Stochastic Bandits



Kirthevasan Kandasamy

Carnegie Mellon University

University of Moratuwa, Sri Lanka August 17, 2017

Slides: www.cs.cmu.edu/~kkandasa/misc/mora-slides.pdf

Slides are up on my webpage:

www.cs.cmu.edu/~kkandasa

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publications

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misc.

kirthevasan kandasamy

PhD Student. Carnegie Mellon University

[CV] [Google Scholar] [GitHub] [Contact]



I am a fourth year Machine Learning PhD student (now ABD) in the School of Computer Science at Carnegie Mellon University. I am co-advised by Jeff Schneider and Barnabas Poczos. I am a member of the Auton Lab and the StatML Group, Prior to CMU, I completed my B.Sc in Electronics & Telecommunications Engineering at the University of Moratuwa, Sr Lanka.

My research interests lie in the intersection of statistical and algorithmic Machine Learning. My current research spans bandit problems, Bayesian optimisation, Gaussian processes, nonparametric statistics and graphical models. As of late, I have also hopped on the deep learning bandwagon.

I am generously supported by a Facebook PhD fellowship (2017) and a CMU Presidential fellowship (2015).

Preprints



Asynchronous Parallel Bayesian Optimisation via Thompson Sampling <u>Kirthevasan Kandasamy</u>, Akshay Krishnamurthy, Jeff Schneider, Barnabas Poczos [arxiv] [AutoML slides] [Moratuwa slides]

Multi-fidelity Gaussian Process Bandit Optimisation Kirthevasan Kandasamy, Gautam Dasarathy, Junier Oliva, Jeff Schneider, Barnabas Poczos [arxiv] [code] [UCL stides]

Influence Functions for Machine Learning: Nonparametric Estimators for Entropies, Divergences and Mutual Informations
Kirtheyasan Kandasamy, Akshay Krishnamurthy, Barnahas Pozzos, Larry Wasserman, James Rohins

On-line advertising



You are given a pool of 250 ads.

Task:

- ▶ You can display one ad at a time, (say for 10⁶ times).
- ▶ You wish to maximise the cumulative number of clicks, i.e. identify ads with the highest click-through-rate and display them most of the time.

The Stochastic Multi-armed Bandit

(Robbins, 1952)

- ▶ You are given K arms, $\mathcal{X} = \{1, ..., K\}$.
- ► At every round you "play/pull" an arm.

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- ▶ **Goal:** Maximise the cumulative sum of expected rewards,

$$\mathbb{E}\left[\sum_{t=1}^n y_t\right] = \sum_{t=1}^n f(x_t).$$

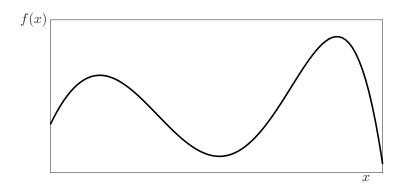
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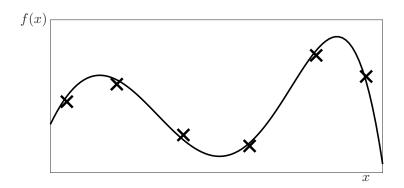
▶ Goal: An algorithm (policy/strategy) which achieves "small" cumulative regret,

$$R_n = \sum_{t=1}^n f(x_\star) - \sum_{t=1}^n f(x_t) = \sum_{t=1}^n \left(f(x_\star) - f(x_t) \right).$$
 where, $x_\star = \operatorname{argmax}_{x \in \mathcal{X}} f(x)$.

 $f:\mathcal{X}\to\mathbb{R}$ is a black-box function that is accessible only via noisy evaluations. \mathcal{X} is a metric space, e.g. \mathbb{R}^d .

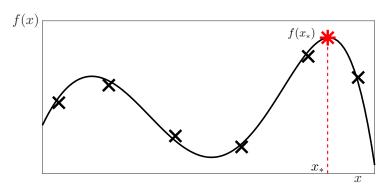


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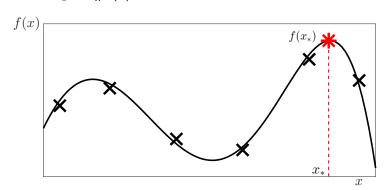


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Let $x_{\star} = \operatorname{argmax}_{x} f(x)$.



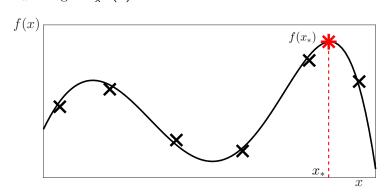
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Cumulative Regret after n evaluations

$$R_n = \sum_{t=1}^n f(x_*) - f(x_t).$$

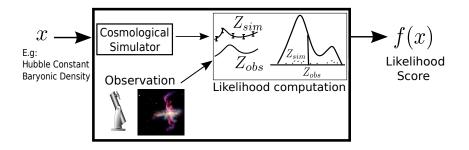
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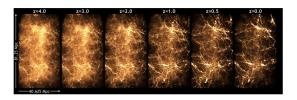


Simple Regret after n evaluations

$$S_n = f(x_*) - \max_{t=1,\ldots,n} f(x_t).$$

Applications





Applications

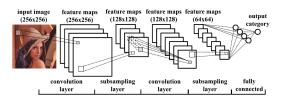


parameters

Neural Network

cross validation accuracy

- Train NN using given hyper-parameters
- Compute accuracy on validation set



Applications



Expensive Blackbox Function

 $\longrightarrow f(x)$

Other Examples:

- Pre-clinical Drug Discovery
- Optimal policy in Autonomous Driving
- Synthetic gene design







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N.B: Pulling/playing an arm = experiment = function evaluation

Outline

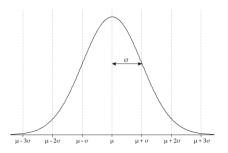
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Gaussian (Normal) distribution

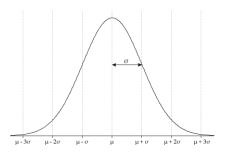




- A probability distribution for real valued random variables.
- ▶ Mean μ and variance σ^2 completely characterises distribution.

Gaussian (Normal) distribution



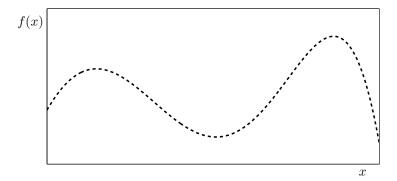


- ▶ A probability distribution for real valued random variables.
- ▶ Mean μ and variance σ^2 completely characterises distribution.
- For samples X_1, \ldots, X_n , let $\hat{\mu} = \frac{1}{n} \sum_i X_i$ be the sample mean. Then, $\hat{\mu} \pm 1.96 \frac{\sigma}{\sqrt{n}}$ is a 95% confidence interval for μ .
- ► Can draw samples (e.g. in Matlab: mu + sigma * randn()).

 $\mathcal{GP}(\mu,\kappa)$: A distribution over functions from \mathcal{X} to \mathbb{R} . Completely characterised by mean function $\mu:\mathcal{X}\to\mathbb{R}$, and covariance kernel $\kappa:\mathcal{X}\times\mathcal{X}\to\mathbb{R}$.

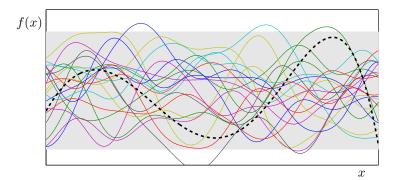
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Functions with no observations



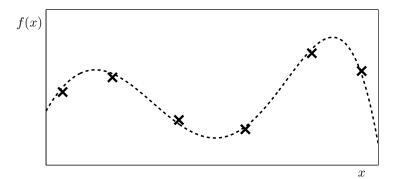
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Prior \mathcal{GP}



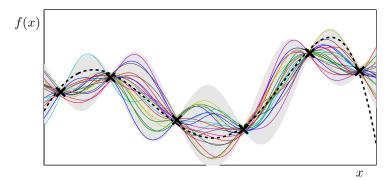
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Observations



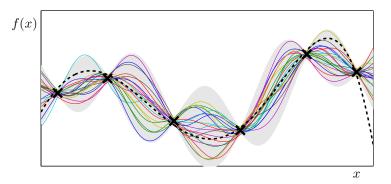
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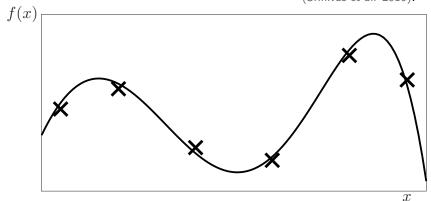


After t observations, $f(x) \sim \mathcal{N}(\mu_t(x), \sigma_t^2(x))$.

Model $f \sim \mathcal{GP}(\mathbf{0}, \kappa)$.

Gaussian Process Upper Confidence Bound (GP-UCB)

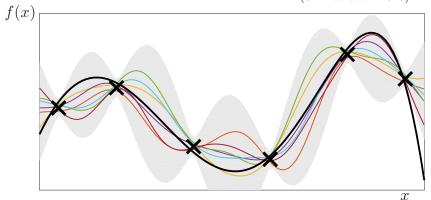
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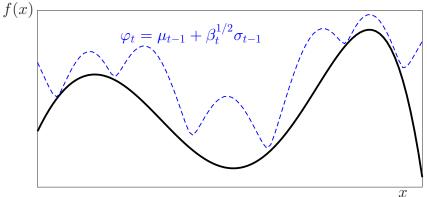
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Gaussian Process Upper Confidence Bound (GP-UCB)

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Construct upper conf. bound: $\varphi_t(x) = \mu_{t-1}(x) + \beta_t^{1/2} \sigma_{t-1}(x)$.

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Gaussian Process Upper Confidence Bound (GP-UCB)

(Srinivas et al. 2010). f(x) $\varphi_t = \mu_{t-1} + \beta_t^{1/2} \sigma_{t-1}$ x

Maximise upper confidence bound.

GP-UCB

$$x_t = \underset{x}{\operatorname{argmax}} \ \mu_{t-1}(x) + \beta_t^{1/2} \sigma_{t-1}(x)$$

- $\blacktriangleright \mu_{t-1}$: Exploitation
- $ightharpoonup \sigma_{t-1}$: Exploration
- β_t controls the tradeoff. $\beta_t \approx \log t$.

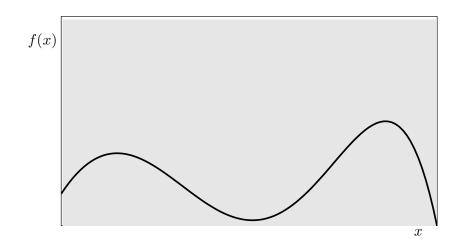
GP-UCB

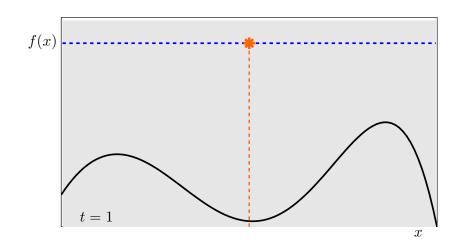
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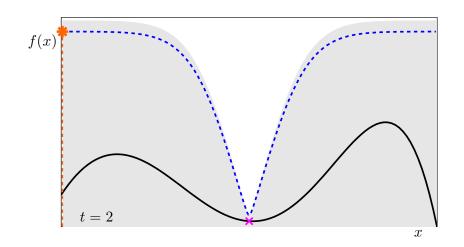
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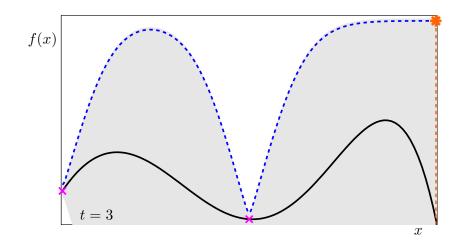
GP-UCB,
$$\kappa$$
 is an SE kernel (Srinivas et al. 2010) w.h.p $S_n = f(x_\star) - \max_{t=1,\ldots,n} f(x_t) \lesssim \sqrt{\frac{\log(n)^d \operatorname{vol}(\mathcal{X})}{n}}$

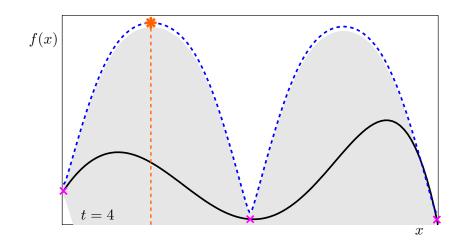
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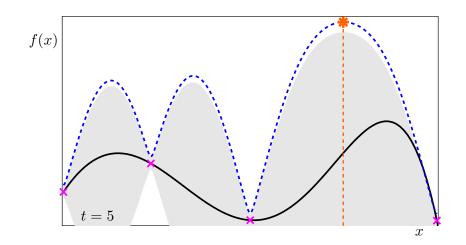


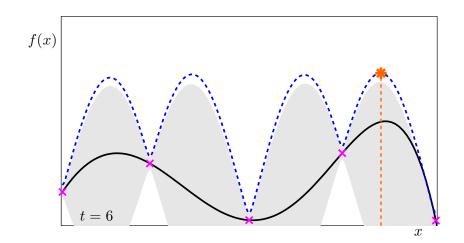


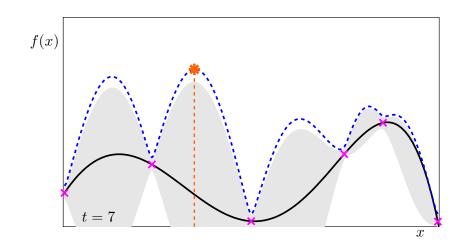


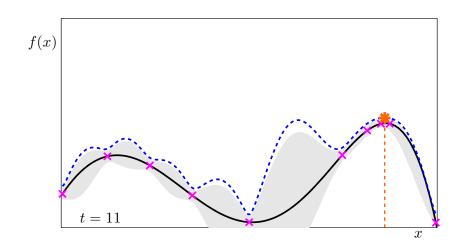


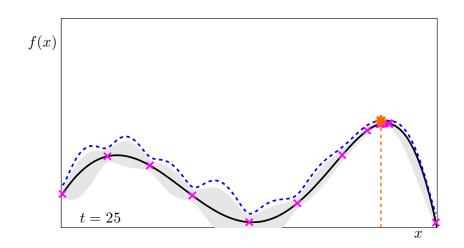










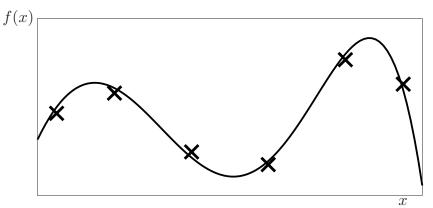


Algorithm 2: Thompson Sampling in GP Bandits

Model $f \sim \mathcal{GP}(\mathbf{0}, \kappa)$.

Thompson Sampling (TS)

(Thompson, 1933).

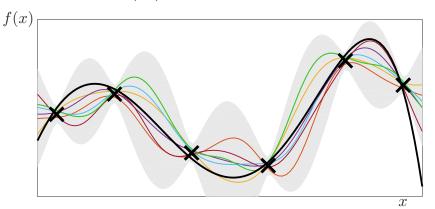


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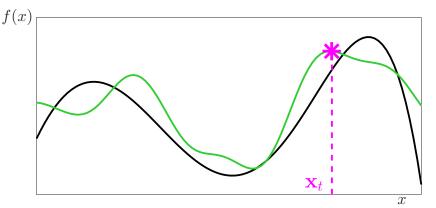


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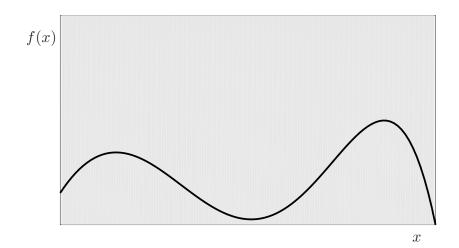
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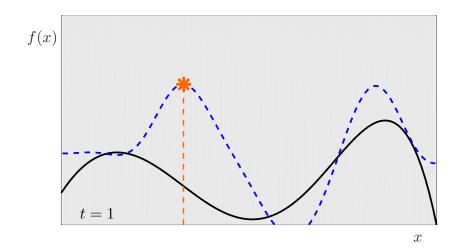
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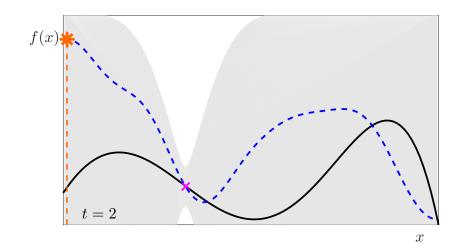
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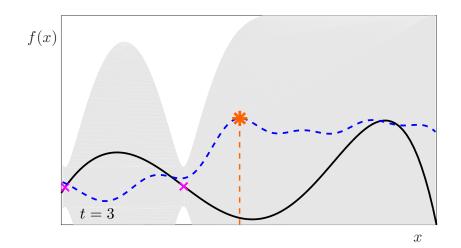


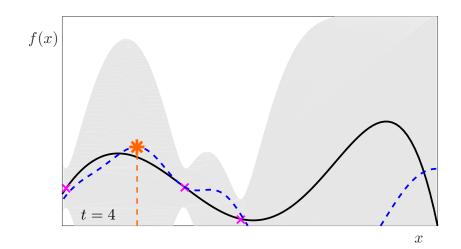
Draw sample g from posterior. Choose $x_t = \operatorname{argmax}_x g(x)$.

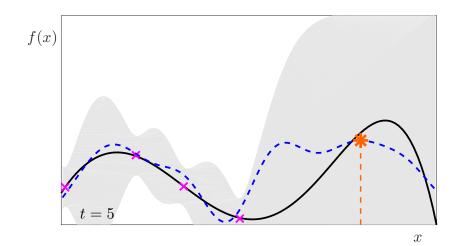


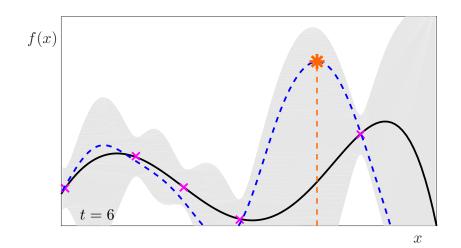


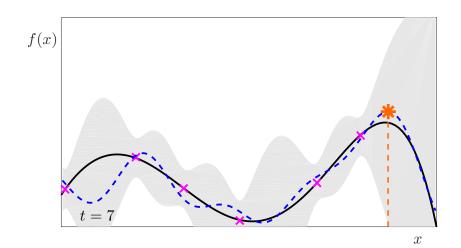


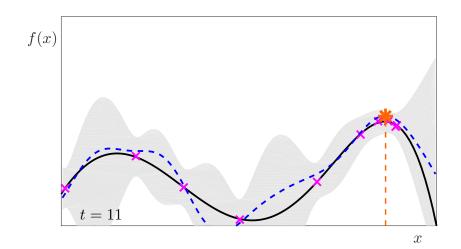


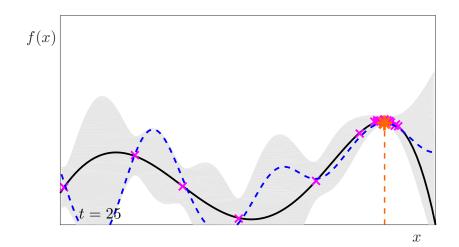












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SL2College



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- Looking to study abroad?
 - We connect Sri Lankan students who are seeking higher education abroad with undergraduates and graduates from respective universities, who would guide them through the process
 - Willing to go that 'Extra Mile' with your research?
 - Submitting a paper for an International Journal / Conference is not easy. Join our Research Collaboration programme to find a qualified. experienced mentor, whose expertise will take you there.

- Looking for Professional / Vocational Qualifications?
 - We can guide you on Professional and Vocational qualifications required to excel in your career path

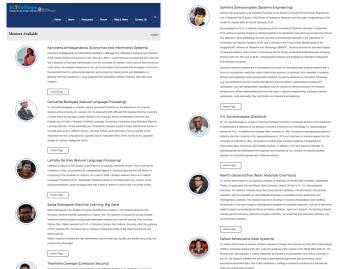
Moving to Sri Lanka?

Planning to return to your motherland? Seeking job opportunities in Sri Lanka? We provide guidance and information for everyone who want to return or migrate to Sri Lanka.

www.sl2college.org

SL2College Research Collaboration Program

-Ashwin de Silva



www.sl2college.org/research-collab research-collab@sl2college.org

SL2College Research Collaboration Program

How it works

We have a pool of doctoral/post-doctoral/professorial mentors (all Sri Lankan).

We connect Sri Lankan undergrads to mentors, who will guide the students on a research project.

Aim: Publish a paper (at a good venue) within a 9-15 month time frame.

Application Process

- ► Fill out the application form on our webpage: www.sl2college.org/research-collab
 - mention areas of interests and preferred mentors.
- ... and email your CV to research-collab@sl2college.org.

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- \blacktriangleright If we decide to proceed, we ask you to submit a ~ 1 page research statement,
 - your research interests & future plans
 - why you are interested in working with aforesaid mentor.

Application Process

- ► Fill out the application form on our webpage: www.sl2college.org/research-collab
 - mention areas of interests and preferred mentors.
- ... and email your CV to research-collab@sl2college.org.
- If we decide to proceed, we ask you to submit a ~ 1 page research statement,
 - your research interests & future plans
 - why you are interested in working with aforesaid mentor.
- We send your CV & statement to the mentor. If he/she is interested, we initiate a collaboration.
- ▶ You report to us once every 3 months.

SL2College Research Collaboration Team







Ashwin

Nuwan

Rajitha





Umashanthi

Kirthevasan

www.sl2college.org/research-collab research-collab@sl2college.org

Outline

- ► Part I: Stochastic bandits (cont'd)
 - 1. Gaussian processes for smooth bandits
 - 2. Algorithms: Upper Confidence Bound (UCB) & Thompson Sampling (TS)
- ► Digression: SL2College Research Collaboration Program
- ▶ Part II: My research
 - Multi-fidelity bandit: cheap approximations to an expensive experiments
 - 2. Parallelising arm pulls

Part 2.1: Multi-fidelity Bandits

Motivating question:

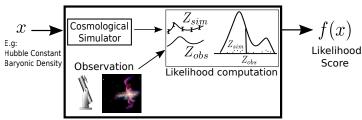
What if we have cheap approximations to f?

Part 2.1: Multi-fidelity Bandits

Motivating question:

What if we have cheap approximations to f?

1. Computational astrophysics and other scientific experiments: simulations and numerical computations with less granularity.

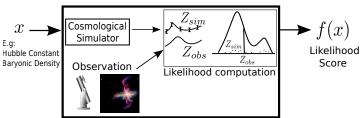


Part 2.1: Multi-fidelity Bandits

Motivating question:

What if we have cheap approximations to f?

1. Computational astrophysics and other scientific experiments: simulations and numerical computations with less granularity.



- 2. Hyper-parameter tuning: Train & validate with a subset of the data.
- 3. Robotics & autonomous driving: computer simulation vs real world experiment.

Multi-fidelity Methods

For specific applications,

```
    ► Industrial design (Forrester et al. 2007)
    ► Hyper-parameter tuning (Agarwal et al. 2011, Klein et al. 2015, Li et al. 2016)
    ► Active learning (Zhang & Chaudhuri 2015)
    ► Robotics (Cutler et al. 2014)
```

Multi-fidelity bandits & optimisation (Huang et al. 2006, Forrester et al. 2007, March & Wilcox 2012, Poloczek et al. 2016)

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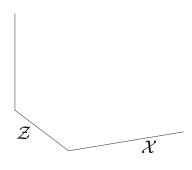
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Multi-fidelity bandits & optimisation (Huang et al. 2006,
Forrester et al. 2007, March & Wilcox 2012, Poloczek et al. 2016)
... with theoretical guarantees (Kandasamy et al. NIPS 2016a&b,
Kandasamy et al. ICML 2017)
```

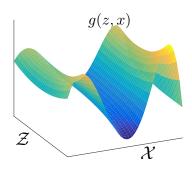
(Kandasamy et al. ICML 2017)



A fidelity space ${\mathcal Z}$ and domain ${\mathcal X}$

 $\mathcal{Z} \leftarrow \mathsf{all} \; \mathsf{granularity} \; \mathsf{values}$

 $\mathcal{X} \leftarrow \mathsf{space} \; \mathsf{of} \; \mathsf{cosmological} \; \mathsf{parameters}$



(Kandasamy et al. ICML 2017)

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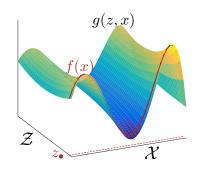
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 $g(z,x) \leftarrow$ likelihood score when performing integrations on a grid of size z at cosmological parameters x.

(Kandasamy et al. ICML 2017)



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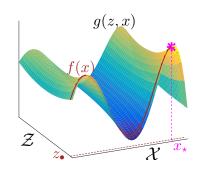
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Denote
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 $z_{\bullet} = \text{highest grid size}.$

(Kandasamy et al. ICML 2017)



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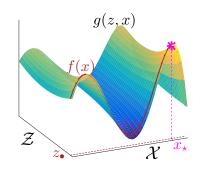
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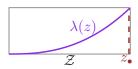
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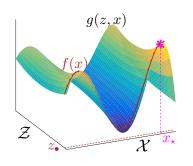
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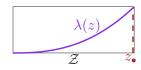
A cost function, $\lambda: \mathcal{Z} \to \mathbb{R}_+$. $\lambda(z) = \mathcal{O}(z^p)$ (say).



Multi-fidelity Simple Regret

(Kandasamy et al. ICML 2017)

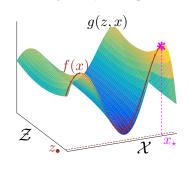


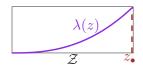


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Multi-fidelity Simple Regret

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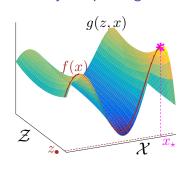
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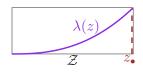
Simple Regret after capital
$$\Lambda$$
: $S(\Lambda) = f(x_*) - \max_{t: z_t = z_{\bullet}} f(x_t)$.

 $\Lambda \leftarrow$ amount of a resource spent, e.g. computation time or money.

Multi-fidelity Simple Regret







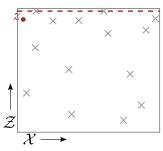
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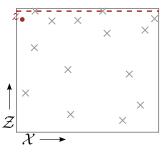
Simple Regret after capital Λ : $S(\Lambda) = f(x_*) - \max_{t: z_t = z_{\bullet}} f(x_t)$.

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No reward for pulling an arm at low fidelities, but use cheap evaluations at $z \neq z_{\bullet}$ to speed up search for x_{\star} .

(Kandasamy et al. ICML 2017)



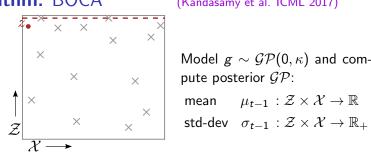


(Kandasamy et al. ICML 2017)

Model $g \sim \mathcal{GP}(0, \kappa)$ and compute posterior \mathcal{GP} :

 $\begin{array}{ll} \mathsf{mean} & \mu_{t-1}: \mathcal{Z} \times \mathcal{X} \to \mathbb{R} \\ \mathsf{std-dev} & \sigma_{t-1}: \mathcal{Z} \times \mathcal{X} \to \mathbb{R}_+ \end{array}$

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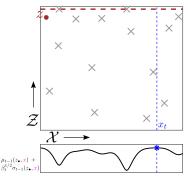


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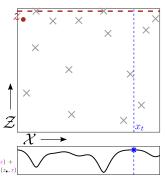


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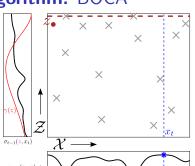
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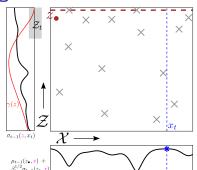
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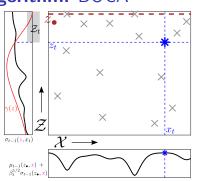
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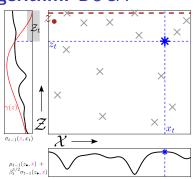


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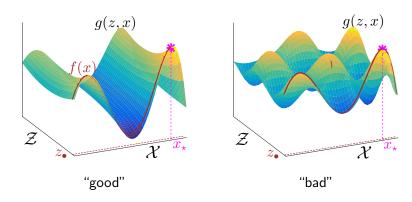
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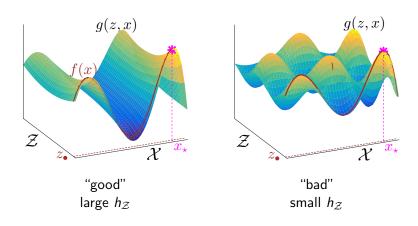


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E.g.: For SE kernels, bandwidth $h_{\mathcal{Z}}$ controls smoothness.

GP-UCB SE kernel, (Srinivas et al. 2010)
$$\text{w.h.p} \qquad S(\Lambda) \; \lesssim \; \sqrt{\frac{\operatorname{vol}(\mathcal{X})}{\Lambda}}$$

BOCA SE kernel, (Kandasamy et al. ICML 2017) w.h.p
$$\forall \alpha > 0$$
, $S(\Lambda) \lesssim \sqrt{\frac{\operatorname{vol}(\mathcal{X}_{\alpha})}{\Lambda}} + \sqrt{\frac{\operatorname{vol}(\mathcal{X})}{\Lambda^{2-\alpha}}}$ $\mathcal{X}_{\alpha} = \left\{x; \ f(x_{\star}) - f(x) \lesssim C_{\alpha} \frac{1}{h_{\pi}}\right\}$

BOCA SE kernel, (Kandasamy et al. ICML 2017)
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If $h_{\mathcal{Z}}$ is large (good approximations), $\operatorname{vol}(\mathcal{X}_{\alpha}) \ll \operatorname{vol}(\mathcal{X})$, and BOCA is much better than GP-UCB.

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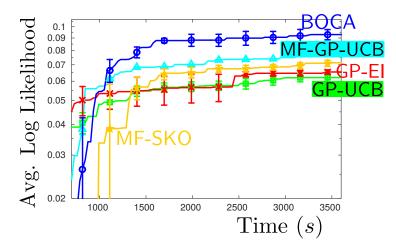
N.B: Dropping constants and polylog terms.

Experiment: Cosmological inference on Type-1a supernovae data

Estimate Hubble constant, dark matter fraction & dark energy fraction by maximising likelihood on $N_{\bullet} = 192$ data. Requires numerical integration on a grid of size $G_{\bullet} = 10^6$. Approximate with $N \in [50, 192]$ or $G \in [10^2, 10^6]$ (2D fidelity space).

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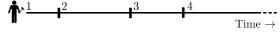


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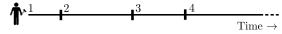
Part 2.2: Parallelising arm pulls

Sequential arm pulls with one worker

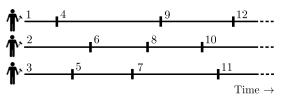


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Sequential arm pulls with one worker

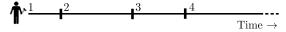


Parallel arm pulls with M workers (Asynchronous)

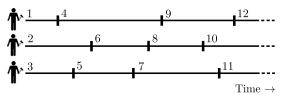


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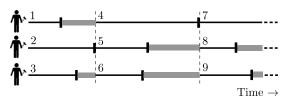
Sequential arm pulls with one worker



Parallel arm pulls with M workers (Asynchronous)



Parallel arm pulls with M workers (Synchronous)



Why parallelisation?

- ► Computational experiments: infrastructure with 100-1000's CPUs or GPUs.
- ▶ Drug discovery: High throughput screening

Why parallelisation?

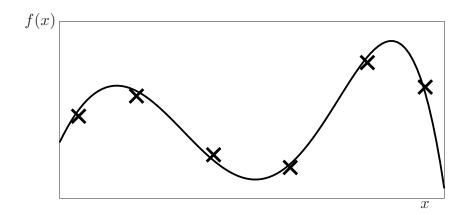
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Prior work: (Ginsbourger et al. 2011, Janusevskis et al. 2012, Wang et al. 2016, González et al. 2015, Desautels et al. 2014, Contal et al. 2013, Shah and Ghahramani 2015, Kathuria et al. 2016, Wang et al. 2017, Wu and Frazier 2016, Hernandez-Lobato et al. 2017)

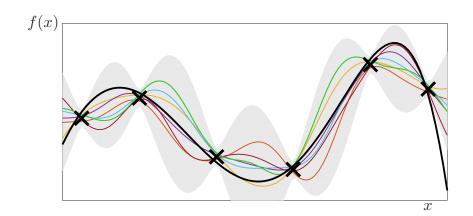
Shortcomings

- Asynchronicity
- Theoretical guarantees
- Computationally & conceptually simple

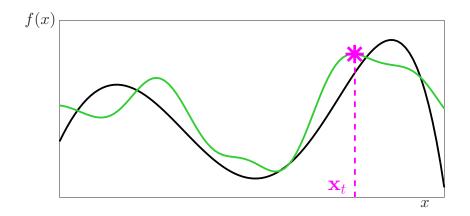
Review: Sequential Thompson Sampling in GP Bandits



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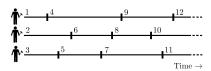


Draw sample g from posterior. Choose $x_t = \operatorname{argmax}_x g(x)$.

Asynchronous: asyTS

At any given time,

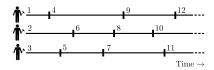
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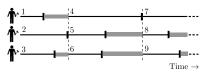
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Synchronous: synTS

At any given time,

- 1. $\{(x'_m, y'_m)\}_{m=1}^M \leftarrow \text{Wait for all workers to finish.}$
- 2. Compute posterior \mathcal{GP} .
- 3. Draw M samples $g_m \sim \mathcal{GP}$, $\forall m$.
- 4. Re-deploy worker m at $\underset{\text{argmax } g_m, \forall m}{\text{w}}$.



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$$\mathbb{E}[S_n] \lesssim \sqrt{\frac{\operatorname{vol}(\mathcal{X})\log(n)^d}{n}}$$

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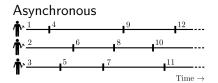
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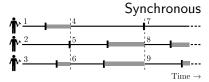
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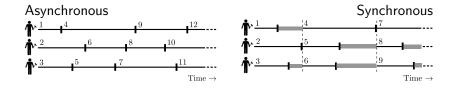
- A sequential algorithm can make use of information from all previous rounds to determine where to evaluate next.
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Theoretical Results: Simple regret with time





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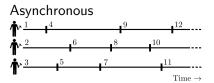
Theorem (Informal)

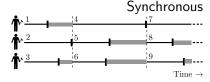
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Otherwise, bounds for asyTS is strictly better than synTS. More the variability in evaluation times, the bigger the difference.

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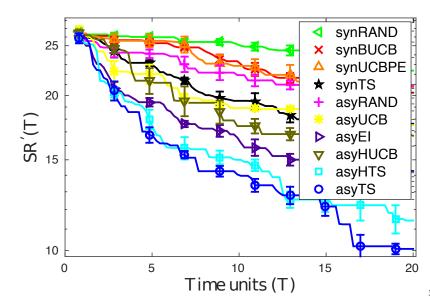
Otherwise, bounds for asyTS is strictly better than synTS. More the variability in evaluation times, the bigger the difference.

- Bounded tail decay: constant factor
- Sub-gaussian tail decay: $\sqrt{\log(M)}$ factor
- Sub-exponential tail decay: log(M) factor

Experiment: Currin-Exponential-14D

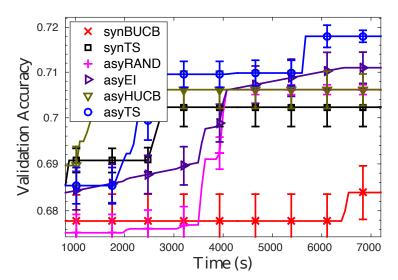
M = 35

Evaluation time sampled from a Pareto-3 distribution



Experiment: Hyper-parameter tuning in Cifar10 M = 4

Tune # filters in in range (32, 256) for each layer in a 6 layer CNN. Time taken for an evaluation: 4 - 16 minutes.



Summary

- Bandits are a framework for studying exploration vs exploitation trade-offs when optimising black-box functions.
- Smooth bandit formulations are more common in practical applications.
- ▶ Several algorithms: UCB, TS, Index based policies, ϵ -greedy etc.

Summary

- Bandits are a framework for studying exploration vs exploitation trade-offs when optimising black-box functions.
- Smooth bandit formulations are more common in practical applications.
- ▶ Several algorithms: UCB, TS, Index based policies, ϵ -greedy etc.
- Multi-fidelity Bandits: Allows us to use cheap approximations to a an expensive experiment to quickly find the optimum.
- ▶ Parallelised TS: Simple and intuitive way to deal with multiple workers.







Barnabás



Gautam



Jeff



Junier

Thank You

Slides: www.cs.cmu.edu/~kkandasa/misc/mora-slides.pdf