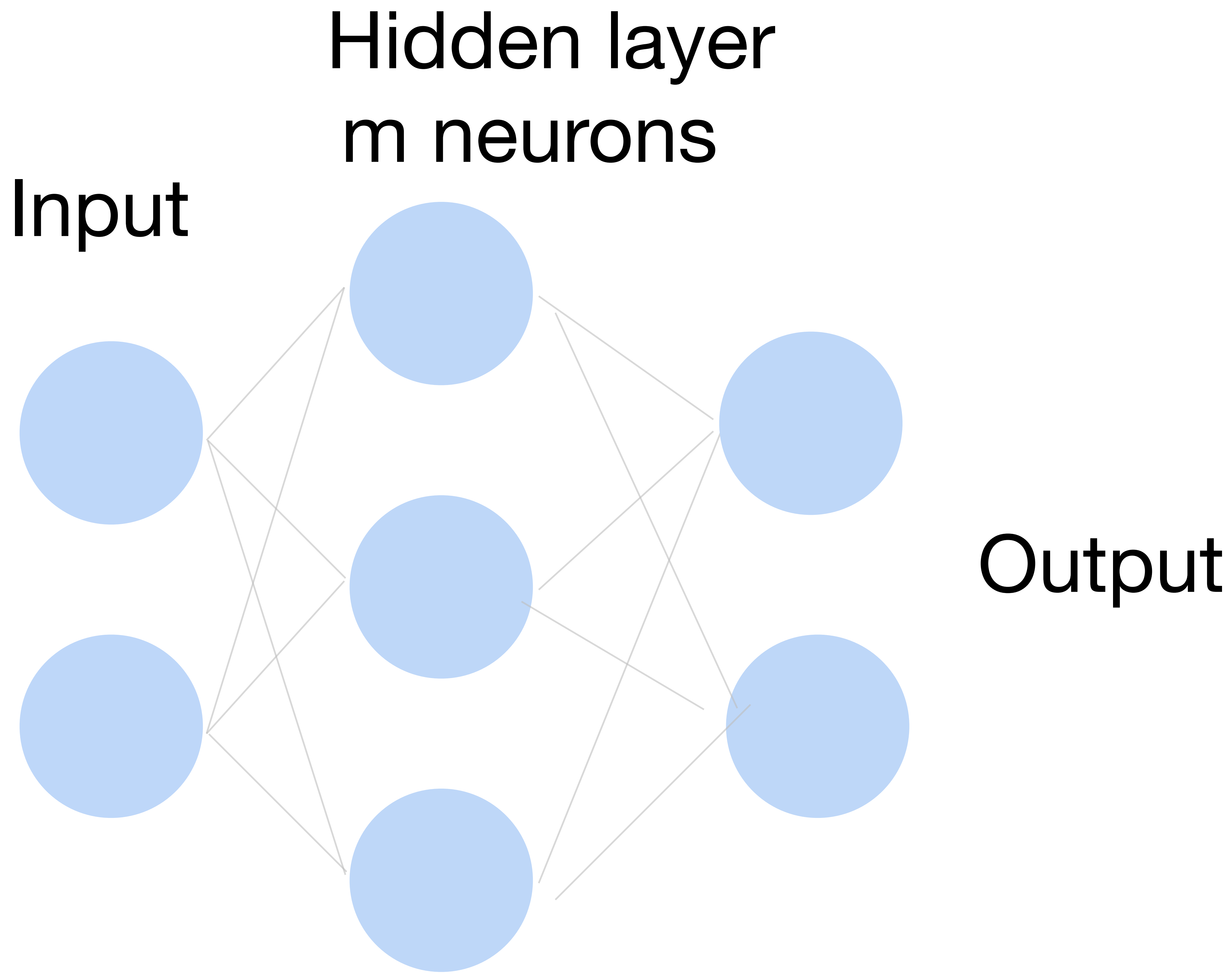
Goal: push **p** and **Y** to be identical

How to train a neural network?

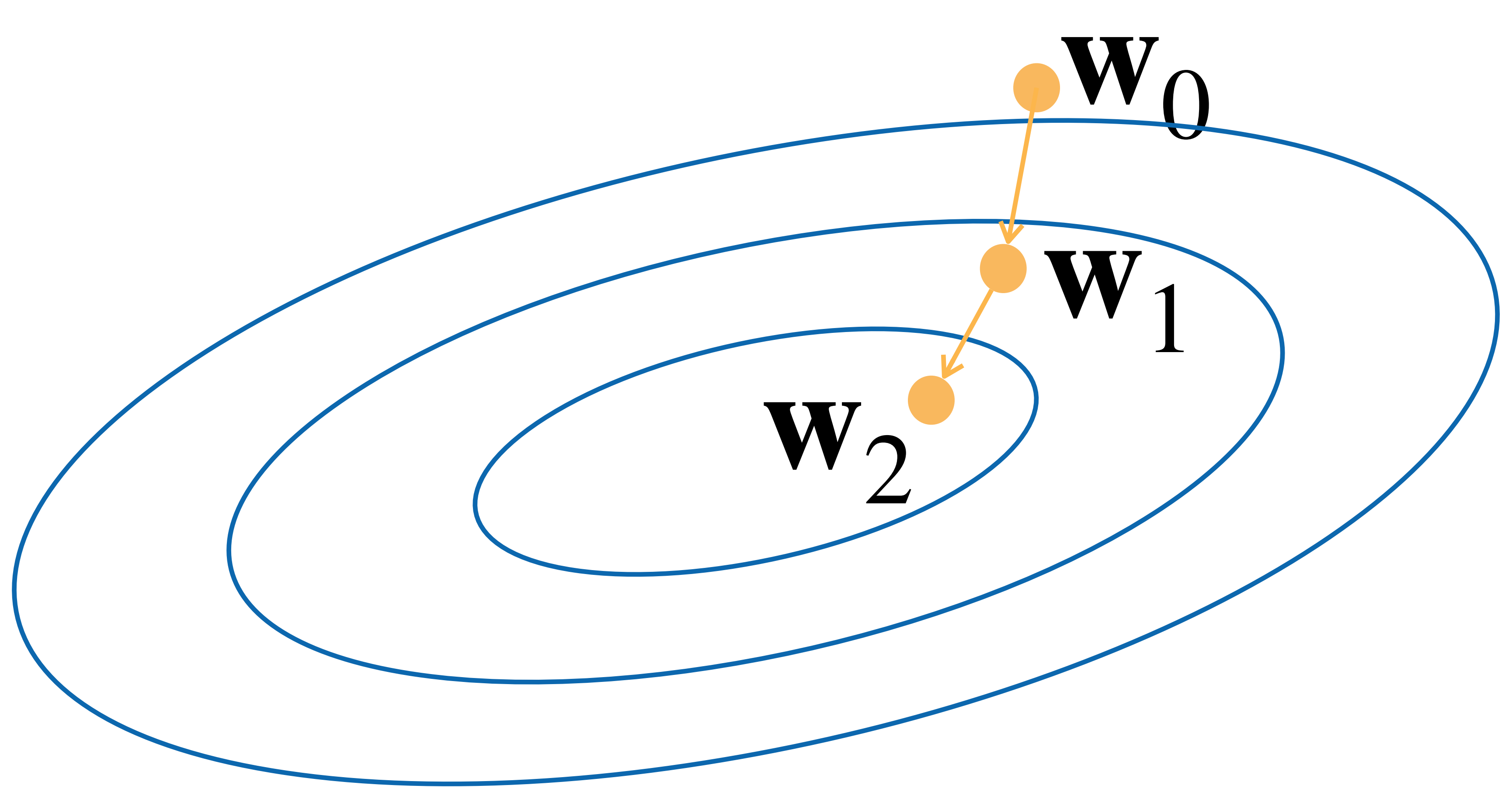
Update the weights W to minimize the loss function

$$L = \frac{1}{|D|} \sum_i \ell(\mathbf{x}_i, y_i)$$

Use gradient descent!



Gradient Descent



- Choose a learning rate $\alpha > 0$
- Initialize the model parameters w_0
- For t =1,2,…
    - Update parameters:

$$\mathbf{w}_t = \mathbf{w}_{t-1} - \alpha \frac{\partial L}{\partial \mathbf{w}_{t-1}}$$

$$= \mathbf{w}_{t-1} - \alpha \frac{1}{|D|} \sum_{\mathbf{x} \in D} \frac{\partial \ell(\mathbf{x}_i, y_i)}{\partial \mathbf{w}_{t-1}}$$

D can
be very large.
Expensive

- Repeat until converges

Minibatch Stochastic Gradient Descent

- Choose a learning rate $\alpha > 0$
- Initialize the model parameters w_0
- For t =1,2,…
  - **Randomly sample a subset (mini-batch)** $B \subset D$
    Update parameters:

$$\mathbf{w}_t = \mathbf{w}_{t-1} - \alpha \frac{1}{|B|} \sum_{\mathbf{x} \in B} \frac{\partial \ell(\mathbf{x}_i, y_i)}{\partial \mathbf{w}_{t-1}}$$

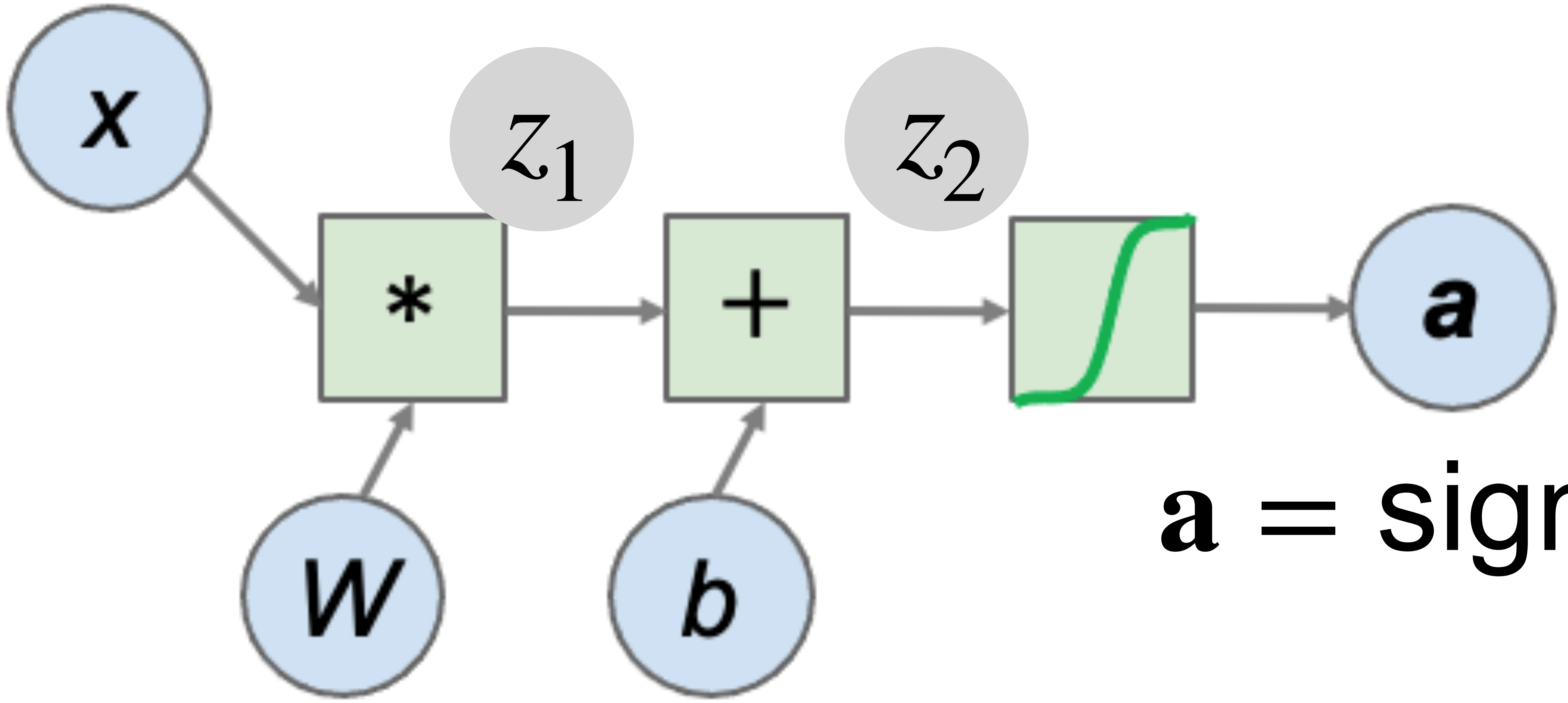
- Repeat

Calculate gradient: backpropagation with chain rule

- Define a loss function L
- Gradient to a variable =

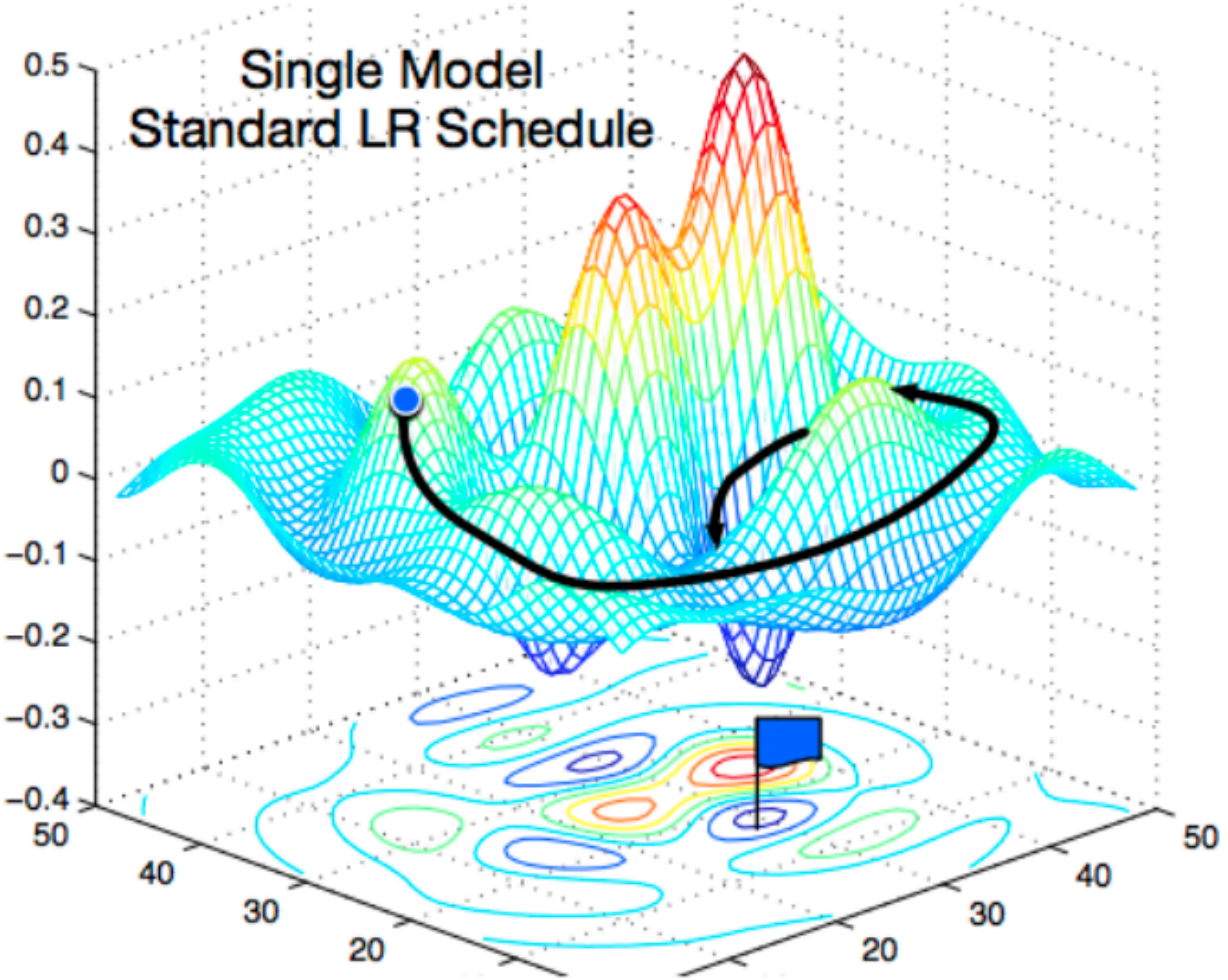
gradient on the top x gradient from the current operation

$$\frac{\partial L}{\partial \boldsymbol{W}} = \frac{\partial L}{\partial \boldsymbol{z}_1} \frac{\partial \boldsymbol{z}_1}{\partial \boldsymbol{W}}$$



$$\mathbf{a} = \text{sigmoid}(\mathbf{W}\mathbf{x} + \mathbf{b})$$

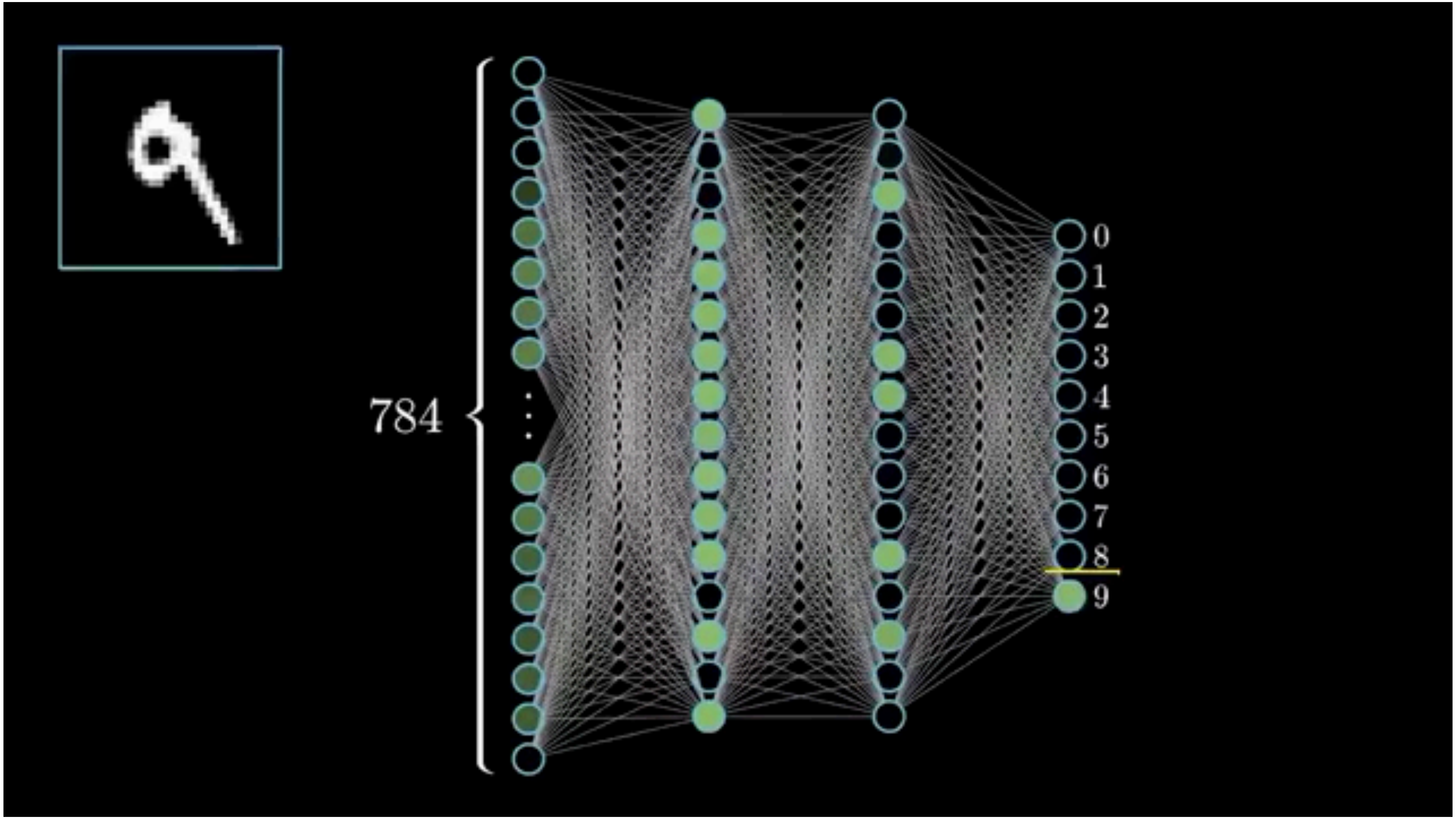
Non-convex Optimization



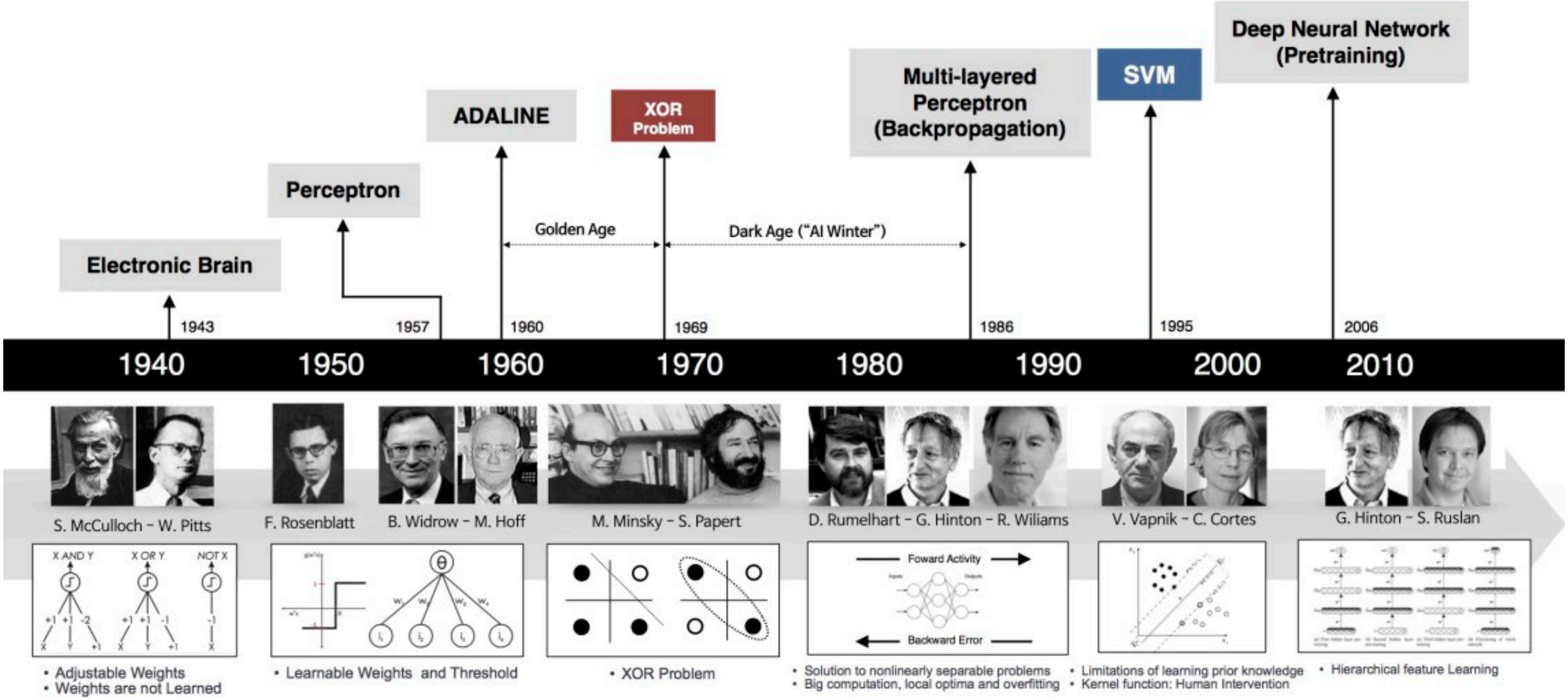
[Gao and Li et al., 2018]

Using SGD in PyTorch (code demo)

Classify MNIST handwritten digits



Brief history of neural networks



How to classify

Cats vs. dogs?

Dual

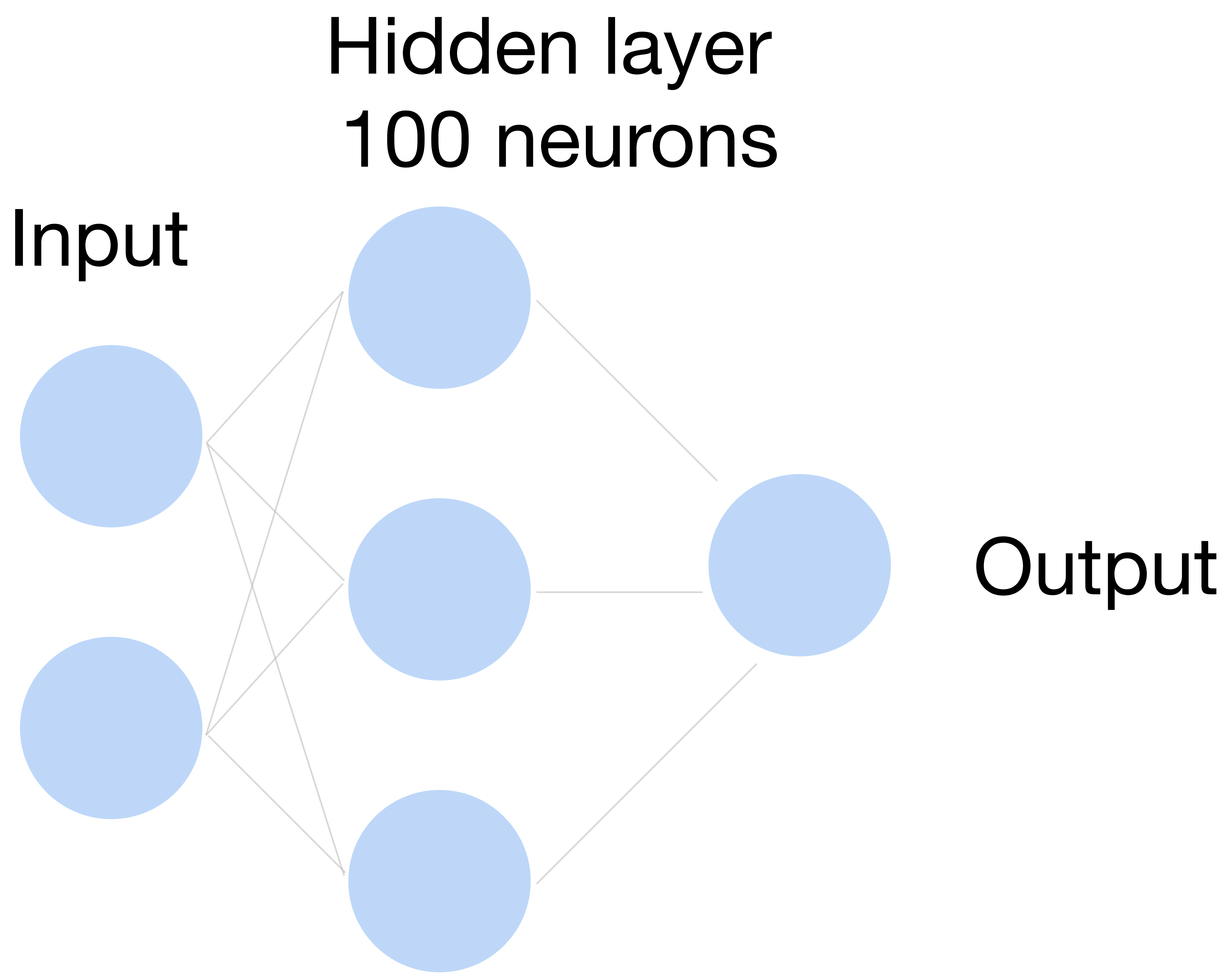
12MP

wide-angle and
telephoto cameras

36M floats in a RGB image!

Fully Connected Networks

Cats vs. dogs?



~ 36M elements x 100 = ~**3.6B** parameters!

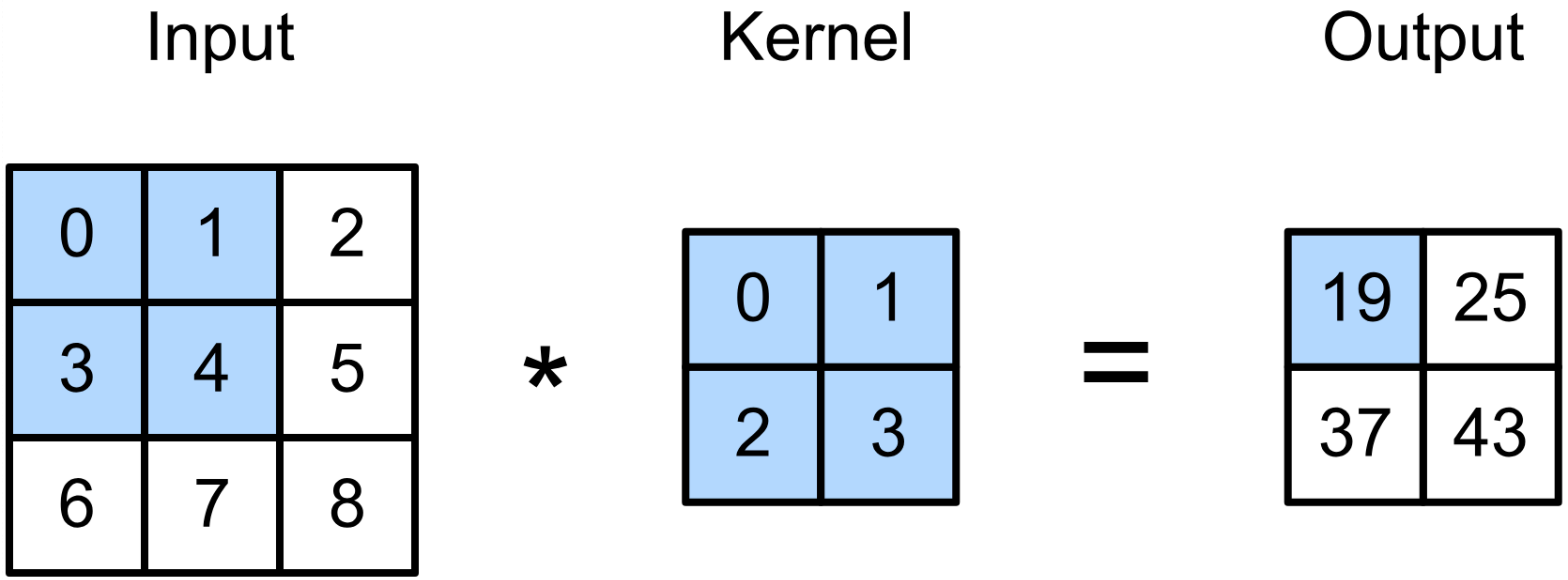
Convolutions come to rescue!

Where is Waldo?

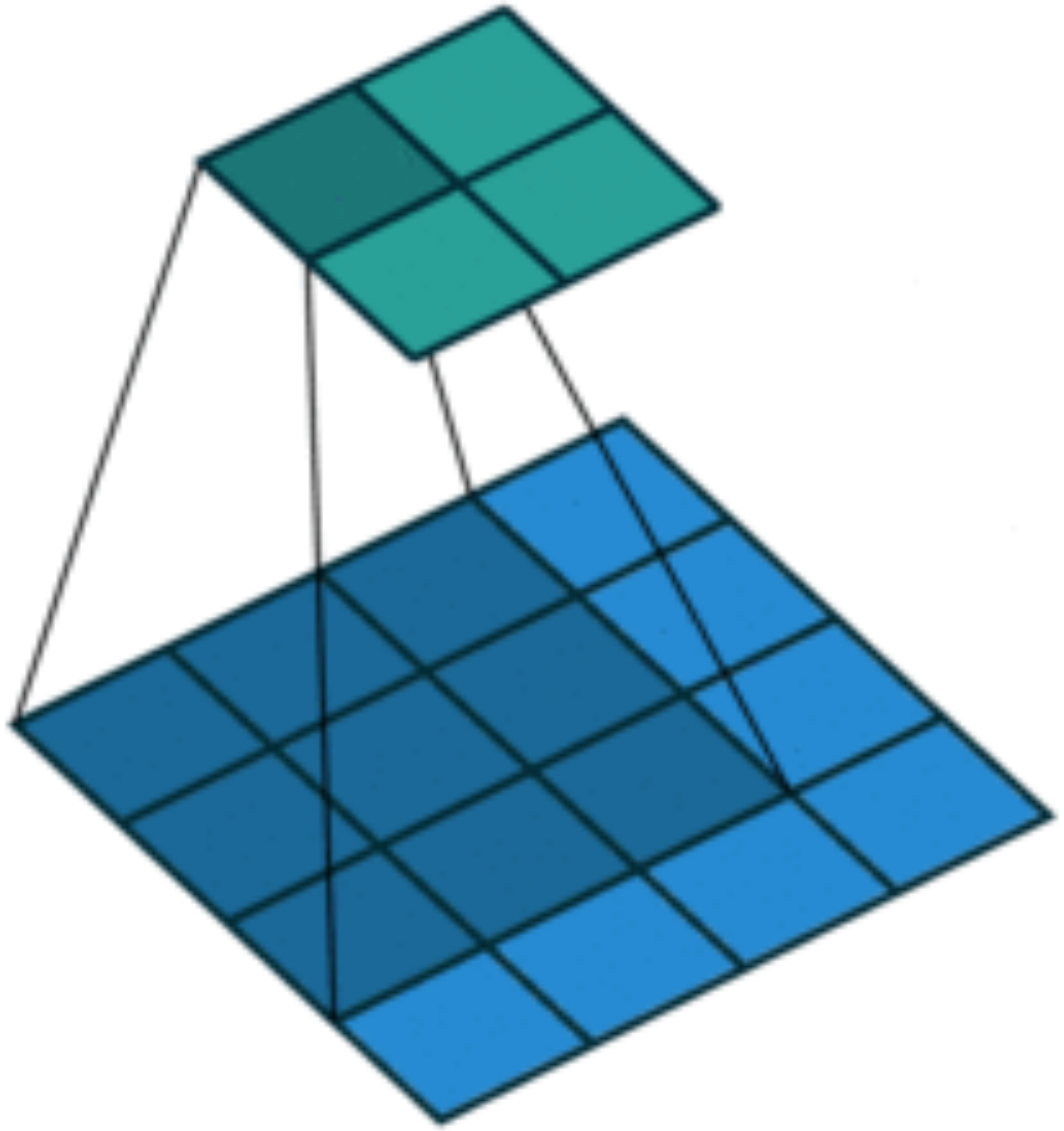
Why Convolution?

- Translation Invariance
- Locality

2-D Convolution

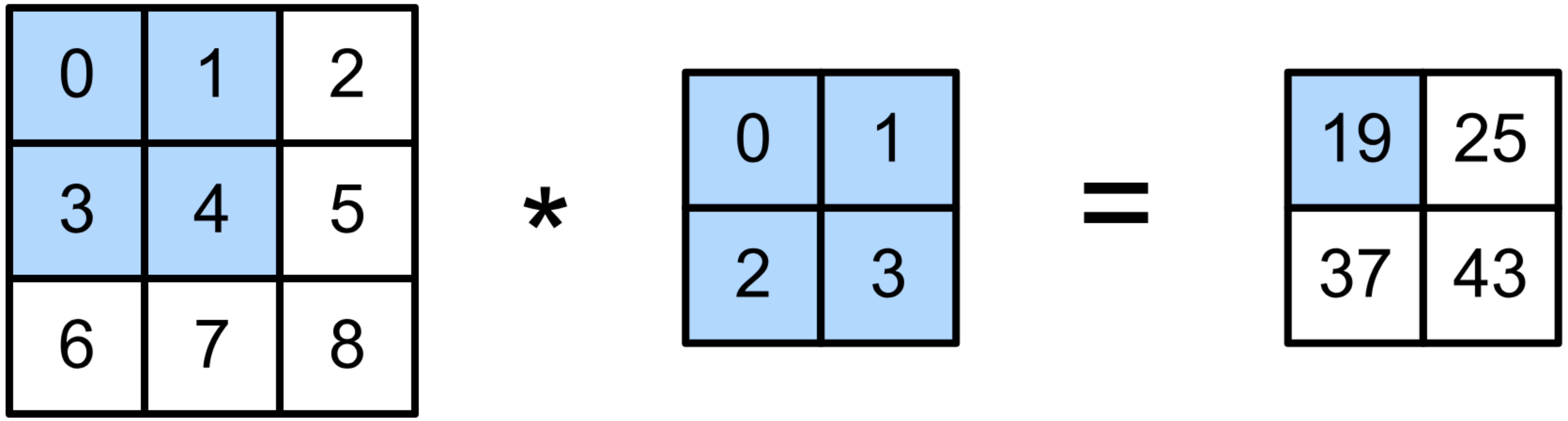


$$0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19,$$
$$1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25,$$
$$3 \times 0 + 4 \times 1 + 6 \times 2 + 7 \times 3 = 37,$$
$$4 \times 0 + 5 \times 1 + 7 \times 2 + 8 \times 3 = 43.$$



(vdumoulin@ Github)

2-D Convolution Layer



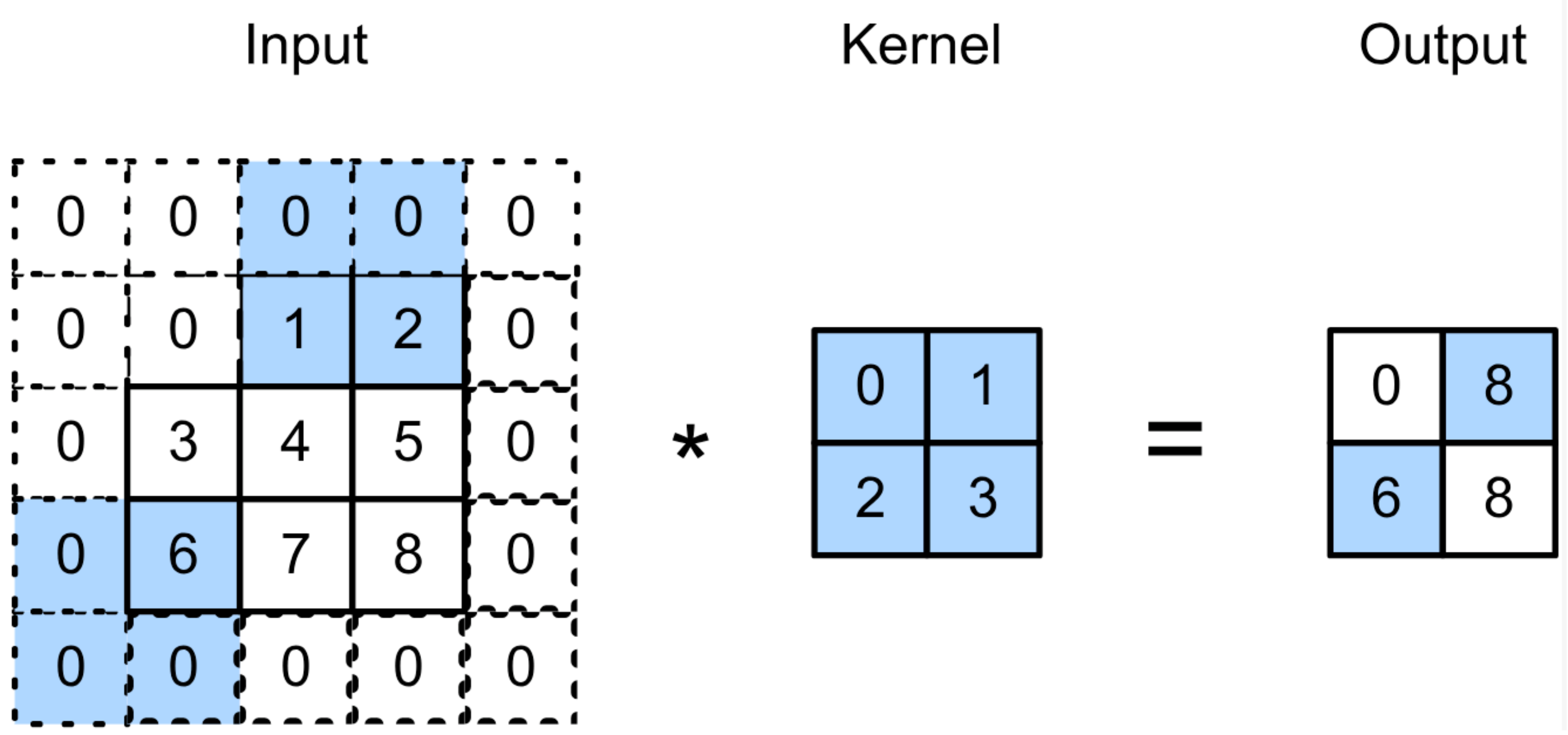
- $\mathbf{X} : n_h \times n_w$ input matrix
- $\mathbf{W} : k_h \times k_w$ kernel matrix
- b: scalar bias
- $\mathbf{Y} : (n_h - k_h + 1) \times (n_w - k_w + 1)$ output matrix

$$\mathbf{Y} = \mathbf{X} \star \mathbf{W} + b$$

- **W** and *b* are learnable parameters

2-D Convolution Layer with Stride and Padding

- Stride is the #rows/#columns per slide
- Padding adds rows/columns around input
- Output shape



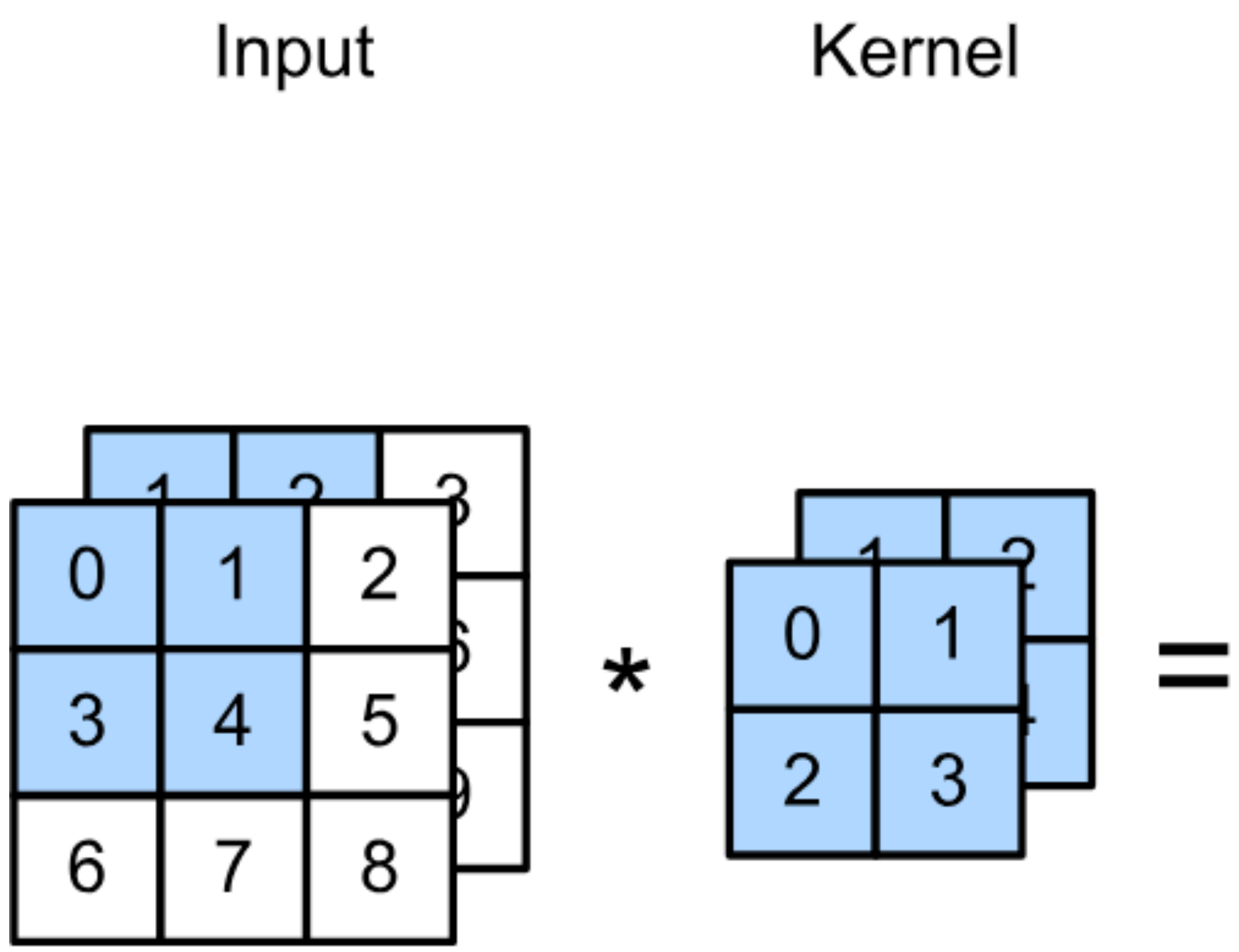
Kernel/filter size

$$\lfloor (n_h - k_h + p_h + s_h)/s_h \rfloor \times \lfloor (n_w - k_w + p_w + s_w)/s_w \rfloor$$

Input size **Pad** **Stride**

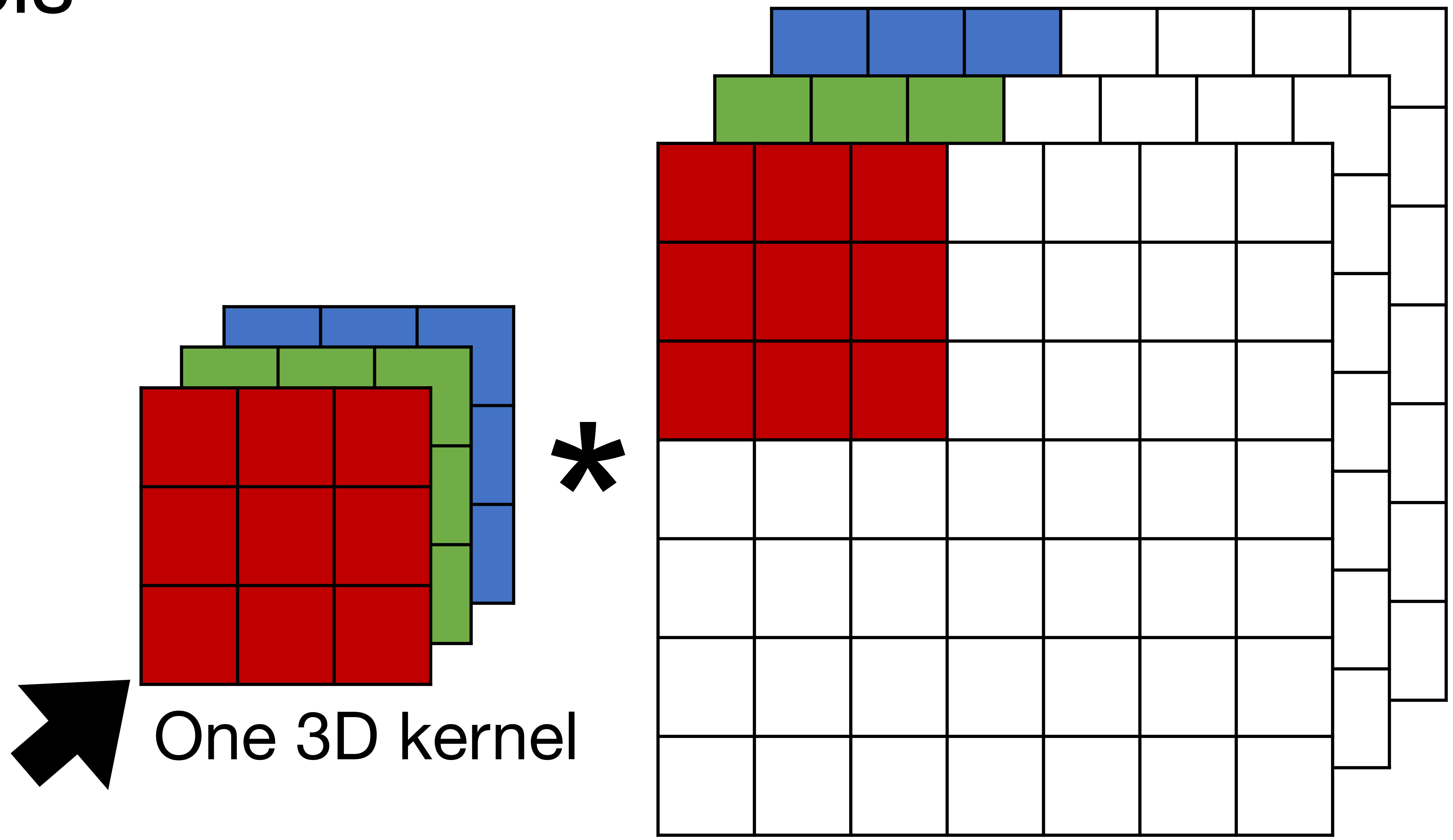
Multiple Input Channels

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a kernel for each channel, and then sum results over channels



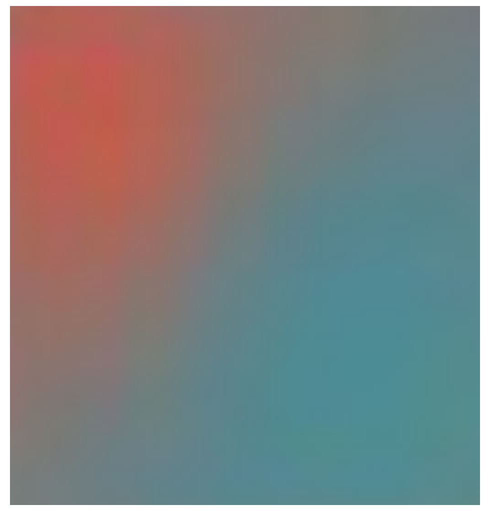
Multiple Input Channels

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a 2D kernel for each channel, and then sum results over channels



Multiple Input Channels

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Also call each 3D kernel a “**filter**”, which produce only **one** output channel (due to summation over channels)



*

One filter
(3 channels)

RGB (3 input channels)

Multiple filters (in one layer)

- Apply multiple filters on the input
- Each filter may learn different features about the input
- Each filter (3D kernel) produces one output channel

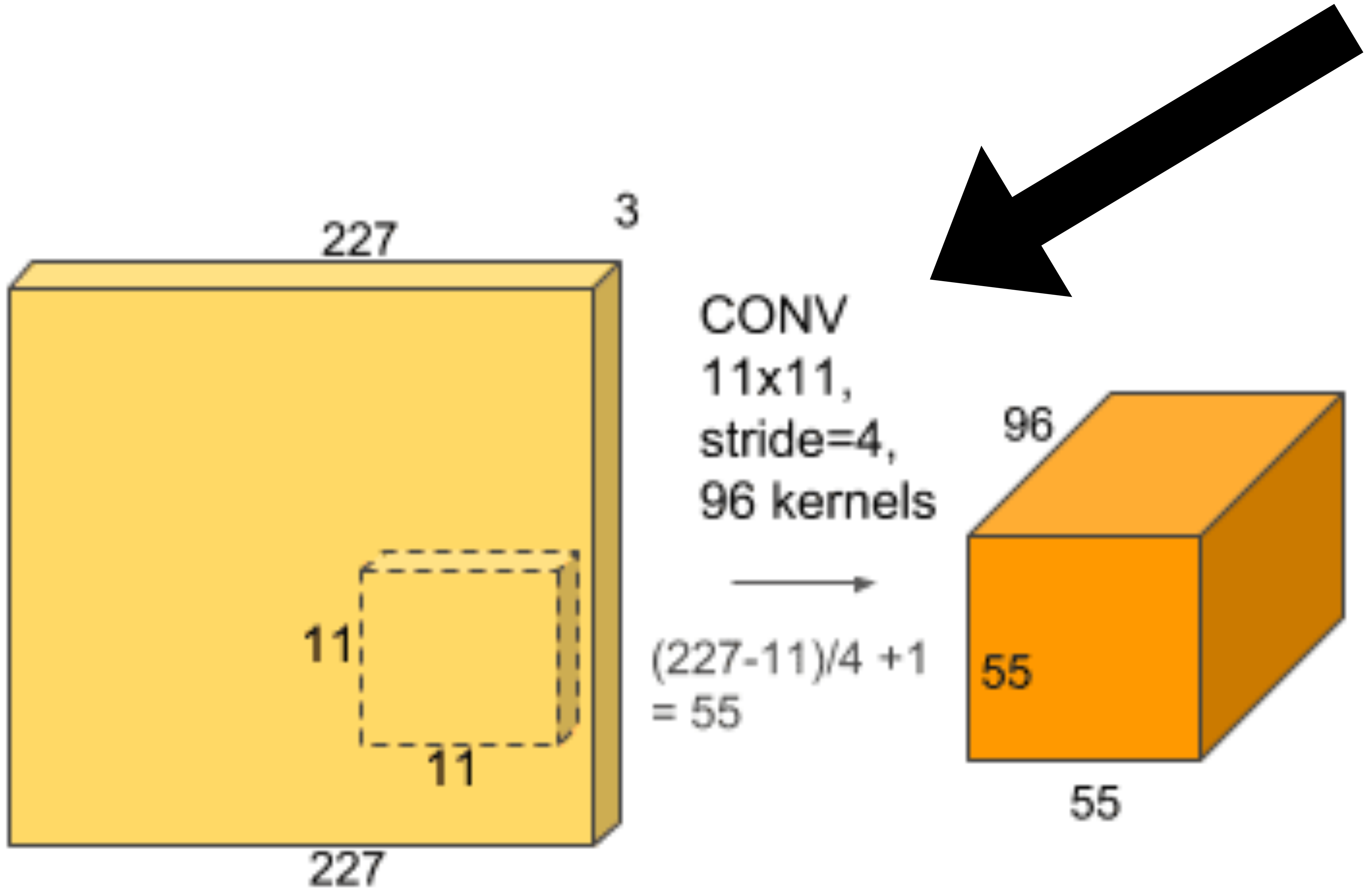
A different filter

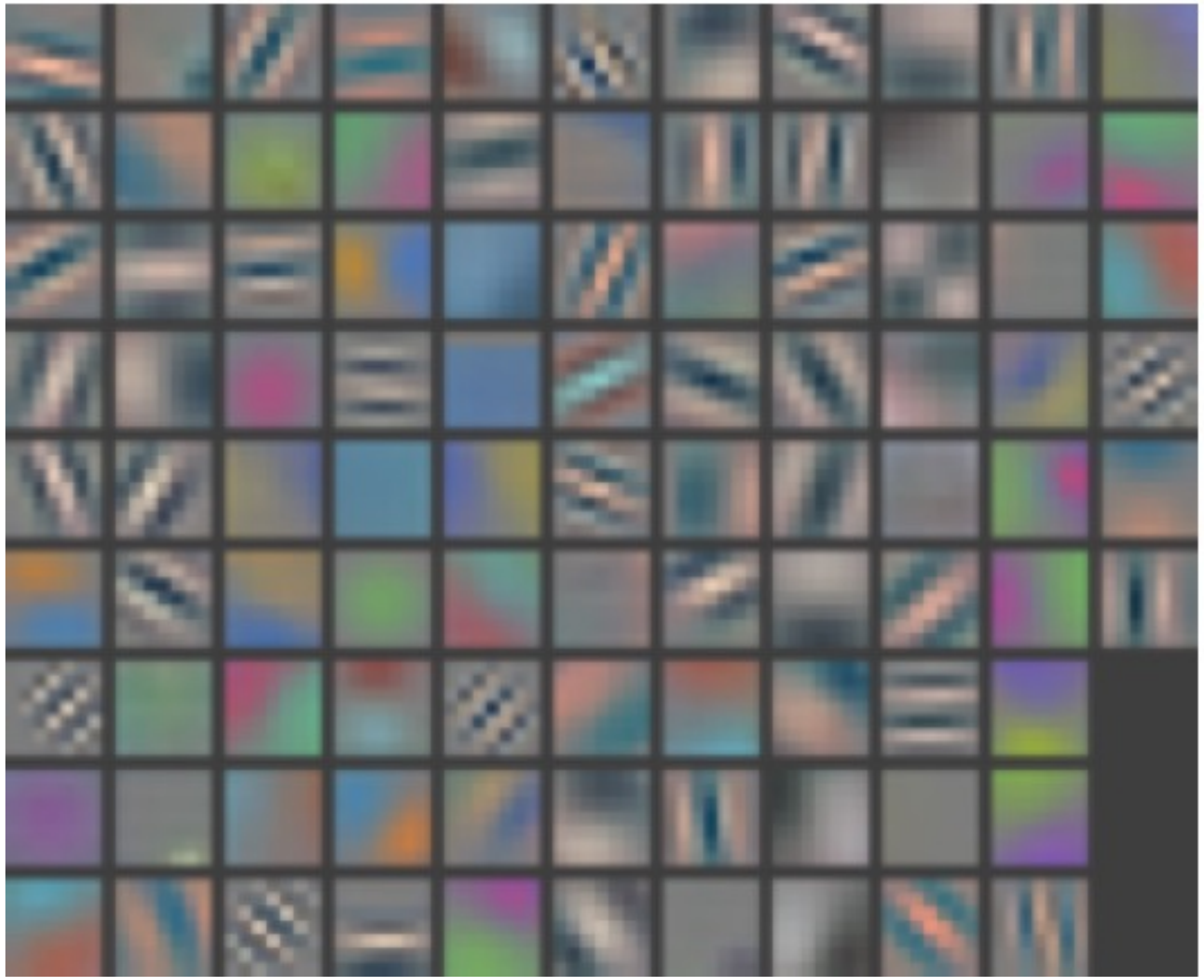
*

RGB (3 input channels)

Conv1 Filters in AlexNet

- 96 filters (each of size 11x11x3)
- Gabor filters





Figures from Visualizing and Understanding Convolutional Networks
by *M. Zeiler and R. Fergus*

Multiple Output Channels

- The # of output channels = # of filters
- Input $\mathbf{X} : c_i \times n_h \times n_w$
- Kernel $\mathbf{W} : c_o \times c_i \times k_h \times k_w$
- Output $\mathbf{Y} : c_o \times m_h \times m_w$

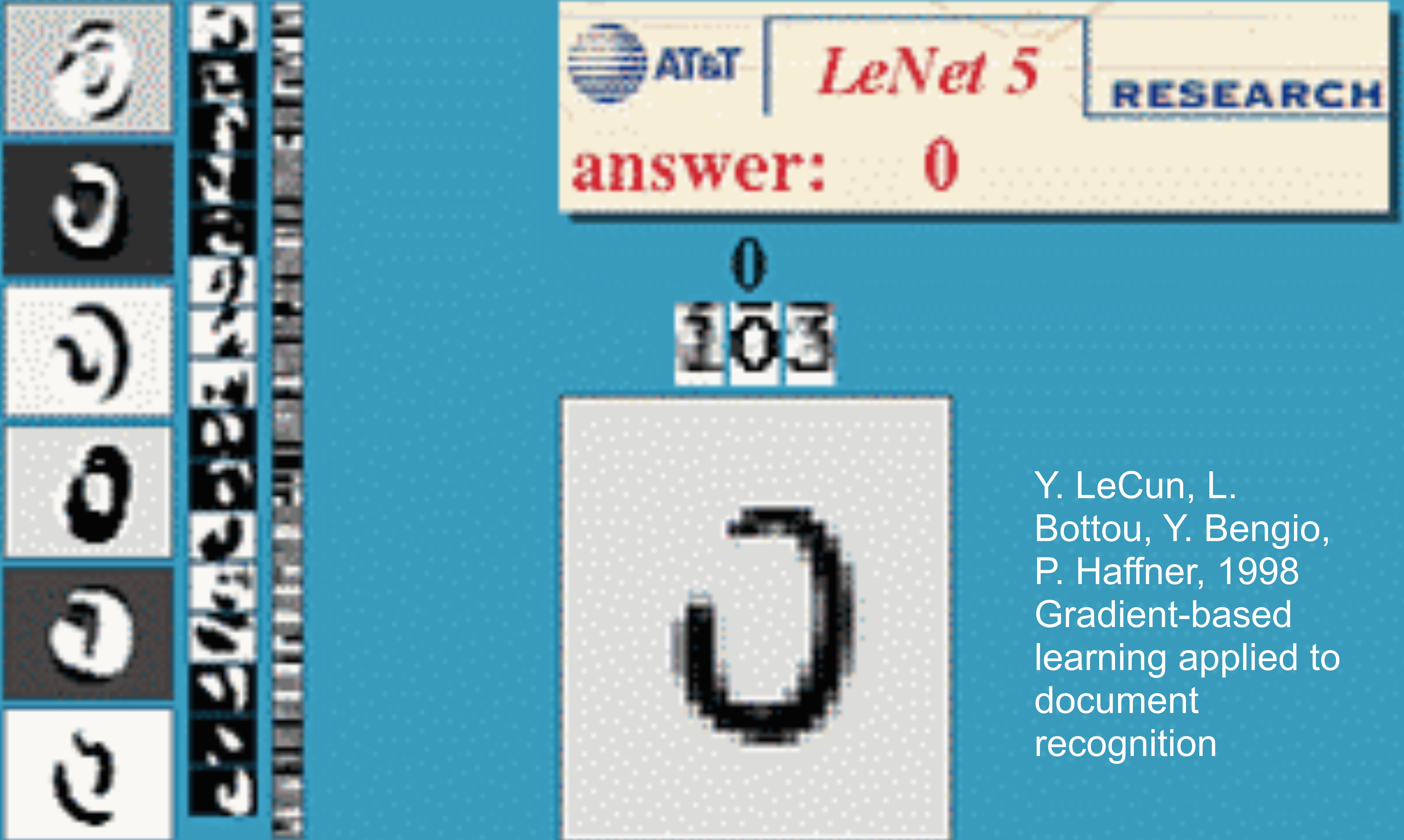
$$\mathbf{Y}_{i,:,:} = \mathbf{X} \star \mathbf{W}_{i,:,:,:}$$

for $i = 1, \ldots, c_o$

Convolutional Neural Networks

LeNet Architecture

gluon-cv.mxnet.io



Y. LeCun, L. Bottou, Y. Bengio, P. Haffner, 1998 Gradient-based learning applied to document recognition

LeNet in Pytorch (HW7)

Connect theory and practice

```
def __init__(self):
    super(LeNet5, self).__init__()
    # Convolution (In LeNet-5, 32x32 images are given as input. Hence padding of 2 is done below)
    self.conv1 = torch.nn.Conv2d(in_channels=1, out_channels=6, kernel_size=5, stride=1, padding=2, bias=True)
    # Max-pooling
    self.max_pool_1 = torch.nn.MaxPool2d(kernel_size=2)
    # Convolution
    self.conv2 = torch.nn.Conv2d(in_channels=6, out_channels=16, kernel_size=5, stride=1, padding=0, bias=True)
    # Max-pooling
    self.max_pool_2 = torch.nn.MaxPool2d(kernel_size=2)
    # Fully connected layer
    self.fc1 = torch.nn.Linear(16*5*5, 120)   # convert matrix with 16*5*5 (= 400) features to a matrix of 120 features (col
    self.fc2 = torch.nn.Linear(120, 84)       # convert matrix with 120 features to a matrix of 84 features (columns)
    self.fc3 = torch.nn.Linear(84, 10)        # convert matrix with 84 features to a matrix of 10 features (columns)
```

Quiz break

Which one of the following is NOT true?

A. LeNet has two convolutional layers

B. The first convolutional layer in LeNet has 5x5x6x3 parameters, in case of RGB input

C. Pooling is performed right after convolution

D. Pooling layer does not have learnable parameters

Quiz break

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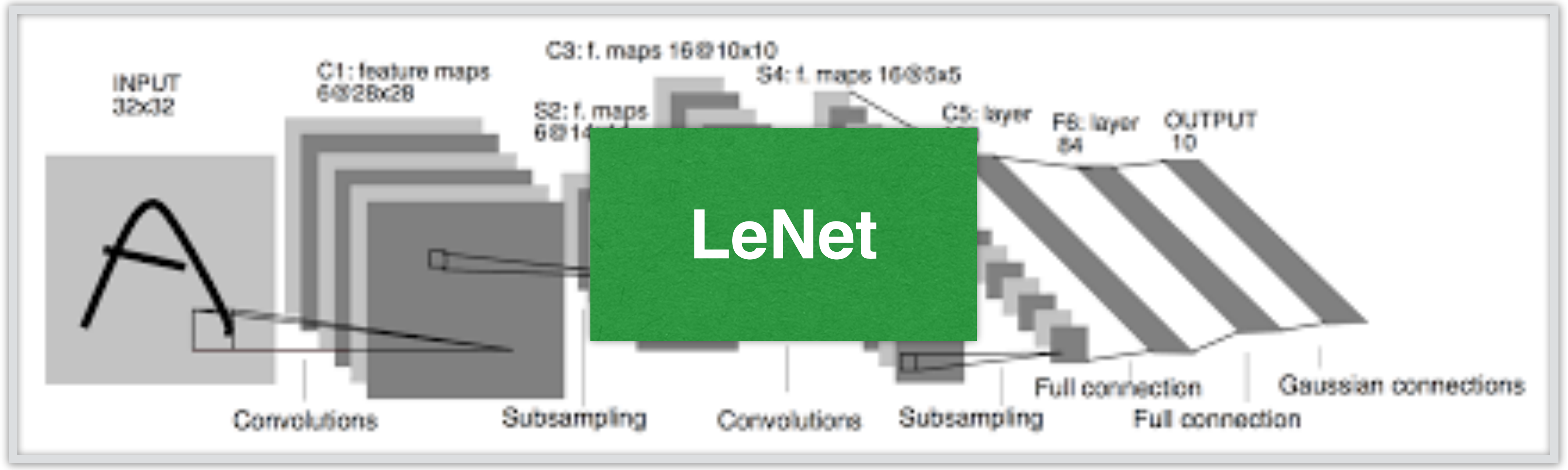
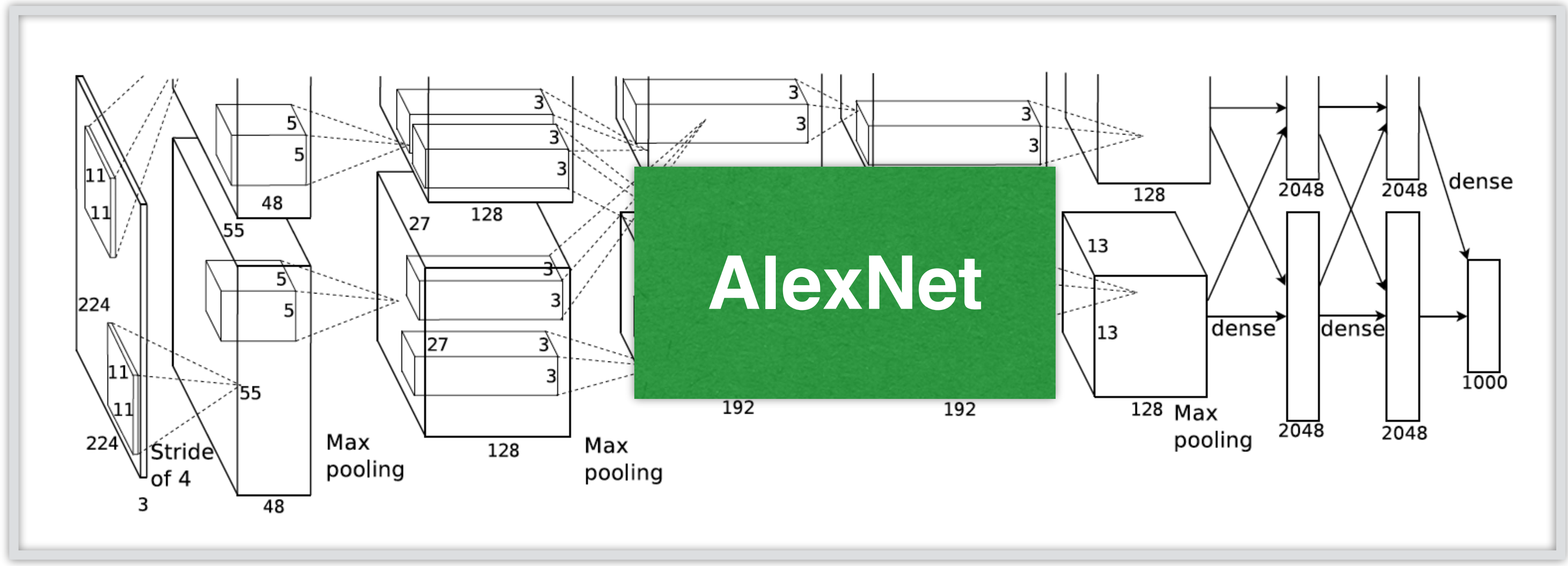
C. Pooling is performed right after convolution

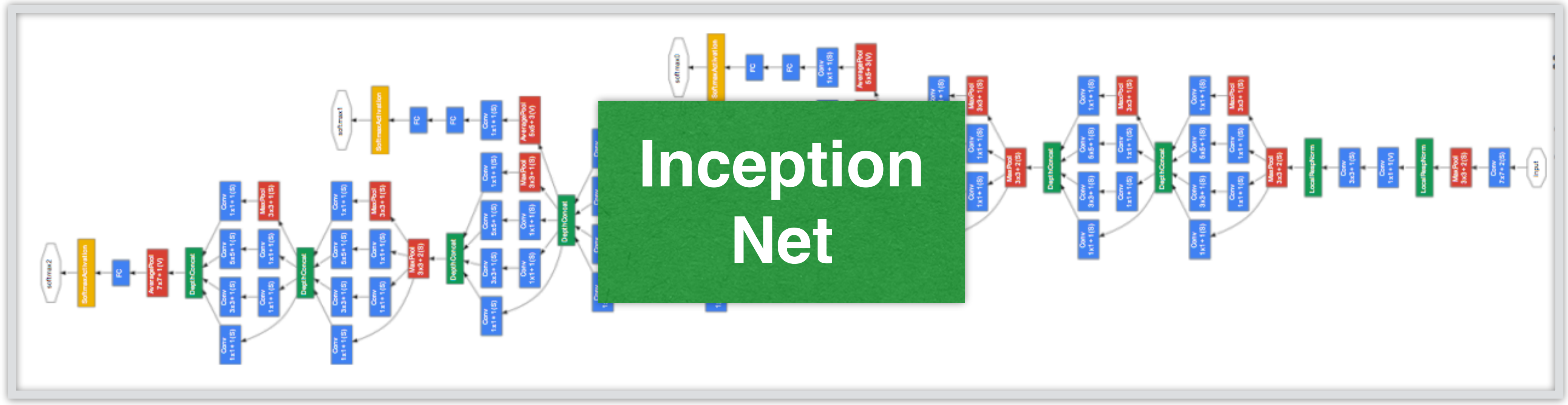
D. Pooling layer does not have learnable parameters

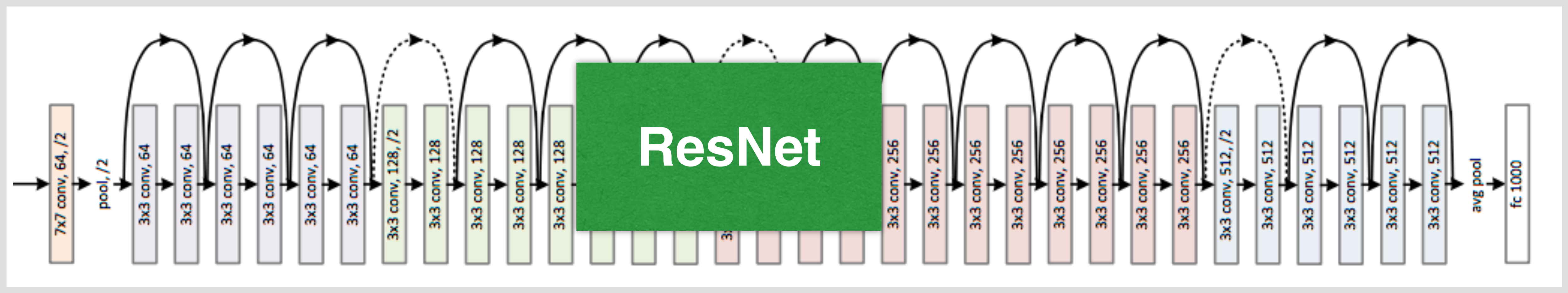
Pooling is performed after ReLU: conv->relu->pooling

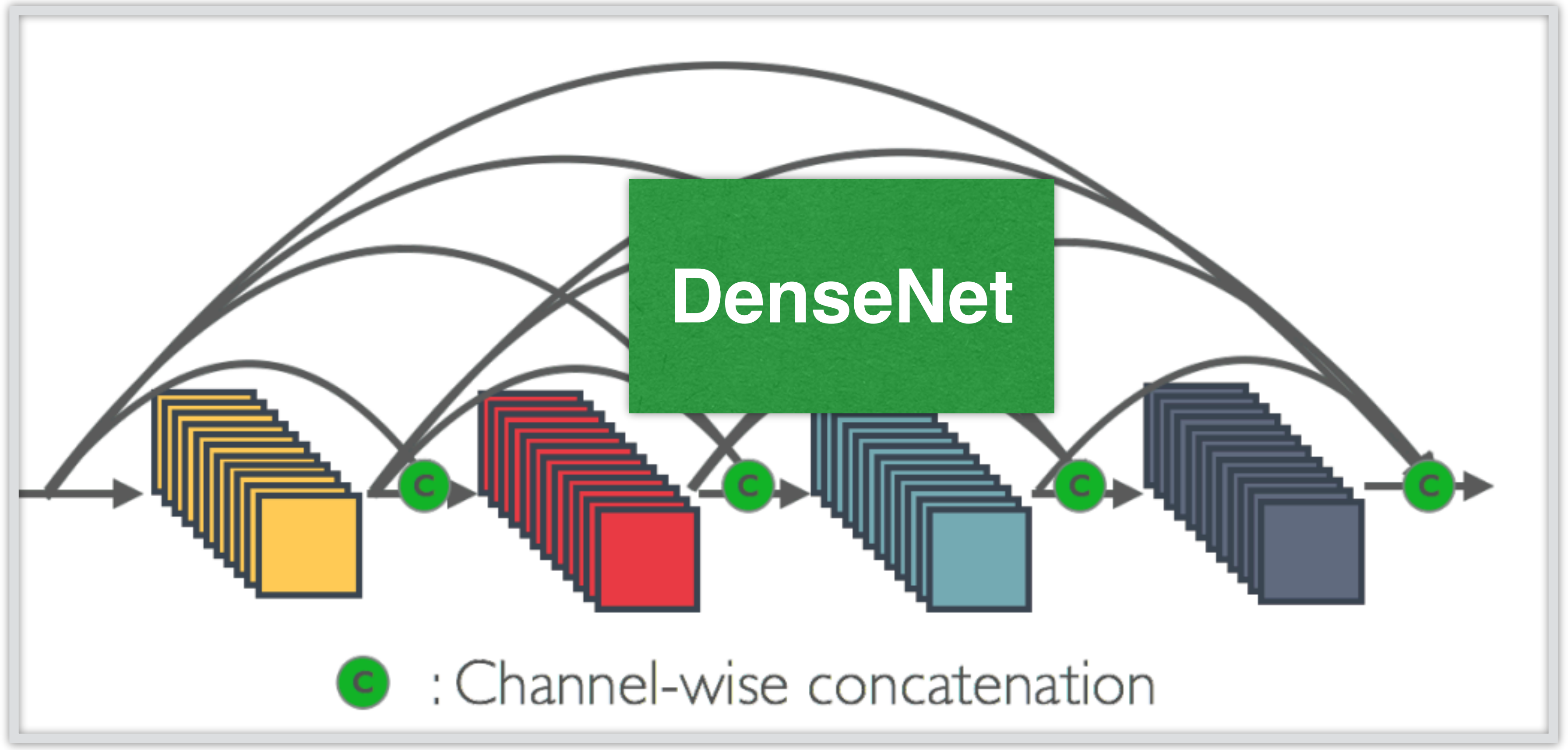
Evolution of neural net architectures





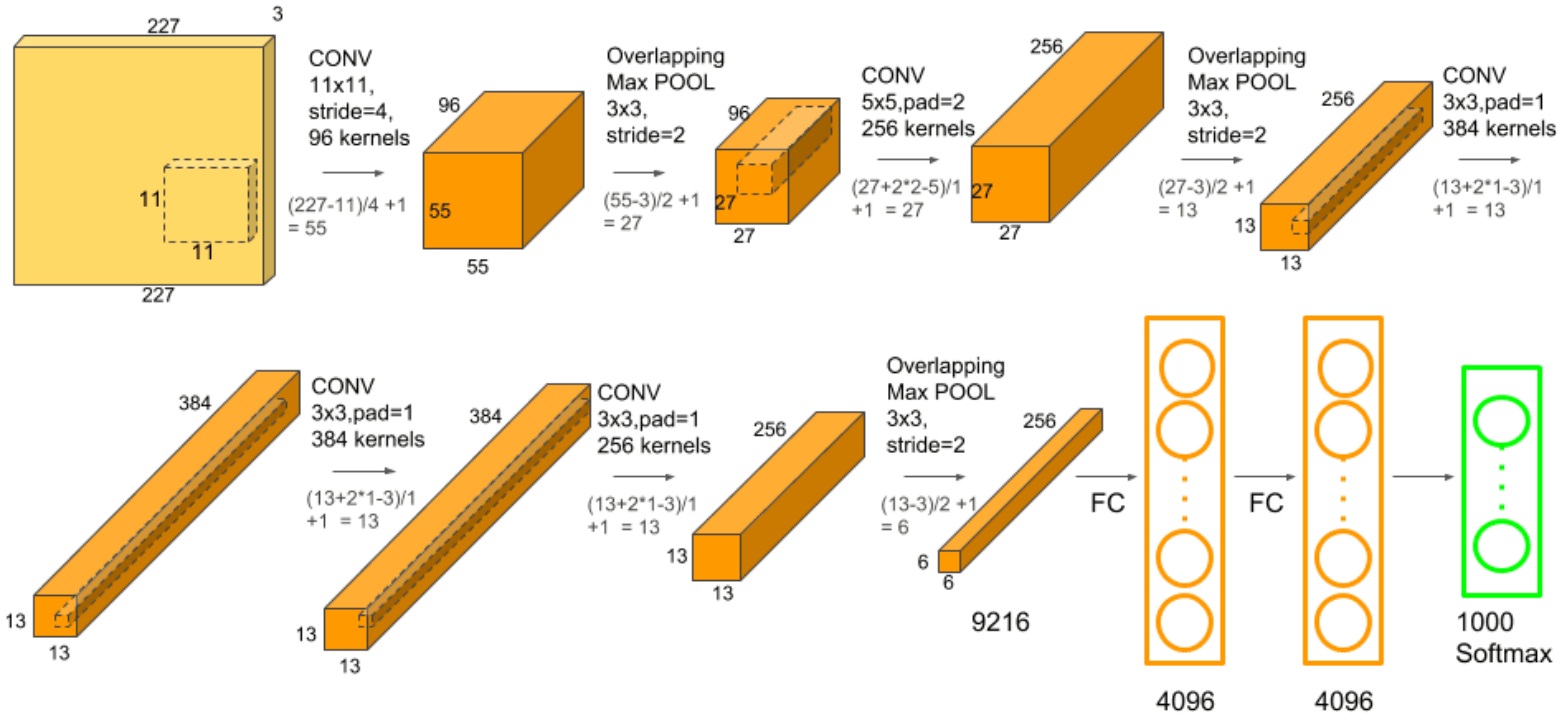






Deng et al. 2009

AlexNet

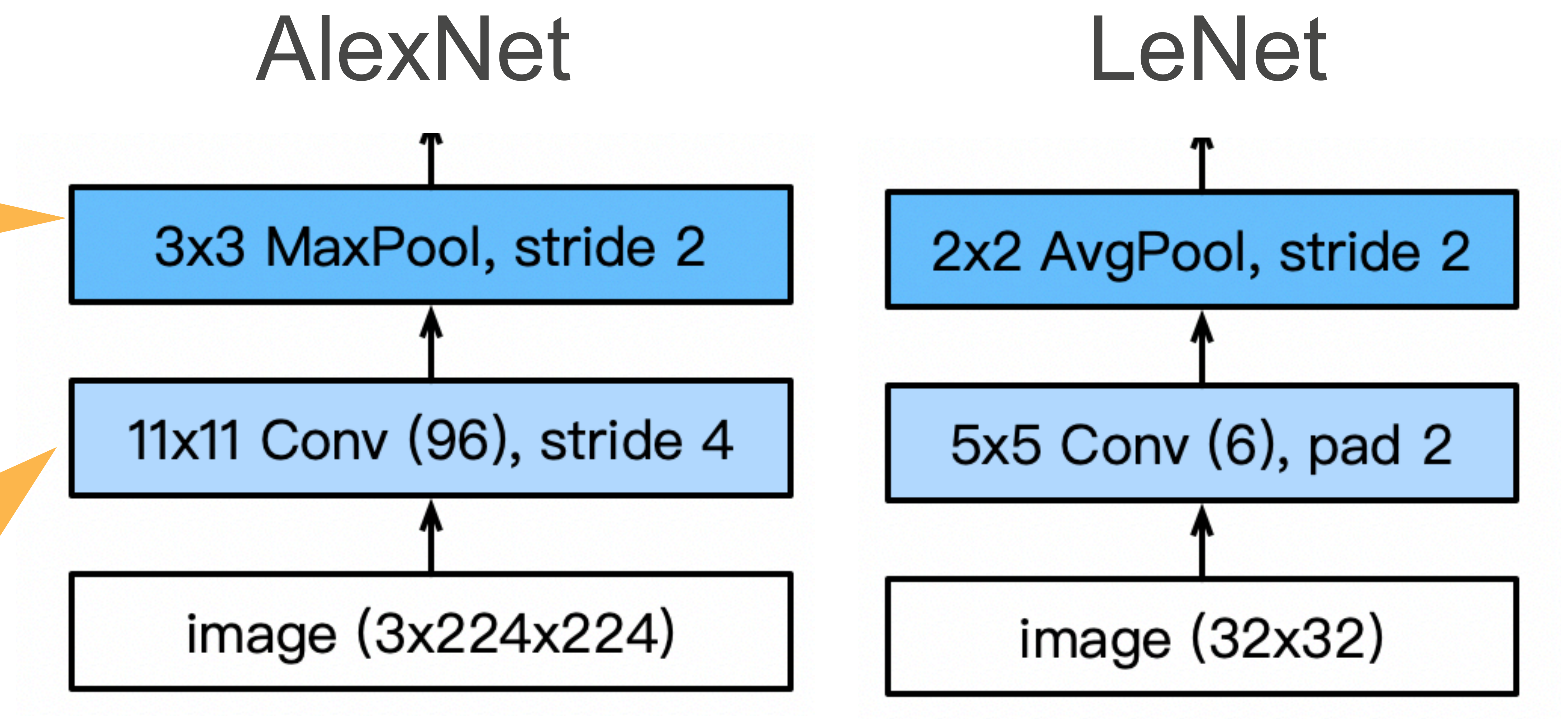


[Krizhevsky et al. 2012]

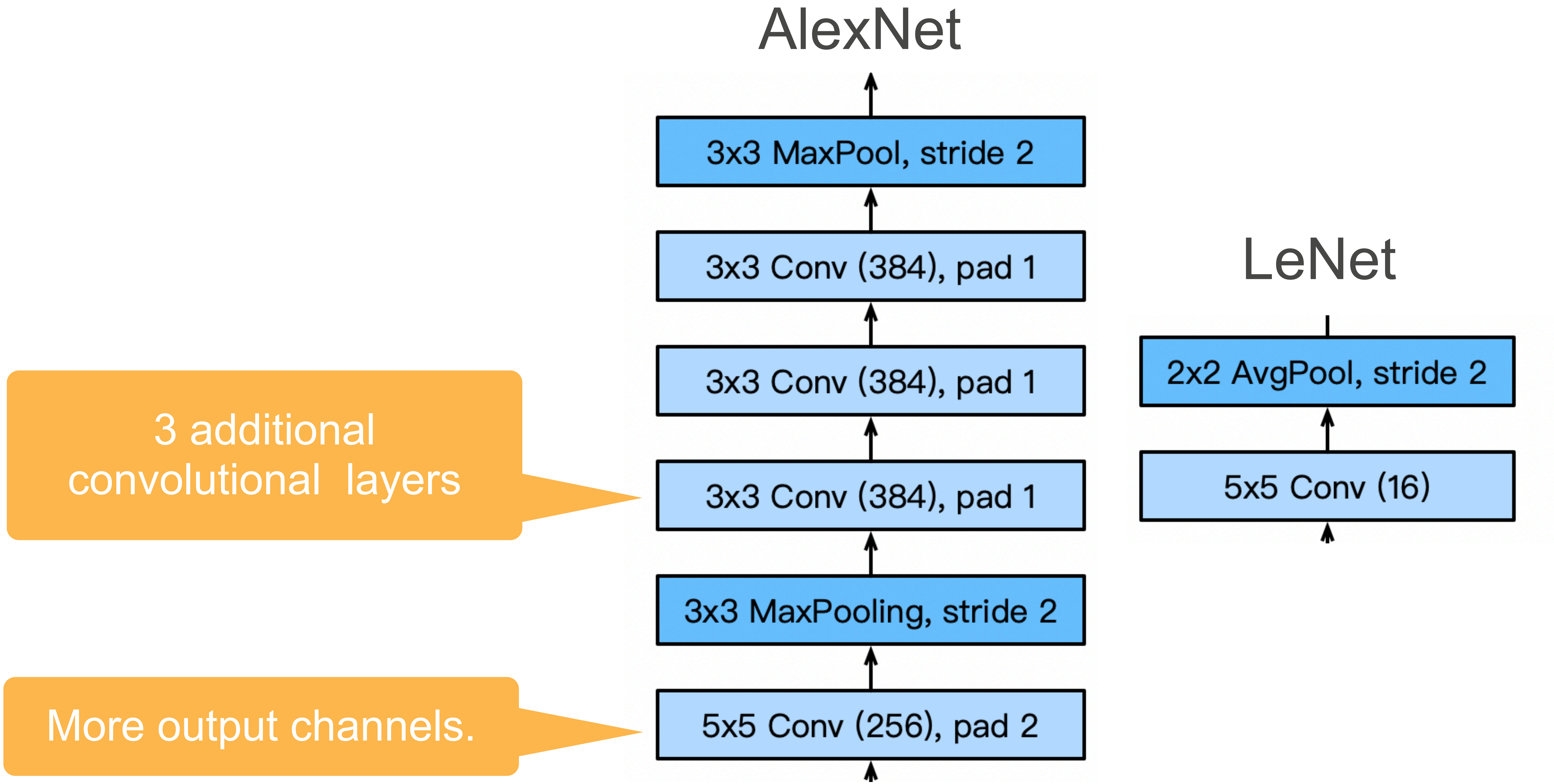
AlexNet vs LeNet Architecture

Larger pool size, change to max pooling

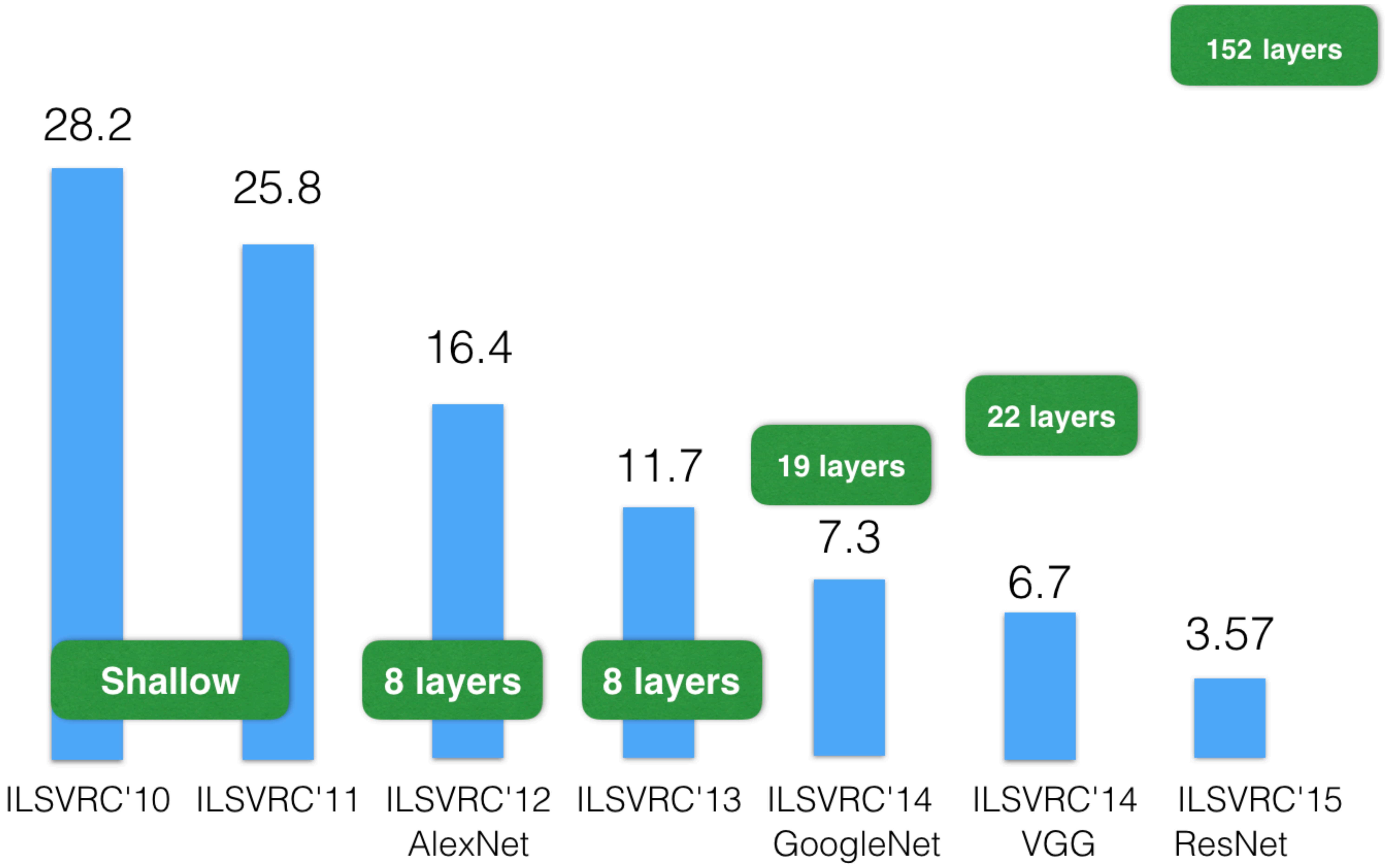
Larger kernel size, stride because of the increased image size, and more output channels.



AlexNet Architecture



ResNet: Going deeper in depth



ImageNet Top-5 error%

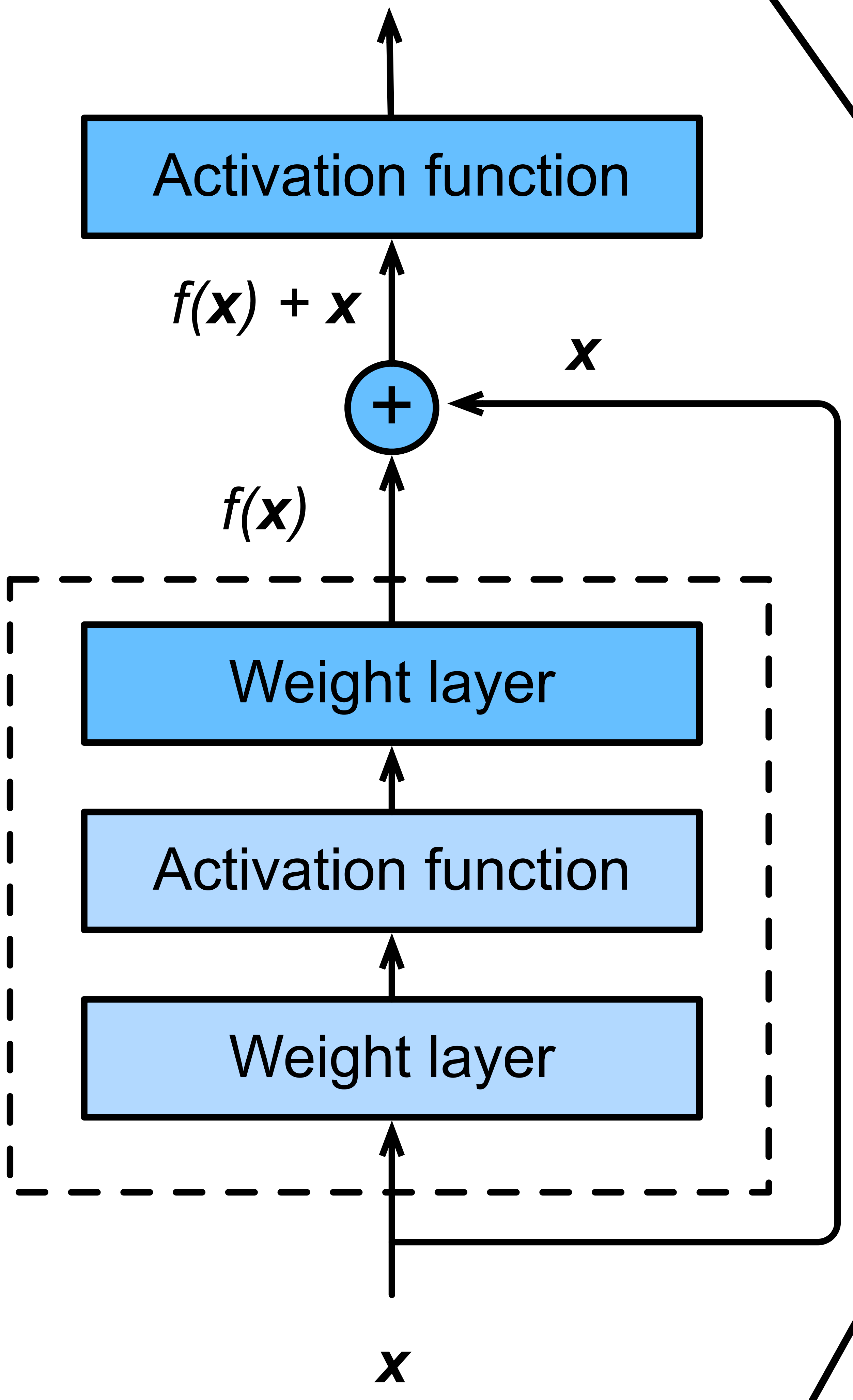
[He et al. 2015]

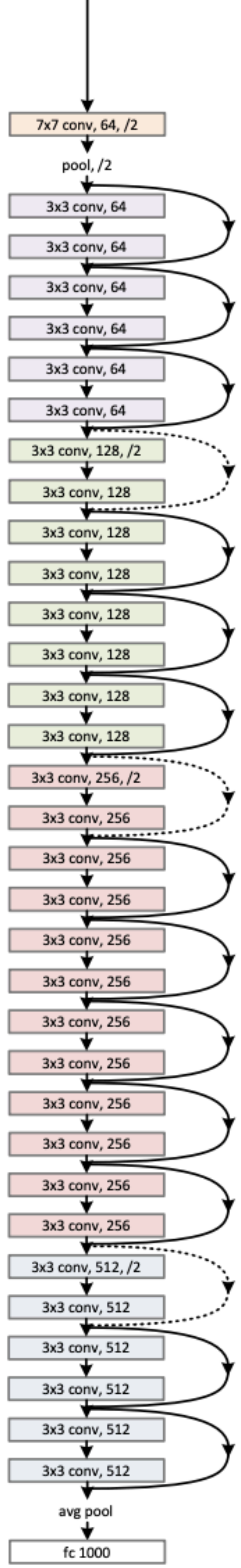
Full ResNet Architecture

[He et al. 2015]

- Stack residual blocks
- Every residual block has two 3x3 conv layers
- Periodically, double # of filters and downsample spatially using stride of 2 (/2 in each dimension)

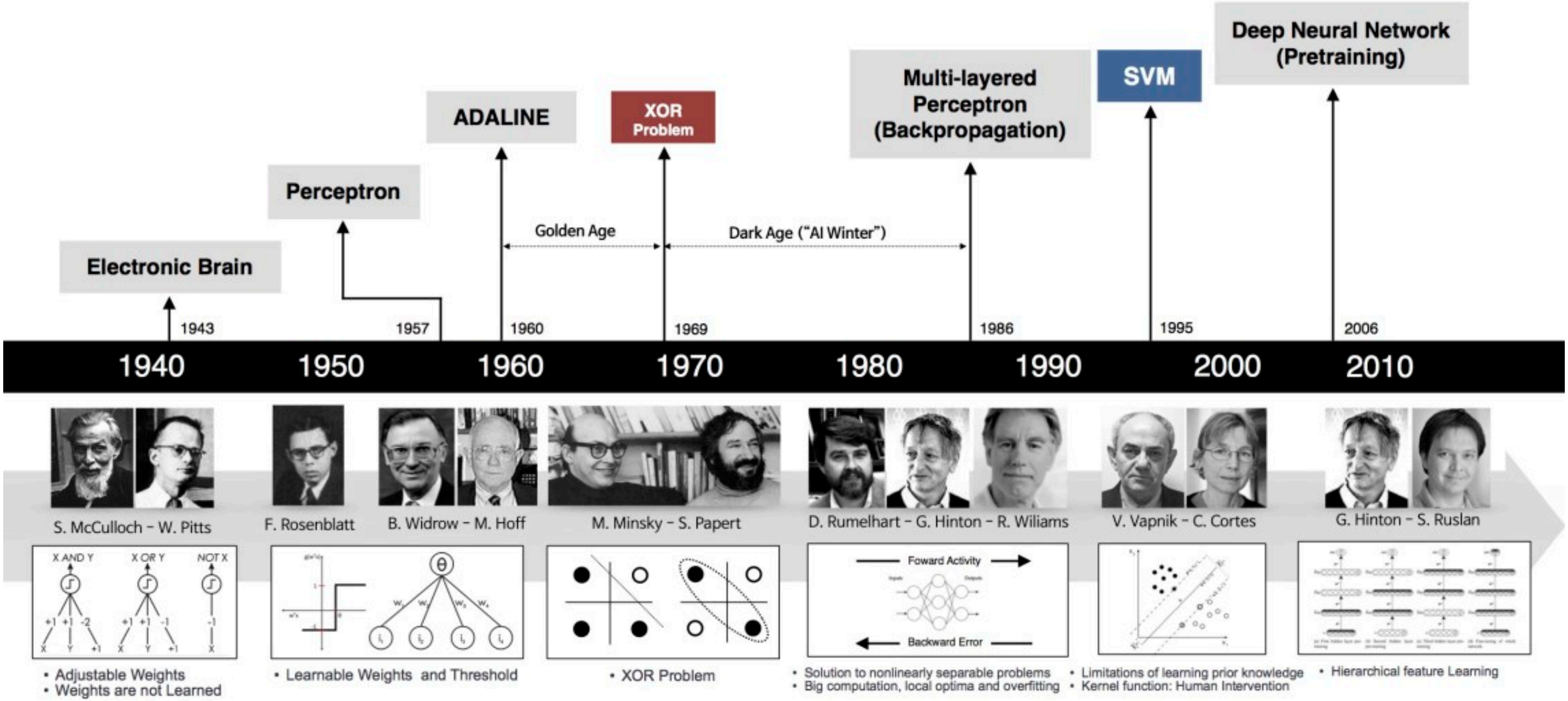
[More advanced topics covered in CS762]





(Figure from Stanford CS231n)

Brief history of neural networks



What we've learned today…

- Modeling a single neuron
  - Linear perceptron
  - Limited power of a single neuron
- Multi-layer perceptron
- Training of neural networks
  - Loss function (cross entropy)
  - Backpropagation and SGD
- Convolutional neural networks
  - Convolution, pooling, stride, padding
  - Basic architectures (LeNet etc.)
  - More advanced architectures (AlexNet, ResNet etc)

Thank you!

Some of the slides in these lectures have been adapted from materials developed by Alex Smola and Mu Li:
https://courses.d2l.ai/berkeley-stat-157/index.html