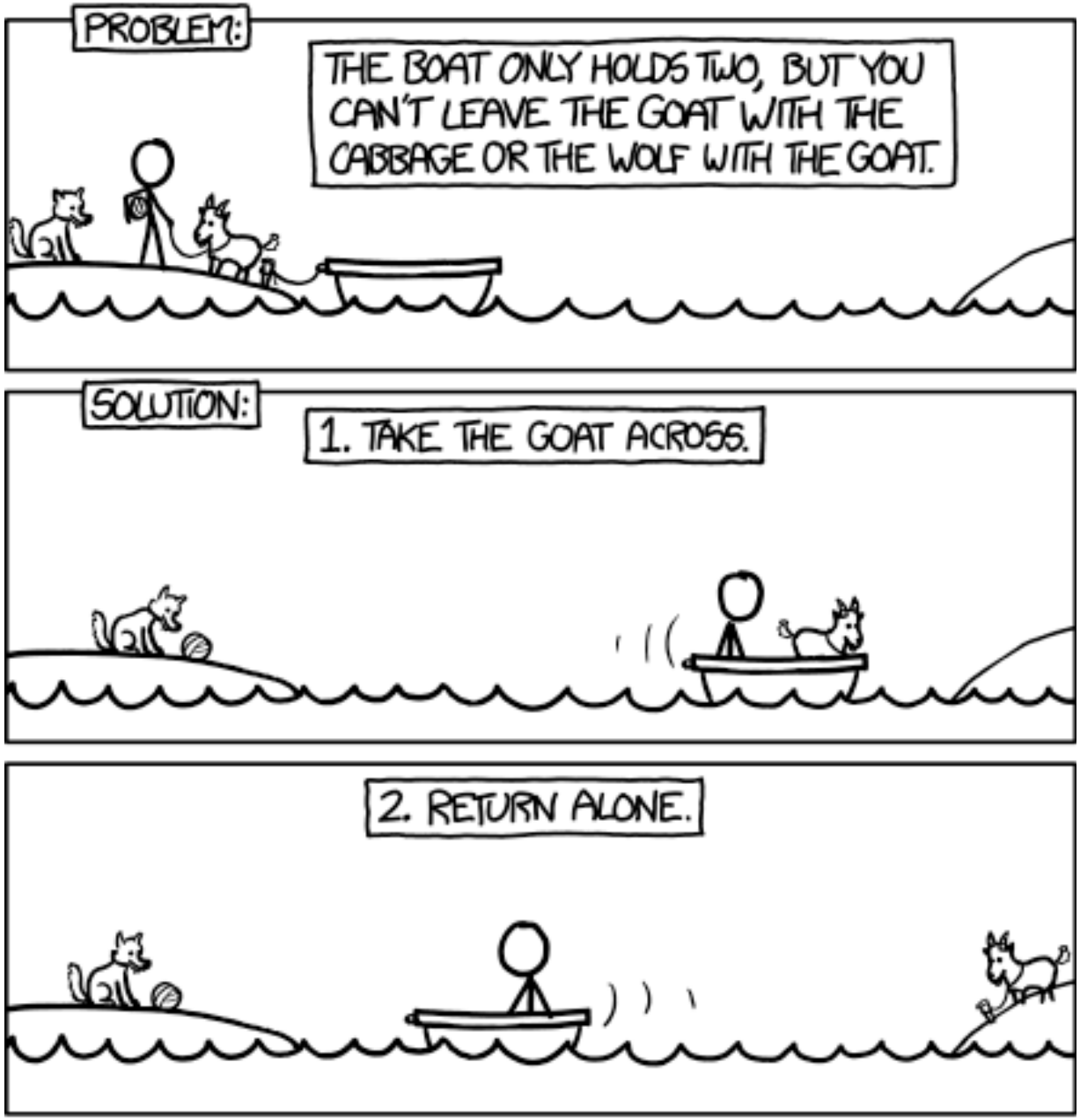
CS540 Intro to AI
Uninformed Search

University of Wisconsin-Madison

Fall 2022

Many AI problems can be formulated as search.



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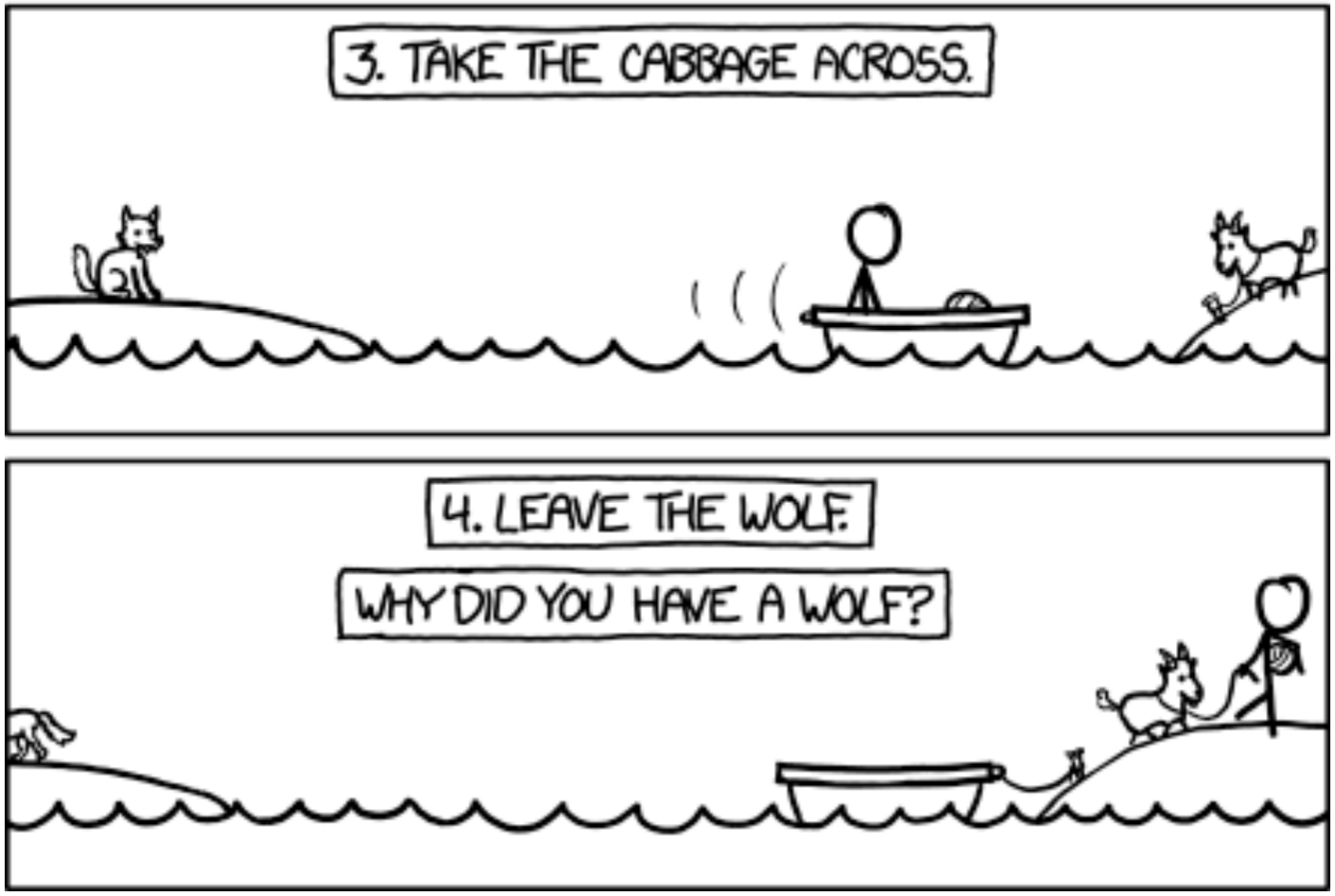
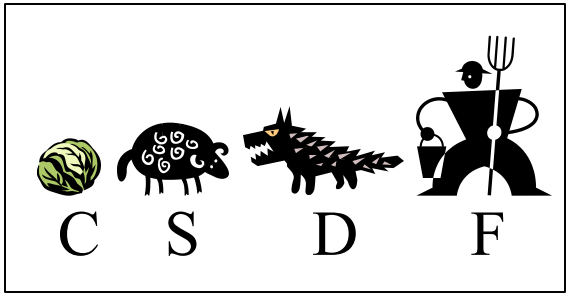
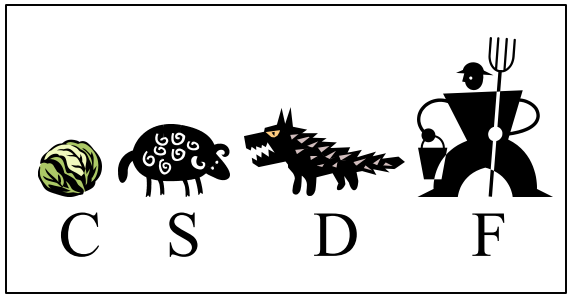


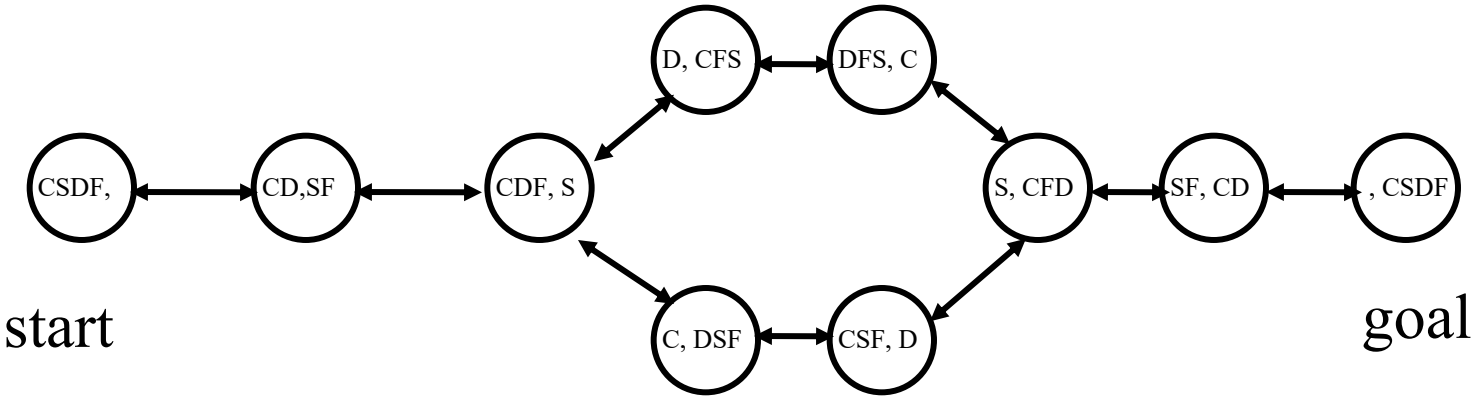
The search problem



- State space S : all valid configurations
- Initial state I={(CSDF,)} $\subseteq S$
- Goal state G={(,CSDF)} $\subseteq S$
- Successor function $succs$(s)$\subseteq S$: states reachable in one step from state s
  - $succs$((CSDF,)) = {(CD, SF)}
  - $succs$((CDF,S)) = {(CD,FS), (D,CFS), (C, DFS)}
- Cost(s,s')=1 for all steps. (weighted later)
- The search problem: find a solution path from a state in I to a state in G.
  - Optionally minimize the cost of the solution.

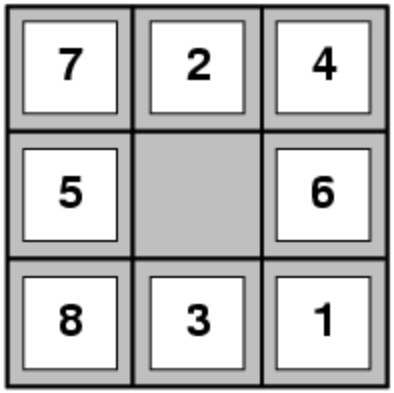
A directed graph in state space



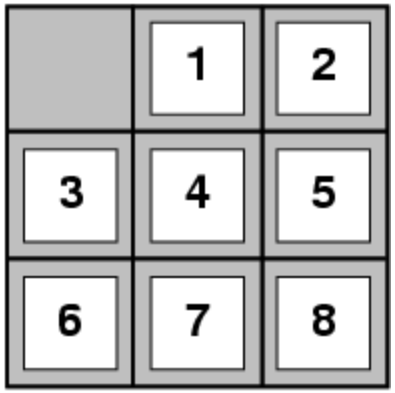


Search examples

- 8-puzzle



Start State

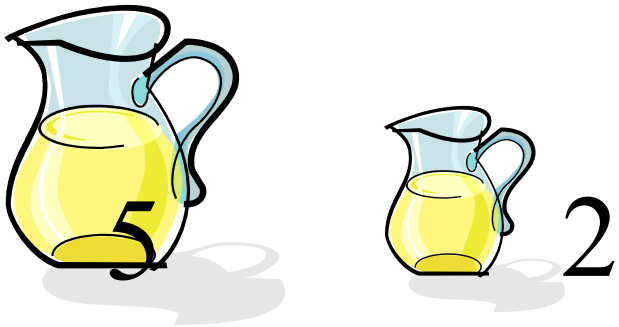


Goal State

- States = 3x3 array configurations
- action = up to 4 kinds of movement
- Cost = 1 for each move

Search examples

- Water jugs: how to get 1?



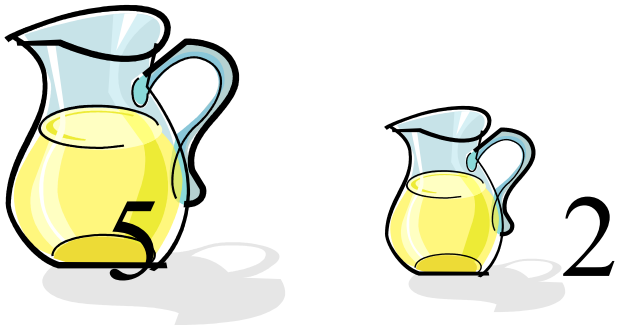
State = (x,y), where x = number of gallons of water in the 5-gallon jug and y is gallons in the 2-gallon jug

Initial State = (5,0)

Goal State = (*,1), where * means any amount

Search examples

- Water jugs: how to get 1?



State = (x,y), where x = number of gallons of water in the 5-gallon jug and y is gallons in the 2-gallon jug

Initial State = (5,0)

Goal State = (*,1), where * means any amount

Operators

(x,y) -> (0,y) ; empty 5-gal jug

(x,y) -> (x,0) ; empty 2-gal jug

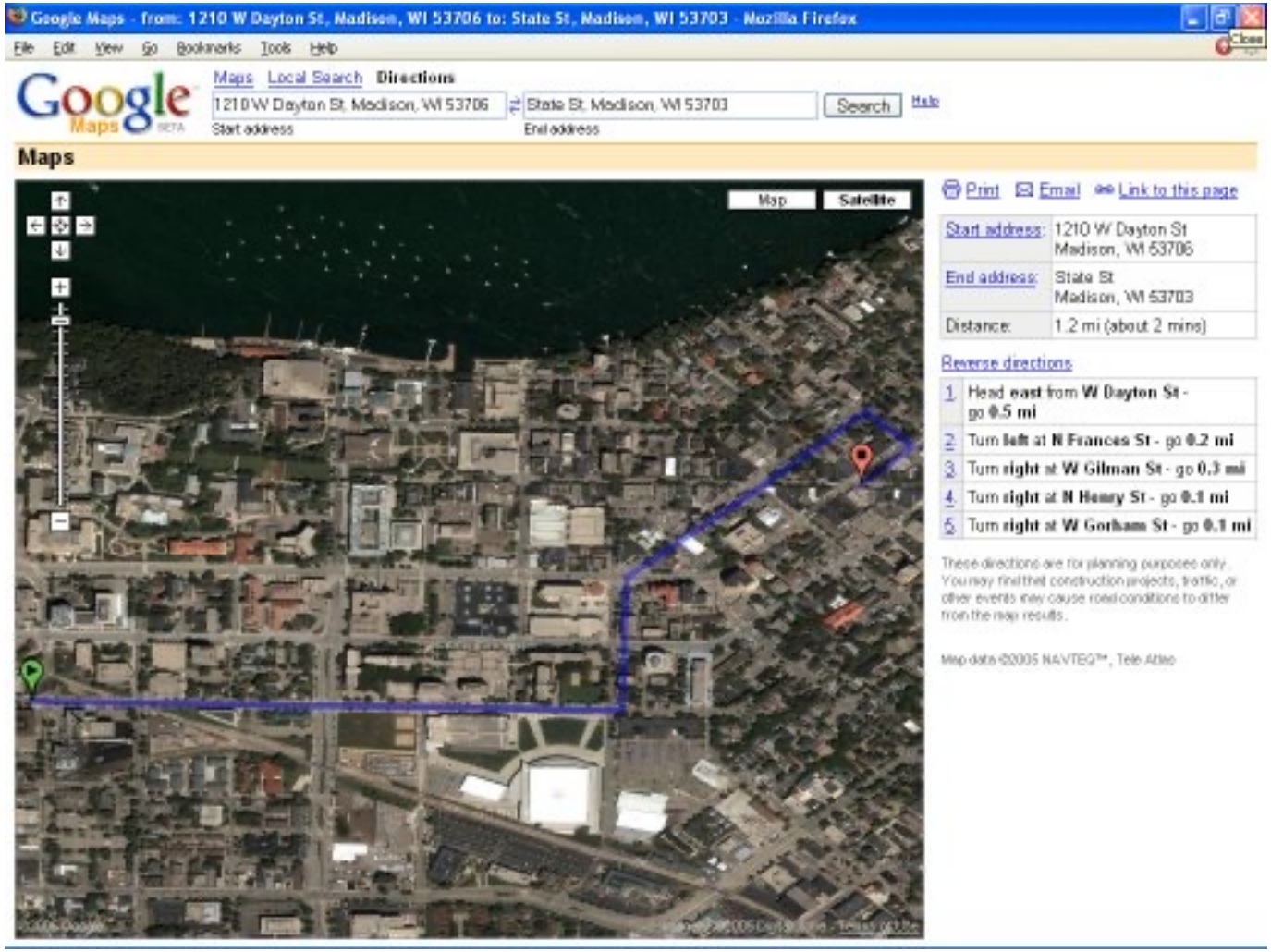
(x,2) and x<=3 -> (x+2,0) ; pour 2-gal into 5-gal

(x,0) and x>=2 -> (x-2,2) ; pour 5-gal into 2-gal

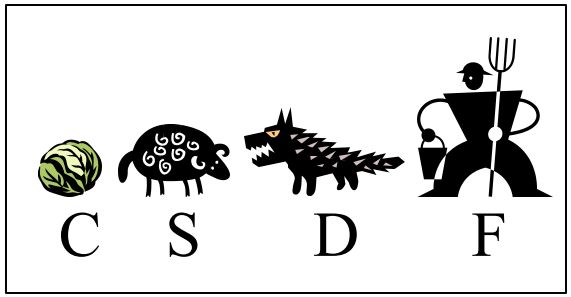
(1,0) -> (0,1) ; empty 5-gal into 2-gal

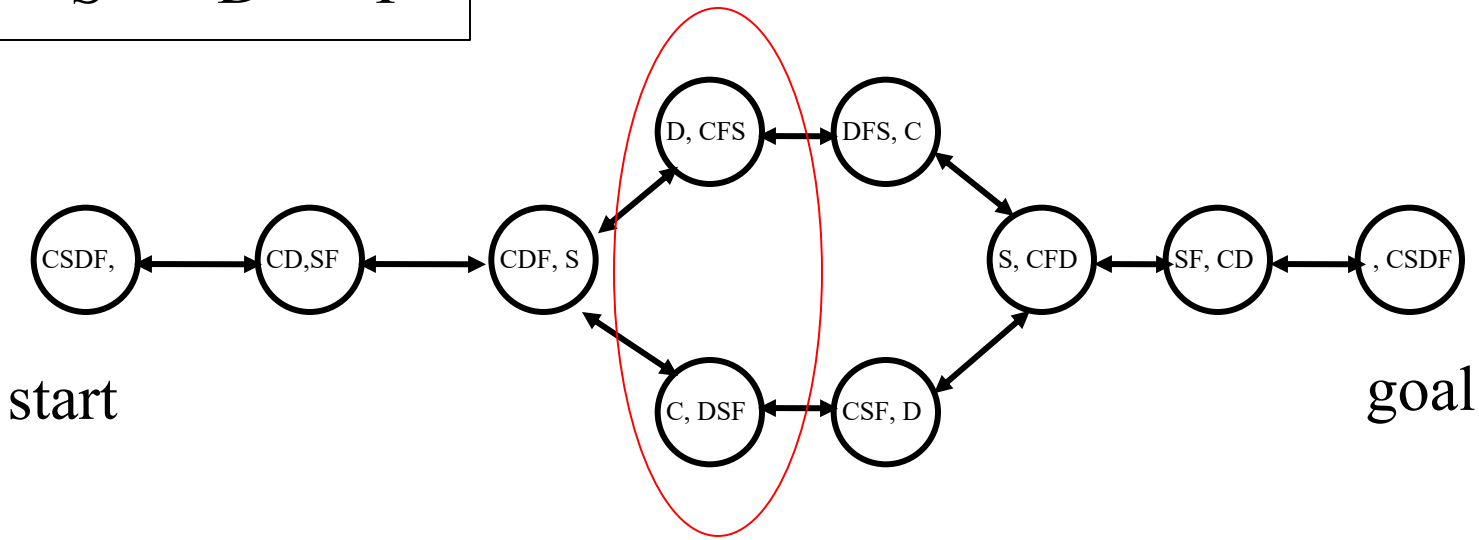
Search examples

- Route finding (State? Successors? Cost weighted)



A directed graph in state space

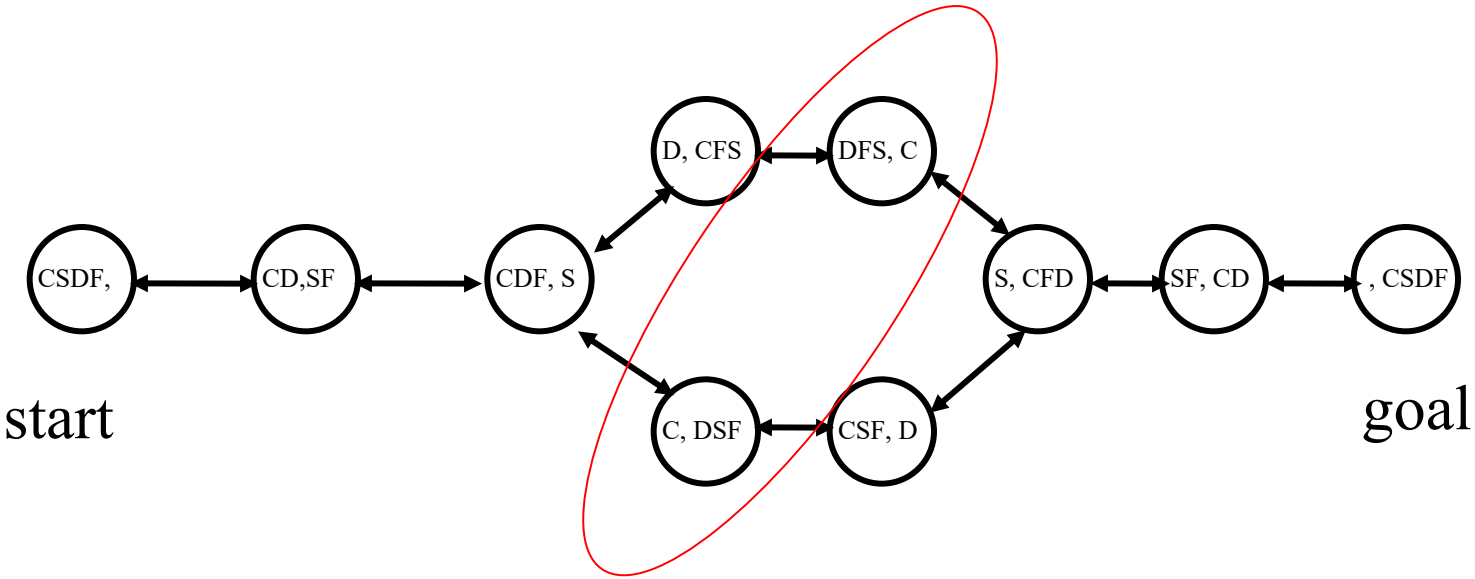




- In general there will be many generated, but un-expanded states at any given time
- One has to choose which one to expand next

Different search strategies

- The generated, but not yet expanded states form the fringe (OPEN).
- The essential difference is which one to expand first.
- Deep or shallow?



Uninformed search on trees

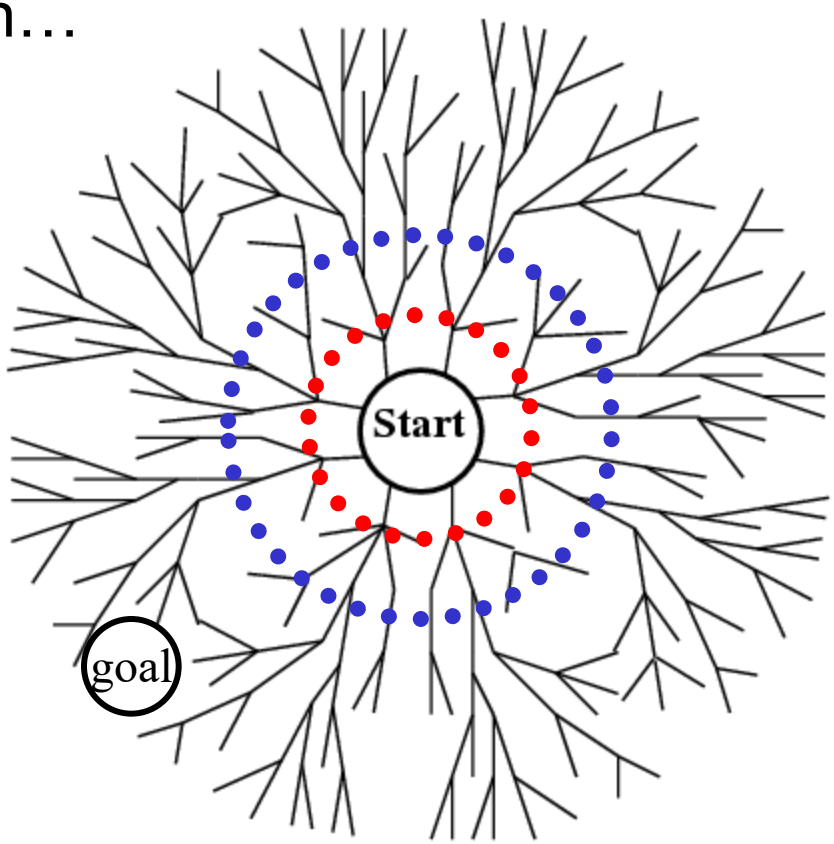
- Uninformed means we only know:
  - The goal test
  - The ***succs***() function
- But not which non-goal states are better: that would be informed search (next topic).
- For now, we also assume ***succs***() graph is a tree.
  - Won't encounter repeated states.
  - We will relax it later.
- Search strategies: BFS, UCS, DFS, IDS
- Differ by what un-expanded nodes to expand

Breadth-first search (BFS)

Expand the shallowest node first

- Examine states **one** step away from the initial states
- Examine states **two** steps away from the initial states
- and so on…

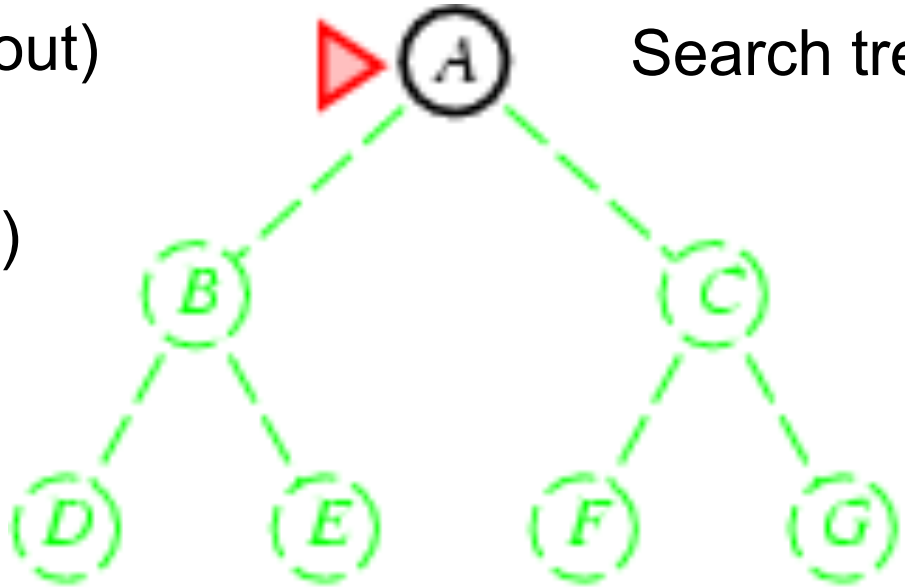
ripple



Breadth-first search (BFS)

Use a queue (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endWhile



Search tree

Initial state: **A**
Goal state: **G**

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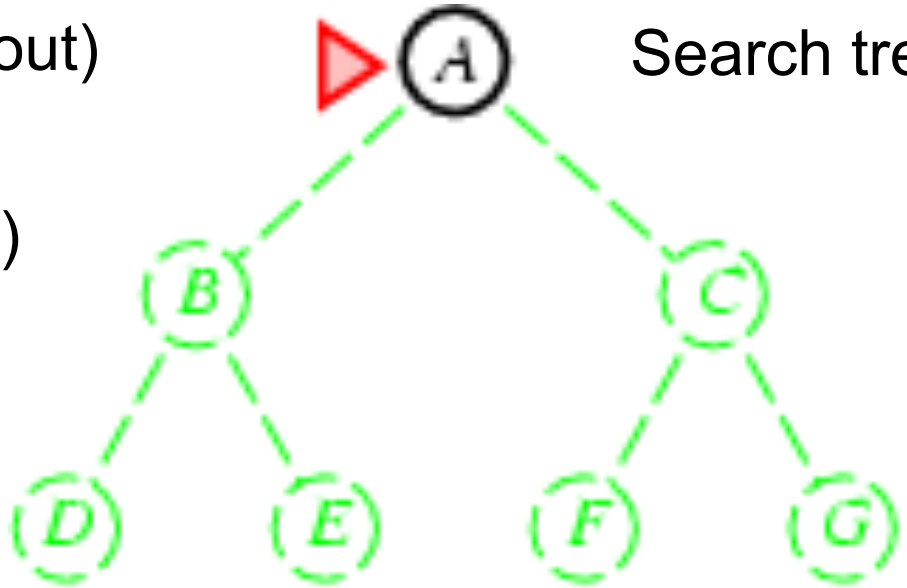

Search tree

queue (fringe, OPEN)
→ [A] →

Initial state: **A**
Goal state: **G**

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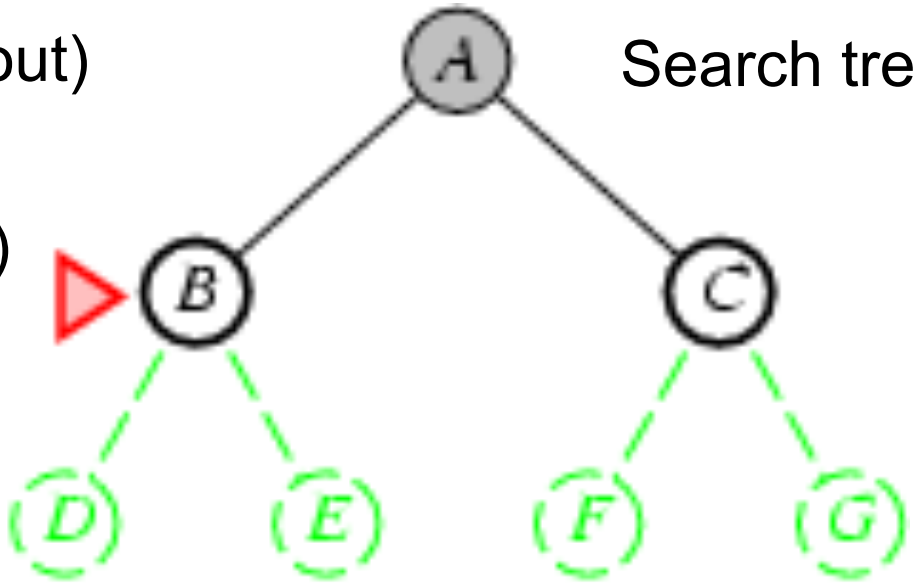

Search tree

queue (fringe, OPEN)
→ [CB] → A

Initial state: **A**
Goal state: **G**

Breadth-first search (BFS)

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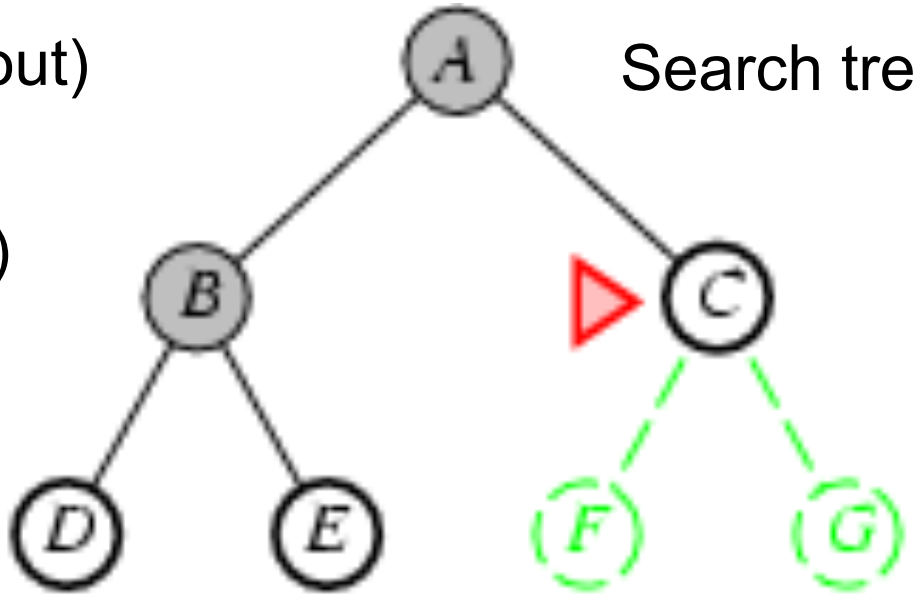

Search tree

queue (fringe, OPEN)
→ [EDC] → B

Initial state: **A**
Goal state: **G**

Breadth-first search (BFS)

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Initial state: **A**
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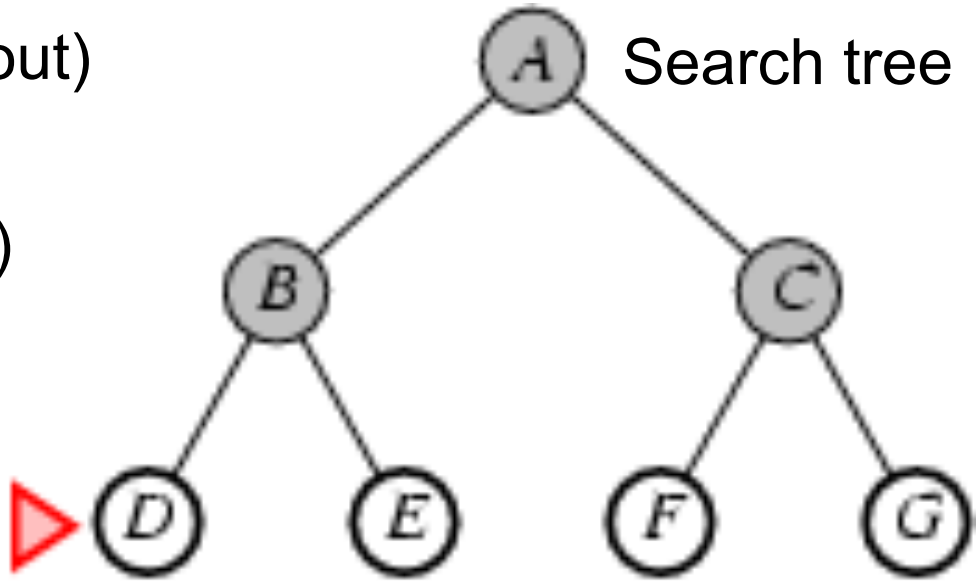

Search tree

queue (fringe, OPEN)
→[GFED] → C

If G is a goal, we've seen it, but we don't stop!

Breadth-first search (BFS)

Use a queue (First-in First-out)

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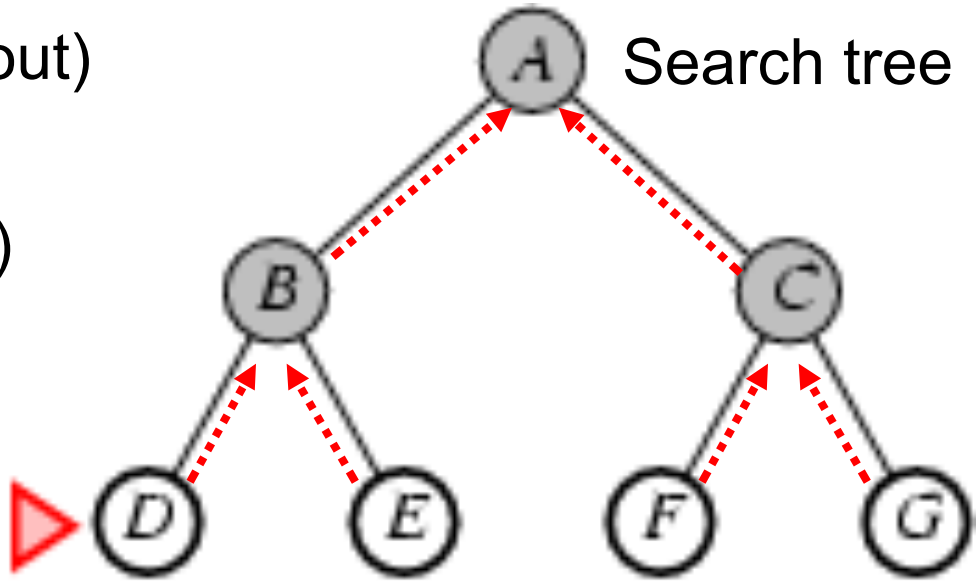

Search tree

queue
→[] →G

... until much later we pop G.

Looking foolish?
Indeed. But let's be consistent…

Breadth-first search (BFS)

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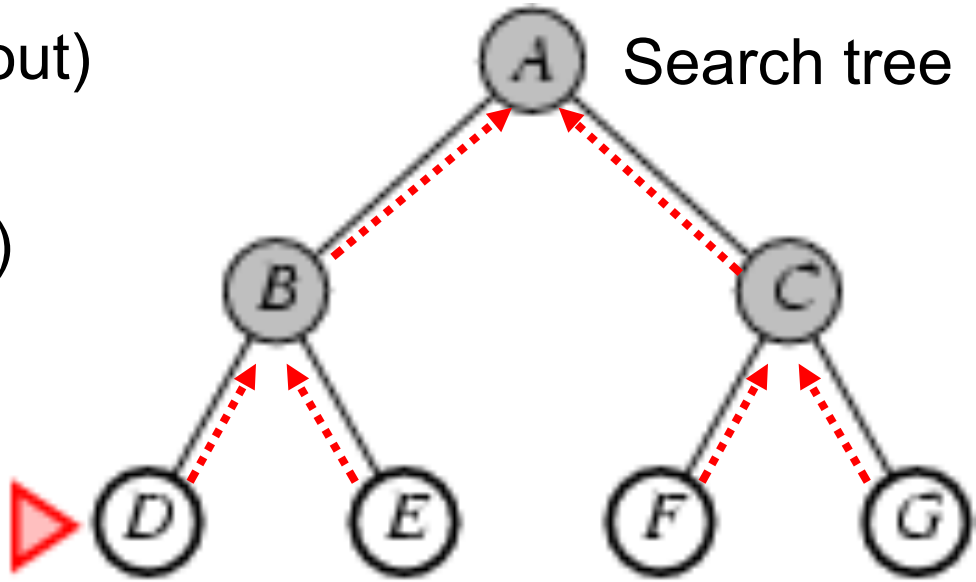


Search tree

queue
→[] →G

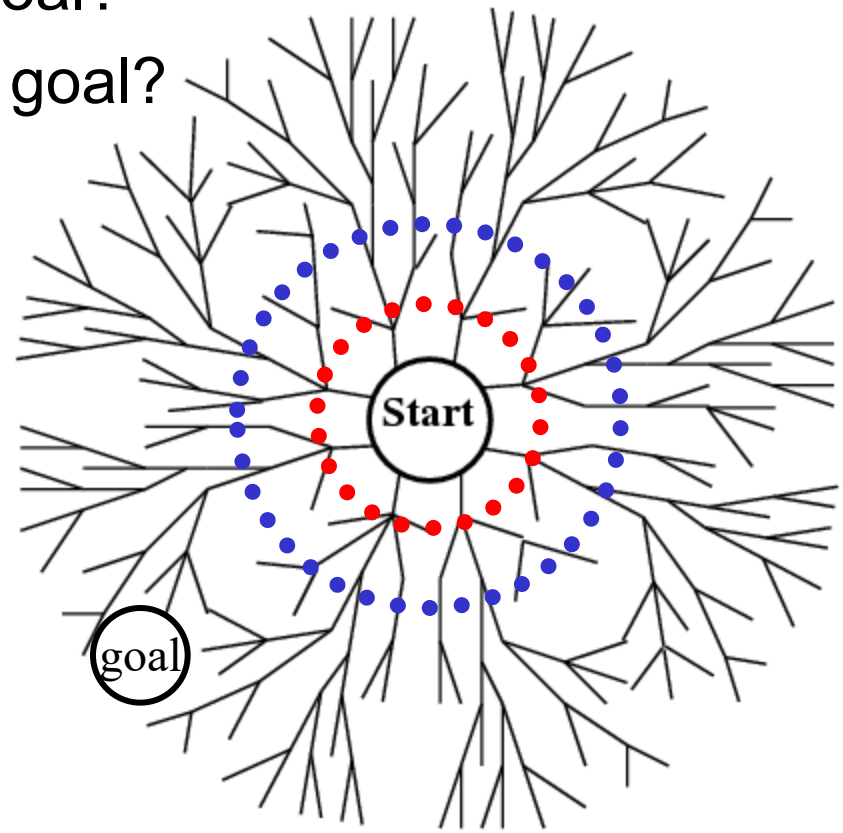
... until much later we pop G.

We need back pointers to recover the solution path.

Looking foolish? Indeed. But let's be consistent…

Performance of BFS

- Assume:
  - the graph may be infinite.
  - Goal(s) exists and is only finite steps away.
- Will BFS find at least one goal?
- Will BFS find the least cost goal?
- Time complexity?
  - # states generated
  - Goal d edges away
  - Branching factor b
- Space complexity?
  - # states stored



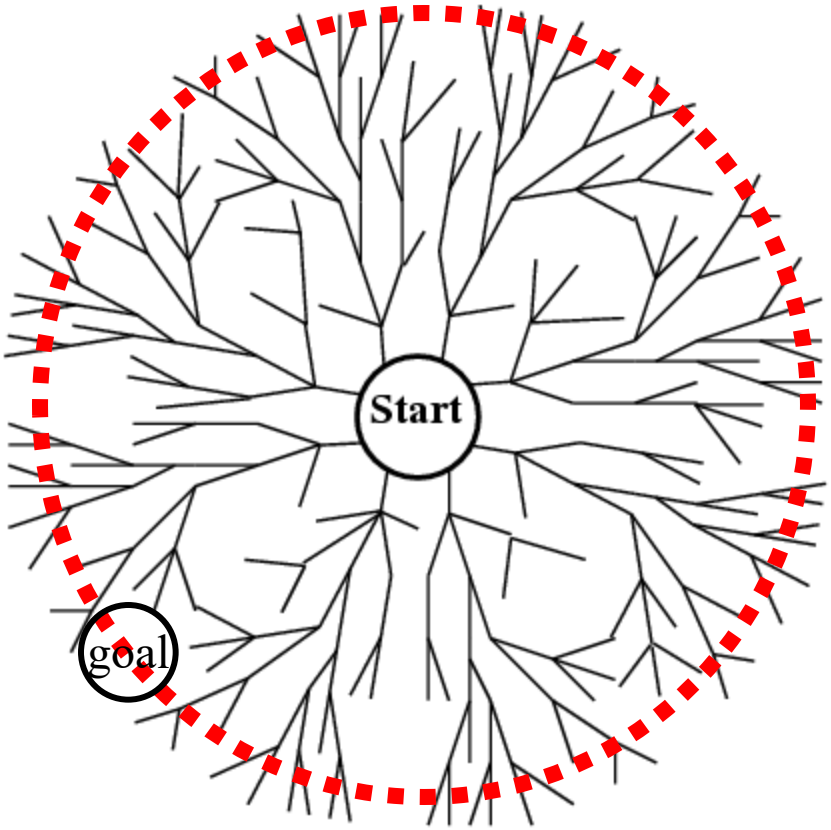
Performance of BFS

Four measures of search algorithms:

- Completeness (not finding all goals): yes, BFS will find a goal.
- Optimality: yes if edges cost 1 (more generally positive non-decreasing in depth), no otherwise.
- Time complexity (worst case): goal is the last node at radius d.
  - Have to generate all nodes at radius d.
  - $b + b^2 + \ldots + b^d \sim O(b^d)$
- Space complexity (bad)
  - Back pointers for all generated nodes $O(b^d)$
  - The queue / fringe (smaller, but still $O(b^d)$)

What's in the fringe (queue) for BFS?

- Convince yourself this is $O(b^d)$



Performance of search algorithms on trees

b: branching factor (assume finite) d: goal depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if [1]	$O(b^d)$	$O(b^d)$

1. Edge cost constant, or positive non-decreasing in depth

Q1-1: You are running BFS on a finite tree-structured state space graph that does not have a goal state. What is the behavior of BFS?

1. Visit all N nodes, then return one at random
2. Visit all N nodes, then stop and return "failure"
3. Visit all N nodes, then return the node farthest from the initial state
4. Get stuck in an infinite loop

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- Space complexity (bad, Figure 3.11)
  - Back points for all generated nodes $O(b^d)$
  - The queue (smaller, but still $O(b^d)$)

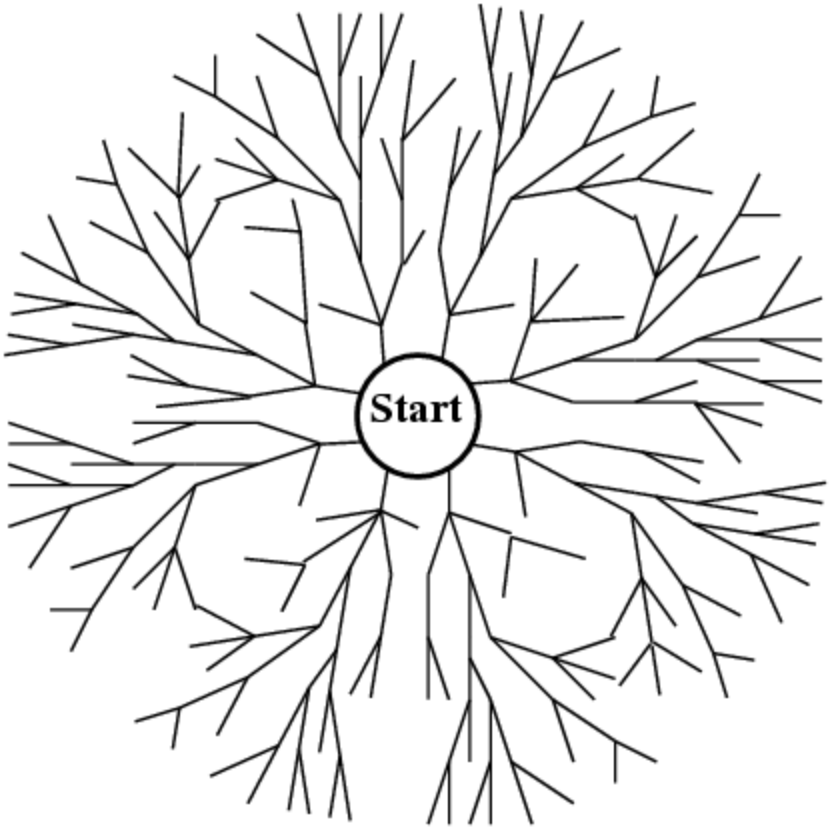
Solution: Uniform-cost search

Uniform-cost search

- Find the least-cost goal
- Each node has a path cost from start (= sum of edge costs along the path).
- Expand the least cost node first.
- Use a priority queue instead of a normal queue
  - Always take out the least cost item

Uniform-cost search (UCS)

- Complete and optimal (if edge costs $\geq \varepsilon > 0$)
- Time and space: can be much worse than BFS
  - Let C^* be the cost of the least-cost goal
  - $O(b^{C^*/\varepsilon})$



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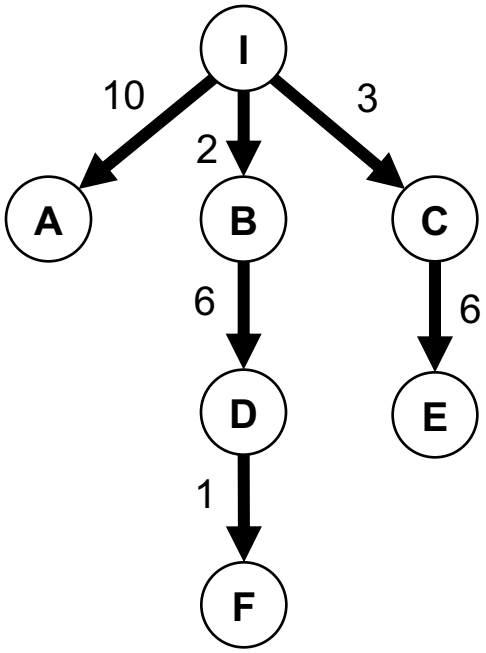
	Complete	optimal	time	space
Breadth-first search	Y	Y, if [1]	$O(b^d)$	$O(b^d)$
Uniform-cost search[2]	Y	Y	$O(b^{C^*/\varepsilon})$	$O(b^{C^*/\varepsilon})$

1. edge cost constant, or positive non-decreasing in depth
2. edge costs $\geq \varepsilon > 0$. C^* is the best goal path cost.

Q1-2: You are running UCS in the state space graph below. You just called the successor function on node D. What is the cost of node F?



1. 2
2. 7
3. 8
4. 9

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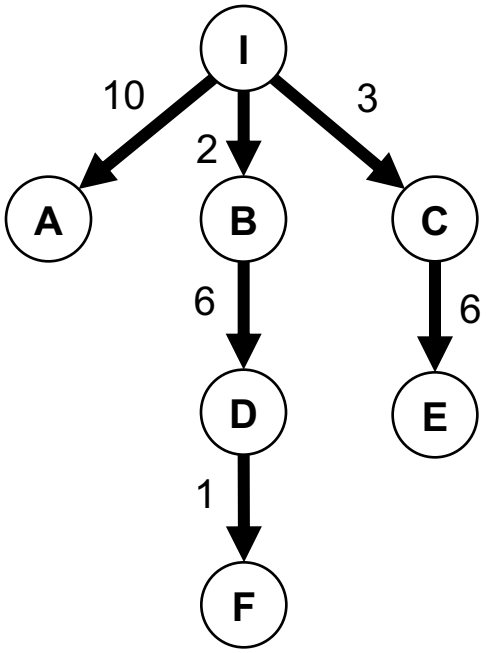

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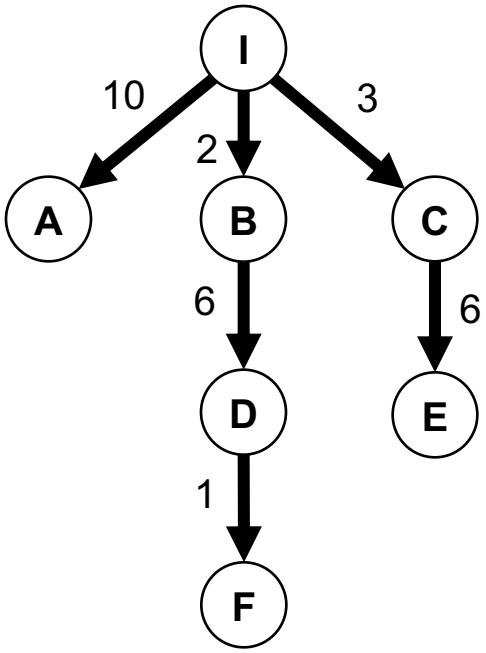
Q1-3: You are running UCS in the state space graph below. You just expanded (visited) node C. What node will the search expand next?



1. A
2. D
3. E
4. F

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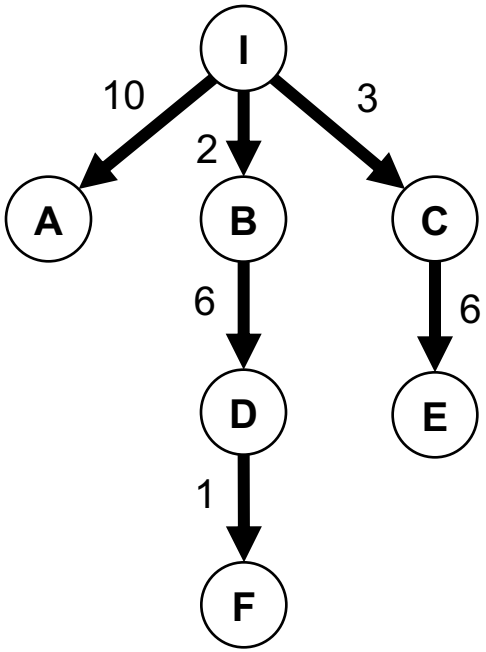

1. A
2. D
3. E
4. F

General State-Space Search Algorithm

```
function general-search(problem, QUEUEING-FUNCTION)
  ;; problem describes the start state, operators, goal test, and
  ;;   operator costs
  ;; queueing-function is a comparator function that ranks two states
  ;; general-search returns either a goal node or "failure"

  nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds then return node
     nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
                                  problem.OPERATORS))
     ;; succ(s)=EXPAND(s, OPERATORS)
     ;; Note: The goal test is NOT done when nodes are generated
     ;; Note: This algorithm does not detect loops
  end
```

Recall the bad space complexity of BFS

Four measures of search algorithms:

- Completeness (not finding all goals): find a goal.
- Optimality: yes if edges cost 1 (more generally positive non-decreasing with depth), no otherwise.
- Time comple): goal is the last node at radius d.
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  - $b + b^2 + \ldots + b^d \sim O(b^d)$
- Space complexity (bad, Figure 3.11)
  - Back points for all generated nodes $O(b^d)$
  - The queue (smaller, but still $O(b^d)$)

Solution: Uniform-cost search

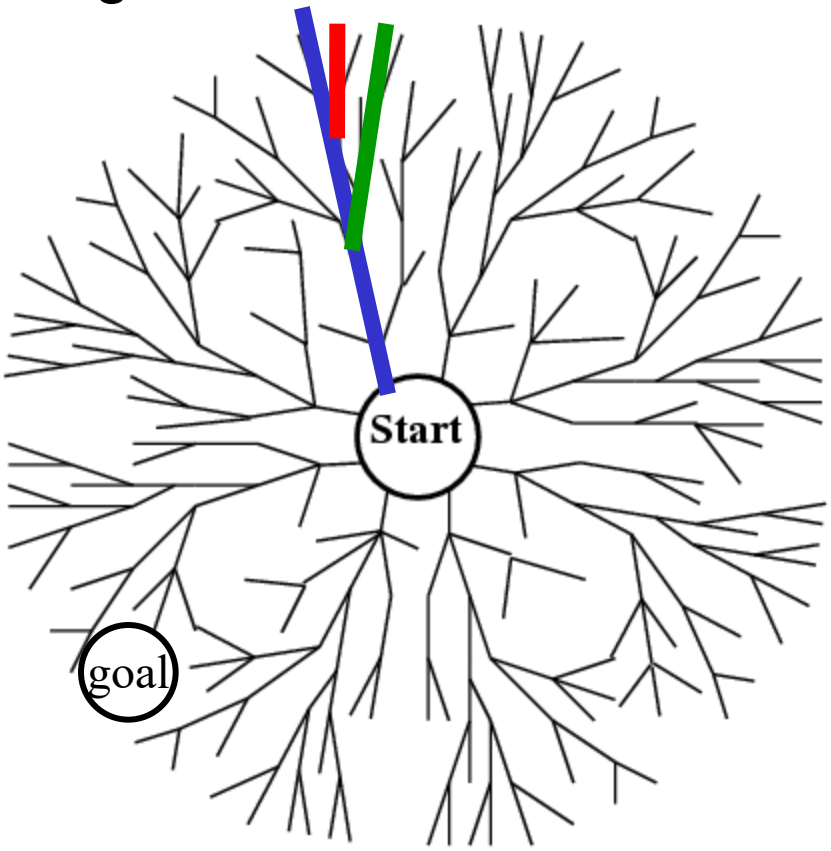
Solution: Depth-first search

Depth-first search

Expand the deepest node first

1. Select a direction, go deep to the end
2. Slightly change the end
3. Slightly change the end some more…

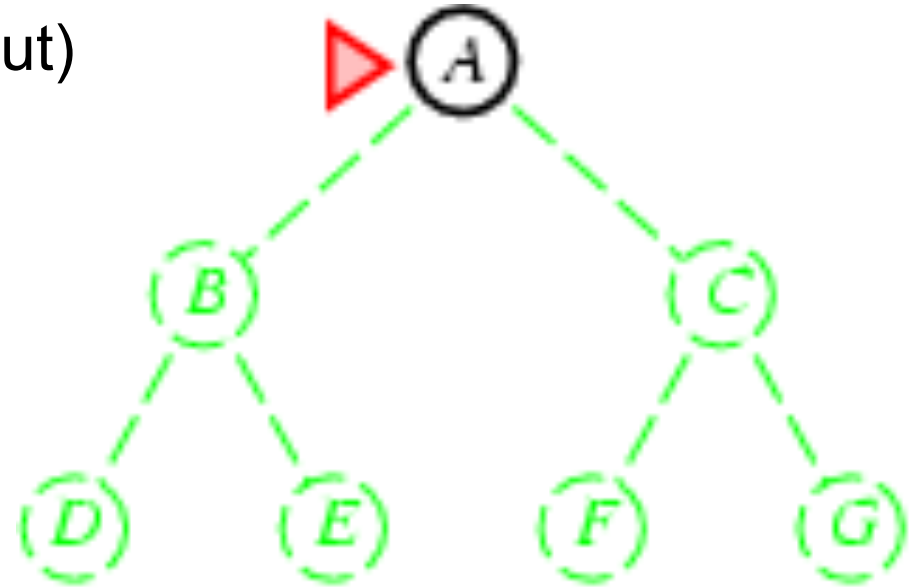
fan



Depth-first search (DFS)

Use a stack (First-in Last-out)

1. push(Initial states)
2. While (stack not empty)
3. s = pop()
4. if (s==goal) success!
5. T = succs(s)
6. push(T)
7. endWhile



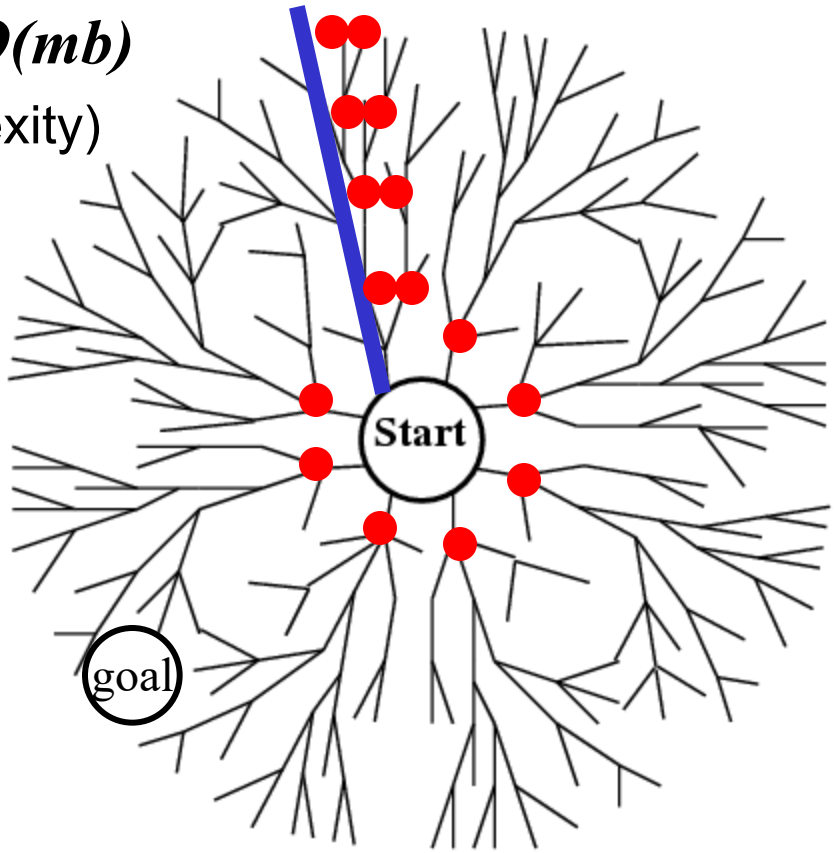
stack (fringe)

[] ⇔

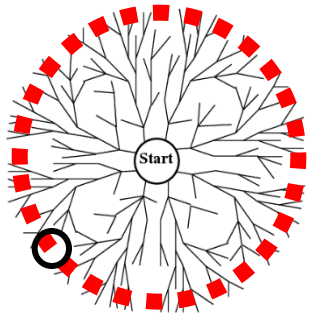
What's in the fringe for DFS?

- m = maximum depth of graph from start
- ***m(b-1) ~ O(mb)***

(Space complexity)



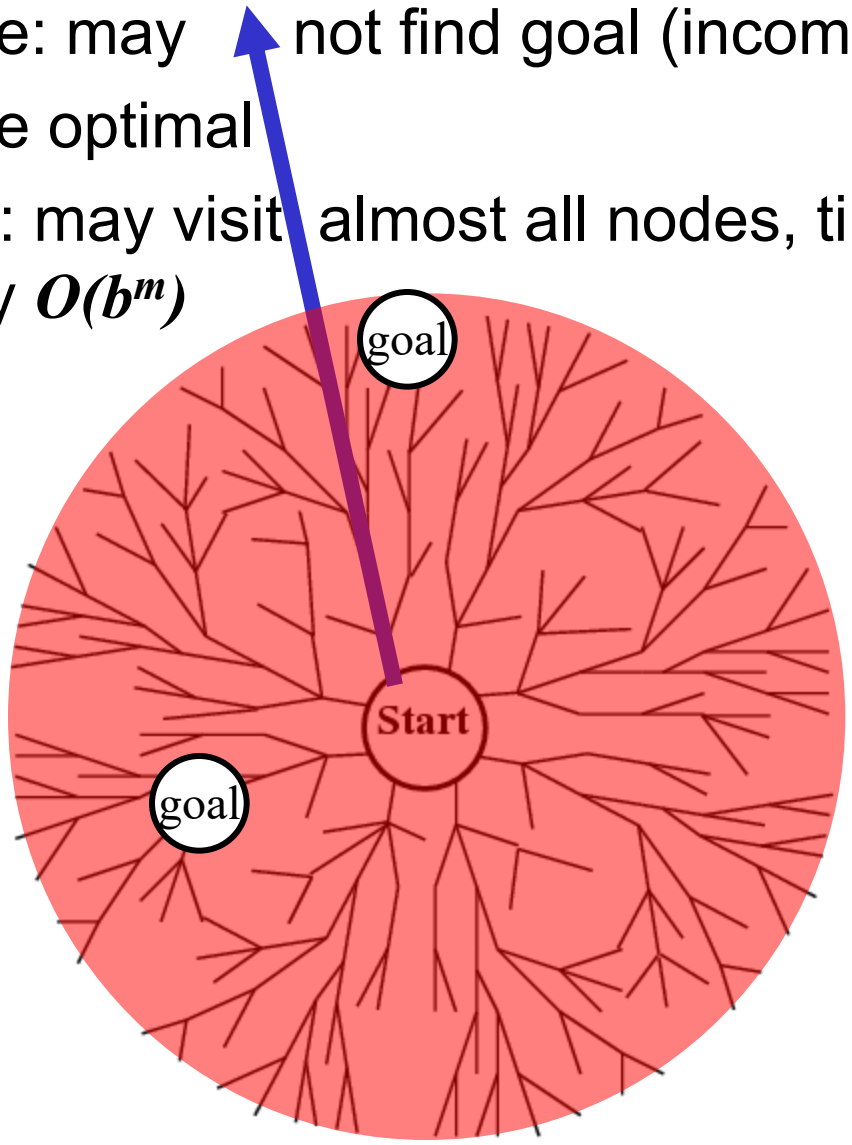
c.f. BFS $O(b^d)$



- "backtracking search" even less space
  - generate siblings (if applicable)

What's wrong with DFS?

- Infinite tree: may not find goal (incomplete)
- May not be optimal
- Finite tree: may visit almost all nodes, time complexity $O(b^m)$



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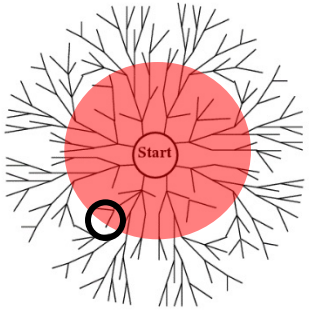

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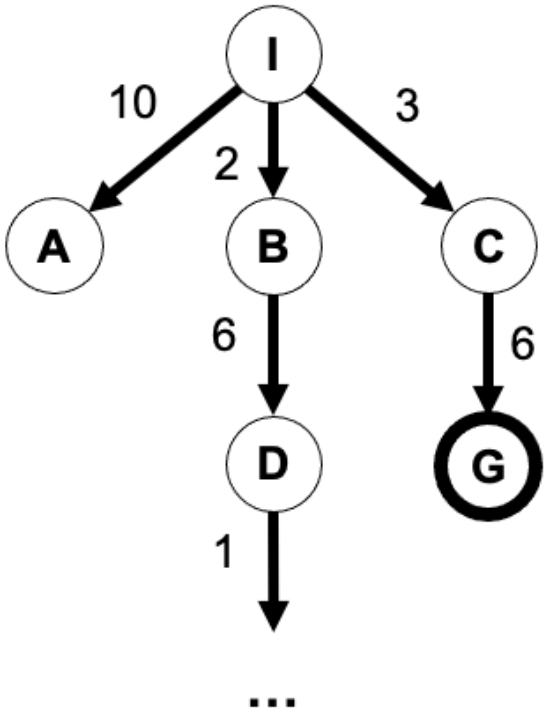
b: branching factor (assume finite) d: goal depth m: graph depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if [1]	$O(b^d)$	$O(b^d)$
Uniform-cost search[2]	Y	Y	$O(b^{C^*/\varepsilon})$	$O(b^{C^*/\varepsilon})$
Depth-first search	N	N	$O(b^m)$	O(bm)

1. edge cost constant, or positive non-decreasing in depth
2. edge costs $\geq \varepsilon > 0$. C* is the best goal path cost.

Q2-1: You are running DFS in the state space graph below. DFS expands nodes left to right. G is the goal state. The state space graph is infinite (the path after D does not terminate). What is the behavior of DFS?



1. Get stuck in an infinite loop
2. Return A
3. Return G
4. Return "failure"

Q2-2: You need to search a randomly generated state space graph with one goal, uniform edges costs, d=2, and m=100. Considering worst case behavior, do you select BFS or DFS for your search?

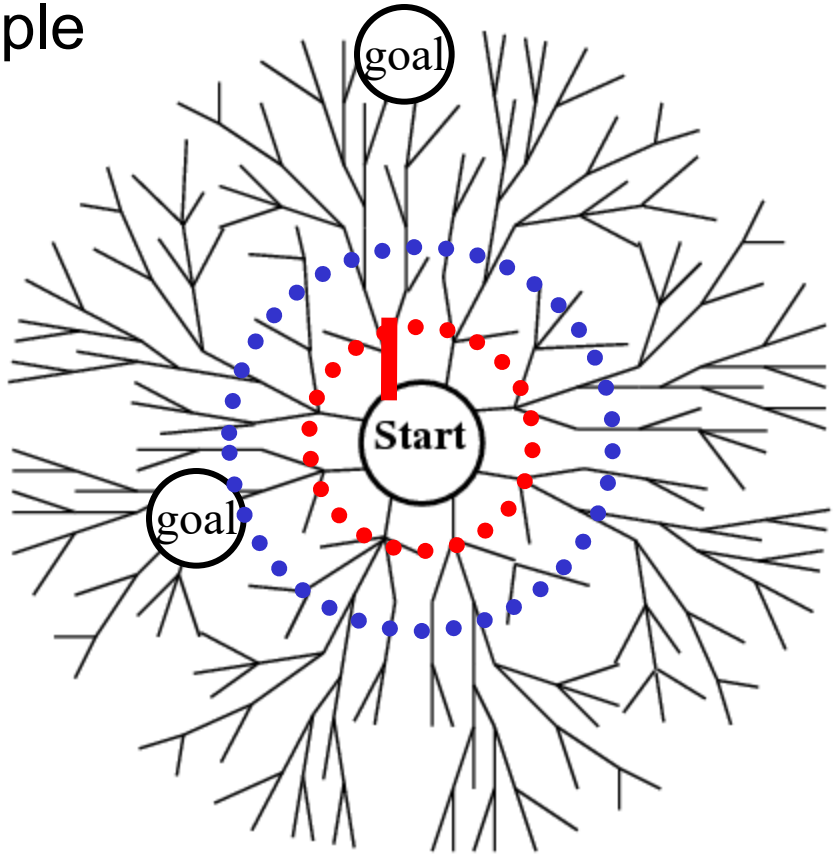
1. BFS 

2. DFS

How about this?

1. DFS, but stop if path length > 1.
2. If goal not found, repeat DFS, stop if path length > 2.
3. And so on…

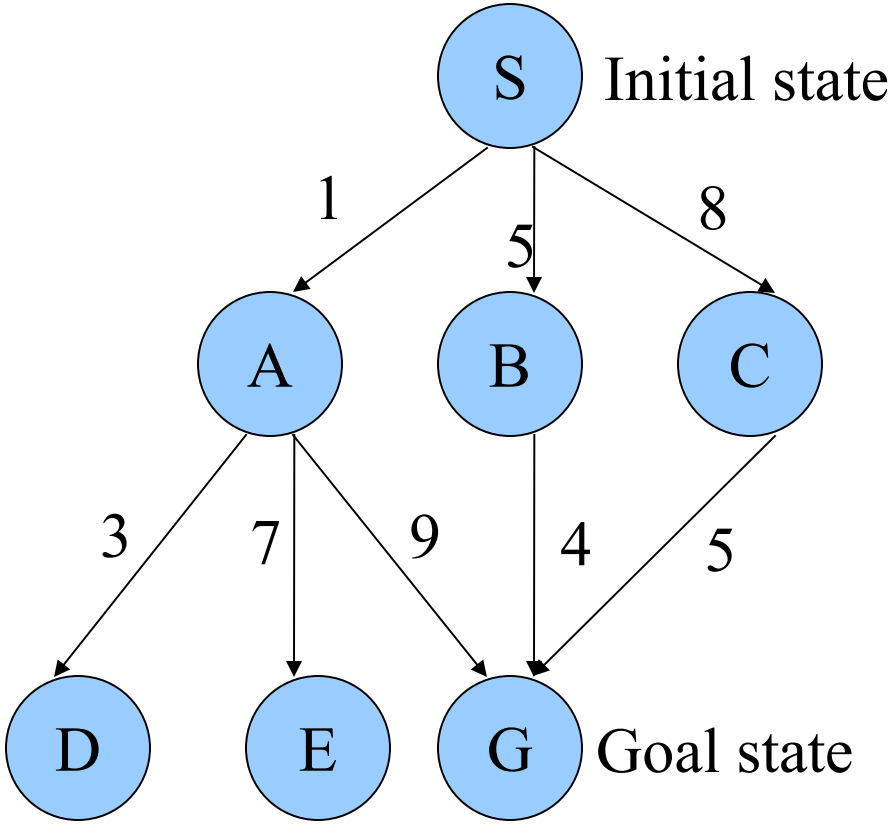
fan within ripple



Iterative deepening

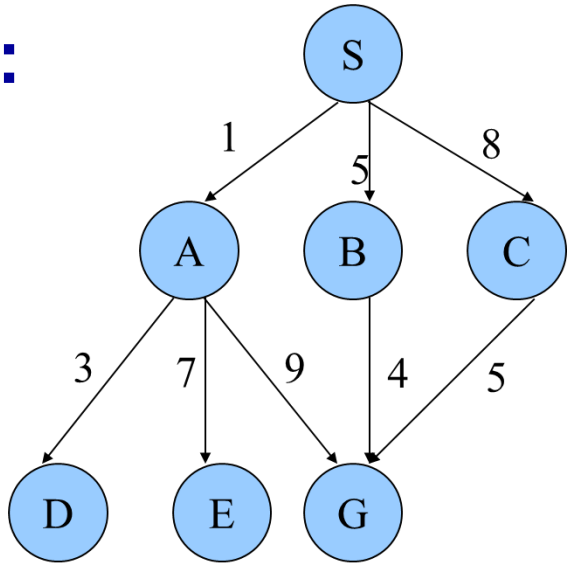
- Search proceeds like BFS, but fringe is like DFS
  - Complete, optimal like BFS
  - Small space complexity like DFS
  - Time complexity like BFS
- Preferred uninformed search method

Example



(All edges are directed, pointing downwards)

Nodes expanded by:



- Breadth-First Search: S A B C D E G

  Solution found: S A G

- Uniform-Cost Search: S A D B C E G

  Solution found: S B G (This is the only uninformed search that worries about costs.)

- Depth-First Search: S A D E G

  Solution found: S A G

- Iterative-Deepening Search: S A B C S A D E G

  Solution found: S A G

Performance of search algorithms on trees

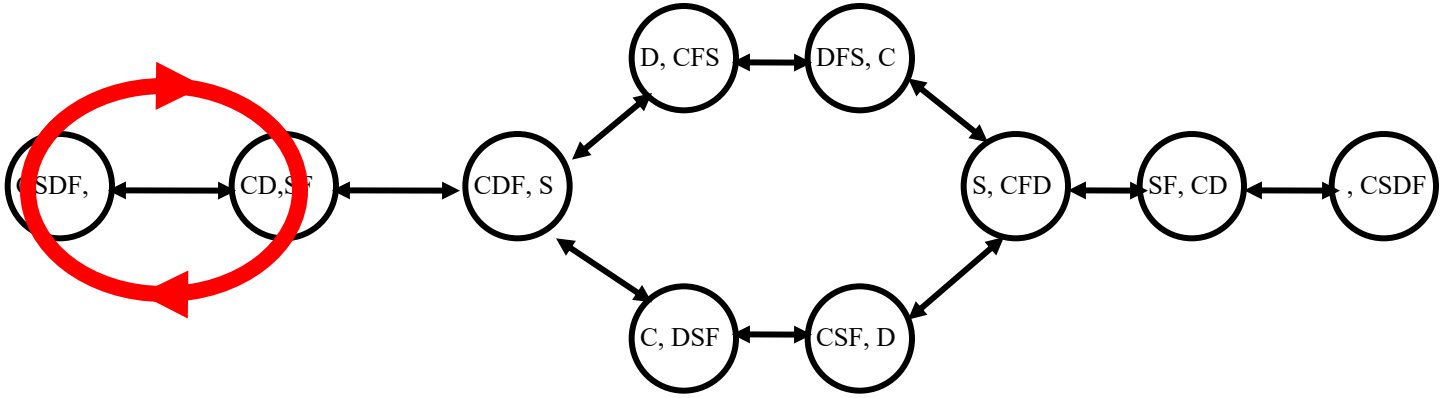
b: branching factor (assume finite) d: goal depth m: graph depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if [1]	$O(b^d)$	$O(b^d)$
Uniform-cost search[2]	Y	Y	$O(b^{C^*/\varepsilon})$	$O(b^{C^*/\varepsilon})$
Depth-first search	N	N	$O(b^m)$	O(bm)
Iterative deepening	Y	Y, if [1]	$O(b^d)$	O(bd)

1. edge cost constant, or positive non-decreasing in depth
2. edge costs $\geq \varepsilon > 0$. C* is the best goal path cost.

If state space graph is not a tree

- The problem: repeated states

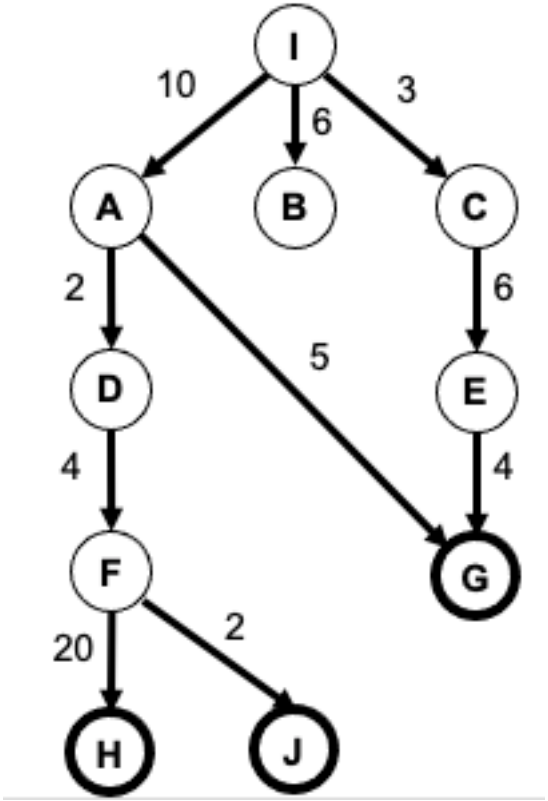


- Ignore the danger of repeated states: wasteful (BFS) or impossible (DFS). Can you see why?
- How to prevent it?

If state space graph is not a tree

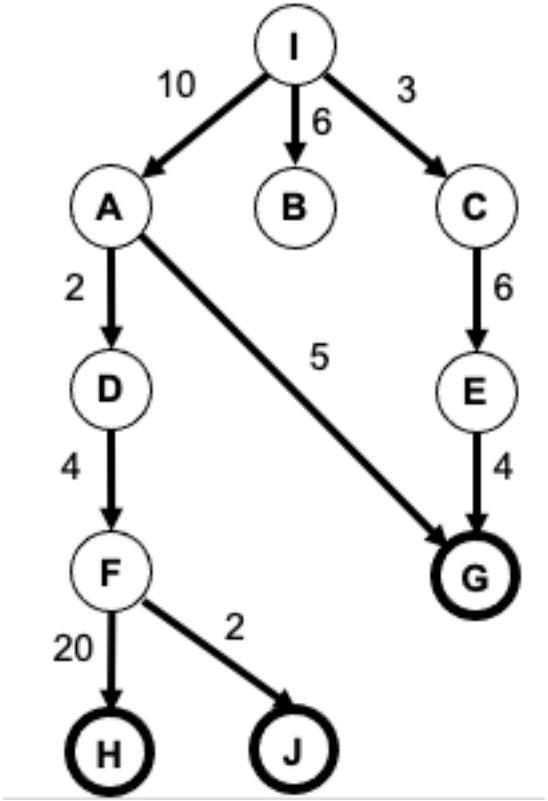
- We have to remember already-expanded states (CLOSED).
- When we take out a state from the fringe (OPEN), check whether it is in CLOSED (already expanded).
  - If yes, throw it away.
  - If no, expand it (add successors to OPEN), and move it to CLOSED.

Q3-1: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by BFS?



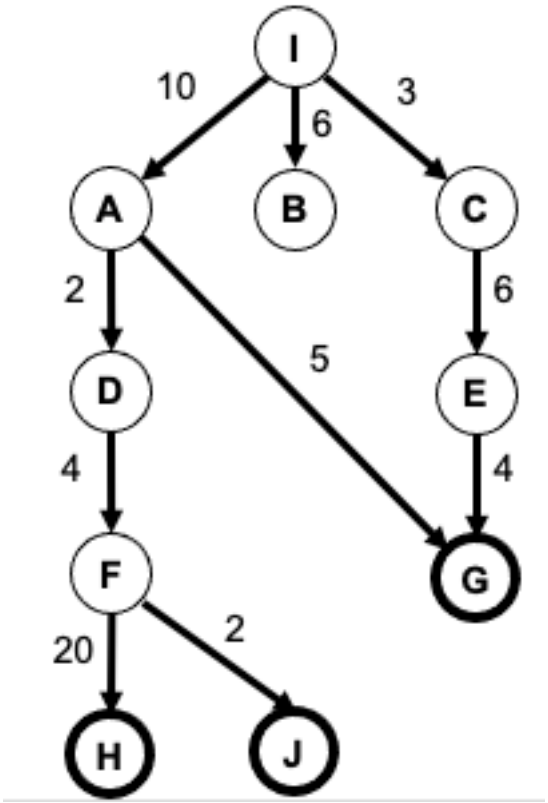
1. IADFH
2. IADFJ
3. IAG
4. ICEG

Q3-2: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by UCS?



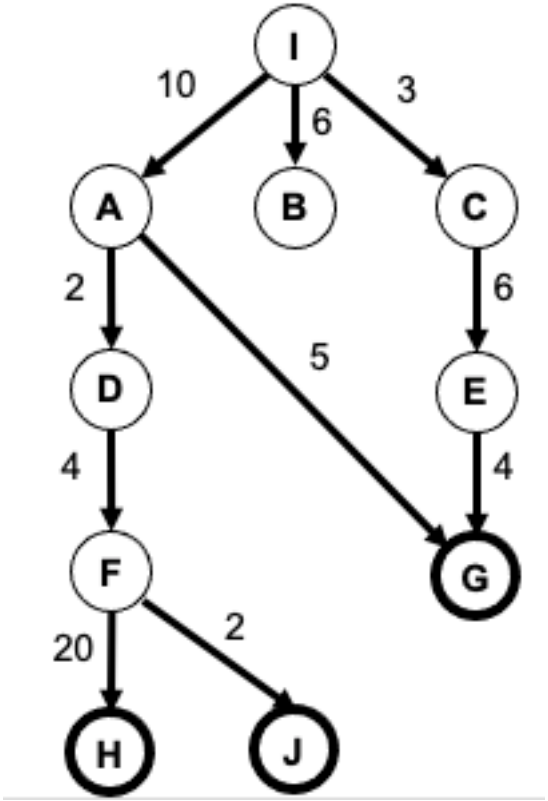
1. IADFH
2. IADFJ
3. IAG
4. ICEG

Q3-3: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by DFS?



1. IADFH
2. IADFJ
3. IAG
4. ICEG

Q3-4: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by IDS?



1. IADFH
2. IADFJ
3. IAG
4. ICEG

What you should know

- Problem solving as search: state, successors, goal test
- Uninformed search
  - Breadth-first search
    - Uniform-cost search
  - Depth-first search
  - Iterative deepening

- Can you unify them using the same algorithm, with different priority functions?
- Performance measures
  - Completeness, optimality, time complexity, space complexity