CS 540 Introduction to Artificial Intelligence
Reinforcement Learning I

University of Wisconsin-Madison

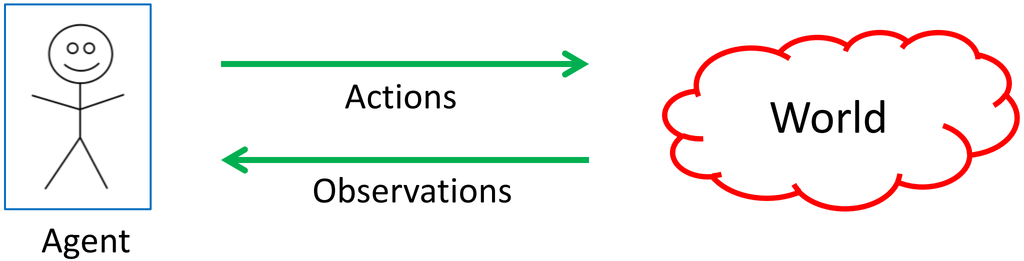
Fall 2022

Outline

- Introduction to reinforcement learning
  - Basic concepts, mathematical formulation, MDPs, policies
- Valuing policies
  - Value functions, Bellman equation, value iteration

Back to Our General Model

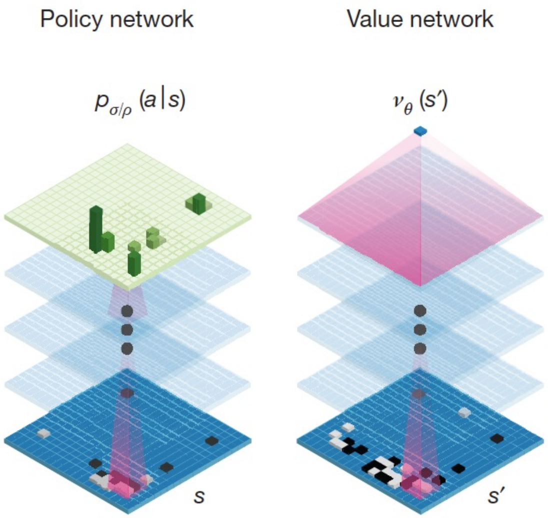
We have an agent interacting with the world



- Agent receives a reward based on state of the world
  - **Goal**: maximize reward / utility **($$$)**
  - Note: **data** consists of actions & observations
    - Compare to unsupervised learning and supervised learning

Examples: Gameplay Agents

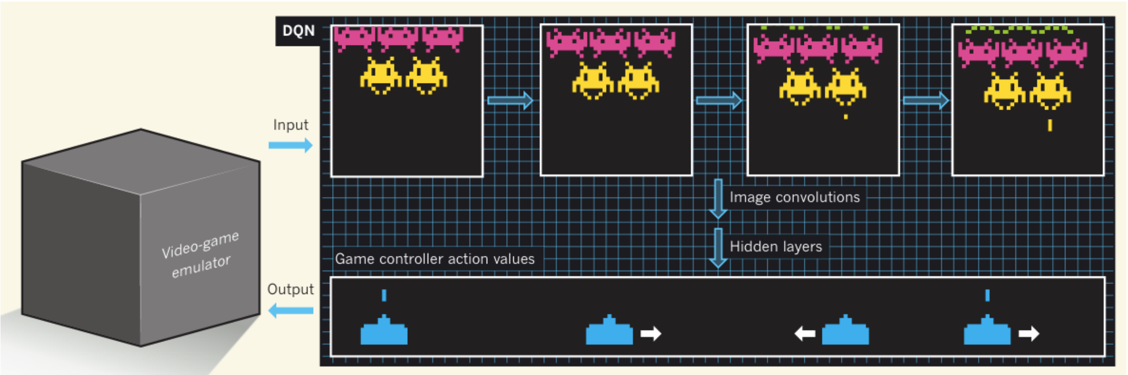
AlphaZero:



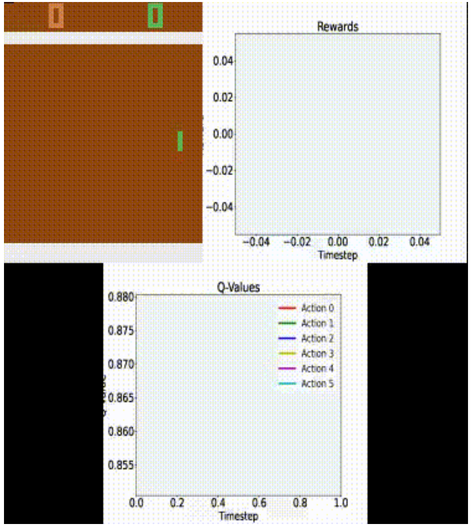
https://deepmind.com/research/alphago/

Examples: Video Game Agents

Pong, Atari



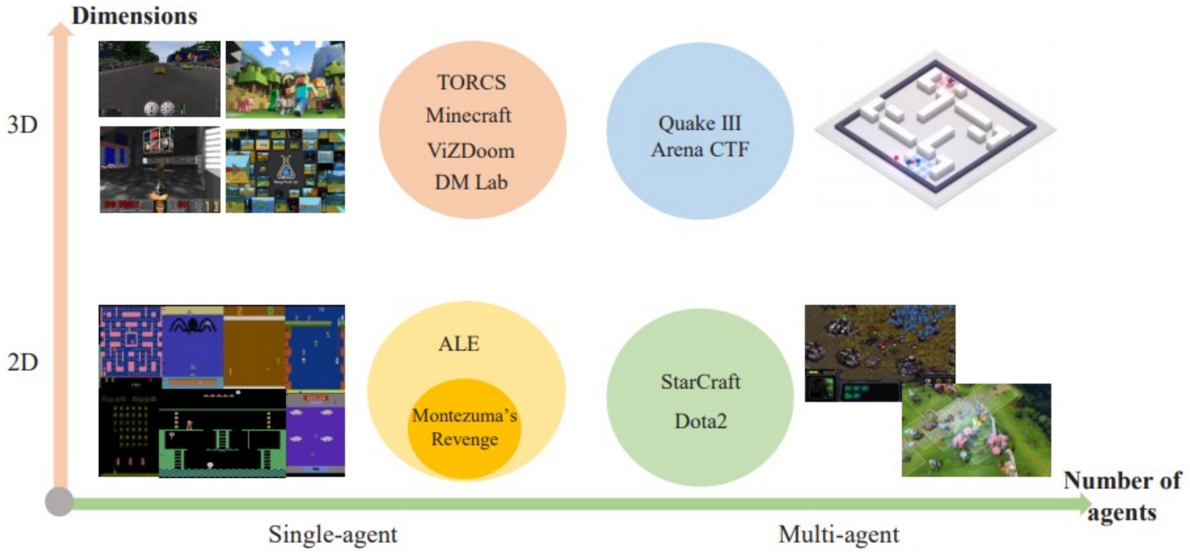
Mnih et al, "Human-level control through deep reinforcement learning"



A. Nielsen

Examples: Video Game Agents

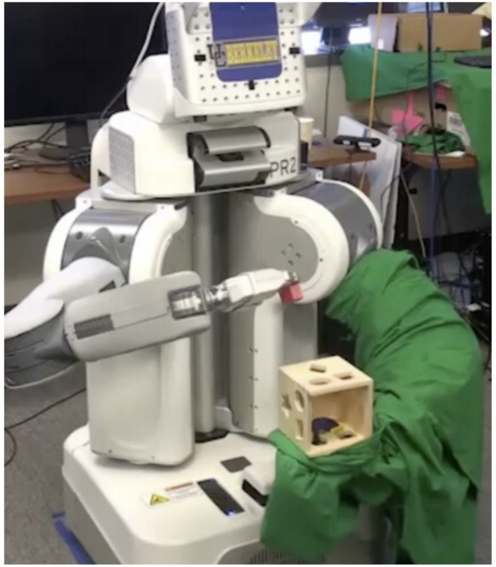
Minecraft, Quake, StarCraft, and more!



Shao et al, "A Survey of Deep Reinforcement Learning in Video Games"

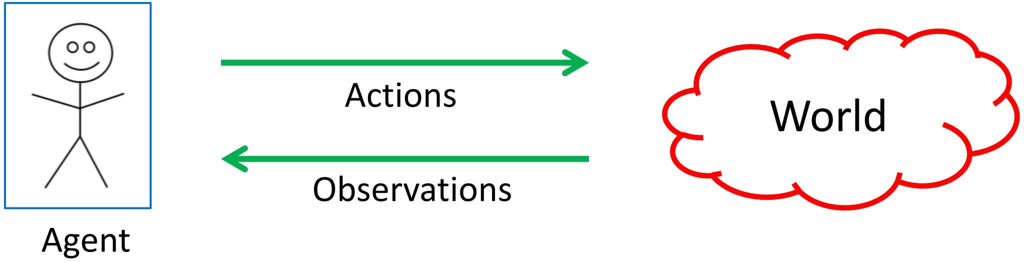
Examples: Robotics

Training robots to perform tasks (e.g., grasp!)



Ibarz et al, " How to Train Your Robot with Deep Reinforcement Learning – Lessons We’ve Learned "

Building The Theoretical Model



Basic setup:

- Set of states, S
- Set of actions A
- Information: at time t, observe state $s_t \in$ S. Get reward r_t
- Agent makes choice $a_t \in$ A. State changes to s_{t+1}, continue

Goal: find a map from **states to actions** maximize rewards.

A “policy”

Markov Decision Process (MDP)

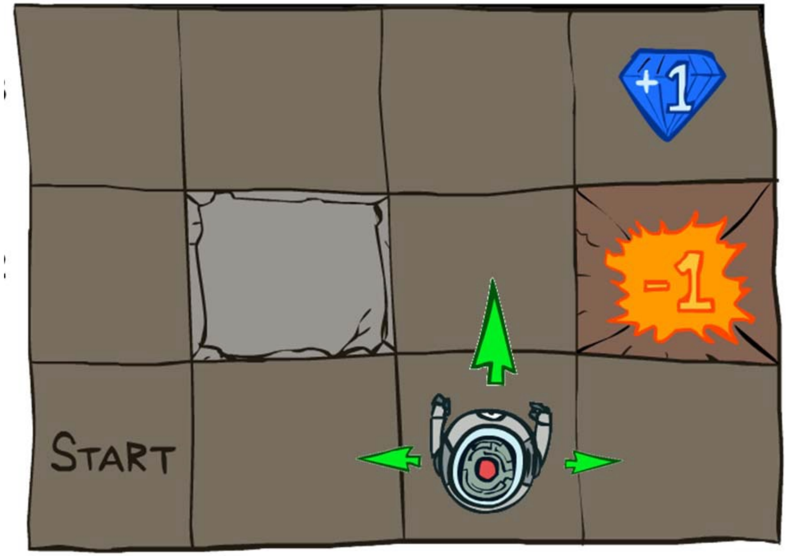
The formal mathematical model:

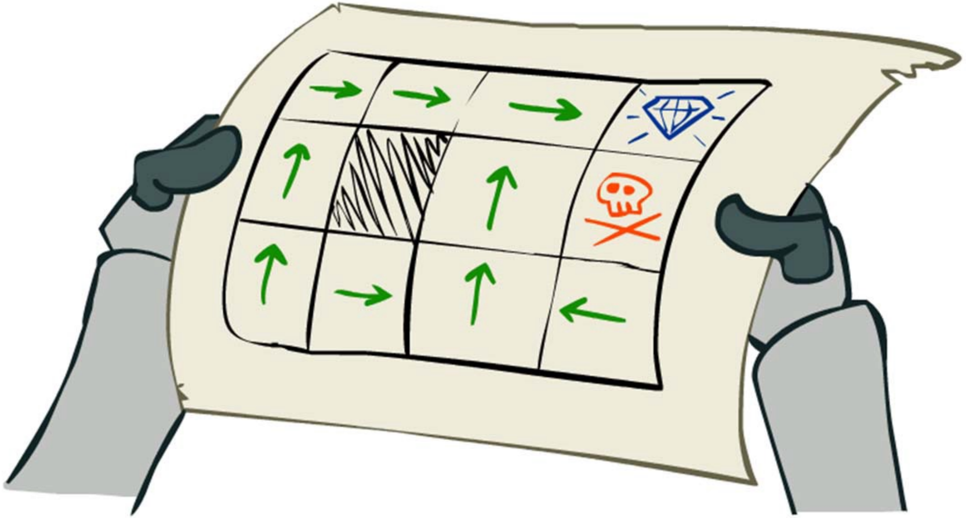
- **State set** S. Initial state s_0. **Action set** A
- **Reward function:** $r(s_t)$
- **State transition model**: $P(s_{t+1} | s_t, a_t)$
  - Markov assumption: transition probability only depends on s_t and a_t, and not earlier history (previous actions or states)
- More generally: $r(s_t, a_t), P(r_t, s_{t+1} | s_t, a_t)$
- **Policy**: $\pi(s) : S \rightarrow A$ action to take at a particular state

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots$$

Example of MDP: Grid World

Robot on a grid; goal: find the best policy

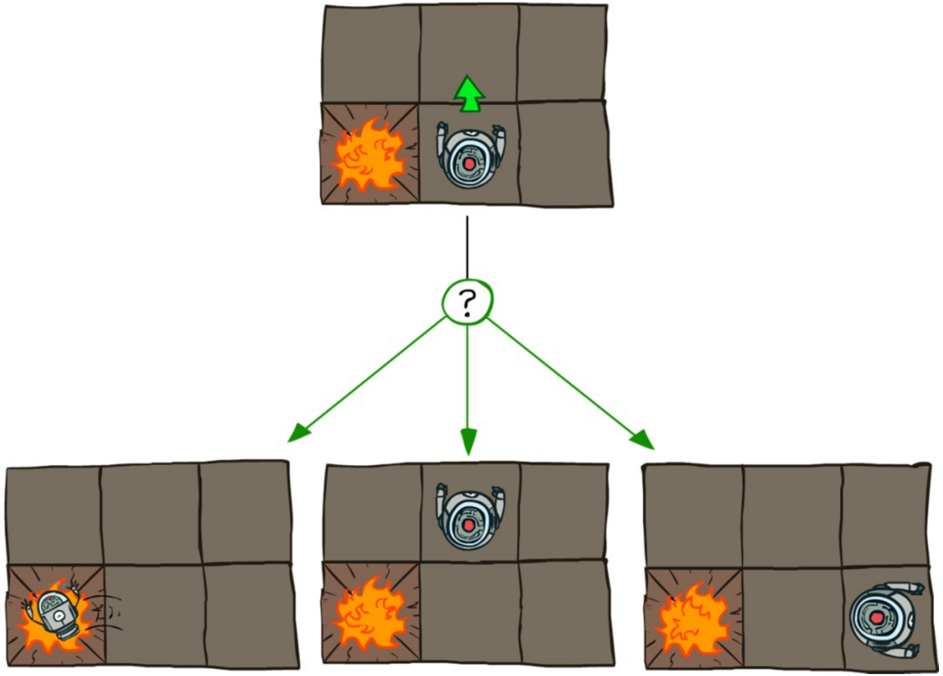


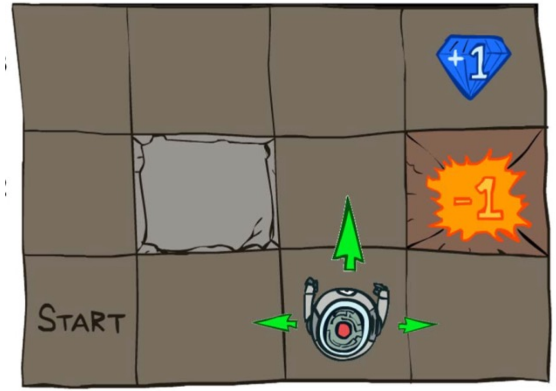


Source: P. Abbeel and D. Klein

Example of MDP: Grid World

Note: (i) Robot is unreliable (ii) Reach target fast

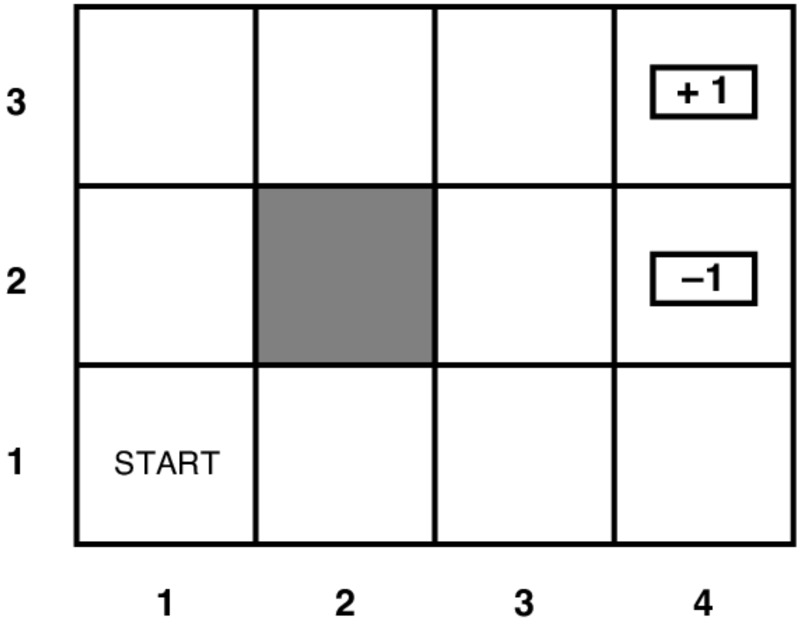


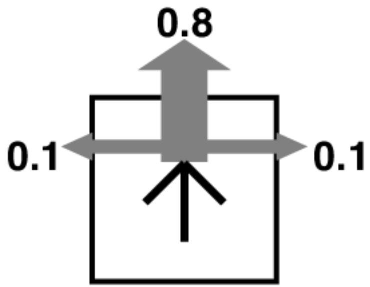


$r(s) = -0.04$ for every non-terminal state

Grid World Abstraction

Note: (i) Robot is unreliable (ii) Reach target fast





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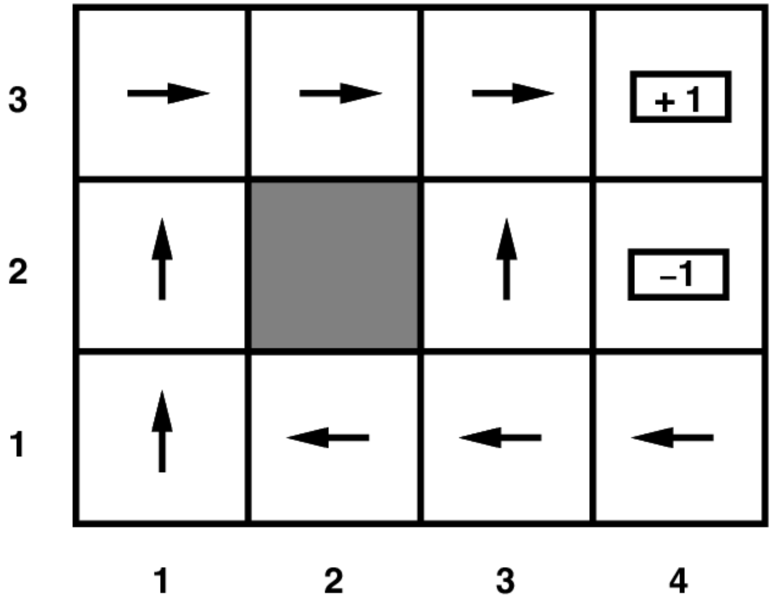
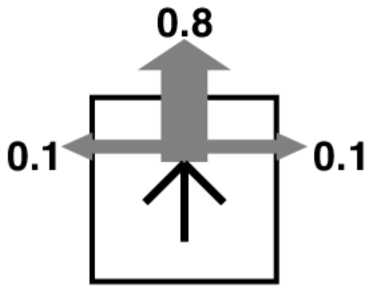
Grid World Optimal Policy

Note: (i) Robot is unreliable (ii) Reach target fast





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Back to MDP Setup

The formal mathematical model:

- **State set** S. Initial state s_0. **Action set** A
- **State transition model:** $P(s_{t+1} | s_t, a_t)$
  - Markov assumption: transition probability only depends on s_t and a_t, and not previous actions or states.
- **Reward function:** $r(s_t)$
- **Policy**: $\pi(s) : S \rightarrow A$ action to take at a particular state.

How do we find the best policy?

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots$$

Break & Quiz

Q 1.1 Which of the following statement about MDP is **not** true?

- A. The reward function must output a scalar value
- B. The policy maps states to actions
- C. The probability of next state can depend on current and previous states
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards

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Break & Quiz

Q 1.1 Which of the following statement about MDP is **not** true?

- A. The reward function must output a scalar value (**True: need to be able to compare**)
- B. The policy maps states to actions (**True: a policy tells you what action to take for each state**).
- **C. The probability of next state can depend on current and previous states** (**False: Markov assumption**).
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards (**True: want to maximize rewards overall**).

Defining the Optimal Policy

For policy π, **expected utility** over all possible state sequences from s_0 produced by following that policy:

$$V^{\pi}(s_0) = \sum_{\substack{\text{sequences} \\ \text{starting from } s_0}} P(\text{sequence})U(\text{sequence})$$

Called the **value function** (for π, s_0)

Discounting Rewards

One issue: these are infinite series. **Convergence**?

- Solution

$$U(s_0, s_1 \ldots) = r(s_0) + \gamma r(s_1) + \gamma^2 r(s_2) + \ldots = \sum_{t \geq 0} \gamma^t r(s_t)$$

- Discount factor γ between 0 and 1
  - Set according to how important **present** is VS **future**
  - Note: has to be less than 1 for convergence

From Value to Policy

Now that $V^{\pi}(s_0)$ is defined what a should we take?

- First, let π^* be the **optimal** policy for $V^{\pi}(s_0)$, and V^* its expected utility
- What's the expected utility of an action?
  - Specifically, action a in state s?

$$\sum_{s'} P(s'|s, a) V^*(s')$$

All the states we could go to

Transition probability

Expected rewards

Obtaining the Optimal Policy

We know the expected utility of an action

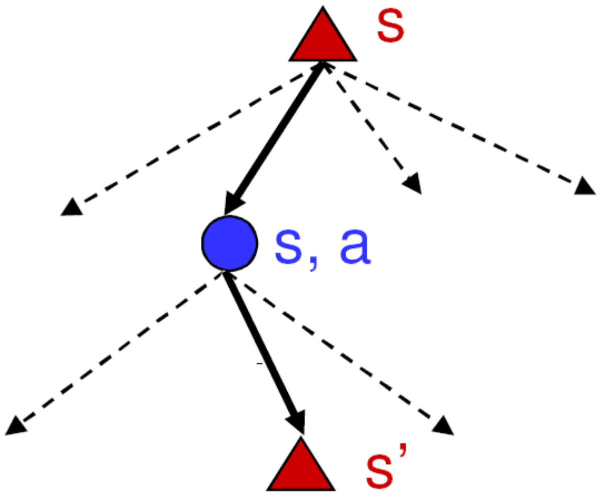
- So, to get the optimal policy, compute

$$\pi^*(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a) V^*(s')$$

All the states we could go to

Transition probability

Expected rewards



Credit L. Lazbenik

Slight Problem…

Now we can get the optimal policy by doing

$$\pi^*(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

- So we need to know $V^*(s)$.
  - But it was defined in terms of the optimal policy!
  - So we need some other approach to get $V^*(s)$.
  - Need some other **property** of the value function!

Bellman Equation

Let's walk over one step for the value function:

$$V^*(s) = r(s) + \underbrace{\gamma \max_a \sum_{s'} P(s'|s,a) V^*(s')}$$

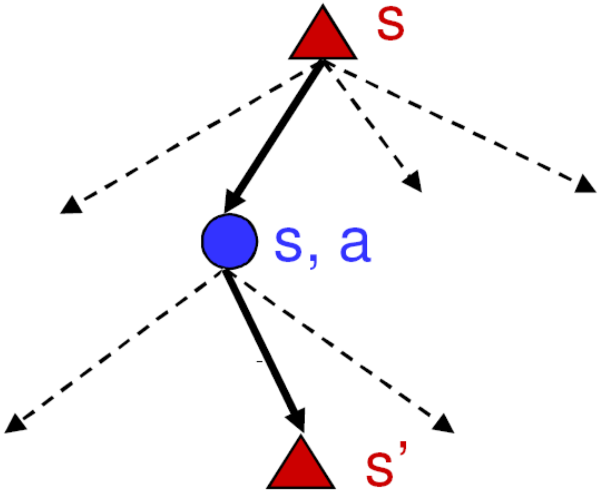
Current state reward

Discounted expected future **rewards**

- Bellman: inventor of dynamic programming

Bellman Equation

Let's walk over one step for the value function:



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$$V^*(s) = r(s) + \gamma \max_a \underbrace{\sum_{s'} P(s'|s,a)V^*(s')}_{\text{Discounted expected future rewards}}$$

Current state reward

Discounted expected future **rewards**

Value Iteration

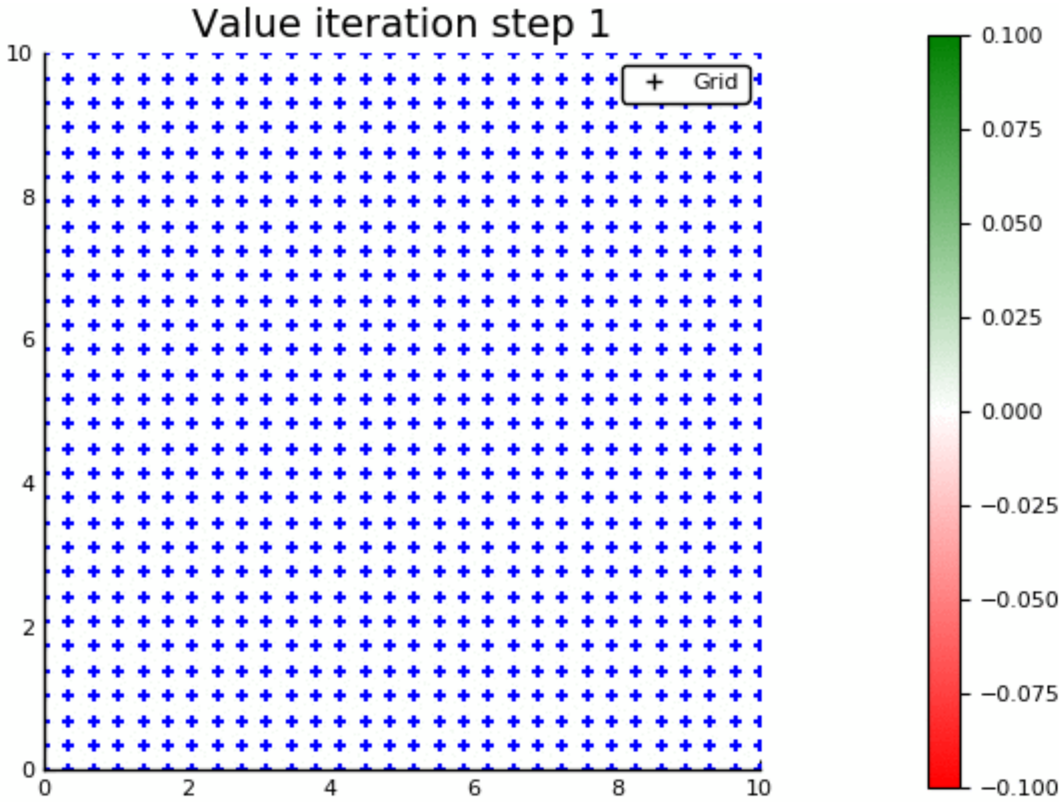
Q: how do we find $V^*(s)$?

- Why do we want it? Can use it to get the best policy
- Know: reward $r(s)$, transition probability $P(s'|s,a)$
  - Knowing r and P is the "planning" problem. In reality r and P must be estimated from interactions : "reinforcement learning"
- Also know $V^*(s)$ satisfies Bellman equation (recursion above)

A: Use the property. Start with $V_0(s)$=0. Then, update

$$V_{i+1}(s) = r(s) + \gamma \max_a \sum_{s'} P(s'|s,a) V_i(s')$$

Value Iteration: Demo



Source: POMDPBGallery Julia Package

Break & Quiz

Q 2.1 Consider an MDP with 2 states {*A, B*} and 2 actions: **"stay"** at current state and "**move**" to other state. Let **r** be the reward function such that **r**(*A*) = 1, **r**(*B*) = 0. Let γ be the discounting factor. Let π: π(*A*) = π(*B*) = **move** (i.e., an "always move" policy). What is the value function $V^{\pi}(A)$?

- A. 0
- B. 1 / (1 -γ)
- C. 1 / (1 -γ^2)
- D. 1

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- A. 0
- B. 1/(1-γ)
- **C. 1/(1-γ^2)** (States: A,B,A,B,… rewards 1,0, γ^2,0, γ^4,0, …)
- D. 1

Summary

- Reinforcement learning setup
- Mathematica formulation: MDP
- Value functions & the Bellman equation
- Value iteration