Course overview and logistics

CS639: Introduction to Game Theory & Mechanism Design

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Game theory

A mathematical framework for modelling interactions between multiple **strategic and selfish** agents (players).

- Agents view the interaction as a *"game"*.
- Each agent is trying to *"win the game"*.

- Agents could be partially competitive and partially cooperative.

- Agents: people, organizations, countries, robots.

Example 1: Prisoner's dilemma

Two robbers are caught after committing a robbery together. They are each given the option to ***betray their partner*** or ***remain silent***.

Prisoner B

Prisoner A		Betray	Remain silent
	Betray	A serves 3 years, B serves 3 years	A goes free, B serves 5 years
	Remain silent	A serves 5 years, B goes free	A serves 1 years, B serves 1 years

Prisoner's dilemma in the real world

1. Individually, countries find it beneficial to develop nuclear weapons.
    - But the mutually better outcome is if no country had nuclear weapons.

2. Competing companies lower prices to attract customers.
    - But best if they collectively keep prices high.
    - **Counter-examples:** OPEC, Cable/wireless companies in the US.

Example 2: Tragedy of the commons

A common resource can be used by some agents.

- Each agent benefits by using the resource.
- It is collectively better for all agents to use less of the resource.
- However, it is better for each individual agent to use more. This leads to socially less desirable outcomes.

Tragedy of the commons in the real world

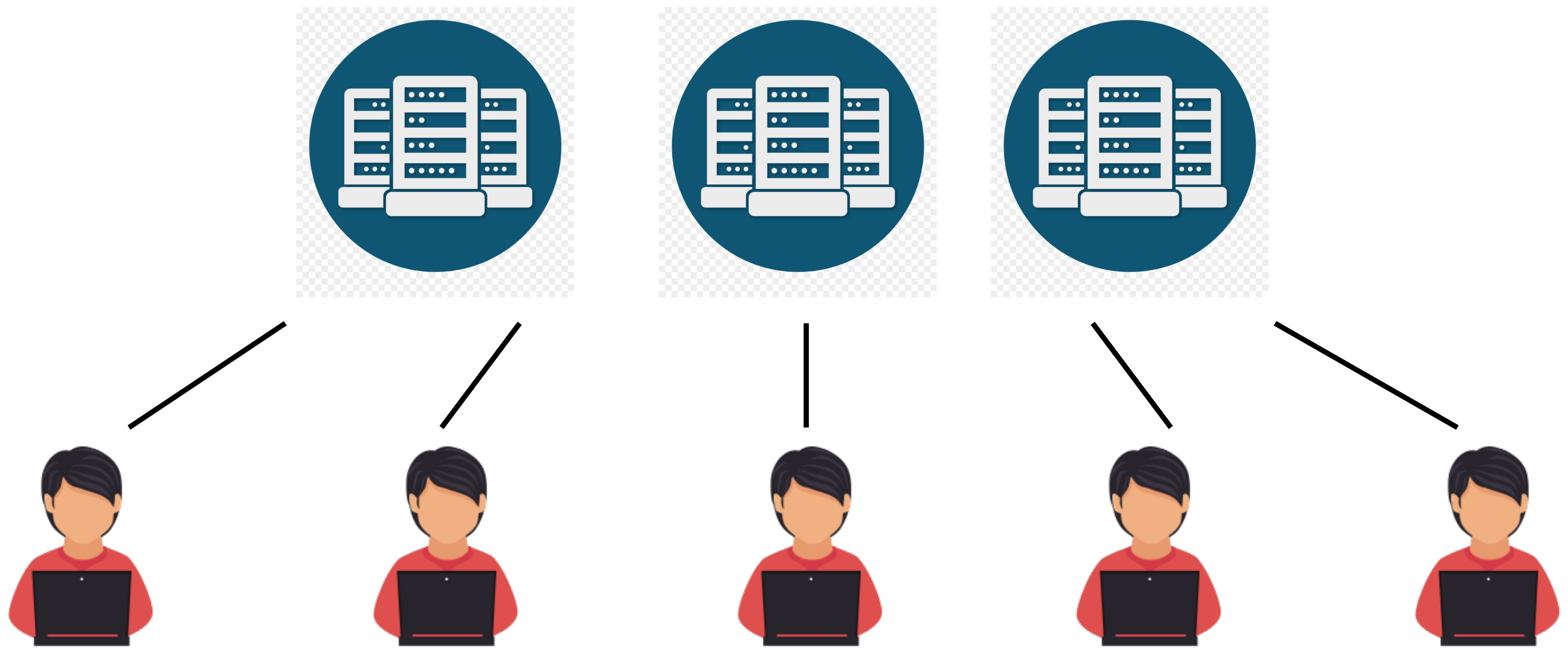
1. Individual fishermen compete to catch as much fish as possible.
   - But over-fishing will deplete the fish population over time.
2. Individually, plastic consumption is convenient and does not affect the environment significantly.
   - But collectively, plastic consumption is not sustainable.
   - **Counter-measures:** Pay for single-use plastic bags.

Mechanism design

The design of games, i.e social interactions, to obtain desirable social outcomes when agents are acting in self-interest.

- “Inverse” game theory
- Ideally, there would be an “obvious” strategy that leads to the desired outcome. (Not always possible)

Example 1: Resource allocation in clusters

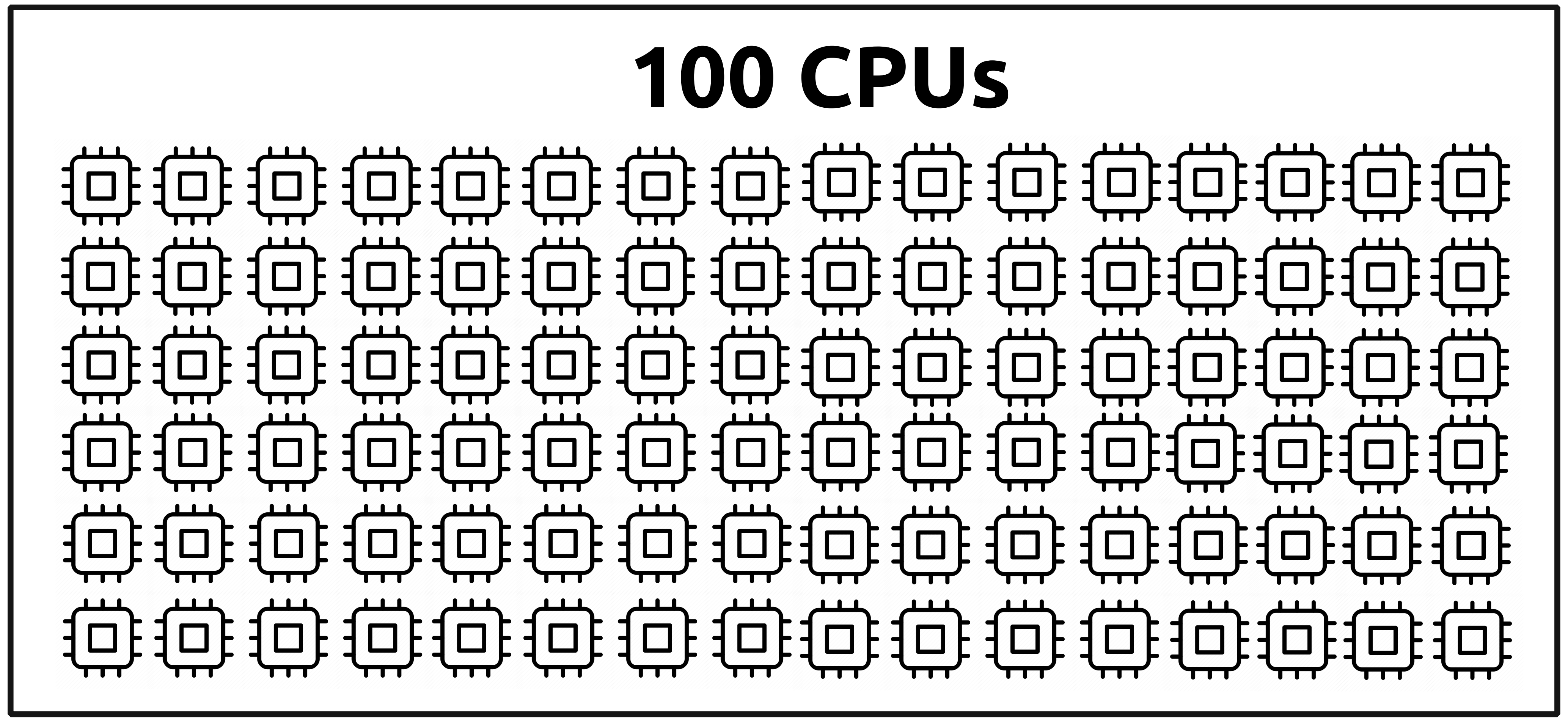


Several users are sharing a compute cluster. How do you allocate the resources among them?

- Users are *competing* for the *finite* amount of resources. Usually demand exceeds supply.
- Users want to “win this game”, i.e try to get more resources for themselves. *E.g “if I ask more, I will get more”.*
- Users wish to be treated fairly.

If managed poorly, this is an example of the tragedy of the commons.

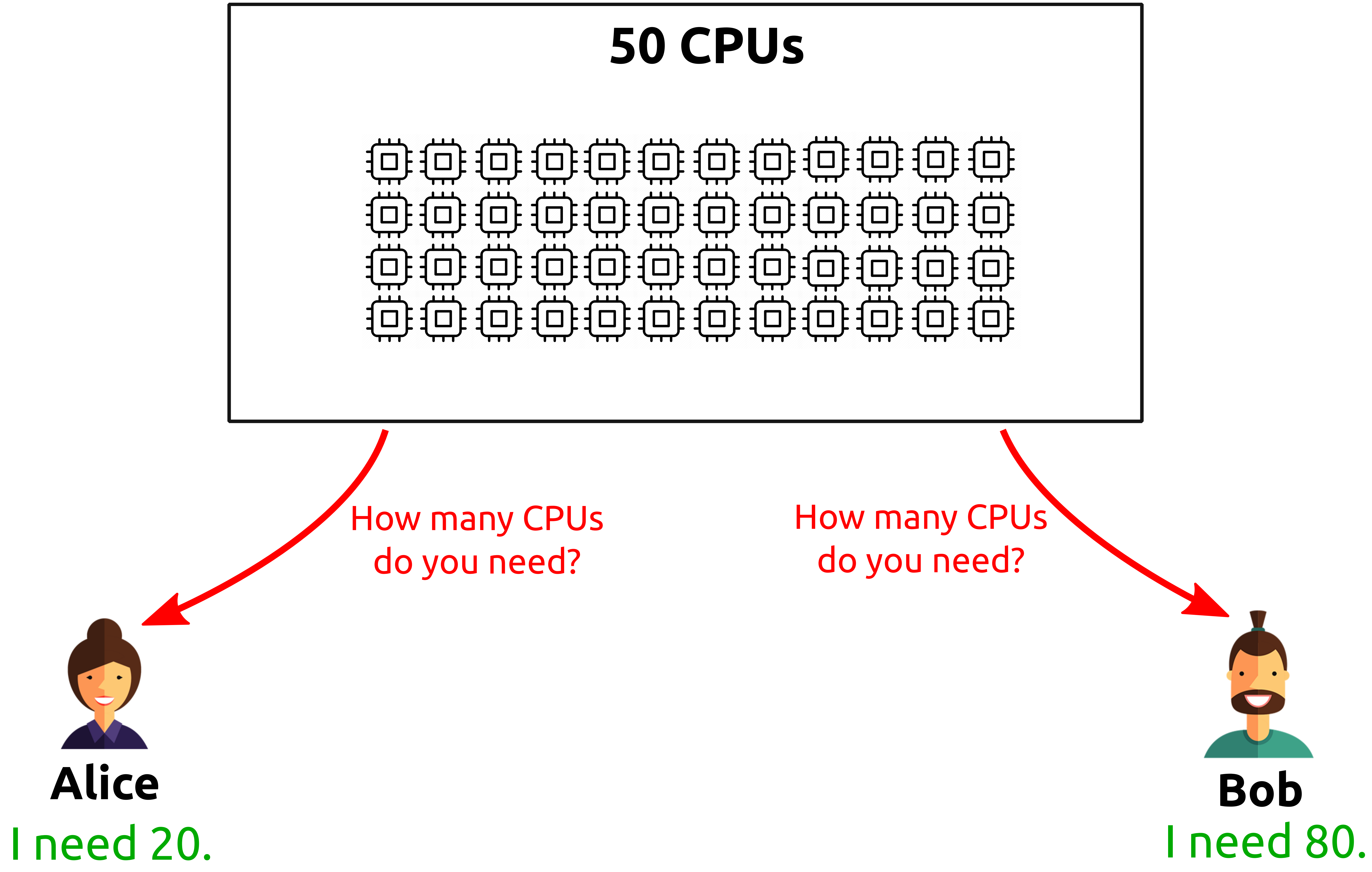
Fair resource allocation in clusters



Alice

Bob

Fair resource allocation under scarcity

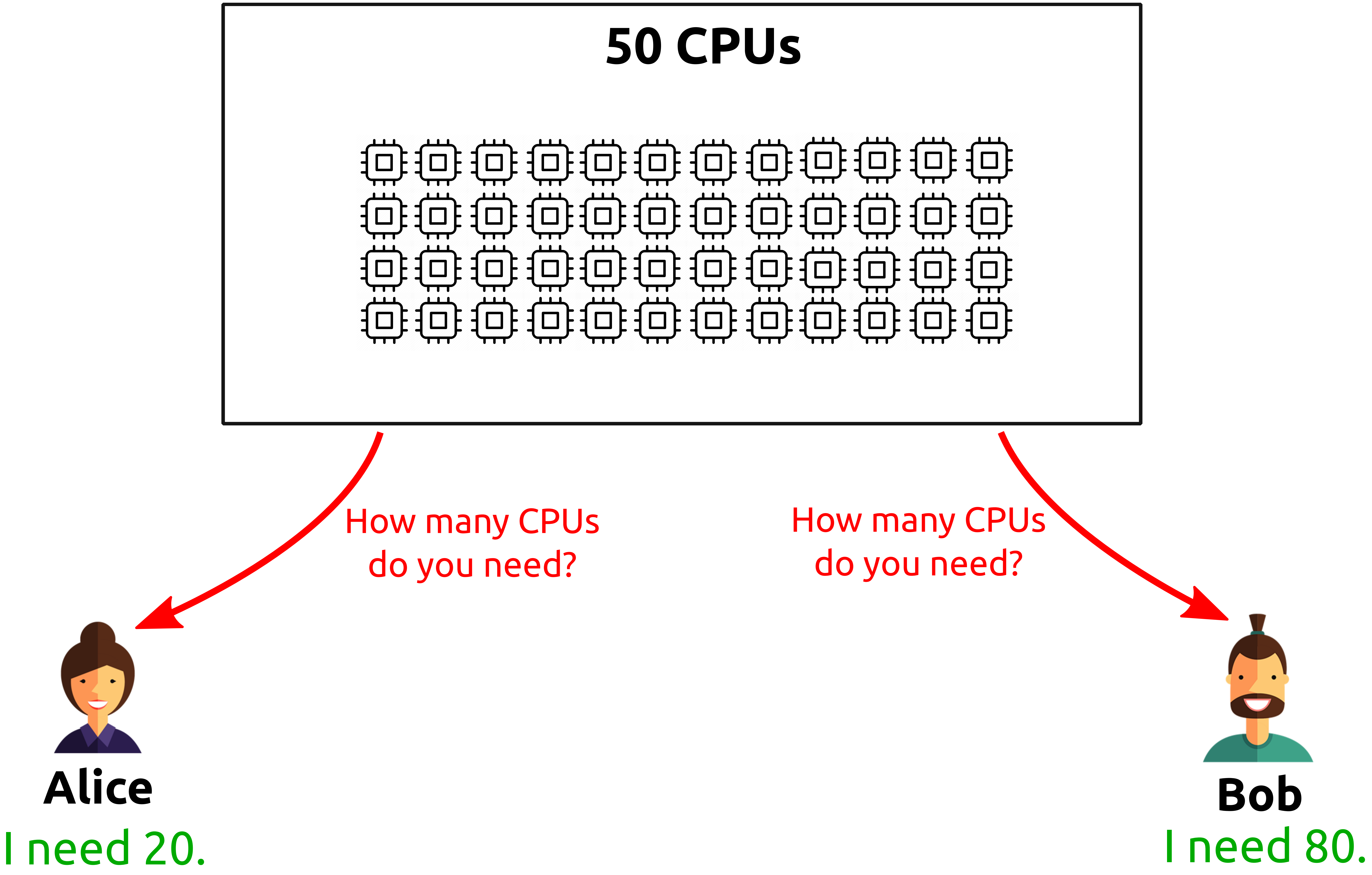


Cluster manager:
Let me try allocating proportionally

Issue 1:
Alice: this is not fair! My fair share is 25 CPUs.

Issue 2:
Bob: If I request more than 80, I will get more than 40.
Alice: I also need to request more.

Are there better solutions?



The Max-min fairness *mechanism*

- we will study this in detail later in class

100 CPUs, 4 Users	 **Carol**	 **Dave**	 **Eve**	 **Frank**
Demands	50	26	10	80
Rnd 1: fair share	25	25	25	25
Allocation			10	
Rnd 2: fair share	30	30	-	30
Allocation		26	-	
Rnd 3: fair share	32	-	-	32
Allocation	32	-	-	32

The Max-min fairness *mechanism*

Properties of this mechanism:

1. No incentives for agents to ask for more.

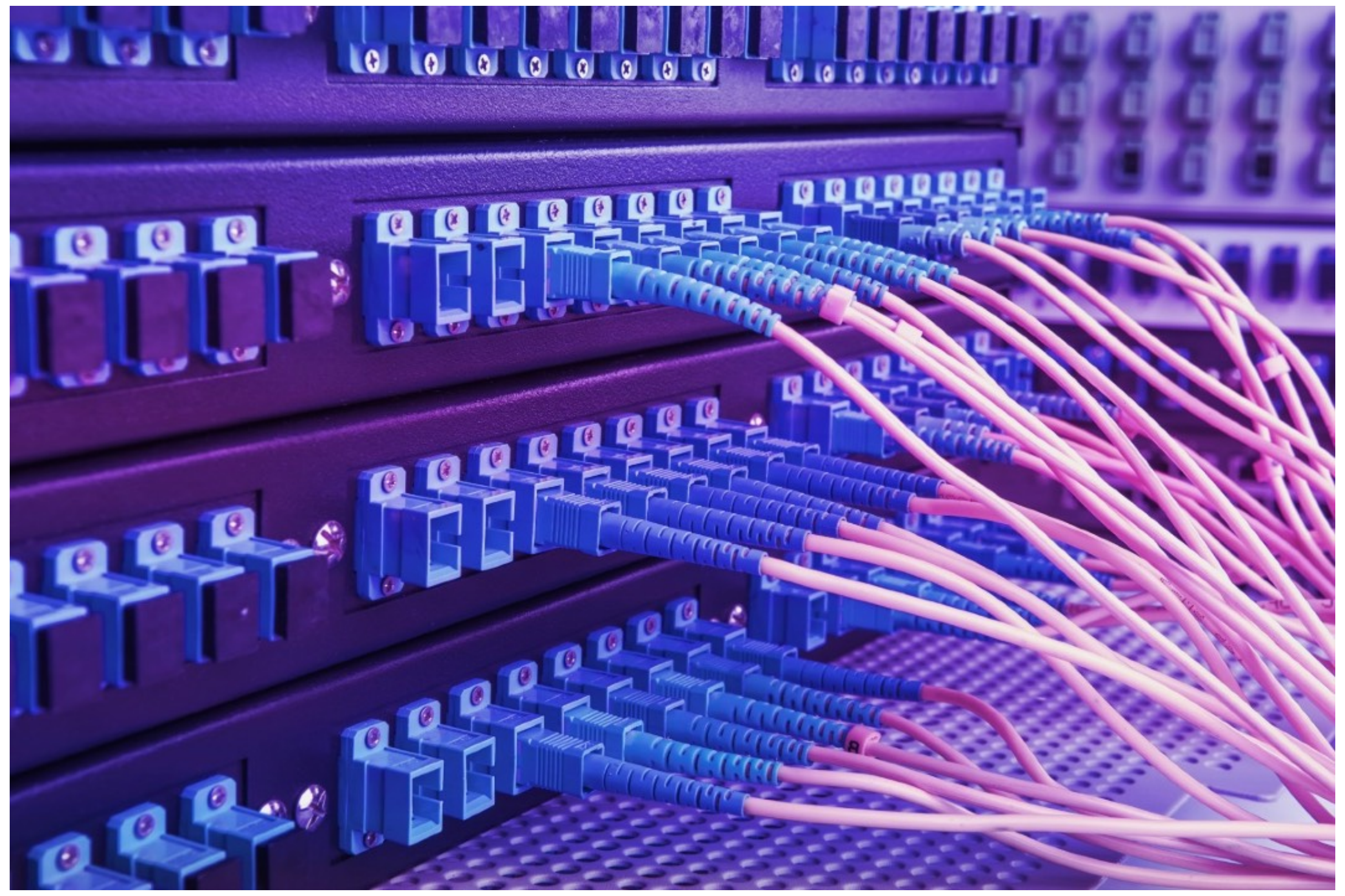
2. Fair *(a user will either fully satisfy her demand or receive at least her fair share)*

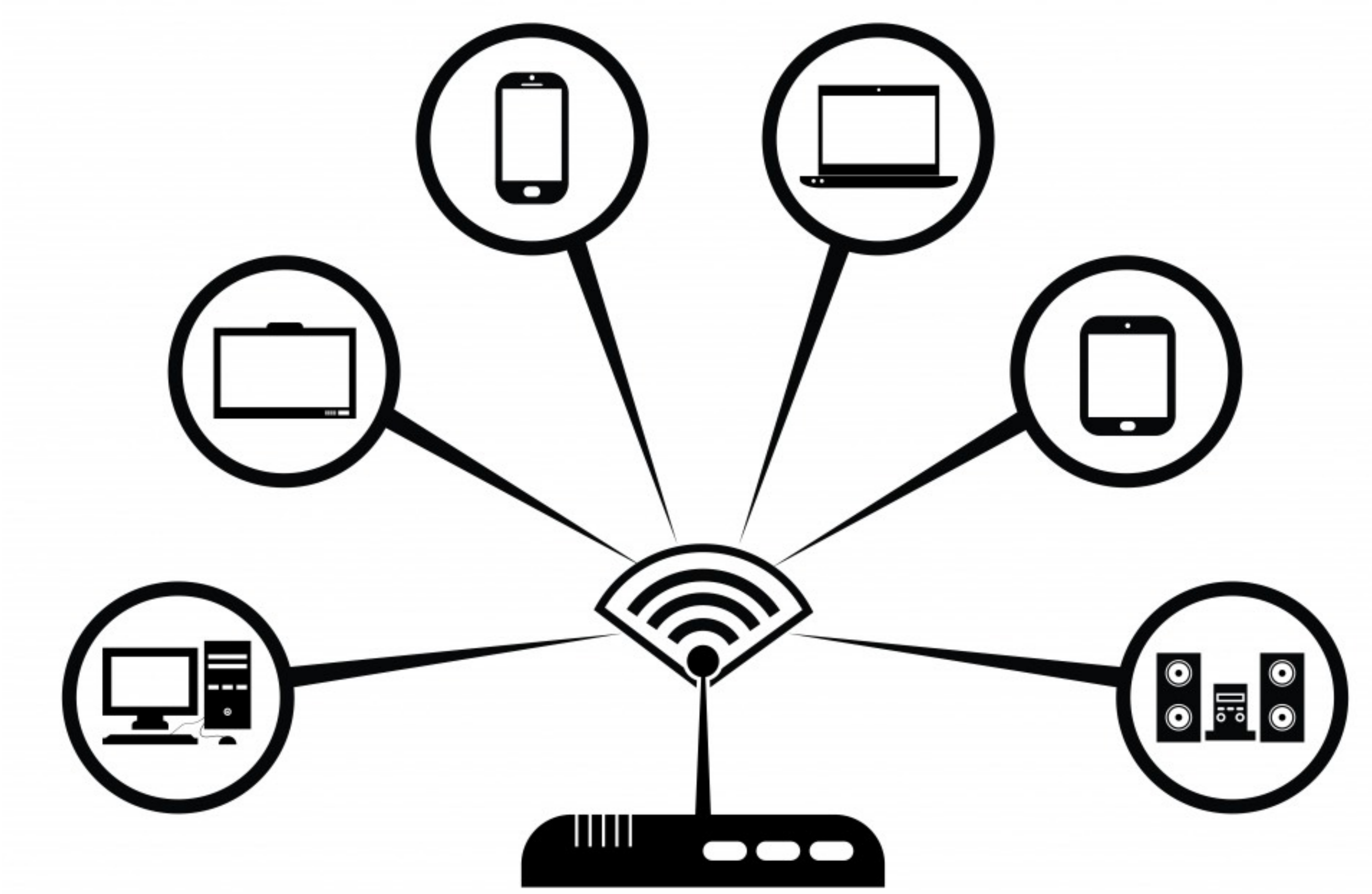
3. No resource wastage

100 CPUs, 4 Users	Carol	Dave	Eve	Frank
Demands	50	26	10	80
Rnd 1: fair share	25	25	25	25
Allocation			10	
Rnd 2: fair share	30	30	-	30
Allocation		26	-	
Rnd 3: fair share	32	-	-	32
Allocation	32	-	-	32

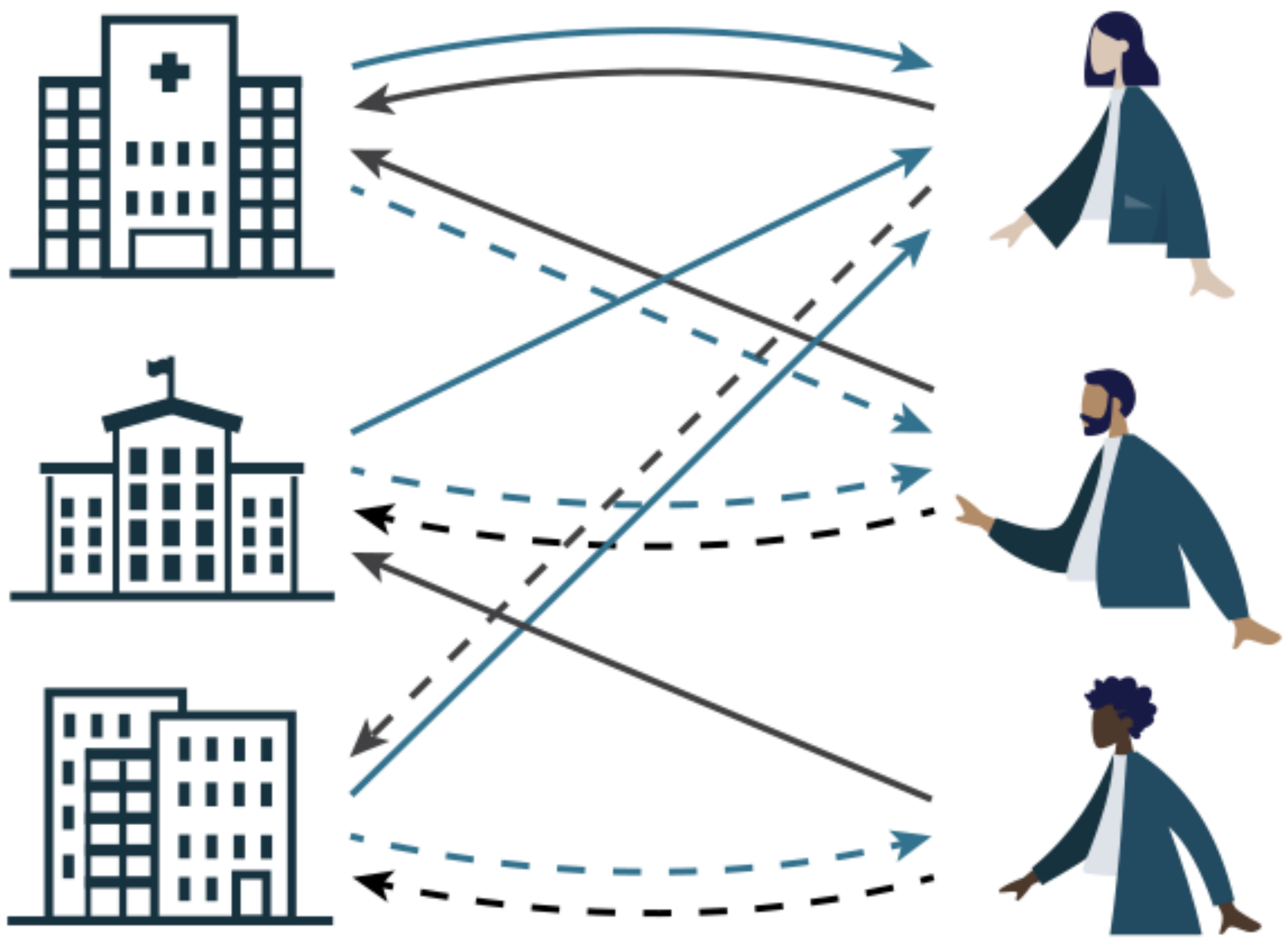
Fair resource allocation in real world cluster managers

Bandwidth allocation in WIFI and Broadband





Example 2: Stable matching



NRMP: National Residency Matching Program

Each year medical school graduates apply to the NRMP.

Hospitals rank the residents.

Residents can rank up to N hospitals (In NRMP, $N = 20$).

Goal: Match residents to hospitals based on submitted rankings.

Cooperative Games & Bargaining

A framework for studying situations where agents can make a mutually beneficial agreement, but have a conflict of interest about which agreement to make.

Example: How to divide wealth created?

Two workers decide to team up to produce tables.

- On their own, they can each make $10 per day.
- If they work together, they can make $100 per day.

Working together is clearly beneficial. But how do they split up the $100 between them?

$50 each seems natural.

Example (cont'd): How to divide wealth created?

A worker and employer decide to team up to produce tables.

- An employer on her own can make $50 per day.
- The worker on her own can make $10 per day.
- If they work together, they can make $100 per day.

Working together is clearly beneficial. But how do they split up the $100 between them?

No, one right solution. But we can rule out bad solutions.

Some desirable properties

Necessary

- *Individual rationality:* Employer should get at least $50, worker should get at least $10. Otherwise, they will not be willing to work together.

Nice to have

- The total allocation should be $100.
- Scale invariance. (E.g allocation should not change if we do the calculation in cents and not dollars.)
- Anonymity: changing labels of agents should not change allocation.

Other nice properties we will explore in class.

Outline

1. Course logistics
2. Syllabus
3. Who should take this class? Prerequisites and expectations

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Logistics: Lectures, Enrollment

- **Lectures**
  - Tuesday & Thursday, 4 - 5.15 pm at Engineering Hall 3355
  - Will be on the whiteboard.

- **Enrollment**
  - At capacity (40). More than 15 on the waitlist.
  - Continue to come to class, some students will likely drop.

Logistics: Recommended material

- Recommended textbook: **Game Theory, Alive** by Anna Karlin & Yuval Peres
- Other useful resources
  - **Game Theory - An Introduction** by Steven Tadelis.
  - Tim Roughgarden's lecture notes, Incentives in Computer Science, Algorithmic Game Theory.
  - Amy Greenwald's lecture notes, Topics In Algorithmic Game Theory.

Logistics: Course staff, Office hours

- **Instructor:** Kirthi Kandasamy
  - OHs: Tue, Thu 5.15 - 6.00 pm, ENGR HALL 3355 (after class)
  - E-mail: kandasamy [at] cs.wisc.edu.

- **Teaching assistant:** Joon Suk Huh
  - OHs:
  - E-mail: jhuh23 [at] cs.wisc.edu.

- **Grader:** Ankur Sonawane

Logistics: Webpages

- **Course website**
  - https://pages.cs.wisc.edu/~kandasamy/courses/24spring-cs639/
  - Information on logistics, syllabus, schedule, and grading

- **Piazza**
  - https://piazza.com/wisc/spring2024/cs639 (**access code: 639wiscgametheory**)
  - Announcements, peer discussions on lectures, homework clarifications

- **Canvas**
  - Homeworks (and some announcements)

Logistics: Contacting the Instructor or TA

- Please read the course website and previous Piazza posts before you ask us.

- If you decide to contact us:

  1. If you think others will benefit from the answer, please ask as a public question on Piazza.

     Public piazza post > OHs > private/anonymous piazza post > email

  2. Keep questions as concise as possible.

Logistics: Homework (50% of grade)

Read course webpage for all details: https://pages.cs.wisc.edu/~kandasamy/courses/24spring-cs639/grading.html

- Homework 0, and another 4-5 Homeworks.
- Should be typeset (no written and scanned homeworks). Five percent extra credit if you LaTeX your solutions.
- Homeworks *will be difficult*.
  - Expect to spend multiple hours/days on some problems.
  - Unless otherwise specified, you *are allowed* to collaborate with up to 2 classmates.
- Do **not** release questions outside of class, or discuss them in public forums.

Logistics: Homework (50% of grade)

- Late homework policy:
  - You have 3 total late days to be used throughout the semester.
  - After that, 50% of score if you submit 2 days after the deadline, and 0% if you are later than 2 days.
  - Extensions are **very unlikely,** and will be *considered* only for documented emergencies.
  - There are additional rules and we will follow them strictly. Please read the course website.

Logistics: Exams

- **Midterm: 25%**
  - Time: TBD, but likely sometime between Mar 4 - Mar 17 (likely Mar 4-10). **Do not make travel plans for now.**
  - Location TBD.
- **Final: 25%**
  - Thursday 05/09/2024 2.45 PM – 4.45 PM.
  - Location TBD.
- **Make-up exams:**
  - Only for *documented* emergencies and conference travel.
  - Otherwise, up to a 20% penalty.

Outline

1. Course logistics
2. **Syllabus**
3. Who should take this class? Prerequisites and expectations

Syllabus: Overview

1. Game theory fundamentals
2. Mechanism design
3. Cooperative game theory
4. Game theory and machine learning

Game theory fundamentals (6-9 lectures)

Basic tools to analyze games and predict behaviour of agents.

- Zero-sum games, minimax theorem
- General-sum games, Nash equilibrium
- Potential games
- Price of anarchy/stability
- Evolutionary and correlated equilibria

Mechanism Design (7-10 lectures)

Designing games to achieve socially desirable outcomes. We will focus on common desiderata such as, individual rationality, incentive compatibility, fairness, and efficiency.

Applications: Auctions, Scoring rules, Fair resource allocation, Stable matching

Cooperative Game Theory (4-5 lectures)

Study, when agents will cooperate when they can create more together than the sum of their parts.

- Coalitional games
  - The core, Shapley value
- Axiomatic bargaining
  - Nash, utilitarian, and egalitarian solutions

Machine Learning and Game Theory (3-4 lectures)

Modern topics in the intersection of ML, Game theory, and mechanism design

- Sequential decision-making as a zero-sum game
- Learning in games and mechanism design

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Target audience for the class

- Advanced undergraduate and early graduate students with a strong background in mathematics

- **Background knowledge**
  - **Formal prerequisites:** CS240, CS475, Econ 301, or Econ 311.
  - Background in calculus, statistics, linear algebra, and probability.
  - Mathematical maturity, should be comfortable with proofs and logical reasoning.

- **No review** of background topics
  - But, if it is a new mathematical concept, I may explain things **once**.

Homework 0

Four questions:

1. Mathematics background **(no collaboration on this question)**
2. Game theory: studying a simple game with tragedy of the commons
3. Cooperative game theory: A bargaining problem

Three Objectives

I. A preview of what's to come

II. Calibrate my teaching/expectations

III. Lets you assess if you are ready to take this class

General advice when taking this class

1. Focus on learning, and not on grades.
    - Class will be challenging. But if you are able to keep up, you will get a good grade.

2. This is a new class, so there will almost surely be some shortcomings. Give me feedback about the course.

3. Be good citizens: attend class, ask questions, answer questions, let others answer/ask questions, respond to questions on piazza.