DATA WITHOUT BORDERS

GAME-THEORETIC CHALLENGES IN DEMOCRATIZING DATA

STATISTICS SEMINAR.   APRIL 8, 2024

KIRTHEVASAN KANDASAMY

DEPARTMENT OF COMPUTER SCIENCES, UW-MADISON

BASED ON JOINT WORK WITH: YIDING CHEN, JERRY ZHU, AND OTHERS

DATA DRIVEN METHODS ARE UBIQUITOUS

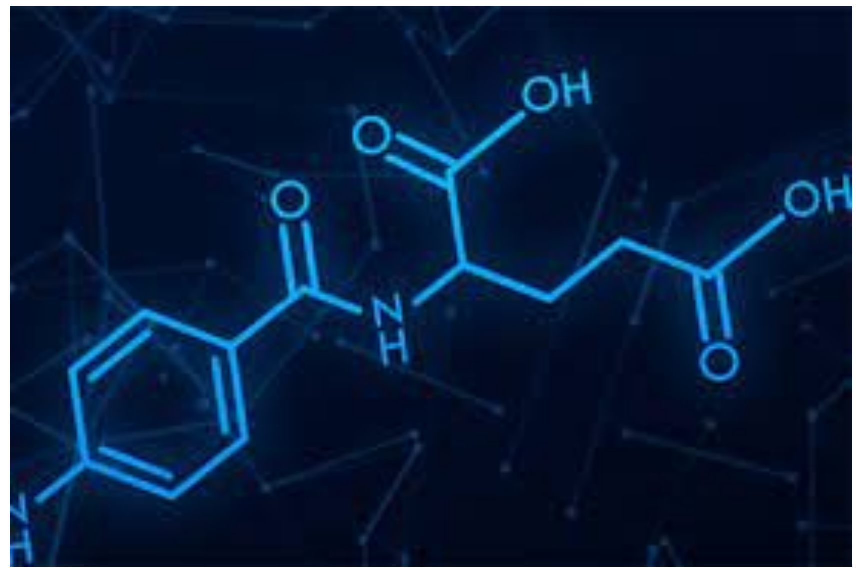
- Consumer facing businesses
- Industrial processes
- Scientific research
- Transport/logistics

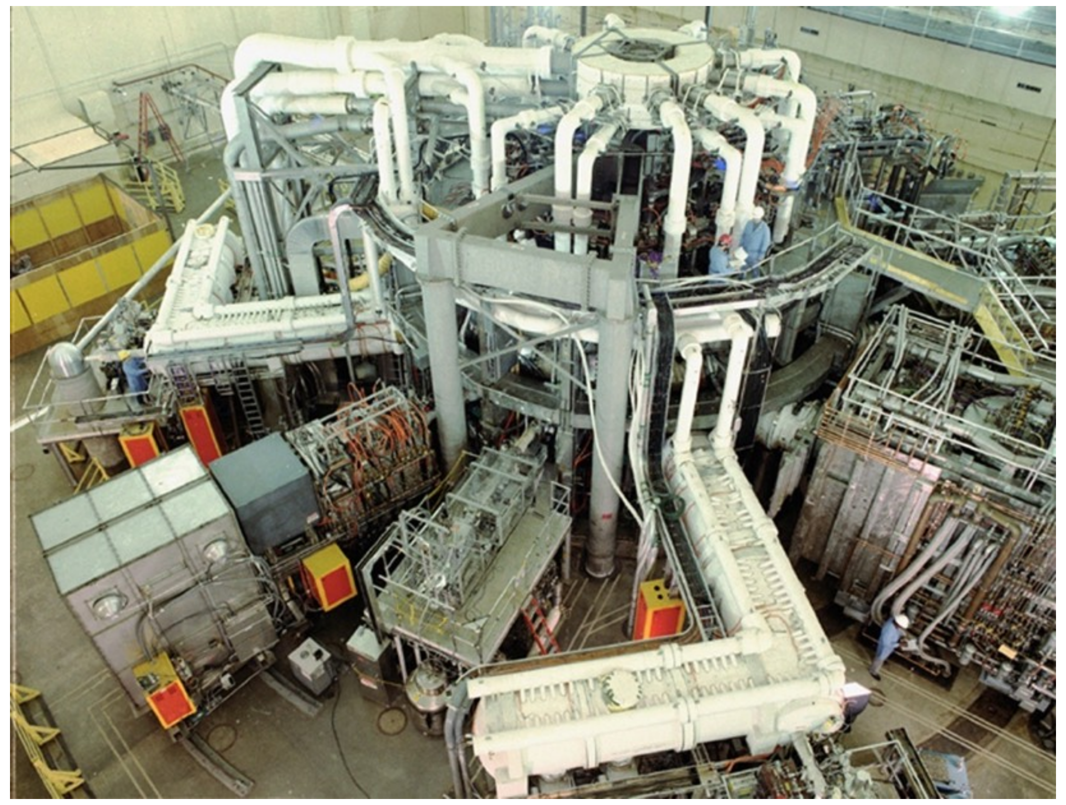
shopify

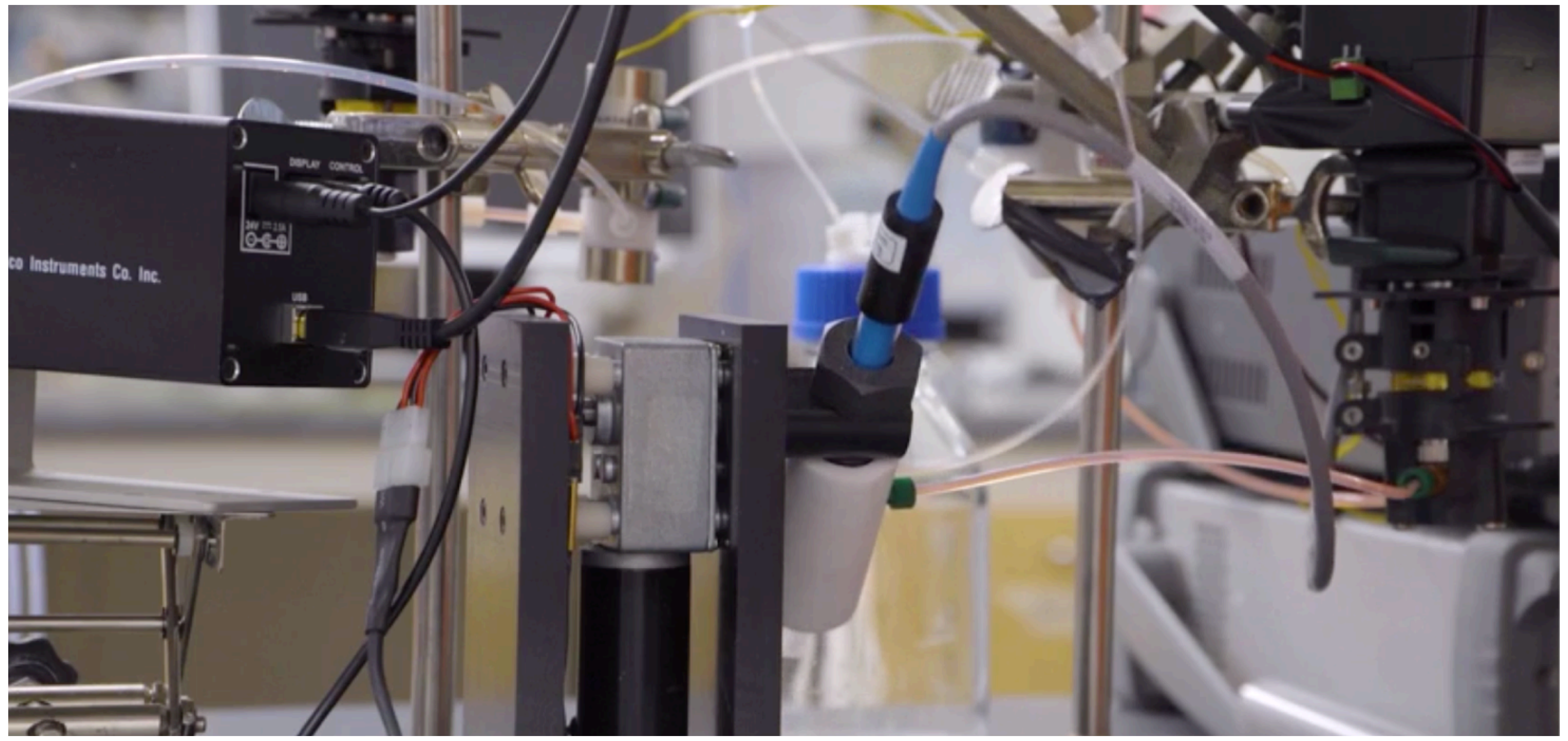
ebay

amazon

DOORDASH







DATA IS AN INVALUABLE RESOURCE

DATA IS AN INVALUABLE RESOURCE

- Data is the *new oil*.
- Data is the *new gold*.

The Economist, NY Times, Forbes, Wired, Deloitte, EY and several more …

DATA IS AN INVALUABLE RESOURCE

- Data is the *new oil*.
- Data is the *new gold*.

The Economist, NY Times, Forbes, Wired, Deloitte, EY and several more …

- But data is different to other types of resources
  - Data is ***costly*** to produce, ***free*** to replicate.

A UTOPIAN GOAL

Everyone collects data, everyone shares their data with others.

- Cost incurred by one organization to produce data can benefit others.
- Better for the organizations, better for society at large.

SHARING WILL DEMOCRATIZE DATA

SHARING WILL DEMOCRATIZE DATA

Small organizations with little data:

A B C D E F

SHARING WILL DEMOCRATIZE DATA

Small organizations with little data:

A B C D E F

Large organization with lots of data:

SHARING WILL DEMOCRATIZE DATA

Small organizations with little data:

A B C D E F

Large organization with lots of data:

SHARING WILL DEMOCRATIZE DATA

Small organizations with little data:

A B C D E F

Large organization with lots of data:

By sharing data with each other, small organizations can compete with larger organizations.

OBSTACLES TO DATA SHARING

OBSTACLES TO DATA SHARING

Ethical/Legal

Privacy

Ownership of data

OBSTACLES TO DATA SHARING

Ethical/Legal

Privacy

Ownership of data

Security

Data breaches

Adversarial attacks

OBSTACLES TO DATA SHARING

Ethical/Legal	Security	Logistical
Privacy	Data breaches	Inter-operability
Ownership of data	Adversarial attacks	Communication costs

OBSTACLES TO DATA SHARING

Ethical/Legal

Privacy

Ownership of data

Security

Data breaches

Adversarial attacks

Logistical

Inter-operability

Communication costs

Incentives

Free-riding

Competition

Data monetization

Data valuation

OBSTACLES TO DATA SHARING

Ethical/Legal

Privacy

Ownership of data

Security

Data breaches

Adversarial attacks

Logistical

Inter-operability

Communication costs

Incentives

Free-riding

Competition

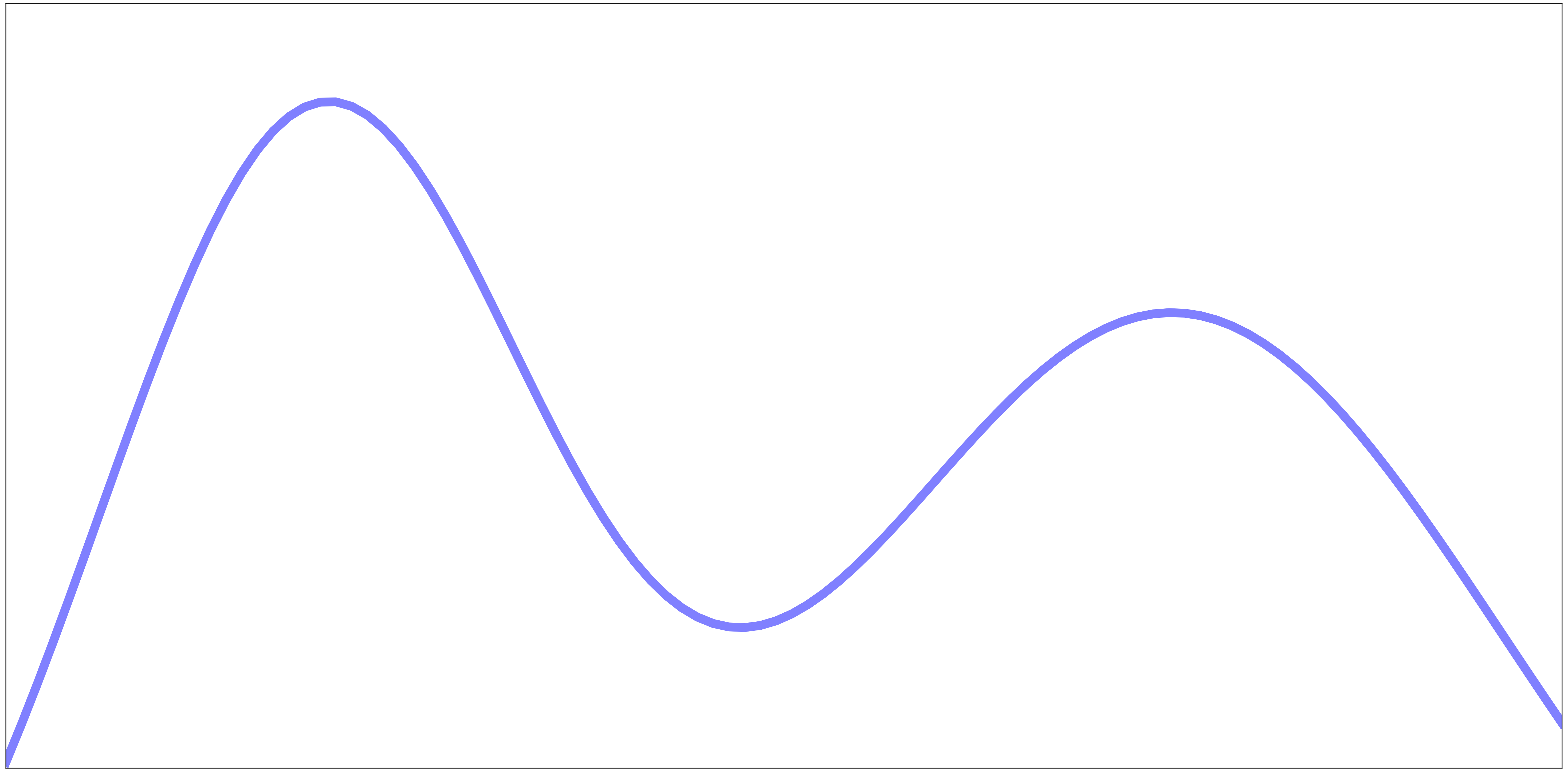
Data monetization

Data valuation

This talk → Free-riding

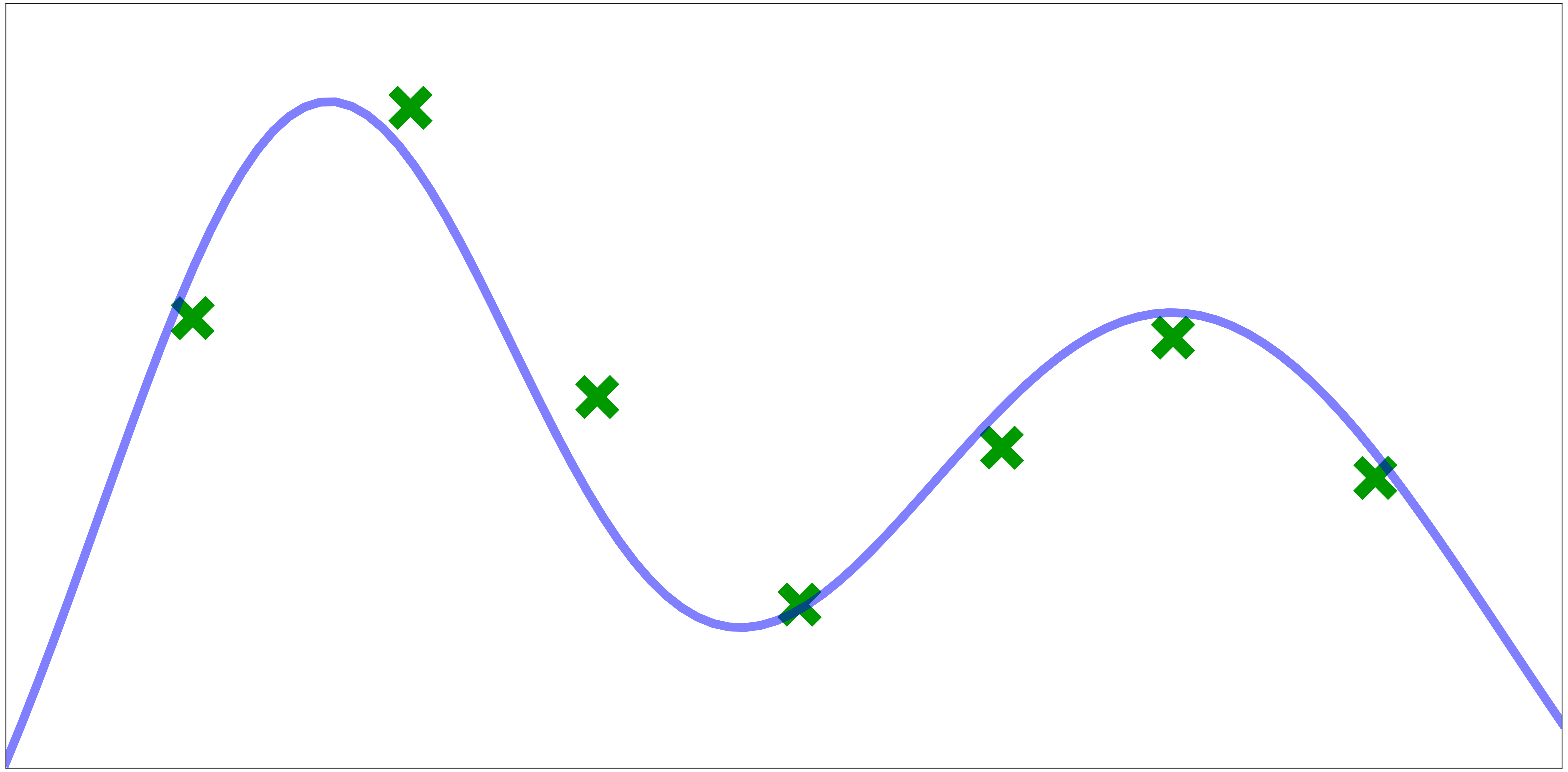
FREE-RIDING (#1/3)

agent's penalty = estimation error + cost of data collection



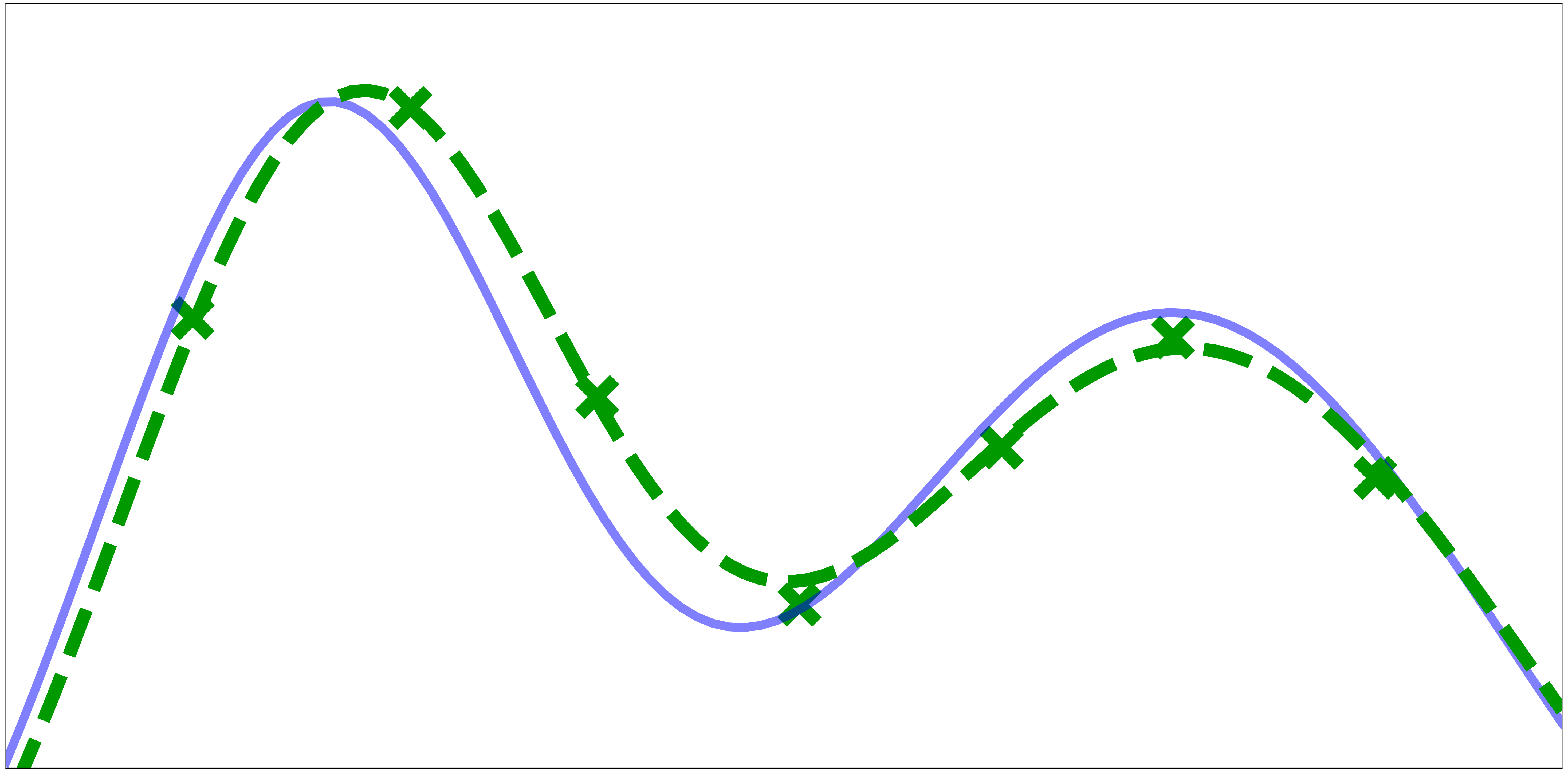
FREE-RIDING (#1/3)

agent's penalty = estimation error + cost of data collection



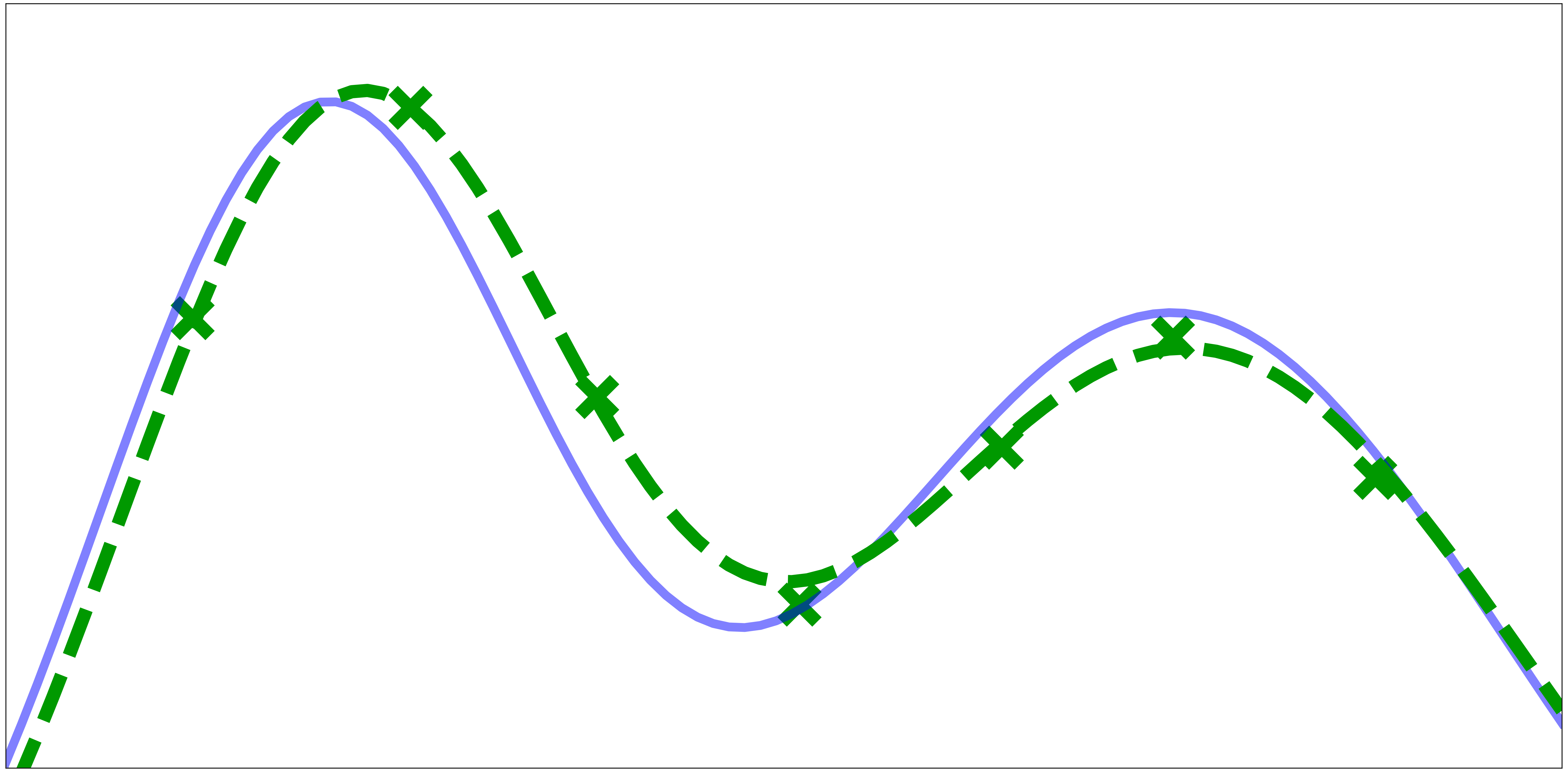
FREE-RIDING (#1/3)

agent's penalty = estimation error + cost of data collection



FREE-RIDING (#1/3)

agent's penalty = estimation error + cost of data collection

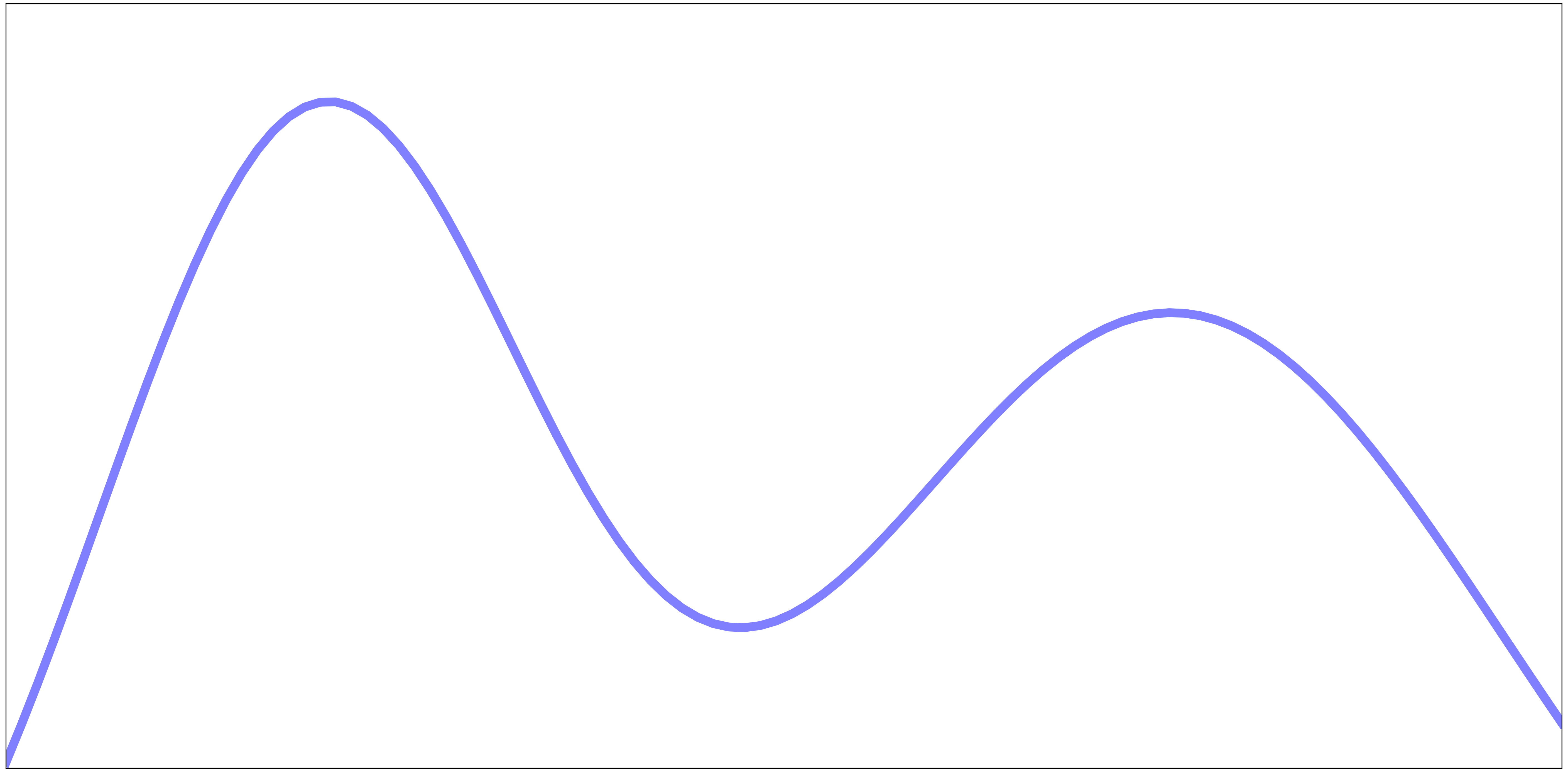


When working on her own, an agent will collect enough data until the cost offsets the (diminishing) increase in value from data.

FREE-RIDING (#2/3)

Multiple agents share data via a *naive* pool-and-share protocol:

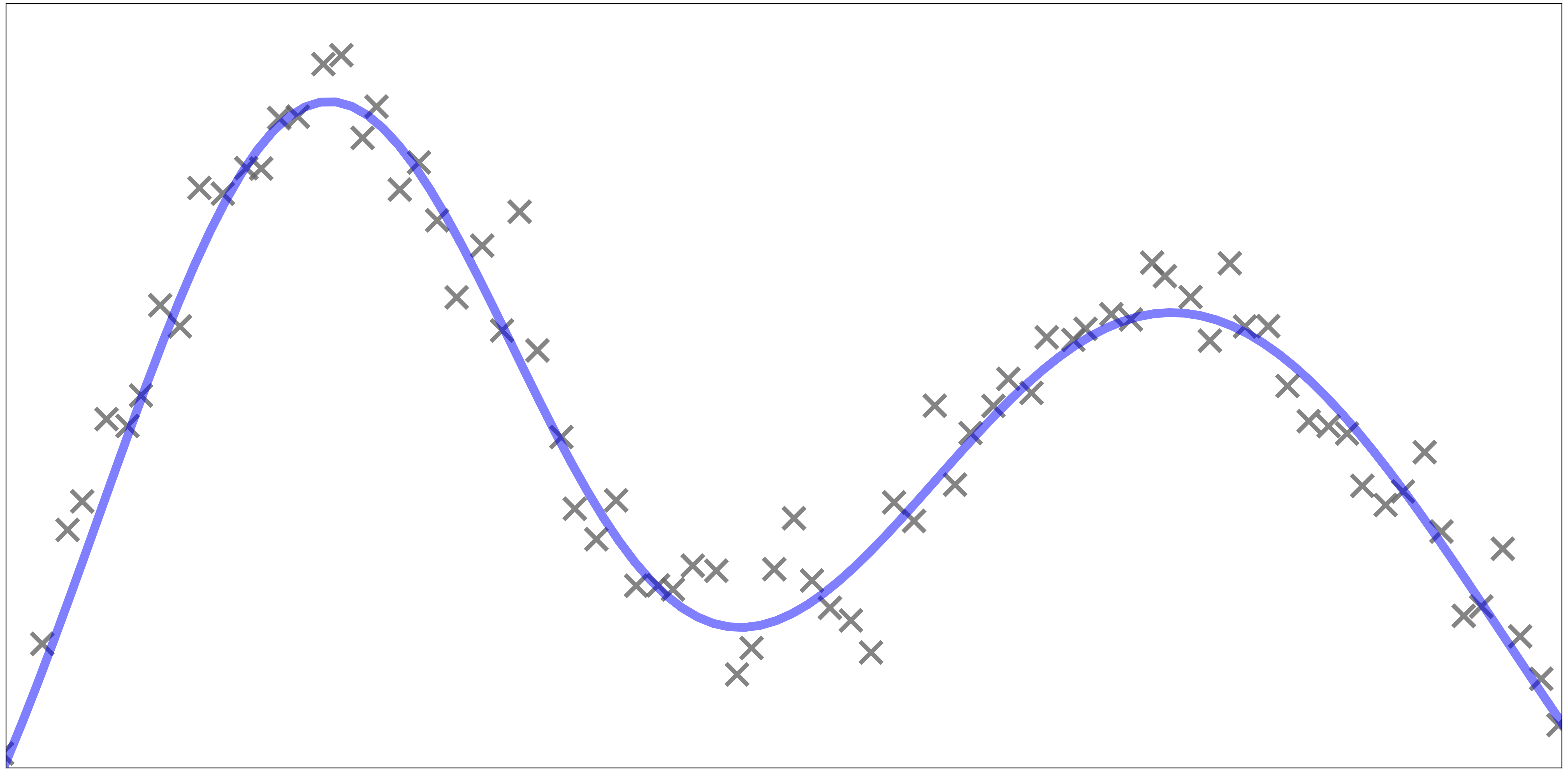
- Everyone collects data, everyone gets a copy of the others' data.



FREE-RIDING (#2/3)

Multiple agents share data via a *naive* pool-and-share protocol:

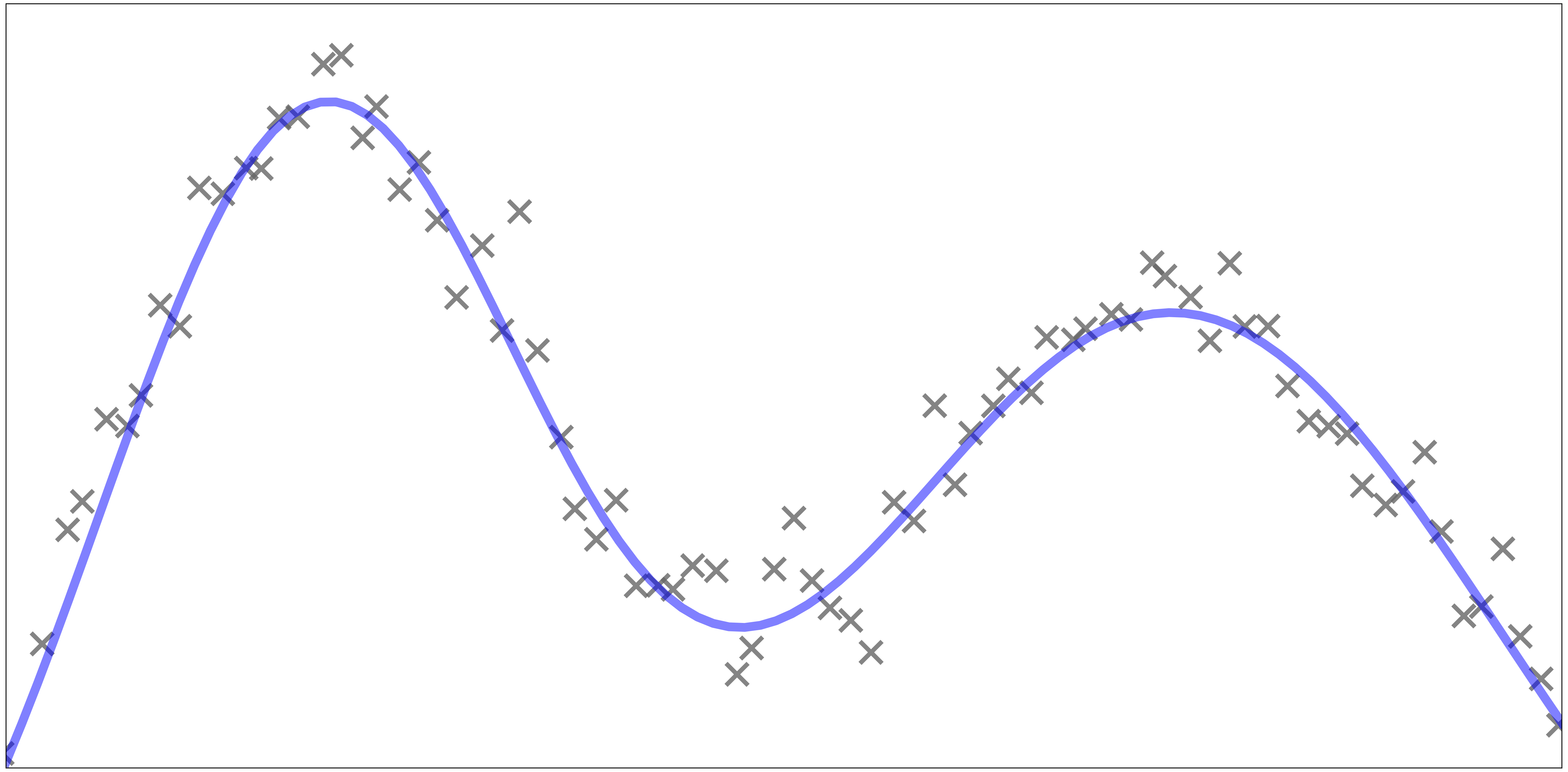
- Everyone collects data, everyone gets a copy of the others' data.



FREE-RIDING (#2/3)

Multiple agents share data via a *naive* pool-and-share protocol:

- Everyone collects data, everyone gets a copy of the others' data.

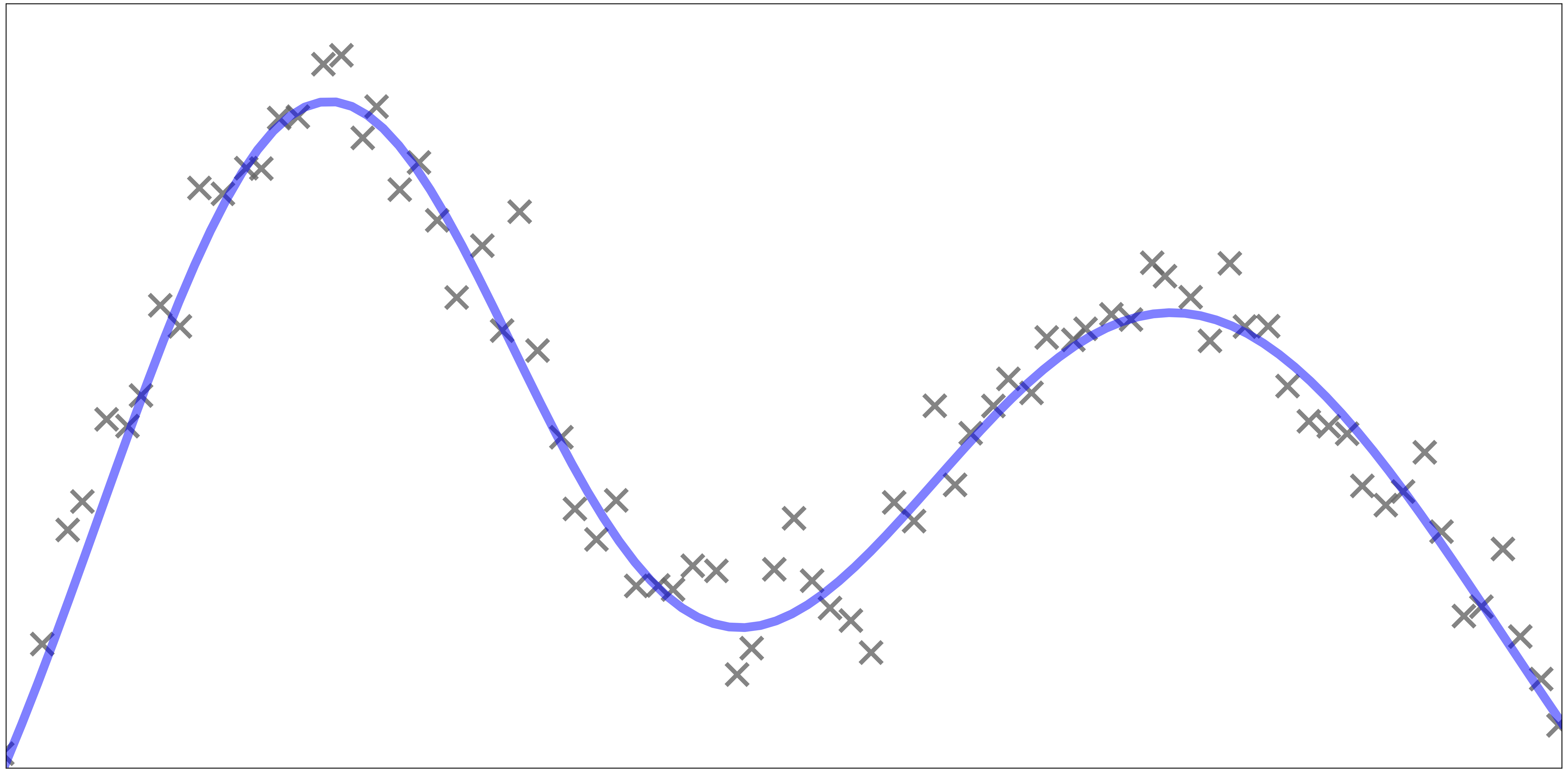


If others are already contributing large amounts of data, an agent has no incentive to collect/contribute data of her own.

FREE-RIDING (#3/3)

A seemingly plausible work-around (but does not work):

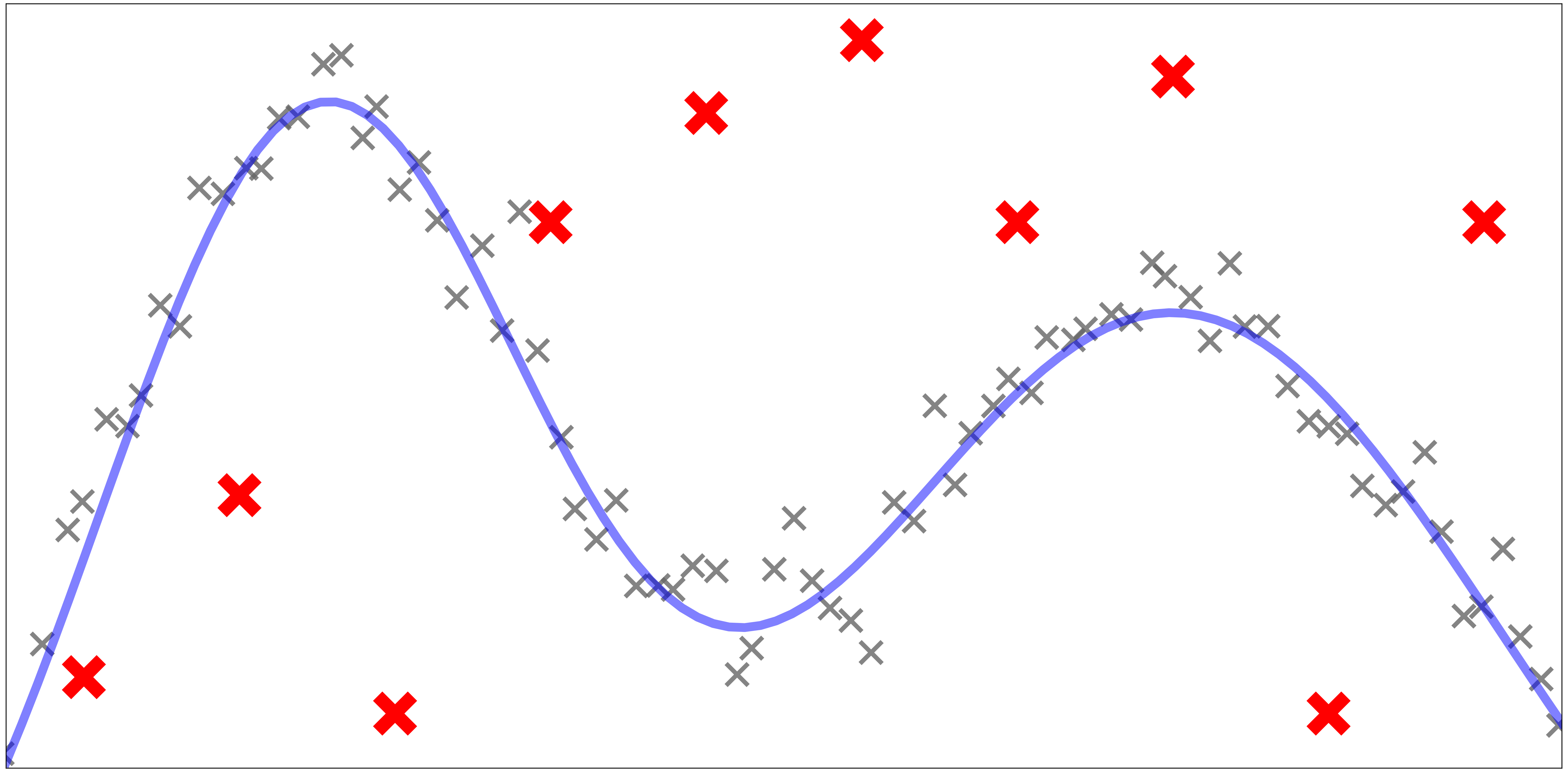
Pool-and-share but only if the agent contributes sufficient data



FREE-RIDING (#3/3)

A seemingly plausible work-around (but does not work):

Pool-and-share but only if the agent contributes sufficient data

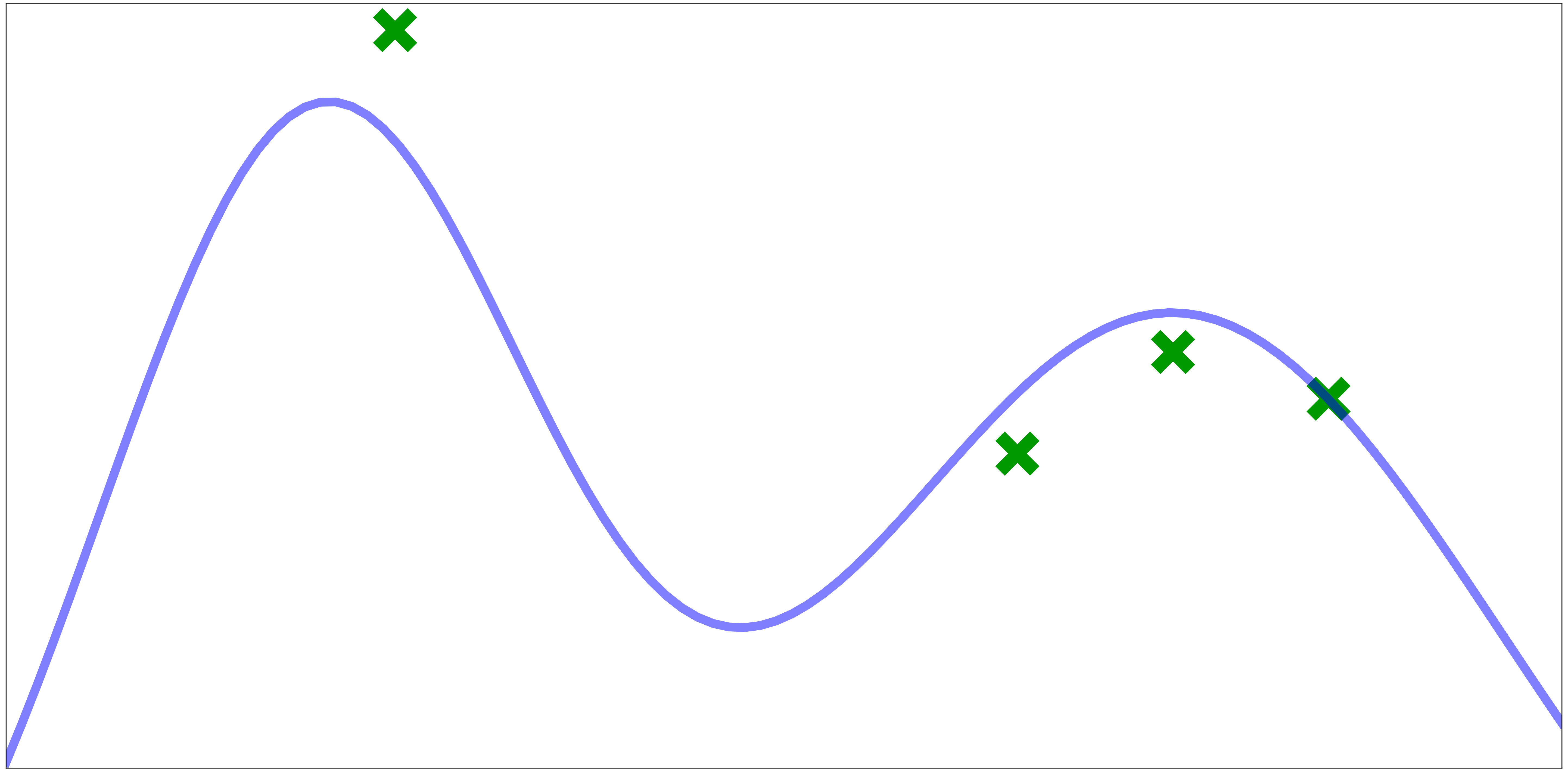


- Agent can submit fabricated data and then discard it when learning.

FREE-RIDING (#3/3)

A seemingly plausible work-around (but does not work):

Pool-and-share but only if the agent contributes sufficient data

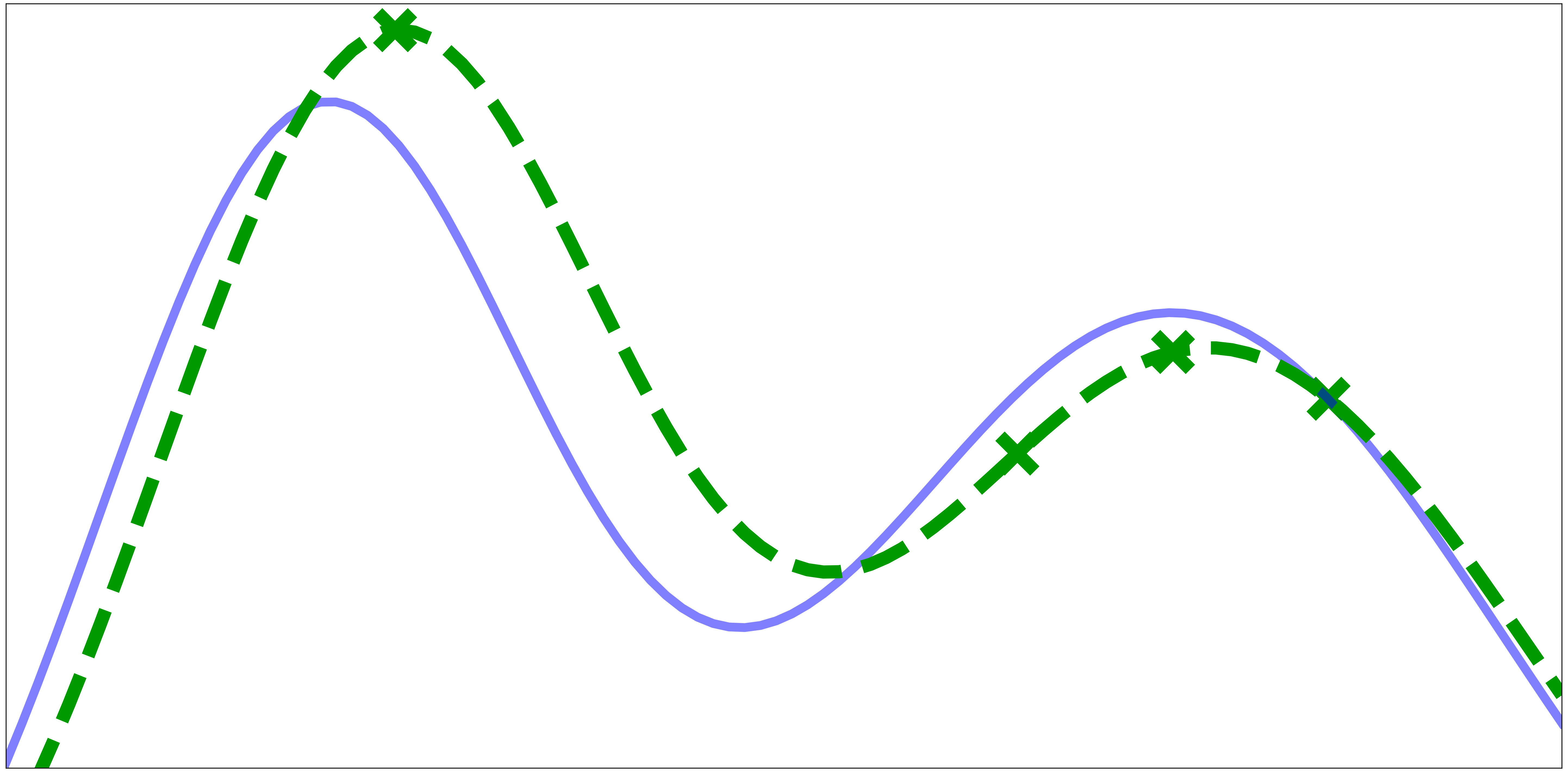


- Agent can submit fabricated data and then discard it when learning.

FREE-RIDING (#3/3)

A seemingly plausible work-around (but does not work):

Pool-and-share but only if the agent contributes sufficient data

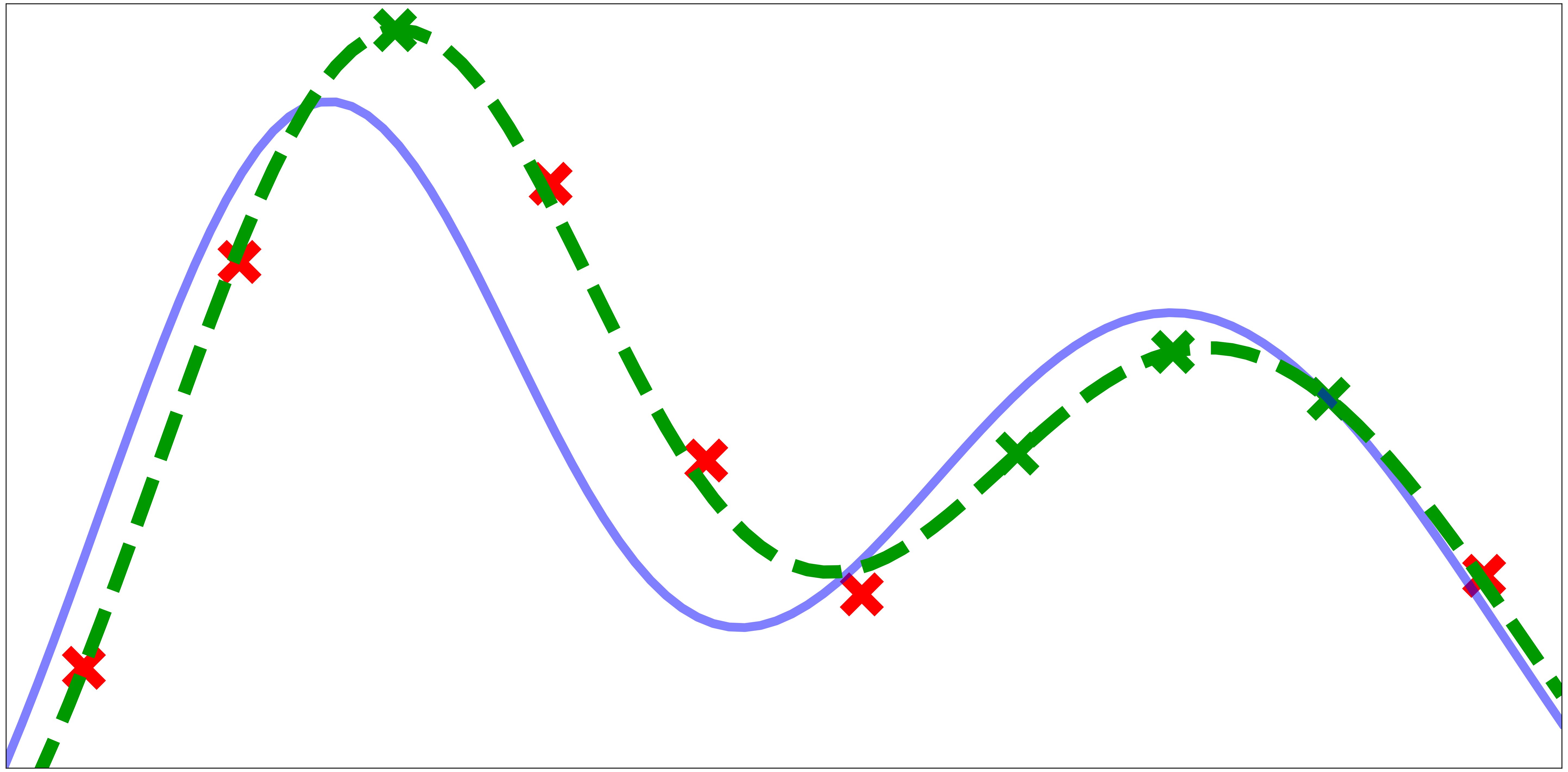


- Agent can submit fabricated data and then discard it when learning.

FREE-RIDING (#3/3)

A seemingly plausible work-around (but does not work):

Pool-and-share but only if the agent contributes sufficient data

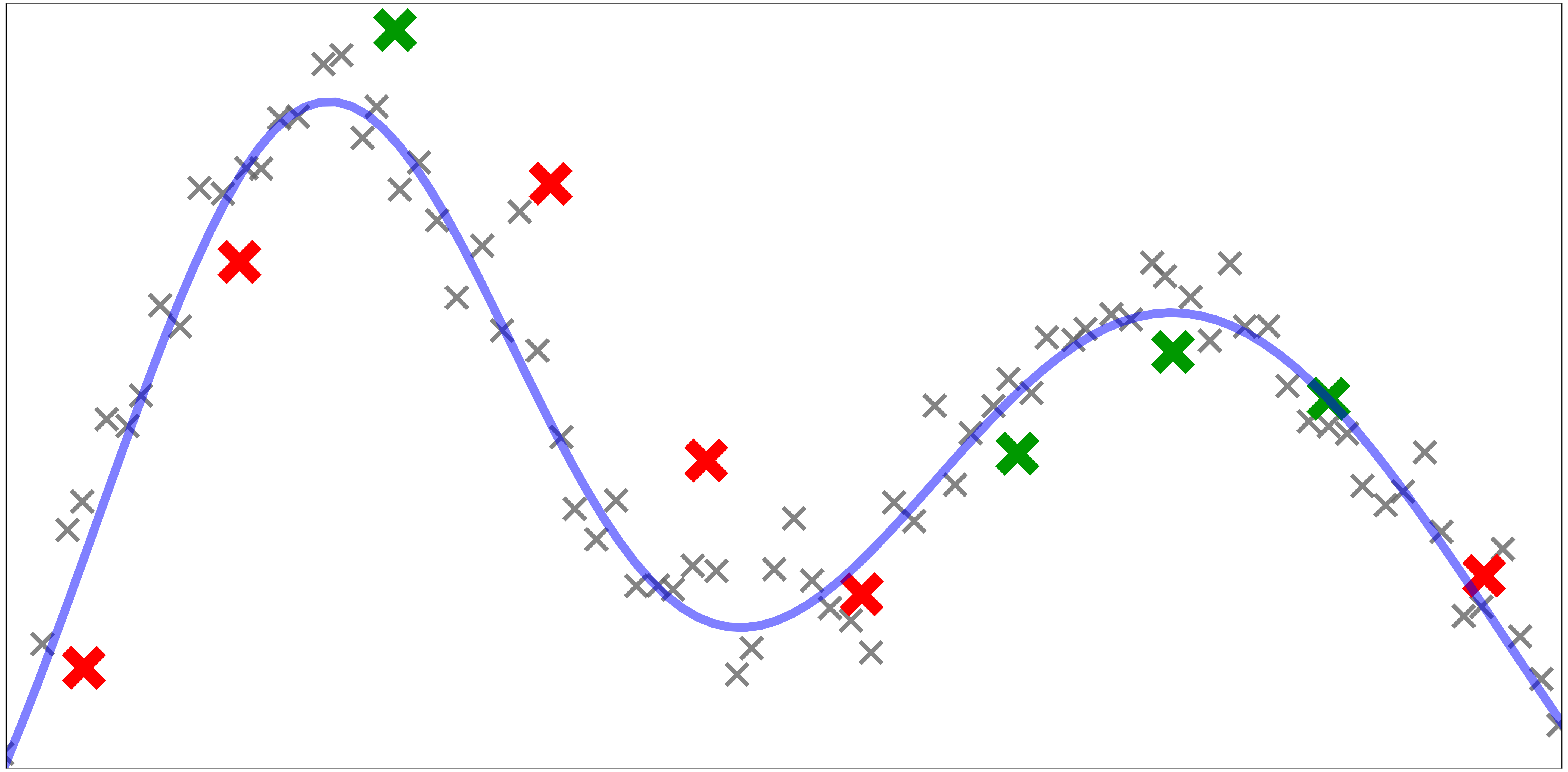


- Agent can submit fabricated data and then discard it when learning.
- Agent may fabricate based on a small sample she has collected, so it may not always be easy to detect.

FREE-RIDING (#3/3)

A seemingly plausible work-around (but does not work):

Pool-and-share but only if the agent contributes sufficient data



- Agent can submit fabricated data and then discard it when learning.
- Agent may fabricate based on a small sample she has collected, so it may not always be easy to detect.

OBSTACLES TO DATA SHARING

Ethical/Legal

Privacy

Ownership of data

Security

Data breaches

Adversarial attacks

Logistical

Inter-operability

Communication costs

Incentives

Free-riding

Competition

Data monetization

Data valuation

BUT THERE IS A DEMAND FOR DATA SHARING IN THE REAL WORLD

Data sharing platforms/consortia

PubChem

PubMed®

ChEMBL

DELTA SHARING

An open standard for secure data sharing

Marketplaces for data and ML models

aws AWS Data Exchange

Ads Data Hub

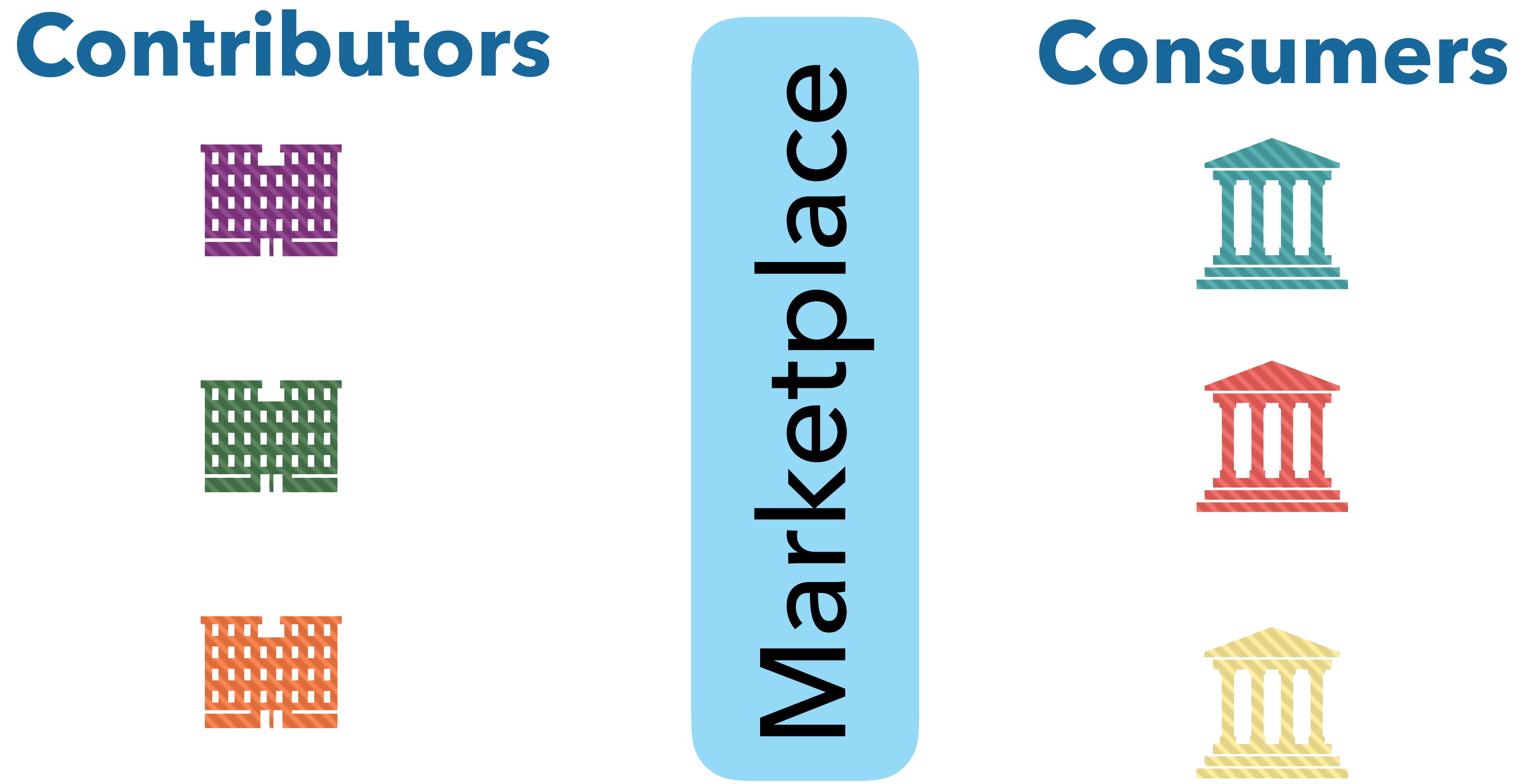
CITRINE INFORMATICS

Azure Data Share

OUR APPROACHES

Mechanisms for data sharing and federated learning

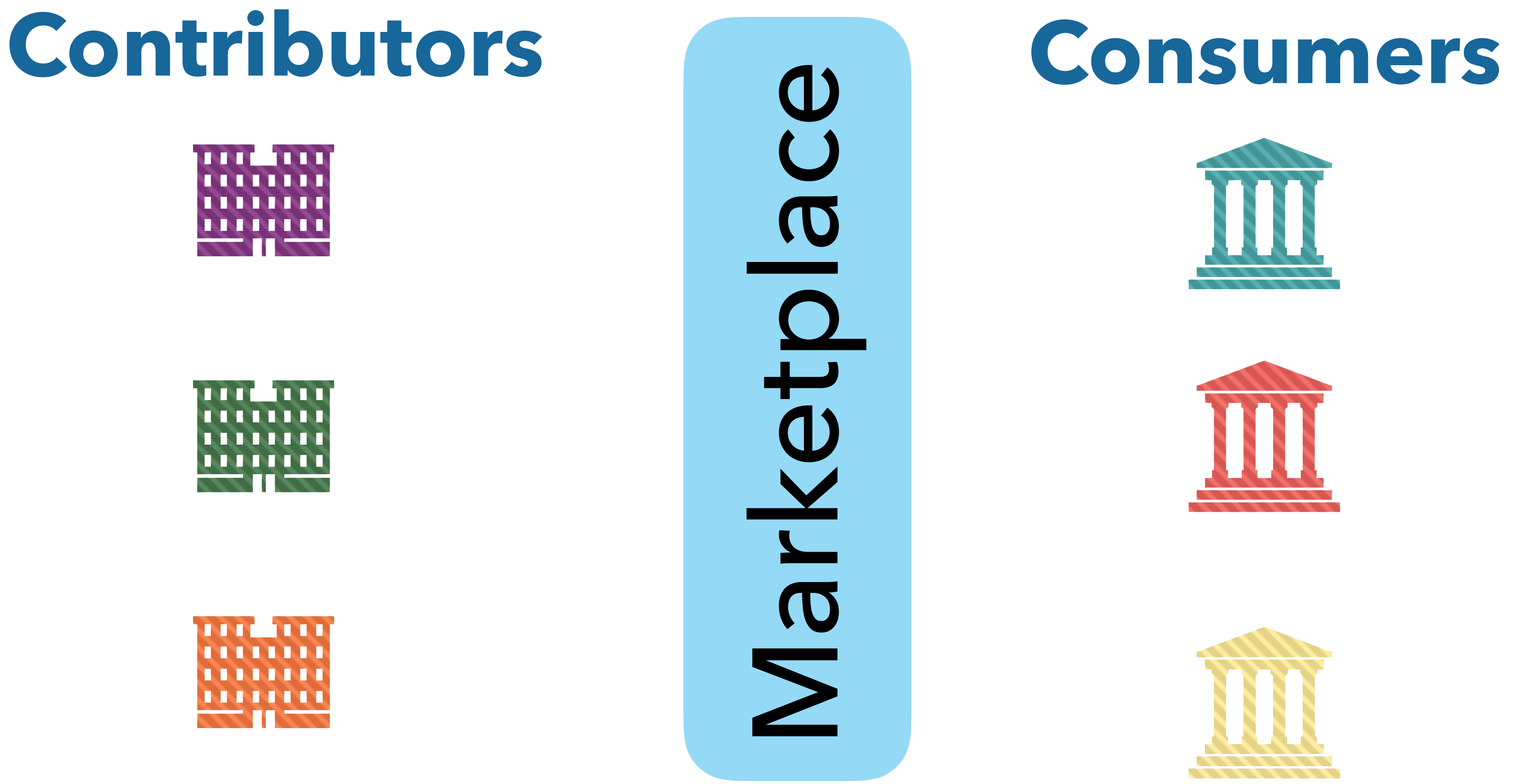
Data marketplaces



Mechanisms for data sharing and federated learning

Goal: Incentivize agents to collect as much data and share it honestly.

Data marketplaces

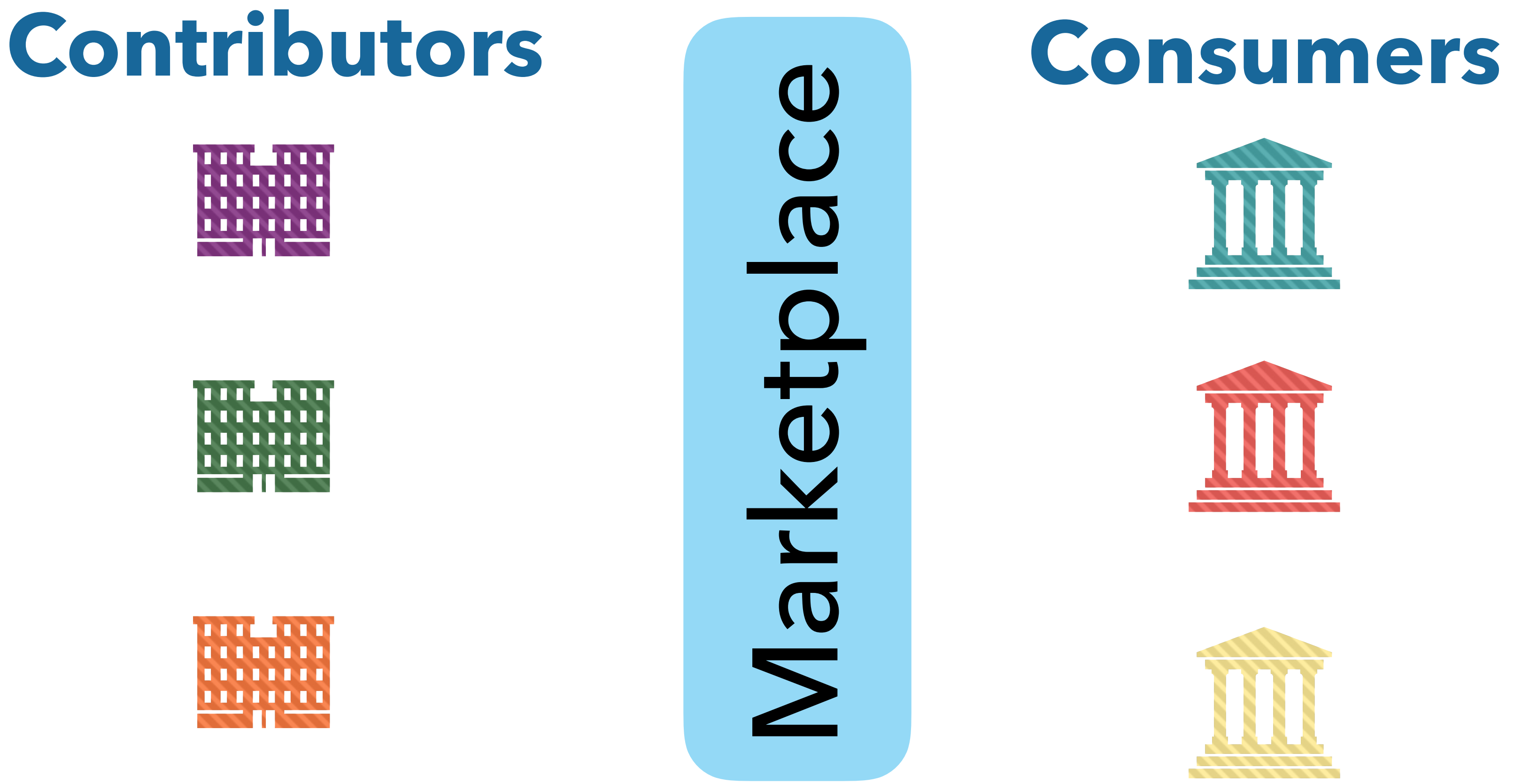


Mechanisms for data sharing and federated learning

Goal: Incentivize agents to collect as much data and share it honestly.

- Do not simply pool and share data!

Data marketplaces

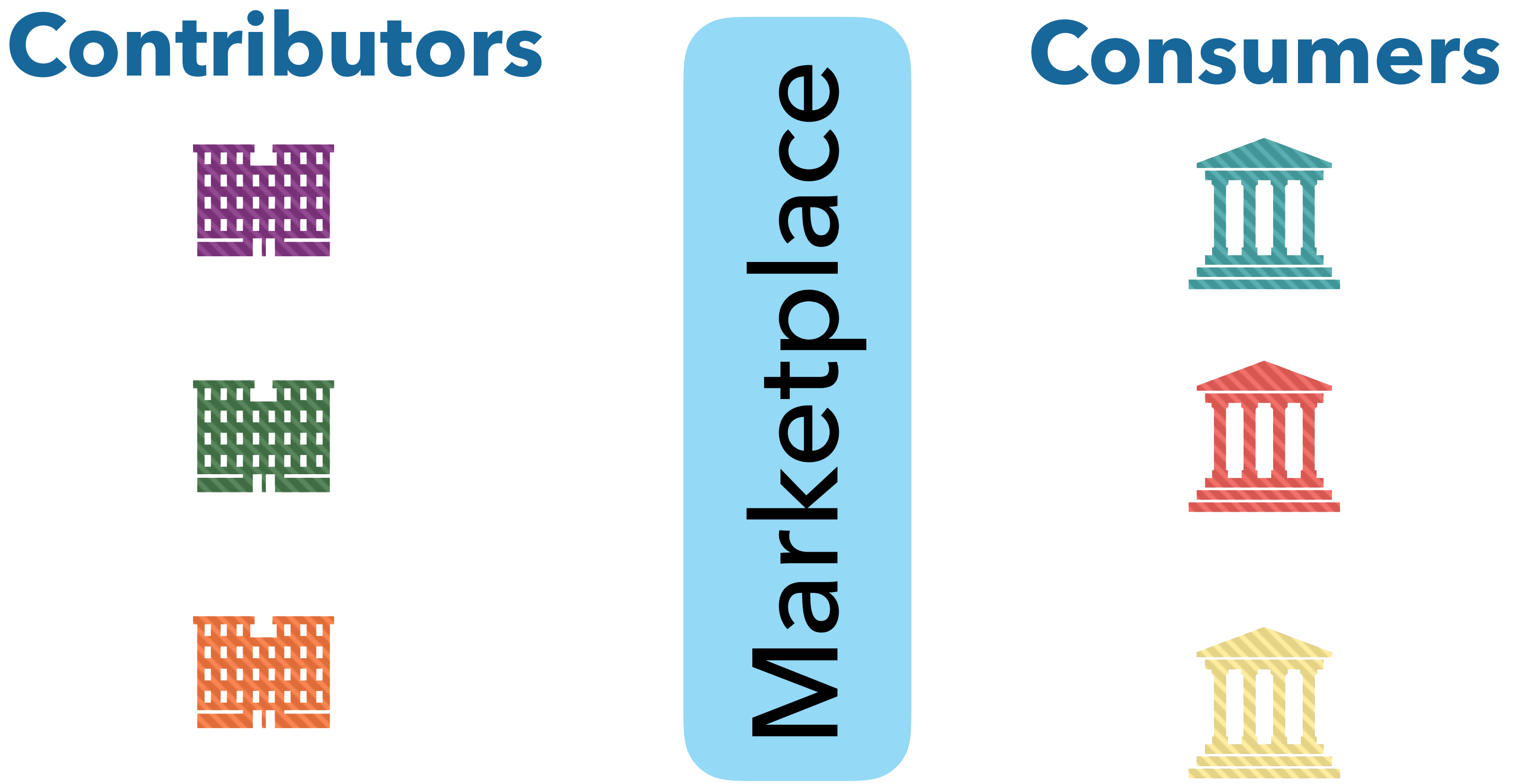


Mechanisms for data sharing and federated learning

Goal: Incentivize agents to collect as much data and share it honestly.

- Do not simply pool and share data!
- Cross-check for quality of the data contributed.

Data marketplaces

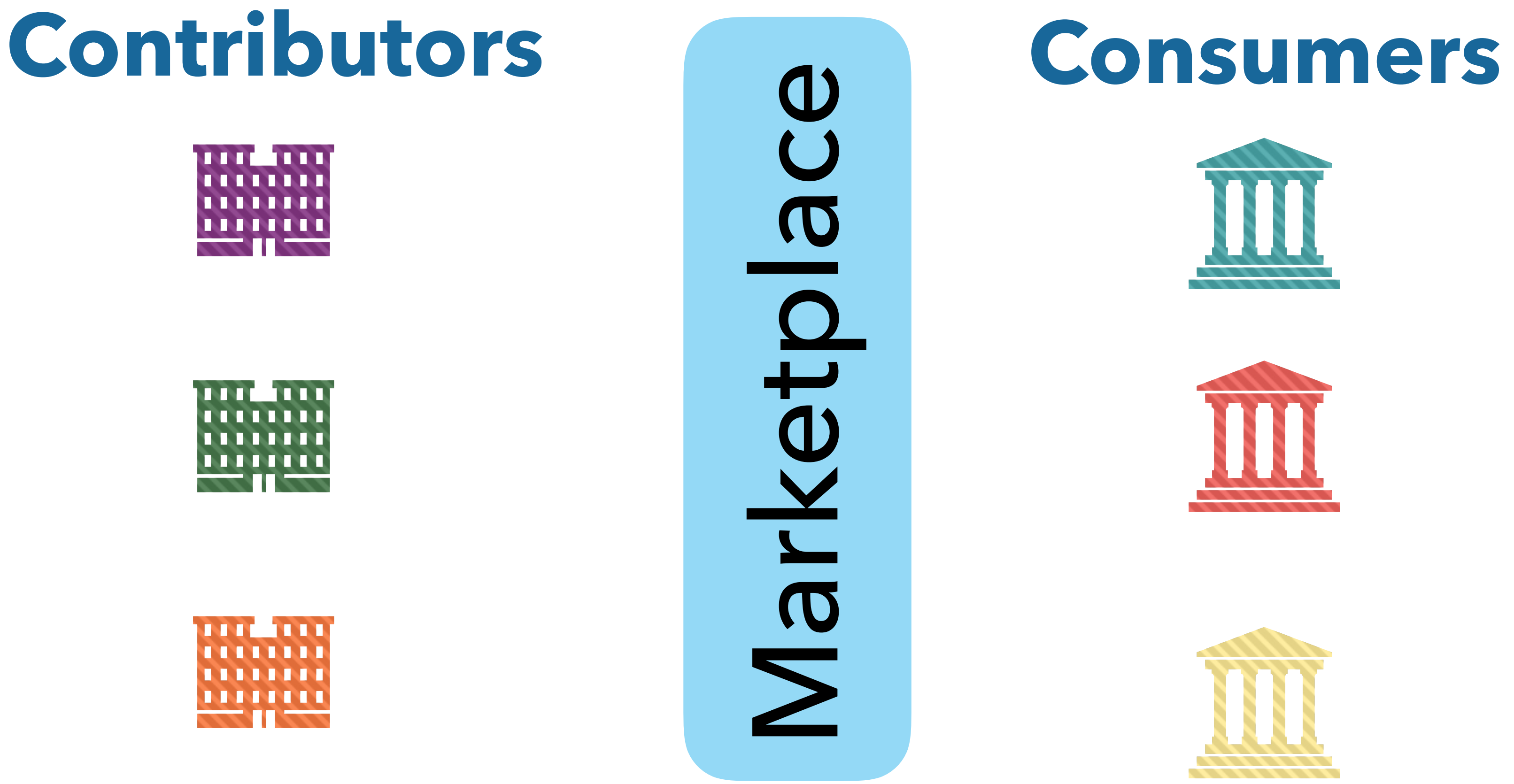


Mechanisms for data sharing and federated learning

Goal: Incentivize agents to collect as much data and share it honestly.

- Do not simply pool and share data!
- Cross-check for quality of the data contributed.
- More/better data contributed ⟹ more/better data received.

Data marketplaces

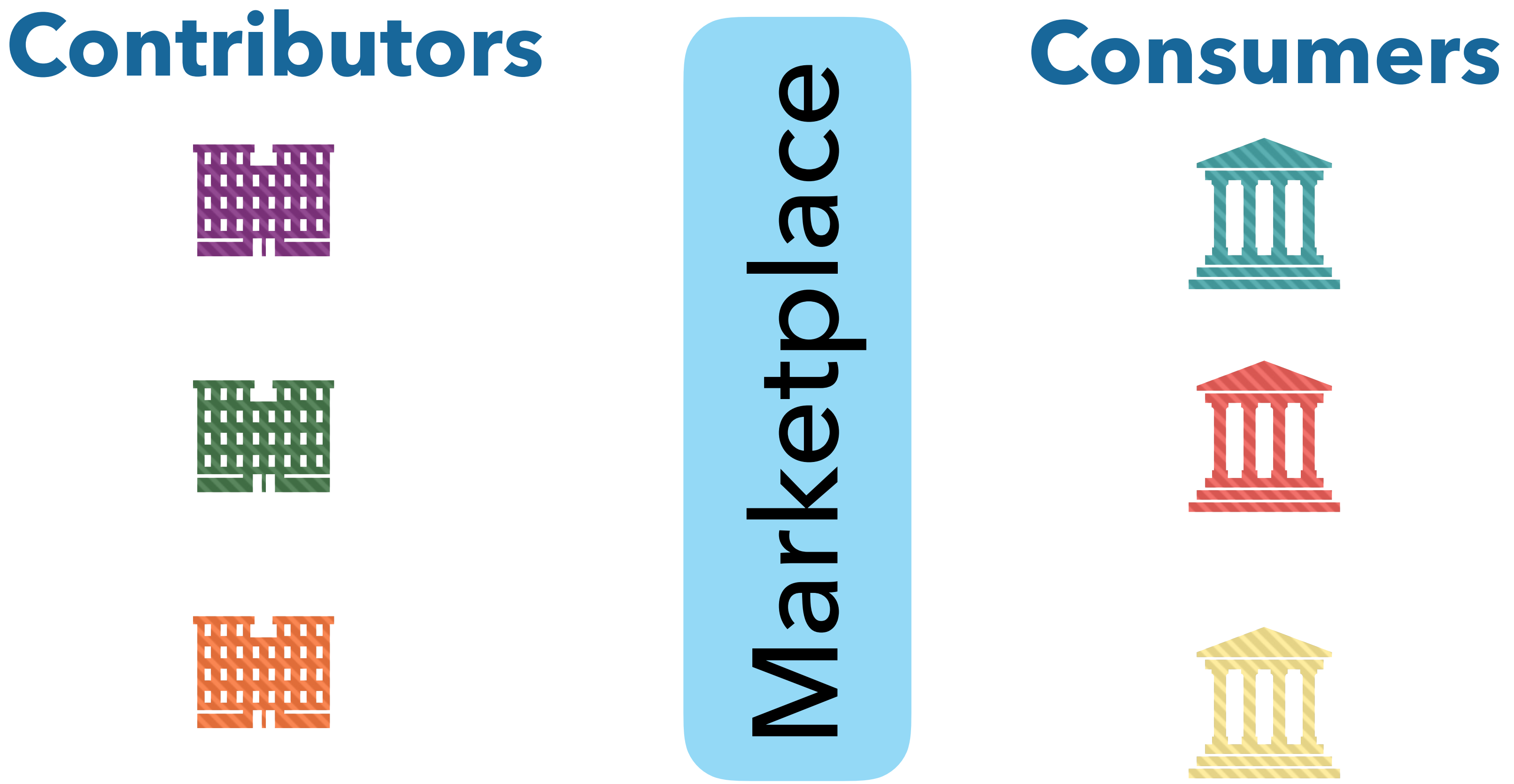


Mechanisms for data sharing and federated learning

Goal: Incentivize agents to collect as much data and share it honestly.

- Do not simply pool and share data!
- Cross-check for quality of the data contributed.
- More/better data contributed $\Longrightarrow$ more/better data received.

Data marketplaces



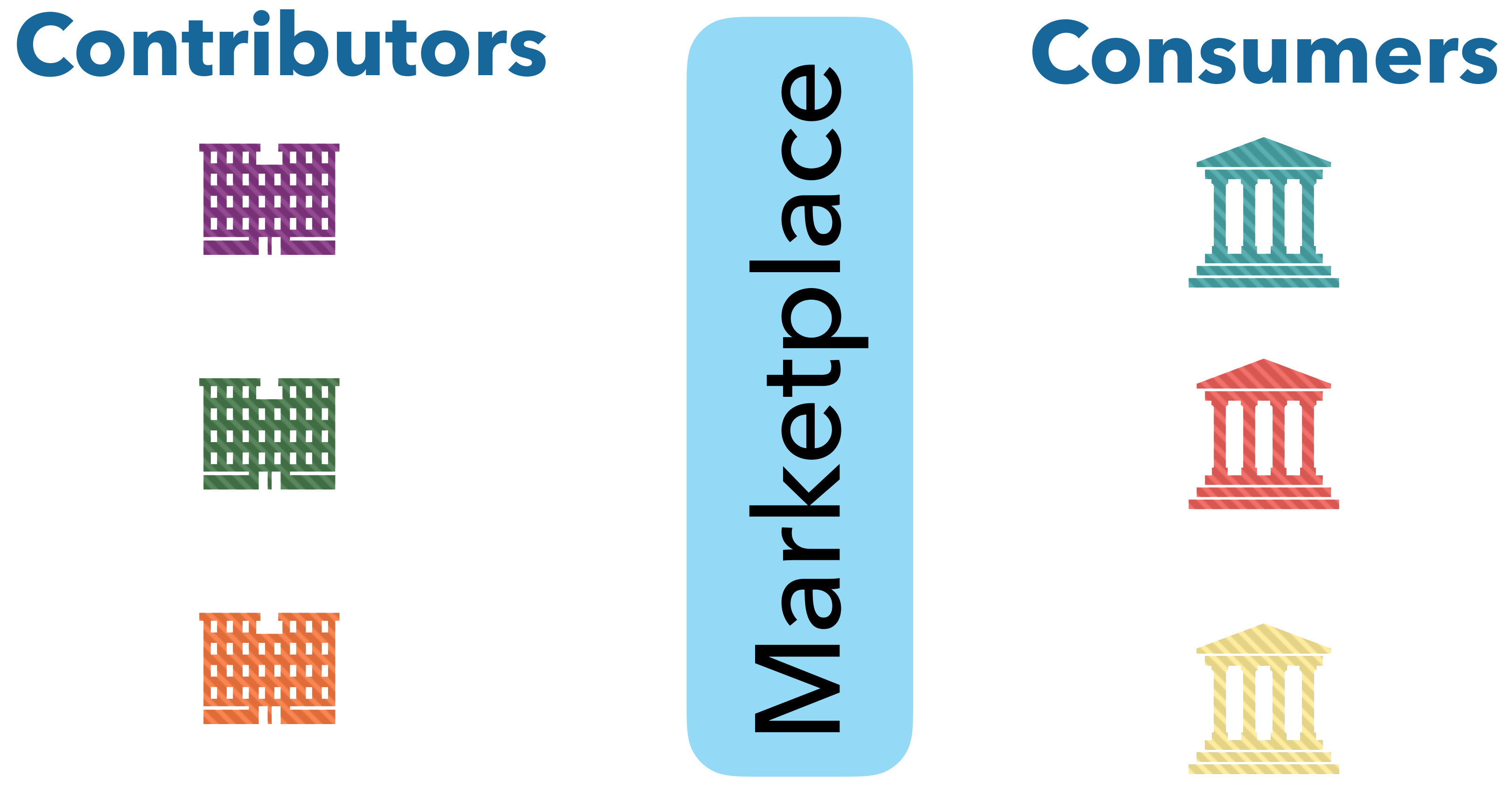
Goal: Incentivize contributors to honestly contribute lots of data. Fairly reward them for effort via payments from consumers.

Mechanisms for data sharing and federated learning

Goal: Incentivize agents to collect as much data and share it honestly.

- Do not simply pool and share data!
- Cross-check for quality of the data contributed.
- More/better data contributed $\Longrightarrow$ more/better data received.

Data marketplaces



Goal: Incentivize contributors to honestly contribute lots of data. Fairly reward them for effort via payments from consumers.

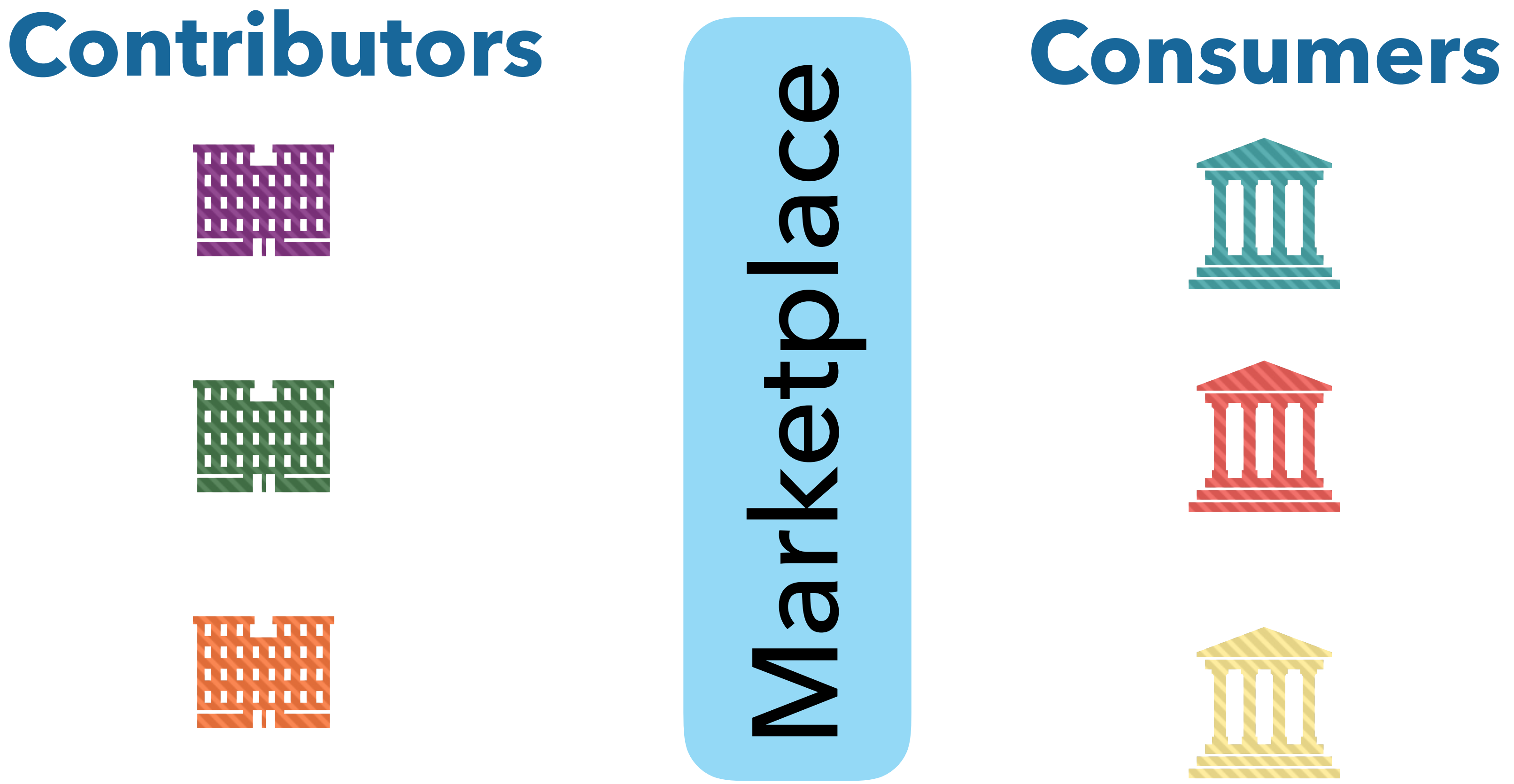
- Higher quality data $\Longrightarrow$ higher revenue for data contributors.

Mechanisms for data sharing and federated learning

Goal: Incentivize agents to collect as much data and share it honestly.

- Do not simply pool and share data!
- Cross-check for quality of the data contributed.
- More/better data contributed $\Longrightarrow$ more/better data received.

Data marketplaces



Goal: Incentivize contributors to honestly contribute lots of data. Fairly reward them for effort via payments from consumers.

- Higher quality data $\Longrightarrow$ higher revenue for data contributors.
- A mediator checks for the quality of the data from contributors

Mechanisms for data sharing and federated learning

Sim, Zhang, Chan, Low 2020
Xu, Lyu, Ma et al 2021
Blum, Haghtalab, Phillips, Shao 2021
Karimireddy, Guo, Jordan 2022
Fraboni, Vidal, Lorenzi 2021
Lin, Du, Liu 2019
Ding, Fang, Huang 2020
Liu, Tian, Chen et al 2022

Data marketplaces

Cai, Daskalakis, Papadimitriou 2015
Agarwal, Dahleh, Sarkar, 2019
Agarwal, Dahleh, Horel, Rui, 2020
Jia, Dao, Wang et al, 2019
Wang, Rausch, Zhang et al 2020

Key difference:

- All these works assume agents will always truthfully submit the data they have, i.e without fabrication/alteration.

OUTLINE

1. **Mechanism design for collaborative normal mean estimation**

   (Chen, Zhu, Kandasamy, *NeurIPS 2023*)

   - **Intuitions, overview of results**
   - **Problem formalism**
   - **Mechanism and theoretical analysis**

2. **Extensions & Future work**

   - **Collaborative supervised learning, design of experiments**
   - **Data marketplaces**

OUTLINE

1. **Mechanism design for collaborative normal mean estimation**

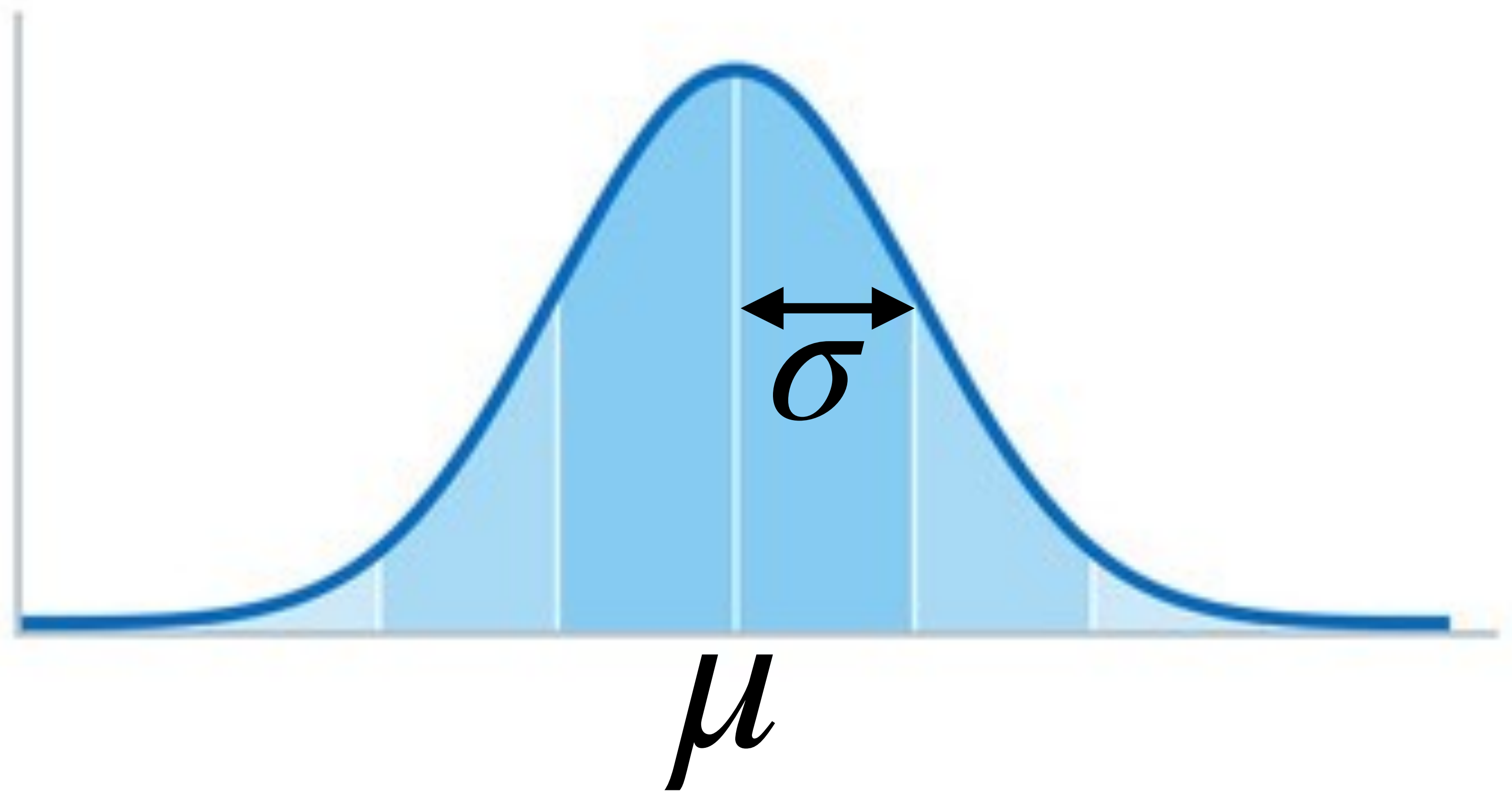
(Chen, Zhu, Kandasamy, *NeurIPS 2023*)

- **Intuitions and Challenges**
- Problem formalism
- Mechanism and theoretical analysis

2. Extensions & Future work

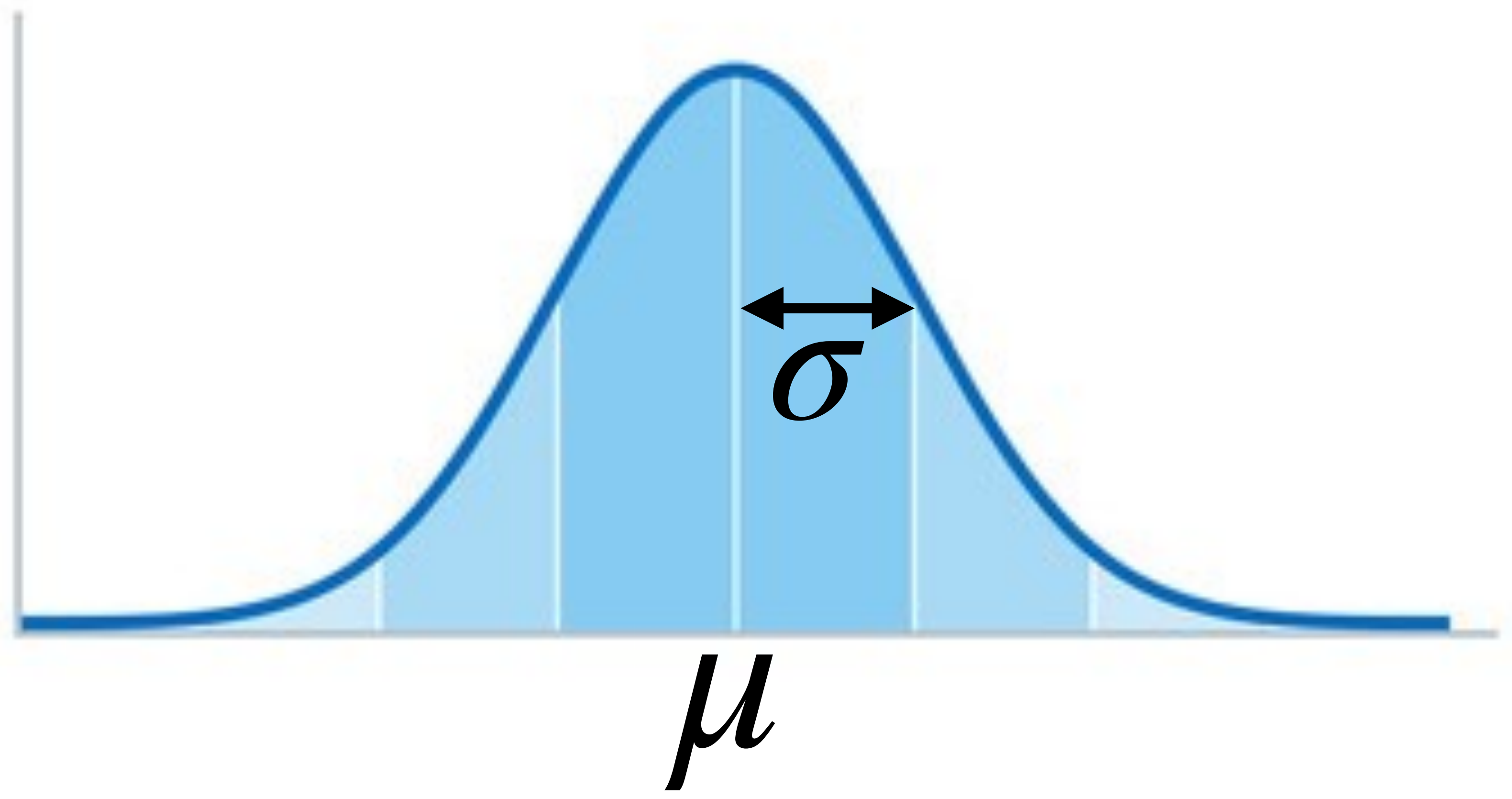
- Collaborative supervised learning, design of experiments
- Data marketplaces

NORMAL MEAN ESTIMATION



NORMAL MEAN ESTIMATION

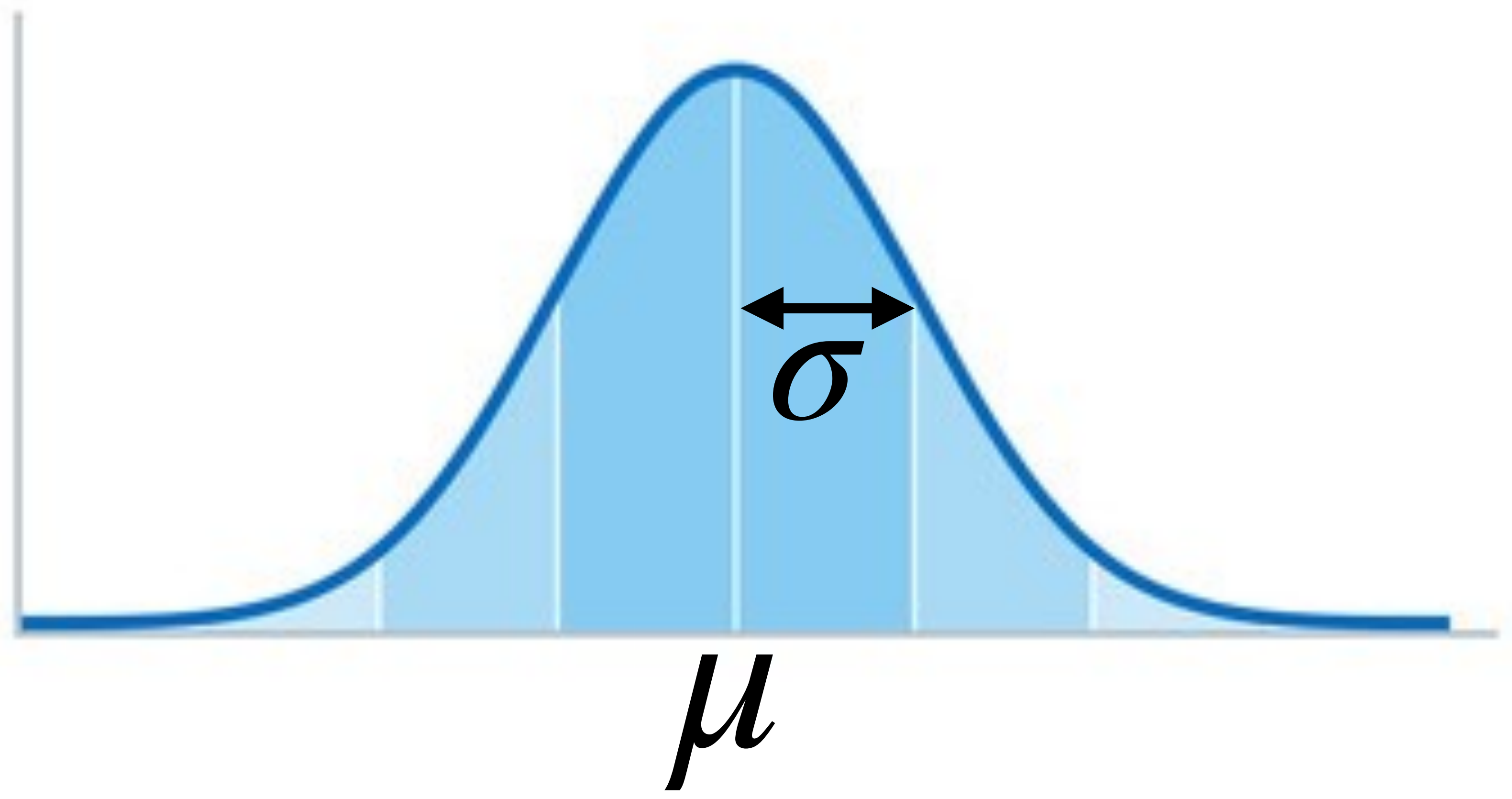
- Estimate the mean μ of a normal distribution with *known* variance σ^2.



NORMAL MEAN ESTIMATION

- Estimate the mean μ of a normal distribution with *known* variance σ^2.

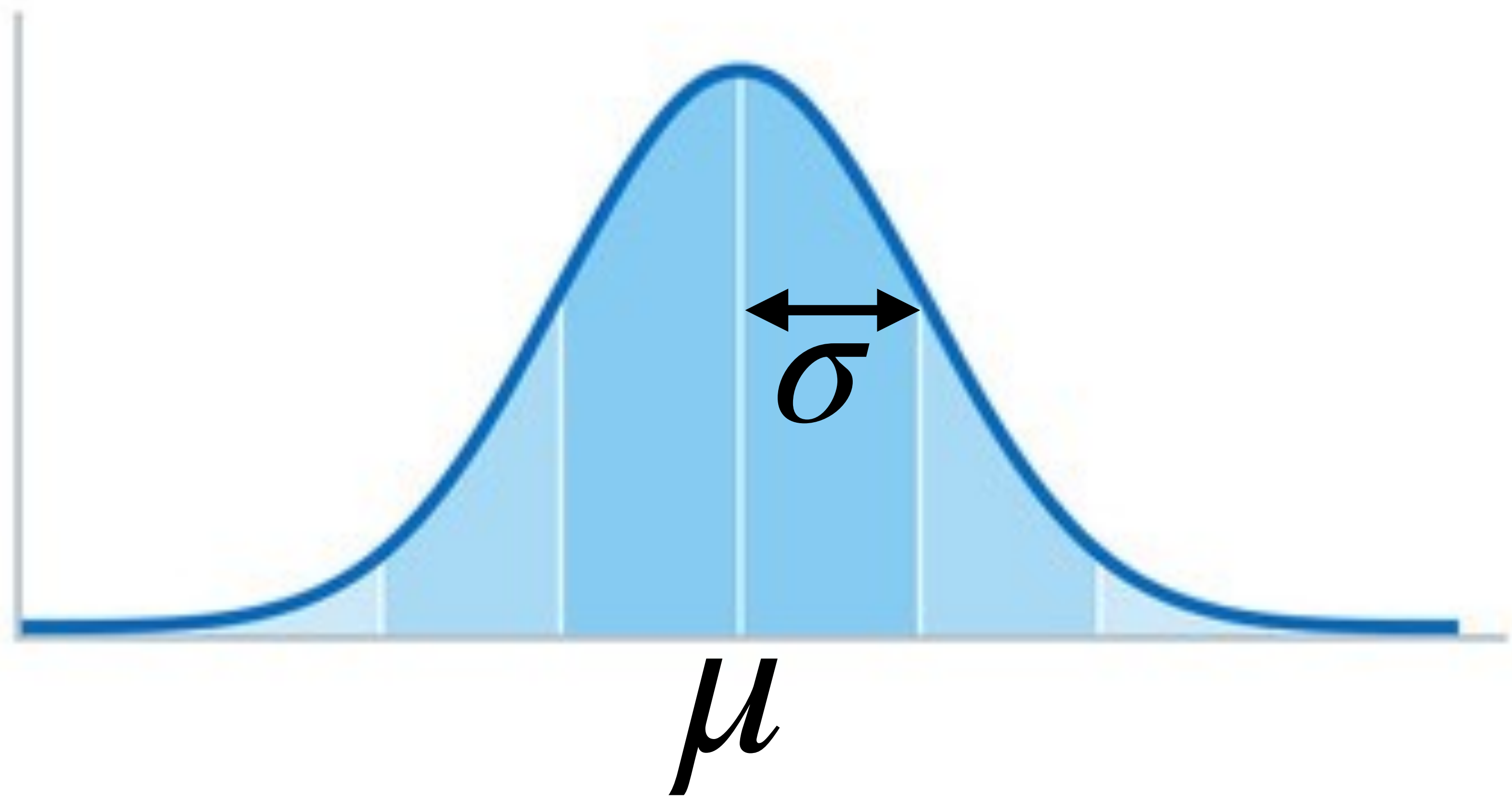
- An agent can collect samples at *known* unit cost c.



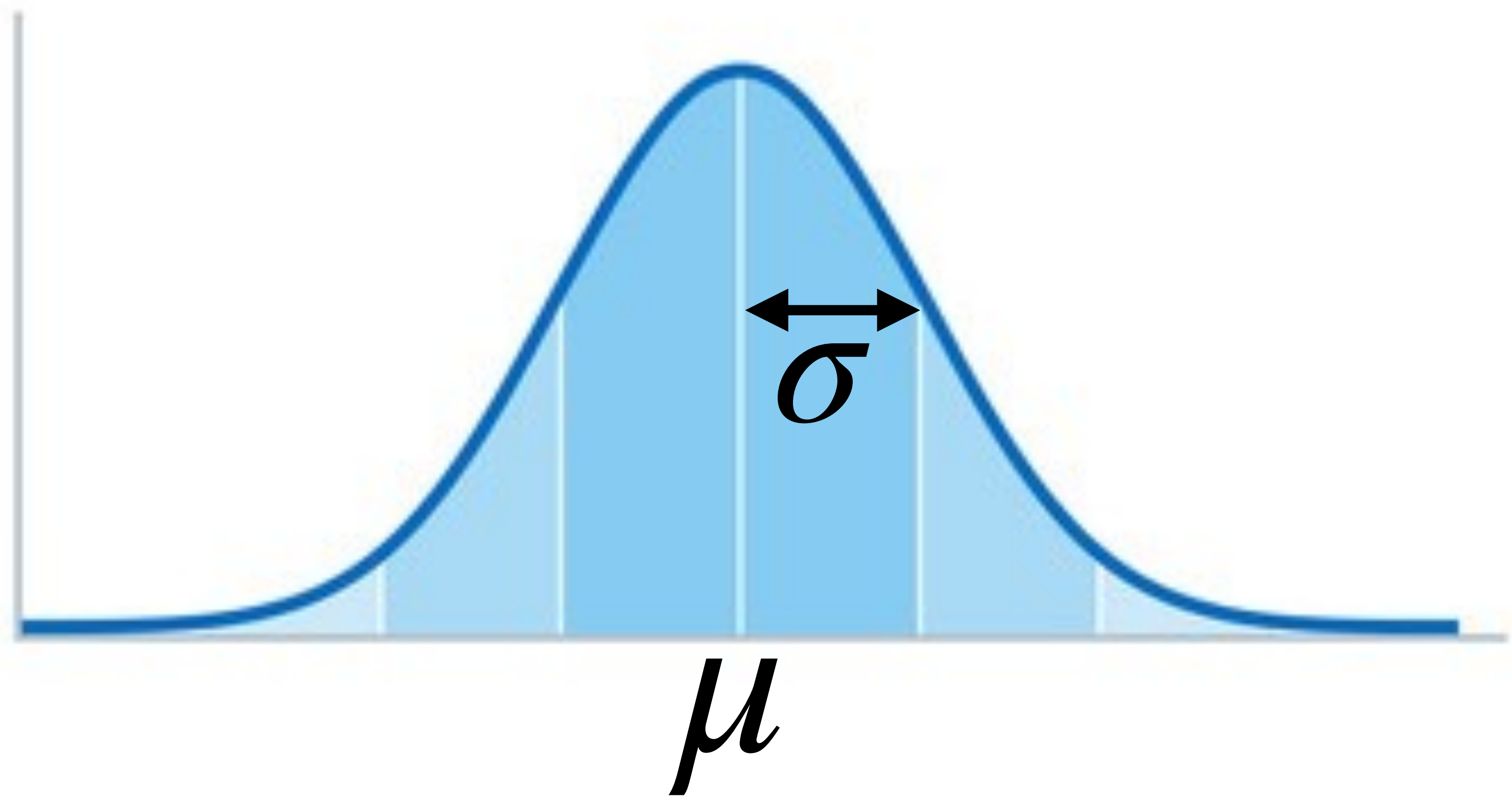
NORMAL MEAN ESTIMATION

- Estimate the mean μ of a normal distribution with *known* variance σ^2.
- An agent can collect samples at *known* unit cost c.
- Each agent wishes to minimize

penalty = estimation error + data collection cost



- Estimate the mean μ of a normal distribution with *known* variance σ^2.
- An agent can collect samples at *known* unit cost c.
- Each agent wishes to minimize

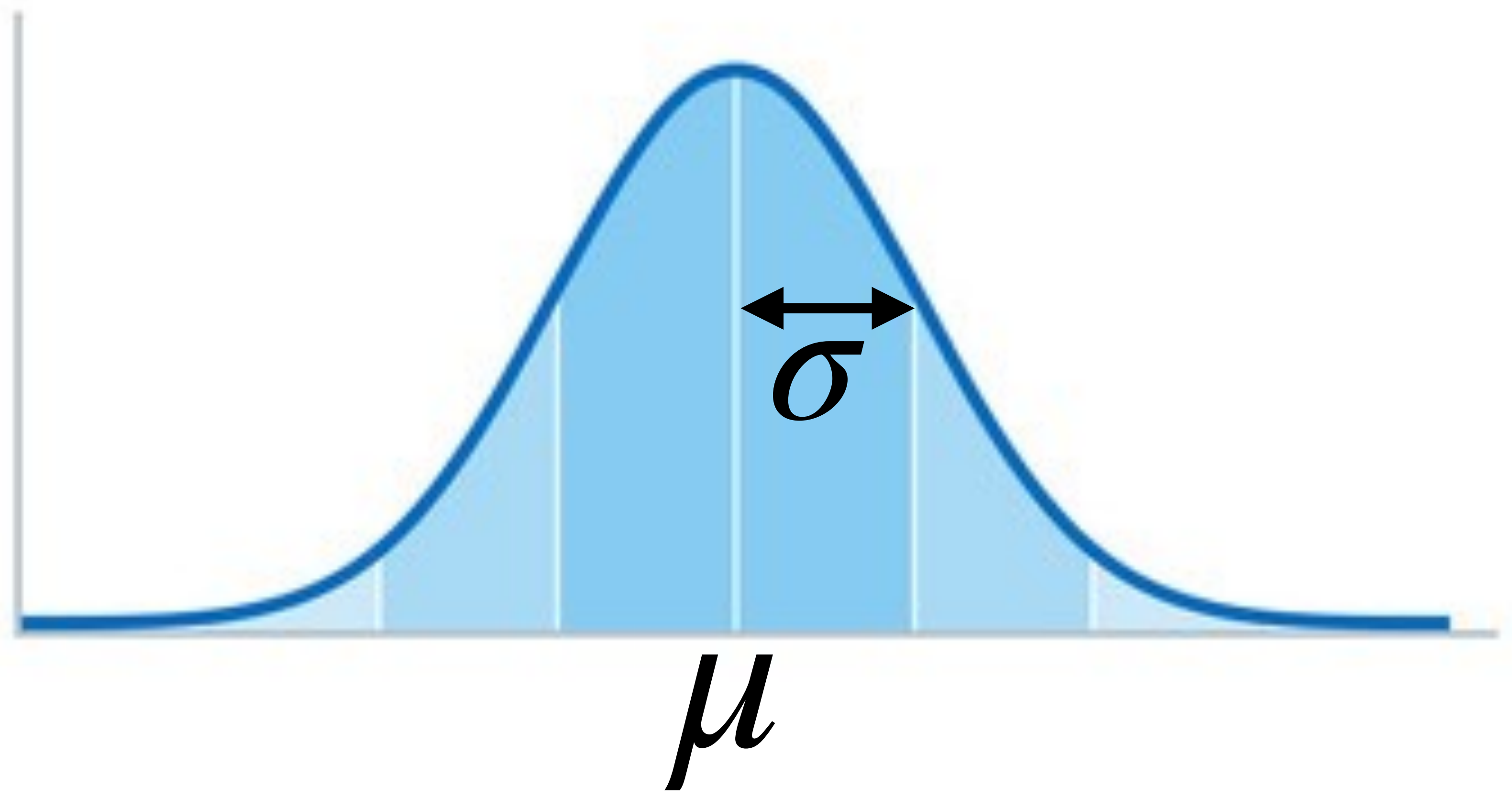


$$\text{penalty} = \text{estimation error} + \text{data collection cost}$$

$$= \frac{\sigma^2}{n} + cn$$

NORMAL MEAN ESTIMATION

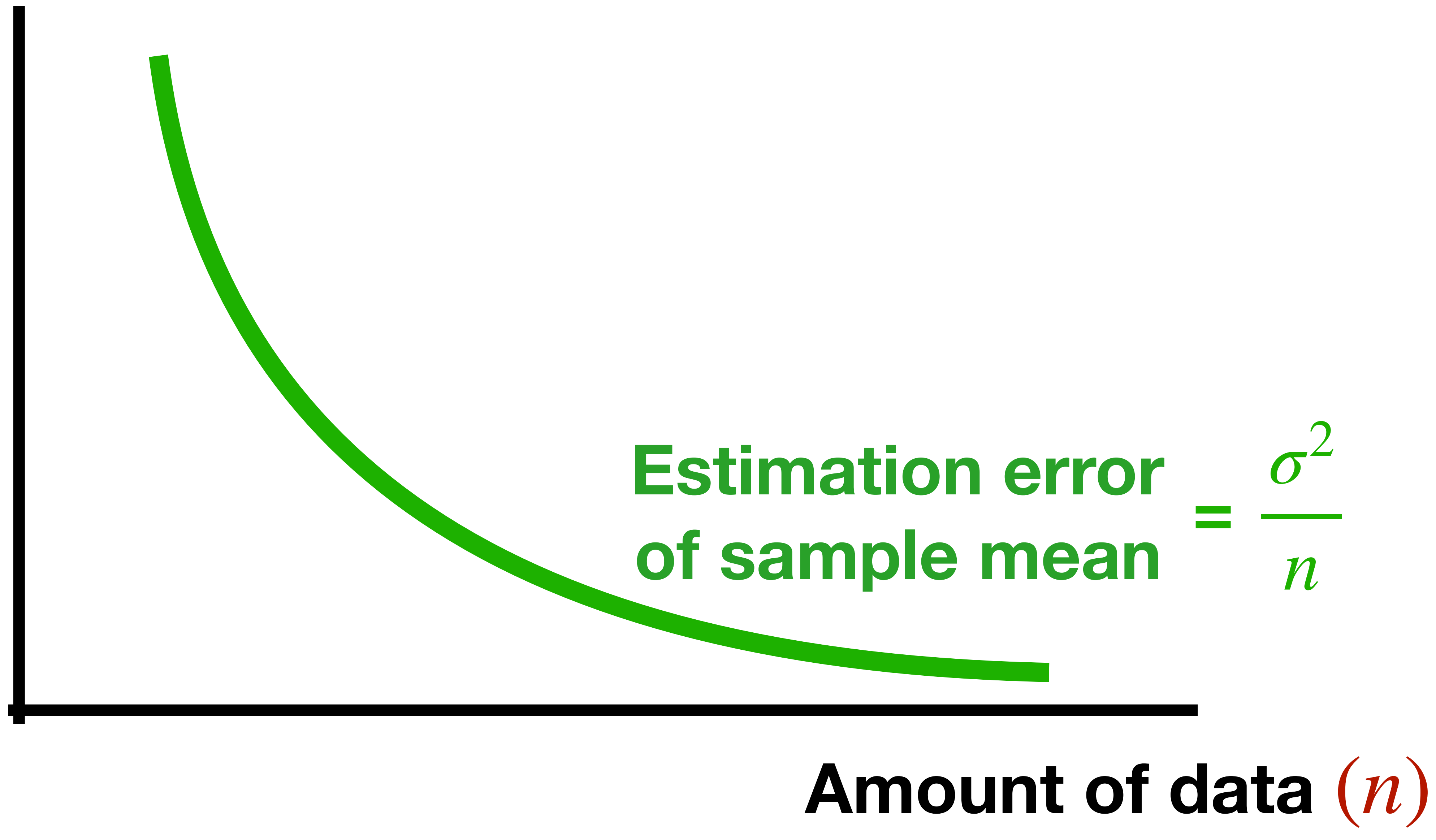
- Estimate the mean μ of a normal distribution with *known* variance σ^2.



- An agent can collect samples at *known* unit cost c.
- Each agent wishes to minimize

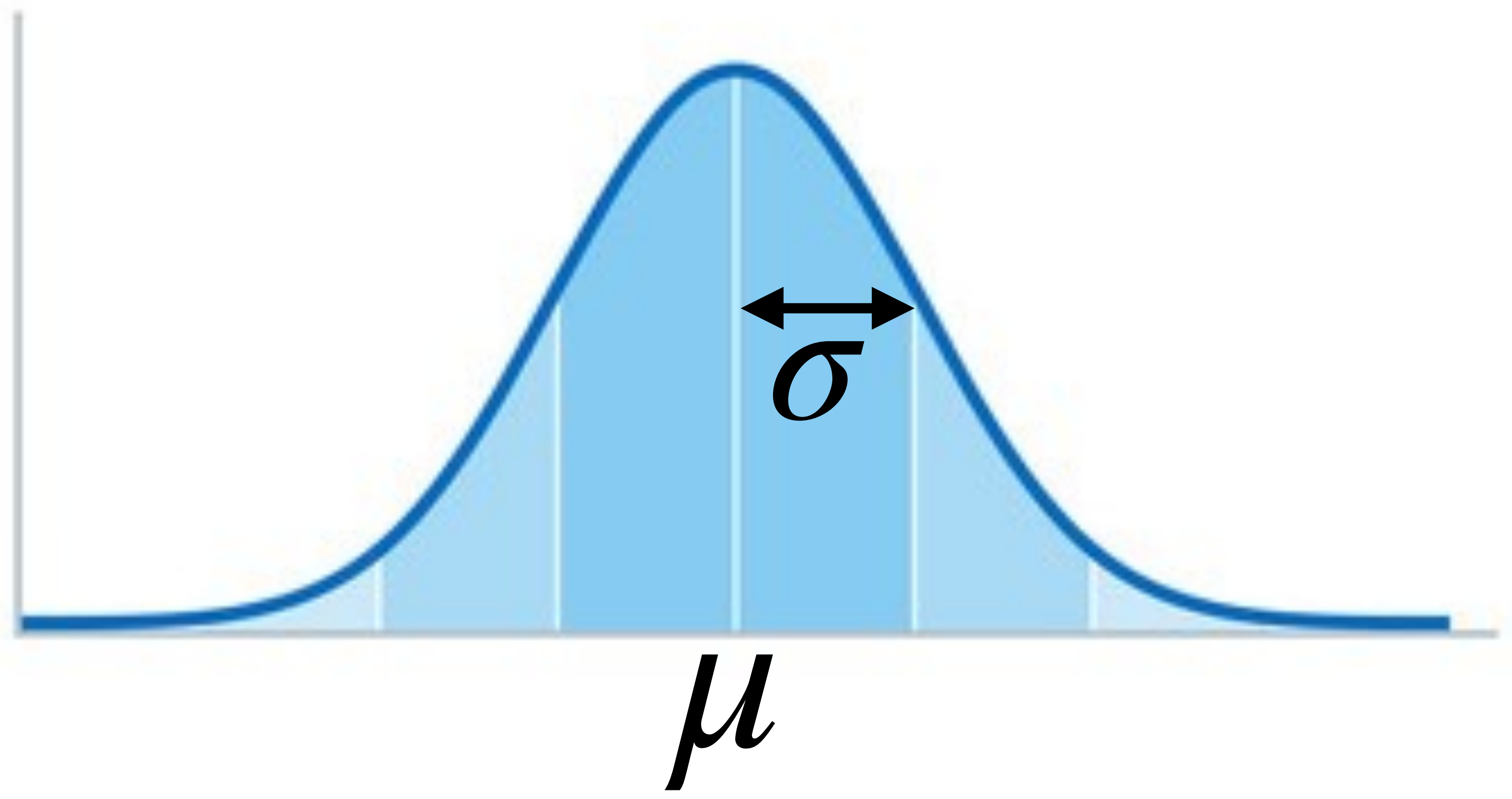
$$\text{penalty} = \text{estimation error} + \text{data collection cost}$$

$$= \frac{\sigma^2}{n} + cn$$



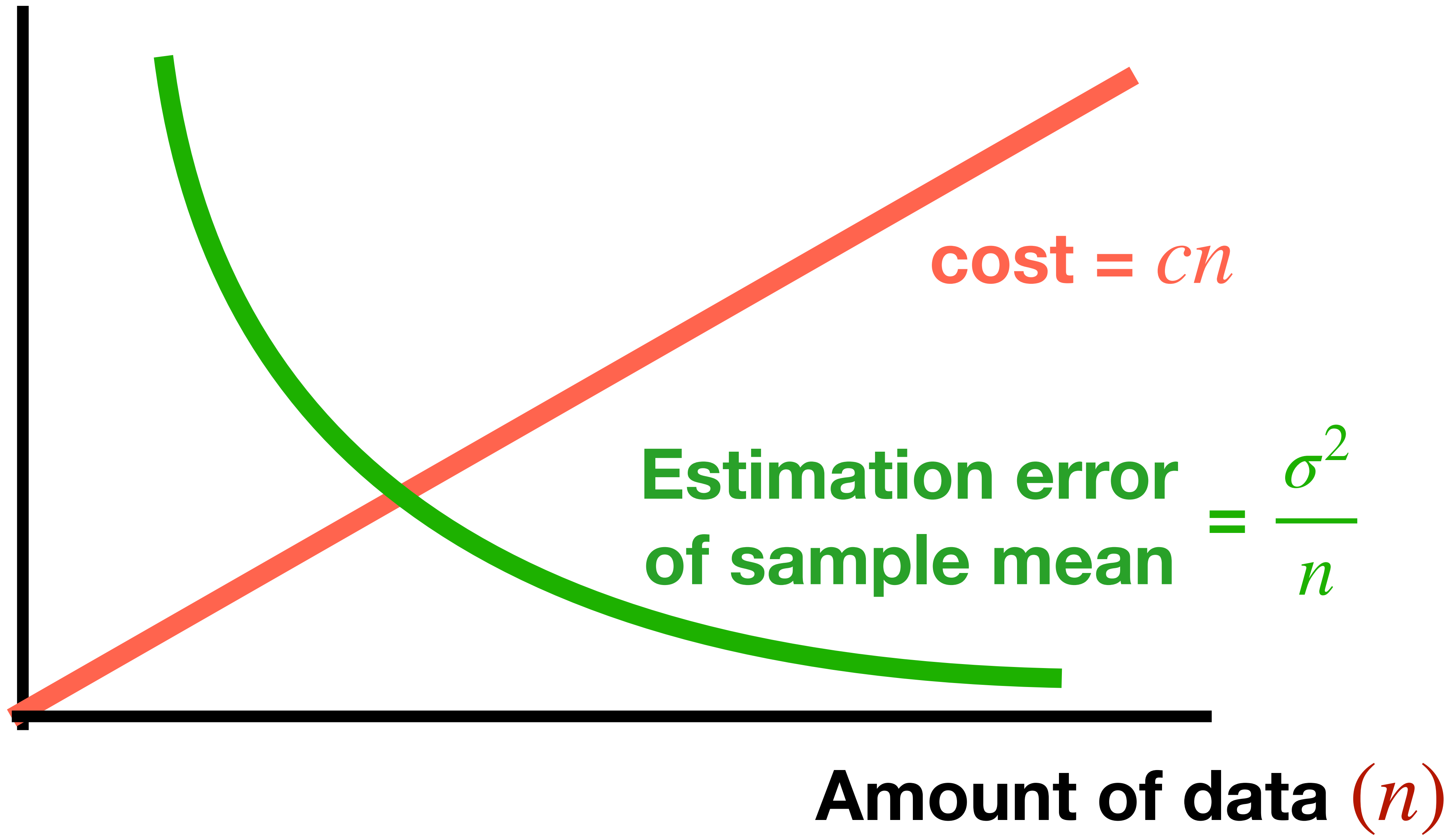
NORMAL MEAN ESTIMATION

- Estimate the mean μ of a normal distribution with *known* variance σ^2.
- An agent can collect samples at *known* unit cost c.
- Each agent wishes to minimize



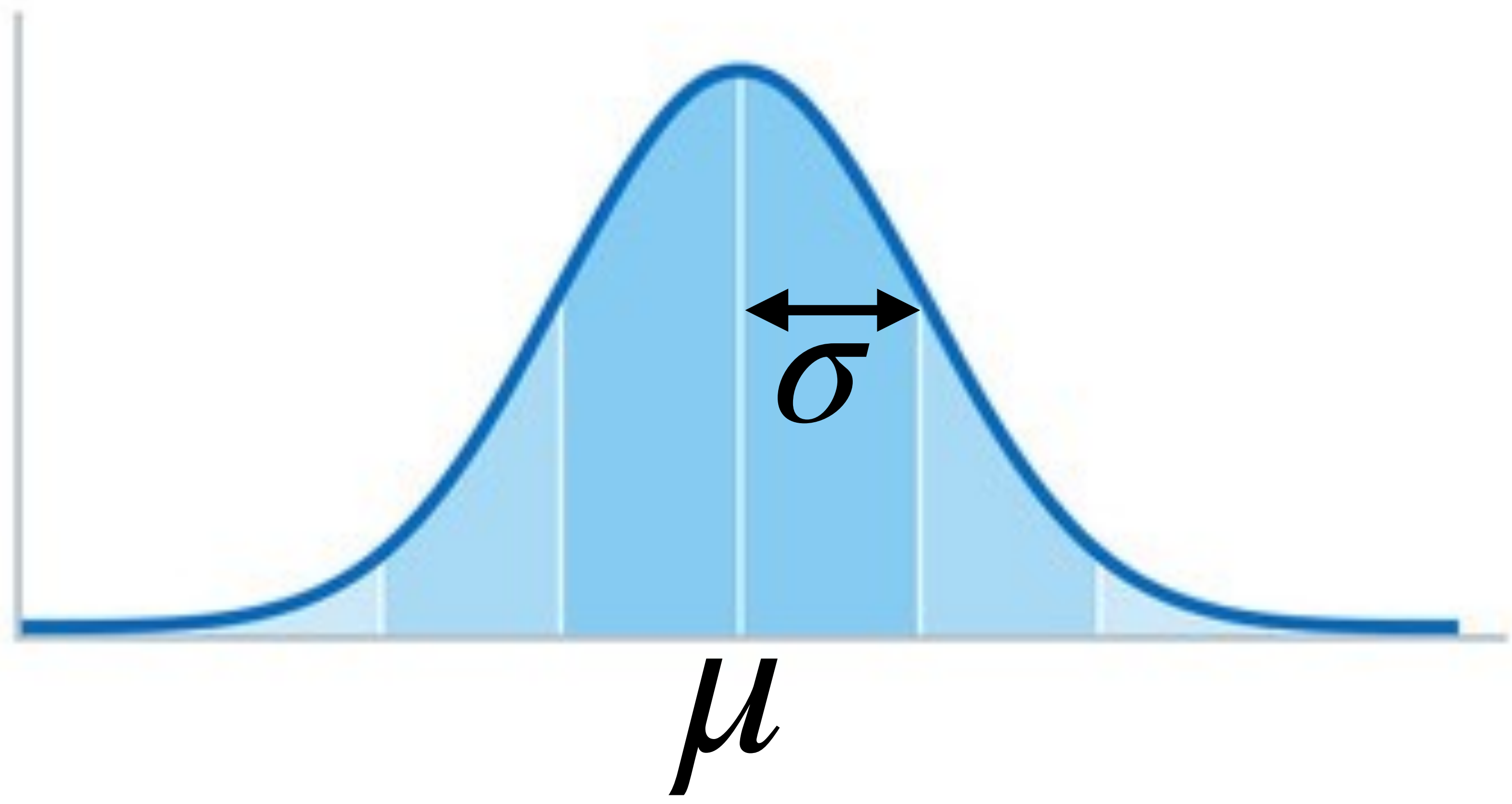
$$\text{penalty} = \text{estimation error} + \text{data collection cost}$$

$$= \frac{\sigma^2}{n} + cn$$

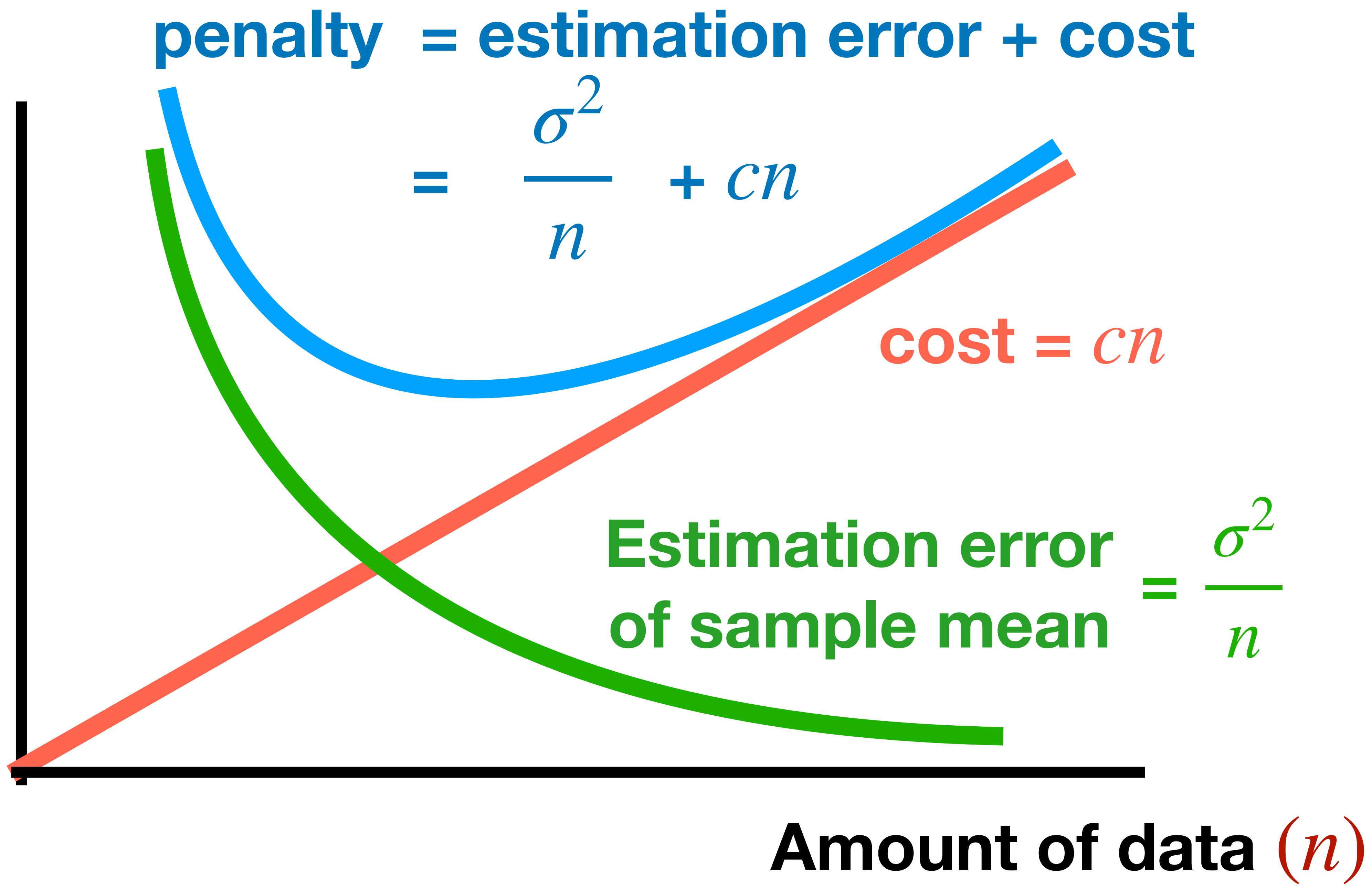


NORMAL MEAN ESTIMATION

- Estimate the mean μ of a normal distribution with *known* variance σ^2.
- An agent can collect samples at *known* unit cost c.
- Each agent wishes to minimize

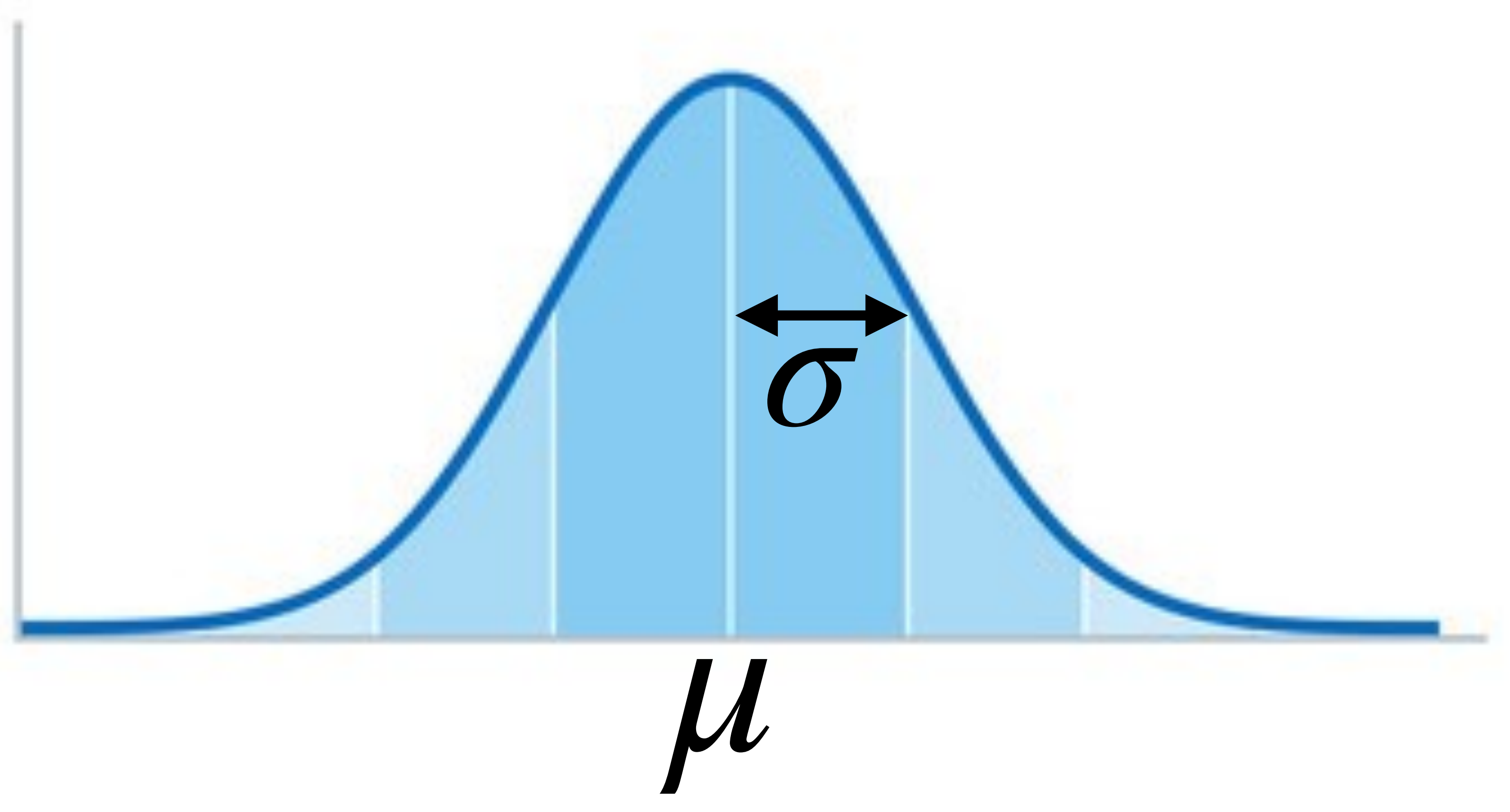


$$\text{penalty} = \text{estimation error} + \text{data collection cost}$$
$$= \frac{\sigma^2}{n} + cn$$



NORMAL MEAN ESTIMATION

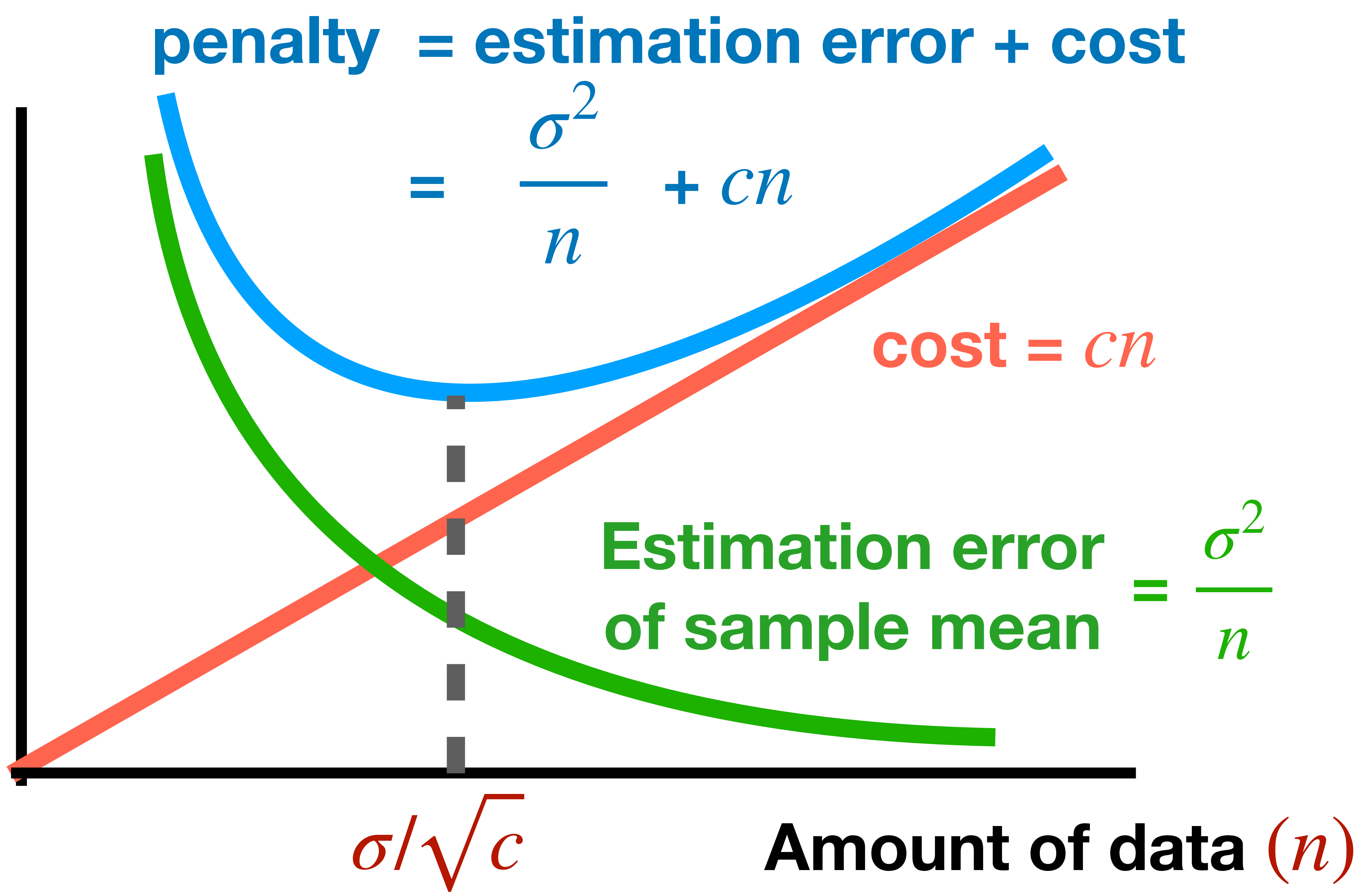
- Estimate the mean μ of a normal distribution with *known* variance σ^2.



- An agent can collect samples at *known* unit cost c.
- Each agent wishes to minimize

$$\text{penalty} = \text{estimation error} + \text{data collection cost}$$
$$= \frac{\sigma^2}{n} + cn$$

- If working on her own, agent will collect $\sigma/\sqrt{c}$ points to minimize penalty.



SEVERAL AGENTS WORKING TOGETHER

SEVERAL AGENTS WORKING TOGETHER

- Now consider m agents collecting and sharing their data.

SEVERAL AGENTS WORKING TOGETHER

- Now consider m agents collecting and sharing their data.
- Social penalty of all m agents if they collectively collect n_{tot} points.

$$\text{social penalty} = \text{ estimation error of all agents } + \text{data collection cost} = m \times \frac{\sigma^2}{n_{\text{tot}}} + c n_{\text{tot}}$$

SEVERAL AGENTS WORKING TOGETHER

- Now consider m agents collecting and sharing their data.

- Social penalty of all m agents if they collectively collect n_{tot} points.

$$\text{social penalty} = \text{ estimation error of all agents } + \text{data collection cost} = m \times \frac{\sigma^2}{n_{\text{tot}}} + cn_{\text{tot}}$$

- To minimize penalty, they should collect $n^\star_{\text{tot}} = \frac{\sigma\sqrt{m}}{\sqrt{c}}$ points.

SEVERAL AGENTS WORKING TOGETHER

- Now consider m agents collecting and sharing their data.
- Social penalty of all m agents if they collectively collect n_{tot} points.

$$\text{social penalty} = \text{ estimation error of all agents } + \text{data collection cost} = m \times \frac{\sigma^2}{n_{\text{tot}}} + cn_{\text{tot}}$$

- To minimize penalty, they should collect $n^{\star}_{\text{tot}} = \frac{\sigma\sqrt{m}}{\sqrt{c}}$ points.
  - Each agent needs to collect only $n^{\star} = \frac{\sigma}{\sqrt{mc}}$ points

    Only $\times\ 1/\sqrt{m}$ when compared to working on her own ($\sigma/\sqrt{c}$ points).

SEVERAL AGENTS WORKING TOGETHER

- Now consider m agents collecting and sharing their data.
- Social penalty of all m agents if they collectively collect n_{tot} points.

$$\text{social penalty} = \text{ estimation error of all agents } + \text{data collection cost} = m \times \frac{\sigma^2}{n_{\text{tot}}} + c n_{\text{tot}}$$

- To minimize penalty, they should collect $n^\star_{\text{tot}} = \frac{\sigma\sqrt{m}}{\sqrt{c}}$ points.
  - Each agent needs to collect only $n^\star = \frac{\sigma}{\sqrt{mc}}$ points

    Only $\times\, 1/\sqrt{m}$ when compared to working on her own ($\sigma/\sqrt{c}$ points).
  - But she has $\times \sqrt{m}$ data.

SEVERAL AGENTS WORKING TOGETHER

	Amount of data she needs to collect (n_i)	Amount of data available to her (n_{tot})	Penalty $\frac{\sigma^2}{n_{\text{tot}}} + cn_i$
Working on her own			
Working together			

SEVERAL AGENTS WORKING TOGETHER

	Amount of data she needs to collect (n_i)	Amount of data available to her (n_{tot})	Penalty $\frac{\sigma^2}{n_{\text{tot}}} + cn_i$
Working on her own	$\frac{\sigma}{\sqrt{c}}$		
Working together	$\frac{\sigma}{\sqrt{cm}}$		

SEVERAL AGENTS WORKING TOGETHER

	Amount of data she needs to collect (n_i)	Amount of data available to her (n_{tot})	Penalty $\frac{\sigma^2}{n_{\text{tot}}} + cn_i$
Working on her own	$\frac{\sigma}{\sqrt{c}}$	$\frac{\sigma}{\sqrt{c}}$	
Working together	$\frac{\sigma}{\sqrt{cm}}$	$\frac{\sigma\sqrt{m}}{\sqrt{c}}$	

SEVERAL AGENTS WORKING TOGETHER

	Amount of data she needs to collect (n_i)	Amount of data available to her (n_{tot})	Penalty $\frac{\sigma^2}{n_{\text{tot}}} + cn_i$
Working on her own	$\frac{\sigma}{\sqrt{c}}$	$\frac{\sigma}{\sqrt{c}}$	$2\sigma\sqrt{c}$
Working together	$\frac{\sigma}{\sqrt{cm}}$	$\frac{\sigma\sqrt{m}}{\sqrt{c}}$	$\frac{2\sigma\sqrt{c}}{\sqrt{m}}$

SEVERAL AGENTS WORKING TOGETHER

	Amount of data she needs to collect (n_i)	Amount of data available to her (n_{tot})	Penalty $\frac{\sigma^2}{n_{tot}} + cn_i$
Working on her own	$\frac{\sigma}{\sqrt{c}}$	$\frac{\sigma}{\sqrt{c}}$	$2\sigma\sqrt{c}$
Working together	$\frac{\sigma}{\sqrt{cm}}$	$\frac{\sigma\sqrt{m}}{\sqrt{c}}$	$\frac{2\sigma\sqrt{c}}{\sqrt{m}}$

Agents can reduce data collection costs, and improve estimation error by sharing data with others.

DESIGNING A MECHANISM FOR COLLABORATIVE NORMAL MEAN ESTIMATION

- Naive mechanism 1: "pool and share"

- Naive mechanism 1: "pool and share"
  - Selfish agents will *free-ride:* not collecting any data, but using the data that the others have contributed.

$$\text{penalty} = \frac{\sigma^2}{n_{\text{tot}}} + c \times n_i$$

- Naive mechanism 1: "pool and share"
  - Selfish agents will *free-ride:* not collecting any data, but using the data that the others have contributed.

$$\text{penalty} = \frac{\sigma^2}{n_{\text{tot}}} + c \times n_i = \frac{\sigma^2}{(m-1) \times \frac{\sigma}{\sqrt{mc}}} + c \times 0$$

- Naive mechanism 1: "pool and share"
  - Selfish agents will *free-ride:* not collecting any data, but using the data that the others have contributed.

$$\text{penalty} = \frac{\sigma^2}{n_{\text{tot}}} + c \times n_i = \frac{\sigma^2}{(m-1) \times \frac{\sigma}{\sqrt{mc}}} + c \times 0 \approx \frac{\sigma\sqrt{c}}{\sqrt{m}}$$

- Naive mechanism 1: "pool and share"
  - Selfish agents will *free-ride:* not collecting any data, but using the data that the others have contributed.

$$\text{penalty} = \frac{\sigma^2}{n_{\text{tot}}} + c \times n_i = \frac{\sigma^2}{(m-1) \times \frac{\sigma}{\sqrt{mc}}} + c \times 0 \approx \frac{\sigma\sqrt{c}}{\sqrt{m}} = \frac{1}{2} \times \underbrace{\frac{2\sigma\sqrt{c}}{\sqrt{m}}}_{\text{penalty for a well-behaved agent}}$$

- Naive mechanism 1: "pool and share"
  - Selfish agents will *free-ride:* not collecting any data, but using the data that the others have contributed.

$$\text{penalty} = \frac{\sigma^2}{n_{\text{tot}}} + c \times n_i = \frac{\sigma^2}{(m-1) \times \frac{\sigma}{\sqrt{mc}}} + c \times 0 \approx \frac{\sigma\sqrt{c}}{\sqrt{m}} = \frac{1}{2} \times \underbrace{\frac{2\sigma\sqrt{c}}{\sqrt{m}}}_{\text{penalty for a well-behaved agent}}$$

- Naive mechanism 2: "pool and share, but only if you contribute enough data"

- Naive mechanism 1: "pool and share"
  - Selfish agents will *free-ride:* not collecting any data, but using the data that the others have contributed.

$$\text{penalty} = \frac{\sigma^2}{n_{\text{tot}}} + c \times n_i = \frac{\sigma^2}{(m-1) \times \frac{\sigma}{\sqrt{mc}}} + c \times 0 \approx \frac{\sigma\sqrt{c}}{\sqrt{m}} = \frac{1}{2} \times \underbrace{\frac{2\sigma\sqrt{c}}{\sqrt{m}}}_{\text{penalty for a well-behaved agent}}$$

- Naive mechanism 2: "pool and share, but only if you contribute enough data"
  - Agents can fabricate data, and then discard it after receiving others' data.

OVERVIEW OF MECHANISM

Each agent i will:

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = \{y_{i,1}, \ldots, y_{i,n'_i}\}$.

Agents may collect any number of points, and lie (e.g withhold, lie, fabricate) about what they collect.

Each agent i will:

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\}$.

Agents may collect any number of points, and lie (e.g withhold, lie, fabricate) about what they collect.

The mechanism:

- To each agent, allocates a noisy version A_i of the others' data. The noise is proportional to how much the agent's submission Y_i differs from the others' submissions $\{Y_j\}_{j \neq i}$.

OVERVIEW OF MECHANISM

Each agent i will:

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\}$.

Agents may collect any number of points, and lie (e.g withhold, lie, fabricate) about what they collect.

The mechanism:

- To each agent, allocates a noisy version A_i of the others' data. The noise is proportional to how much the agent's submission Y_i differs from the others' submissions $\{Y_j\}_{j\neq i}$.

Each agent i will:

- Estimate μ using all the information they have (X_i, Y_i, A_i).

OVERVIEW OF MECHANISM

Each agent i will:

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\}$.

  # Agents may collect any number of points, and lie (e.g withhold, lie, fabricate) about what they collect.

The mechanism:

- To each agent, allocates a noisy version A_i of the others' data. The noise is proportional to how much the agent's submission Y_i differs from the others' submissions $\{Y_j\}_{j \neq i}$.

Each agent i will:

- Estimate μ using all the information they have (X_i, Y_i, A_i).
  - *We design a (minimax) optimal estimator to enforce truthful reporting.*

OVERVIEW OF RESULTS

OVERVIEW OF RESULTS

This mechanism is

OVERVIEW OF RESULTS

This mechanism is

- **Nash Incentive-compatible:** Provided that other agents are well-behaved, the best strategy for an agent is to,

OVERVIEW OF RESULTS

This mechanism is

- **Nash Incentive-compatible:** Provided that other agents are well-behaved, the best strategy for an agent is to,
  - Collect a sufficient amount ($n^\star = \sigma/\sqrt{mc}$) of data.

OVERVIEW OF RESULTS

This mechanism is

- **Nash Incentive-compatible:** Provided that other agents are well-behaved, the best strategy for an agent is to,
    - Collect a sufficient amount ($n^\star = \sigma/\sqrt{mc}$) of data.
    - Submit it truthfully.

OVERVIEW OF RESULTS

This mechanism is

- **Nash Incentive-compatible:** Provided that other agents are well-behaved, the best strategy for an agent is to,
  - Collect a sufficient amount ($n^\star = \sigma/\sqrt{mc}$) of data.
  - Submit it truthfully.
  - Use the recommended minimax-optimal estimator.

OVERVIEW OF RESULTS

This mechanism is

- **Nash Incentive-compatible:** Provided that other agents are well-behaved, the best strategy for an agent is to,
  - Collect a sufficient amount ($n^\star = \sigma/\sqrt{mc}$) of data.
  - Submit it truthfully.
  - Use the recommended minimax-optimal estimator.
- **Individually rational:** Provided that others are well-behaved, an agent does not do worse than the best she could do on her own.

OVERVIEW OF RESULTS

This mechanism is

- **Nash Incentive-compatible:** Provided that other agents are well-behaved, the best strategy for an agent is to,
  - Collect a sufficient amount ($n^\star = \sigma/\sqrt{mc}$) of data.
  - Submit it truthfully.
  - Use the recommended minimax-optimal estimator.
- **Individually rational:** Provided that others are well-behaved, an agent does not do worse than the best she could do on her own.
- **Efficient:** Social penalty at the Nash strategies is at most a factor 2 of the global minimum.

OUTLINE

1. **Mechanism design for collaborative normal mean estimation**

(Chen, Zhu, Kandasamy, *Neurips 2023*)

- Intuitions and Challenges
- **Problem formalism**
- Mechanism and theoretical analysis

2. Extensions & Future work

- Collaborative supervised learning, design of experiments
- Data marketplaces

FORMALISM 1/4: MECHANISM

FORMALISM 1/4: MECHANISM

A mechanism M receives a dataset from each agent, and returns an *allocation* A_i to each agent.

FORMALISM 1/4: MECHANISM

A mechanism M receives a dataset from each agent, and returns an *allocation* A_i to each agent.

- The mechanism designer can choose a space of allocations $\mathscr{A}$ to obtain desirable outcomes.

FORMALISM 1/4: MECHANISM

A mechanism M receives a dataset from each agent, and returns an *allocation* A_i to each agent.

- The mechanism designer can choose a space of allocations $\mathscr{A}$ to obtain desirable outcomes.

  E.g. A larger dataset, $\mathscr{A} = \bigcup_{k \geq 0} \mathbb{R}^k$

We can write the space of mechanisms $\mathscr{M}$ as,

FORMALISM 1/4: MECHANISM

A mechanism M receives a dataset from each agent, and returns an *allocation* A_i to each agent.

- The mechanism designer can choose a space of allocations $\mathscr{A}$ to obtain desirable outcomes.

E.g. A larger dataset, $\mathscr{A} = \bigcup_{k\geq 0} \mathbb{R}^k$

We can write the space of mechanisms $\mathscr{M}$ as,

$$\mathscr{M} = \left\{ M = (\mathscr{A}, b); \quad \mathscr{A} \subset \text{universal set}, \quad b : \left(\bigcup_{n\geq 0} \mathbb{R}^n \right)^m \to \mathscr{A}^m \right\}$$

A mechanism M receives a dataset from each agent, and returns an *allocation* A_i to each agent.

- The mechanism designer can choose a space of allocations $\mathscr{A}$ to obtain desirable outcomes.

E.g. A larger dataset, $\mathscr{A} = \bigcup_{k\geq 0}\mathbb{R}^k$

Datasets received from the m agents.

We can write the space of mechanisms $\mathscr{M}$ as,

$$\mathscr{M} = \left\{ M = (\mathscr{A}, b); \quad \mathscr{A} \subset \text{universal set}, \quad b : \left(\bigcup_{n\geq 0}\mathbb{R}^n\right)^m \to \mathscr{A}^m \right\}$$

FORMALISM 2/4: AGENT'S STRATEGY SPACE

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$.

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$.
- Submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\} = f_i(X_i)$.

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$.
- Submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\} = f_i(X_i)$.
  - f_i maps the dataset collected to possibly altered dataset of a potentially different size (e.g fabrication, withholding, altering etc).

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$.
- Submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\} = f_i(X_i)$.
  - f_i maps the dataset collected to possibly altered dataset of a potentially different size (e.g fabrication, withholding, altering etc).
- On receiving her allocation A_i, she will estimate μ via an estimator $h_i(X_i, Y_i, A_i)$.

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$.
- Submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\} = f_i(X_i)$.
  - f_i maps the dataset collected to possibly altered dataset of a potentially different size (e.g fabrication, withholding, altering etc).
- On receiving her allocation A_i, she will estimate μ via an estimator $h_i(X_i, Y_i, A_i)$.
  - An agent need not use the "straightforward" (e.g sample mean) estimator.

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$.
- Submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\} = f_i(X_i)$.
  - f_i maps the dataset collected to possibly altered dataset of a potentially different size (e.g fabrication, withholding, altering etc).
- On receiving her allocation A_i, she will estimate μ via an estimator $h_i(X_i, Y_i, A_i)$.
  - An agent need not use the “straightforward" (e.g sample mean) estimator.

An agent's strategy $s_i = (n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$

FORMALISM 2/4: AGENT'S STRATEGY SPACE

After the mechanism is published an agent will

- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$.
- Submit $Y_i = \{y_{i,1}, \ldots, y_{i,n_i'}\} = f_i(X_i)$.
  - f_i maps the dataset collected to possibly altered dataset of a potentially different size (e.g fabrication, withholding, altering etc).
- On receiving her allocation A_i, she will estimate μ via an estimator $h_i(X_i, Y_i, A_i)$.
  - An agent need not use the "straightforward" (e.g sample mean) estimator.

An agent's strategy $s_i = (n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$

$$\mathcal{F} = \text{submission functions} = \left\{ f : \bigcup_{n \geq 0} \mathbb{R}^n \to \bigcup_{n \geq 0} \mathbb{R}^n \right\}, \qquad \mathcal{H} = \text{estimators} = \left\{ h : \bigcup_{n \geq 0} \mathbb{R}^n \times \bigcup_{n \geq 0} \mathbb{R}^n \times \mathcal{A} \to \mathbb{R} \right\}$$

FORMALISM 3/4: AGENT'S PENALTY (NEGATIVE UTILITY)

FORMALISM 3/4: AGENT'S PENALTY (NEGATIVE UTILITY)

An agent's penalty p_i in a mechanism M under a strategy profile $s = (s_1, \ldots, s_m)$,

FORMALISM 3/4: AGENT'S PENALTY (NEGATIVE UTILITY)

An agent's penalty p_i in a mechanism M under a strategy profile $s = (s_1, \ldots, s_m)$,

$$p_i(M, s) = \text{estimation error} + \text{data collection cost}$$

$$= \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i(X_i, Y_i, A_i) - \mu \right)^2 \right] + cn_i$$

FORMALISM 3/4: AGENT'S PENALTY (NEGATIVE UTILITY)

An agent's penalty p_i in a mechanism M under a strategy profile $s = (s_1, \ldots, s_m)$,

$$p_i(M, s) = \text{estimation error} + \text{data collection cost}$$

$$= \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i(X_i, Y_i, A_i) - \mu \right)^2 \right] + cn_i$$

- We take a $\sup_{\mu \in \mathbb{R}} \ldots$ since μ is unknown. Makes the problem well-defined.

FORMALISM 3/4: AGENT'S PENALTY (NEGATIVE UTILITY)

An agent's penalty p_i in a mechanism M under a strategy profile $s = (s_1, \ldots, s_m)$,

$$p_i(M, s) = \quad \text{estimation error} \quad + \quad \text{data collection cost}$$

$$= \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i(X_i, Y_i, A_i) - \mu \right)^2 \right] + cn_i$$

- We take a $\sup_{\mu \in \mathbb{R}} \ldots$ since μ is unknown. Makes the problem well-defined.
  - Otherwise, when $\mu = \mu'$, an agent can choose $n_i = 0$ and $h_i(\cdot, \cdot, \cdot) = \mu'$ to achieve zero penalty.

FORMALISM 3/4: AGENT'S PENALTY (NEGATIVE UTILITY)

An agent's penalty p_i in a mechanism M under a strategy profile $s = (s_1, \ldots, s_m)$,

$$p_i(M, s) = \text{estimation error} + \text{data collection cost}$$
$$= \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i(X_i, Y_i, A_i) - \mu \right)^2 \right] + cn_i$$

- We take a $\sup_{\mu \in \mathbb{R}} \ldots$ since μ is unknown. Makes the problem well-defined.
  - Otherwise, when $\mu = \mu'$, an agent can choose $n_i = 0$ and $h_i(\cdot, \cdot, \cdot) = \mu'$ to achieve zero penalty.
  - But this works only if agent knows $\mu = \mu'$ a priori.

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

A mechanism will also publish a *recommended strategy profile* $s^\star = \{s_i^\star\}_{i\in[n]}$.

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

A mechanism will also publish a *recommended strategy profile* $s^\star = \{s_i^\star\}_{i\in[n]}$.

Desiderata:

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

A mechanism will also publish a *recommended strategy profile* $s^\star = \{s_i^\star\}_{i \in [n]}$.

Desiderata:

1. **Nash Incentive-compatible (NIC):** $s^\star$ is a Nash equilibrium, i.e $p_i(M, (s_i^\star, s_{-i}^\star)) \leq p_i(M, (s_i', s_{-i}^\star))$ for all agents i and all other strategies s_i'.

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

A mechanism will also publish a *recommended strategy profile* $s^\star = \{s_i^\star\}_{i\in[n]}$.

Desiderata:

1. **Nash Incentive-compatible (NIC):** $s^\star$ is a Nash equilibrium, i.e $p_i(M,(s_i^\star, s_{-i}^\star)) \leq p_i(M,(s_i', s_{-i}^\star))$ for all agents i and all other strategies s_i'.
2. **Individually rational:** An agent's penalty at $s^\star$ is no worse than the lowest penalty she could achieve on her own, i.e $p_i(M, s^\star) \leq 2\sigma/\sqrt{c}$.

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

A mechanism will also publish a *recommended strategy profile* $s^\star = \{s_i^\star\}_{i\in[n]}$.

Desiderata:

1. **Nash Incentive-compatible (NIC):** $s^\star$ is a Nash equilibrium, i.e $p_i(M,(s_i^\star, s_{-i}^\star)) \le p_i(M,(s_i', s_{-i}^\star))$ for all agents i and all other strategies s_i'.
2. **Individually rational:** An agent's penalty at $s^\star$ is no worse than the lowest penalty she could achieve on her own, i.e $p_i(M, s^\star) \le 2\sigma/\sqrt{c}$.
3. **Approximately efficient:** The social penalty $P(M, s^\star) = \sum_i p_i(M, s^\star)$ is at most a constant factor of the global minimum, i.e

$$P(M, s^\star) \le C \cdot \min_{M', s'} p(M', s')$$

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

A mechanism will also publish a *recommended strategy profile* $s^\star = \{s_i^\star\}_{i \in [n]}$.

Desiderata:

1. **Nash Incentive-compatible (NIC):** $s^\star$ is a Nash equilibrium, i.e $p_i(M, (s_i^\star, s_{-i}^\star)) \leq p_i(M, (s_i', s_{-i}^\star))$ for all agents i and all other strategies s_i'.
2. **Individually rational:** An agent's penalty at $s^\star$ is no worse than the lowest penalty she could achieve on her own, i.e $p_i(M, s^\star) \leq 2\sigma/\sqrt{c}$.
3. **Approximately efficient:** The social penalty $P(M, s^\star) = \sum_i p_i(M, s^\star)$ is at most a constant factor of the global minimum, i.e

$$P(M, s^\star) \leq C \cdot \min_{M', s'} p(M', s')$$

min without NIC, IR constraints

FORMALISM 4/4: DESIDERATA FOR A MECHANISM

A mechanism will also publish a *recommended strategy profile* $s^\star = \{s_i^\star\}_{i \in [n]}$.

Desiderata:

1. **Nash Incentive-compatible (NIC):** $s^\star$ is a Nash equilibrium, i.e $p_i(M, (s_i^\star, s_{-i}^\star)) \leq p_i(M, (s_i', s_{-i}^\star))$ for all agents i and all other strategies s_i'.
2. **Individually rational:** An agent's penalty at $s^\star$ is no worse than the lowest penalty she could achieve on her own, i.e $p_i(M, s^\star) \leq 2\sigma/\sqrt{c}$.
3. **Approximately efficient:** The social penalty $P(M, s^\star) = \sum_i p_i(M, s^\star)$ is at most a constant factor of the global minimum, i.e

$$P(M, s^\star) \leq C \cdot \underbrace{\min_{M', s'} p(M', s')}_{= \; 2\sigma\sqrt{mc}}$$

min without NIC, IR constraints

(pool-and-share)

OUTLINE

1. **Mechanism design for collaborative normal mean estimation**
   (Chen, Zhu, Kandasamy, Neurips 2023)
   - Intuitions and Challenges
   - Problem formalism
   - **Mechanism and theoretical analysis**

2. Extensions & Future work
   - Collaborative supervised learning, design of experiments
   - Data marketplaces

MECHANISM DESIGN FOR COLLABORATIVE NORMAL MEAN ESTIMATION

Each agent i will

MECHANISM DESIGN FOR COLLABORATIVE NORMAL MEAN ESTIMATION

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

- For each agent i:

MECHANISM DESIGN FOR COLLABORATIVE NORMAL MEAN ESTIMATION

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' submissions Y_{-i}.

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' submissions Y_{-i}.
  - Set noise variance $\eta_i^2 = \alpha^2 \left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$ # Variance proportional to difference

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' submissions Y_{-i}.
  - Set noise variance $\eta_i^2 = \alpha^2 \left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$ # Variance proportional to difference
  - $Z_i \leftarrow \left\{ z + \epsilon_z, \quad \text{for all } z \in Y_{-i} \backslash Z_i, \quad \text{where } \epsilon_z \sim \mathcal{N}(0, \eta_i^2) \right\}$.

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' submissions Y_{-i}.
  - Set noise variance $\eta_i^2 = \alpha^2 \left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$ # Variance proportional to difference
  - $Z_i \leftarrow \left\{ z + \epsilon_z, \quad \text{for all } z \in Y_{-i} \backslash Z_i, \quad \text{where } \epsilon_z \sim \mathcal{N}(0, \eta_i^2) \right\}$.
  - Set allocation to each agent, $A_i \leftarrow (Z_i, Z_i', \eta_i^2)$.

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' submissions Y_{-i}.
  - Set noise variance $\eta_i^2 = \alpha^2 \left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$ # Variance proportional to difference
  - $Z_i \leftarrow \left\{ z + \epsilon_z, \quad \text{for all } z \in Y_{-i} \backslash Z_i, \quad \text{where } \epsilon_z \sim \mathcal{N}(0, \eta_i^2) \right\}$.
  - Set allocation to each agent, $A_i \leftarrow (Z_i, Z_i', \eta_i^2)$.

Each agent i will

Each agent i will

- Choose their strategy $s_i = (n_i, f_i, h_i)$
- Collect n_i points $X_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$ and submit $Y_i = f_i(X_i)$.

Mechanism

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' submissions Y_{-i}.
  - Set noise variance $\eta_i^2 = \alpha^2 \left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$ # Variance proportional to difference
  - $Z_i \leftarrow \left\{ z + \epsilon_z, \quad \text{for all } z \in Y_{-i} \backslash Z_i, \quad \text{where } \epsilon_z \sim \mathcal{N}(0, \eta_i^2) \right\}$.
  - Set allocation to each agent, $A_i \leftarrow (Z_i, Z_i', \eta_i^2)$.

Each agent i will

- Compute their estimate $h_i(X_i, Y_i, A_i)$

RECOMMENDED STRATEGIES

Mechanisms recommends that agents follow $s_i = s_i^\star = (n_i^\star, f_i^\star, h_i^\star)$,

RECOMMENDED STRATEGIES

Mechanisms recommends that agents follow $s_i = s_i^\star = (n_i^\star, f_i^\star, h_i^\star)$,

$$n_i^\star = \frac{\sigma}{\sqrt{cm}},$$

$$f_i^\star = \text{identity},$$

$$h_i^\star\Big(X_i, Y_i, \underbrace{(Z_i, Z_i', \eta_i^2)}_{A_i}\Big) = \frac{\sum_{u \in X_i \cup Z_i} u + \frac{1}{1+\eta_i^2/\sigma^2} \sum_{u \in Z_i'} u}{|X_i \cup Z_i| + \frac{1}{1+\eta_i^2/\sigma^2} |Z_i'|}$$

RECOMMENDED STRATEGIES

Mechanisms recommends that agents follow $s_i = s_i^\star = (n_i^\star, f_i^\star, h_i^\star)$,

$$n_i^\star = \frac{\sigma}{\sqrt{cm}},$$

$$f_i^\star = \text{identity},$$

$$h_i^\star\left(X_i, Y_i, \underbrace{(Z_i, Z_i', \eta_i^2)}_{A_i}\right) = \frac{\sum_{u \in X_i \cup Z_i} u + \frac{1}{1+\eta_i^2/\sigma^2} \sum_{u \in Z_i'} u}{|X_i \cup Z_i| + \frac{1}{1+\eta_i^2/\sigma^2} |Z_i'|}$$

That is collect a sufficient amount of data $n_i^\star$, submit it truthfully $f_i^\star$, and use a weighted average estimator $h_i^\star$.

Mechanisms recommends that agents follow $s_i = s_i^\star = (n_i^\star, f_i^\star, h_i^\star)$,

$$n_i^\star = \frac{\sigma}{\sqrt{cm}},$$

$$f_i^\star = \text{identity},$$

$$h_i^\star\Big(X_i, Y_i, \underbrace{(Z_i, Z_i', \eta_i^2)}_{A_i}\Big) = \frac{\sum_{u \in X_i \cup Z_i} u + \frac{1}{1 + \eta_i^2/\sigma^2} \sum_{u \in Z_i'} u}{|X_i \cup Z_i| + \frac{1}{1 + \eta_i^2/\sigma^2} |Z_i'|}$$

Z_i' is the corrupted dataset.

That is collect a sufficient amount of data $n_i^\star$, submit it truthfully $f_i^\star$, and use a weighted average estimator $h_i^\star$.

Theorem: At the recommended strategy profile $s^\star$, the mechanism is Nash incentive-compatible, individually rational, and approximately efficient with $P(M, s^\star) \leq 2 \cdot \inf_{M,s} P(M, s)$.

Theorem: At the recommended strategy profile $s^\star$, the mechanism is Nash incentive-compatible, individually rational, and approximately efficient with $P(M, s^\star) \leq 2 \cdot \inf_{M,s} P(M, s)$.

Theorem (High-dimensional distributions with bounded variance): The recommended strategy profile $s^\star$ is an $\mathcal{O}(1/m)$-approximate Nash equilibrium. Moreover, the mechanism is approximately efficient with $P(M, s^\star) \leq \big(2 + \mathcal{O}(1/m)\big) \cdot \inf_{M,s} P(M, s)$.

PROOF OF NASH INCENTIVE-COMPATIBILITY

PROOF OF NASH INCENTIVE-COMPATIBILITY

We need to show that $s^\star = \{(n_i^\star, f_i^\star, h_i^\star)\}_i$ is a Nash equilibrium, i.e

$$p_i(M, (s_i^\star, s_{-i}^\star)) \leq p_i\left(M, (s_i, s_{-i}^\star)\right) \quad \text{for all agents } i \text{ and all deviations } s_i$$

PROOF OF NASH INCENTIVE-COMPATIBILITY

We need to show that $s^\star = \{(n_i^\star, f_i^\star, h_i^\star)\}_i$ is a Nash equilibrium, i.e

$$p_i(M, (s_i^\star, s_{-i}^\star)) \le p_i\left(M, (s_i, s_{-i}^\star)\right) \quad \text{for all agents } i \text{ and all deviations } s_i$$

Step 1: First, we will show that for any amount of data collected n_i, submitting it truthfully and using the recommended estimator minimizes the penalty, i.e

$$p_i\left(M, \left((n_i, f_i^\star, h_i^\star), s_{-i}^\star\right)\right) \le p_i\left(M, \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) \quad \text{for all } (n_i, f_i, h_i) \in \mathbb{N} \times \mathscr{F} \times \mathscr{H}$$

PROOF OF NASH INCENTIVE-COMPATIBILITY

We need to show that $s^\star = \{(n_i^\star, f_i^\star, h_i^\star)\}_i$ is a Nash equilibrium, i.e

$$p_i(M, (s_i^\star, s_{-i}^\star)) \leq p_i\left(M, (s_i, s_{-i}^\star)\right) \quad \text{for all agents } i \text{ and all deviations } s_i$$

Step 1: First, we will show that for any amount of data collected n_i, submitting it truthfully and using the recommended estimator minimizes the penalty, i.e

$$p_i\left(M, \left((n_i, f_i^\star, h_i^\star), s_{-i}^\star\right)\right) \leq p_i\left(M, \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) \quad \text{for all } (n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$$

Step 2: Then, we will show the agent's penalty is minimized when she collects $n_i^\star$ samples under $(f_i^\star, h_i^\star)$, i.e

$$p_i\left(M, \left((n_i^\star, f_i^\star, h_i^\star), s_{-i}^\star\right)\right) \leq p_i\left(M, \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) \quad \text{for all } n_i \in \mathbb{N}$$

PROOF OF STEP 1

Step 1: First, we will show that for any amount of data collected n_i, submitting it truthfully and using the recommended estimator minimizes the penalty, i.e

$$p_i\left(M, \left((n_i, f_i^\star, h_i^\star), s_{-i}^\star\right)\right) \leq p_i\left(M, \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) \quad \text{for all } (n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$$

PROOF OF STEP 1

Step 1: First, we will show that for any amount of data collected n_i, submitting it truthfully and using the recommended estimator minimizes the penalty, i.e

$$p_i\left(M,\ ((n_i, f_i^\star, h_i^\star), s_{-i}^\star)\right) \leq p_i\left(M,\ ((n_i, f_i, h_i), s_{-i}^\star)\right) \quad \text{for all } (n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$$

We need to show, for all $(n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$,

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu\left[\left(h_i^\star\left(X_i, f_i^\star(X_i), A_i\right) - \mu\right)^2\right] + cn_i \ \leq\ \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right] + cn_i$$

Step 1: First, we will show that for any amount of data collected n_i, submitting it truthfully and using the recommended estimator minimizes the penalty, i.e

$$p_i\Big(M, \big((n_i, f_i^\star, h_i^\star), s_{-i}^\star\big)\Big) \le p_i\Big(M, \big((n_i, f_i, h_i), s_{-i}^\star\big)\Big) \quad \text{for all } (n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$$

We need to show, for all $(n_i, f_i, h_i) \in \mathbb{N} \times \mathcal{F} \times \mathcal{H}$,

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star\left(X_i, f_i^\star(X_i), A_i\right) - \mu\right)^2\right] + cn_i \le \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right] + cn_i$$

Or equivalently,

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star\left(X_i, f_i^\star(X_i), A_i\right) - \mu\right)^2\right] = \inf_{f_i, h_i} \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right]$$

DIGRESSION: MINIMAX NORMAL MEAN ESTIMATION

We are given $X_1^n = \{X_1, \ldots, X_n\}$, drawn from $\mathcal{N}(\mu, \sigma^2)$ where σ^2 is known. Let $\widehat{\mu}(X)$ be an estimator for μ. We wish to show

DIGRESSION: MINIMAX NORMAL MEAN ESTIMATION

We are given $X_1^n = \{X_1, \ldots, X_n\}$, drawn from $\mathcal{N}(\mu, \sigma^2)$ where σ^2 is known. Let $\widehat{\mu}(X)$ be an estimator for μ. We wish to show

$$\text{minimax risk} = \inf_{\widehat{\mu}} \sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[\left(\mu - \widehat{\mu}(X_1^n)\right)^2\right] = \frac{\sigma^2}{n}$$

We are given $X_1^n = \{X_1, \ldots, X_n\}$, drawn from $\mathcal{N}(\mu, \sigma^2)$ where σ^2 is known. Let $\hat{\mu}(X)$ be an estimator for μ. We wish to show

$$\text{minimax risk} = \inf_{\hat{\mu}} \sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[\left(\mu - \hat{\mu}(X_1^n)\right)^2\right] = \frac{\sigma^2}{n}$$

Upper bound via an estimator: We can use the sample mean $\hat{\mu}_{\text{sm}}(X) = (X_1 + \ldots + X_n)/n$.

We are given $X_1^n = \{X_1, \ldots, X_n\}$, drawn from $\mathcal{N}(\mu, \sigma^2)$ where σ^2 is known. Let $\widehat{\mu}(X)$ be an estimator for μ. We wish to show

$$\text{minimax risk} = \inf_{\widehat{\mu}} \sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[\left(\mu - \widehat{\mu}(X_1^n)\right)^2\right] = \frac{\sigma^2}{n}$$

Upper bound via an estimator: We can use the sample mean $\widehat{\mu}_{\text{sm}}(X) = (X_1 + \ldots + X_n)/n$.

$$\text{minimax risk} \leq \sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[\left(\mu - \widehat{\mu}_{\text{sm}}(X_1^n)\right)^2\right] = \frac{\sigma^2}{n}$$

DIGRESSION: MINIMAX RISK FOR NORMAL MEAN ESTIMATION

Lower bound via Bayes' risk: Choose a prior Λ for μ. Then lower bound via the Bayes' risk under Λ.

Lower bound via Bayes' risk: Choose a prior Λ for μ. Then lower bound via the Bayes' risk under Λ.

We will use $\Lambda = \mathcal{N}(0, \tau^2)$. For any estimator $\widehat{\mu}$,

Lower bound via Bayes' risk: Choose a prior Λ for μ. Then lower bound via the Bayes' risk under Λ.

We will use $\Lambda = \mathcal{N}(0,\tau^2)$. For any estimator $\widehat{\mu}$,

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[(\mu - \widehat{\mu}(X_1^n))^2\right] \geq \mathbb{E}_{\mu \sim \Lambda}\left[\mathbb{E}_{X_1^n}\left[(\mu - \widehat{\mu}(X_1^n))^2 \,|\, \mu\right]\right]$$

← sup ≥ avg

Lower bound via Bayes' risk: Choose a prior Λ for μ. Then lower bound via the Bayes' risk under Λ.

We will use $\Lambda = \mathcal{N}(0, \tau^2)$. For any estimator $\hat{\mu}$,

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[(\mu - \hat{\mu}(X_1^n))^2\right] \geq \mathbb{E}_{\mu\sim\Lambda}\left[\mathbb{E}_{X_1^n}\left[(\mu - \hat{\mu}(X_1^n))^2 \mid \mu\right]\right]$$

← sup ≥ avg

$$= \mathbb{E}_{X_1^n}\left[\mathbb{E}_{\mu\sim\Lambda}\left[(\mu - \hat{\mu}(X_1^n))^2 \mid X_1^n\right]\right]$$

← Swap order of expectation

Lower bound via Bayes' risk: Choose a prior Λ for μ. Then lower bound via the Bayes' risk under Λ.

We will use $\Lambda = \mathcal{N}(0, \tau^2)$. For any estimator $\hat{\mu}$,

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[(\mu - \hat{\mu}(X_1^n))^2\right] \geq \mathbb{E}_{\mu \sim \Lambda}\left[\mathbb{E}_{X_1^n}\left[(\mu - \hat{\mu}(X_1^n))^2 \mid \mu\right]\right]$$

← sup ≥ avg

$$= \mathbb{E}_{X_1^n}\left[\mathbb{E}_{\mu \sim \Lambda}\left[(\mu - \hat{\mu}(X_1^n))^2 \mid X_1^n\right]\right]$$

← Swap order of expectation

$$\geq \mathbb{E}_{X_1^n}\left[\frac{\sigma^2}{n + \sigma^2/\tau^2}\right]$$

← By normal conjugacy and choosing $\hat{\mu}$ = posterior mean

DIGRESSION: MINIMAX RISK FOR NORMAL MEAN ESTIMATION

Lower bound via Bayes' risk: Choose a prior Λ for μ. Then lower bound via the Bayes' risk under Λ.

We will use $\Lambda = \mathcal{N}(0, \tau^2)$. For any estimator $\hat{\mu}$,

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_{X_1^n}\left[(\mu - \hat{\mu}(X_1^n))^2\right] \geq \mathbb{E}_{\mu \sim \Lambda}\left[\mathbb{E}_{X_1^n}\left[(\mu - \hat{\mu}(X_1^n))^2 \,|\, \mu\right]\right]$$

← sup ≥ avg

$$= \mathbb{E}_{X_1^n}\left[\mathbb{E}_{\mu \sim \Lambda}\left[(\mu - \hat{\mu}(X_1^n))^2 \,|\, X_1^n\right]\right]$$

← Swap order of expectation

$$\geq \mathbb{E}_{X_1^n}\left[\frac{\sigma^2}{n + \sigma^2/\tau^2}\right]$$

← By normal conjugacy and choosing $\hat{\mu}$ = posterior mean

$$= \frac{\sigma^2}{n + \sigma^2/\tau^2} \quad \to \frac{\sigma^2}{n} \qquad \text{as } \tau^2 \to \infty$$

PROOF OF STEP 1

We will apply the same recipe to show that $f_i^\star, h_i^\star$ are the minimax-optimal submission functions and estimators for the agent.

$$\inf_{f_i, h_i} \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \right] = \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star \left(X_i, f_i^\star(X_i), A_i \right) - \mu \right)^2 \right]$$

PROOF OF STEP 1

We will apply the same recipe to show that $f_i^\star, h_i^\star$ are the minimax-optimal submission functions and estimators for the agent.

$$\inf_{f_i, h_i} \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \right] = \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star \left(X_i, f_i^\star(X_i), A_i \right) - \mu \right)^2 \right]$$

But the data available to the agent is not i.i.d!

- The corruption is data-dependent.

We will apply the same recipe to show that $f_i^\star, h_i^\star$ are the minimax-optimal submission functions and estimators for the agent.

$$\inf_{f_i, h_i} \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \right] = \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star \left(X_i, f_i^\star(X_i), A_i \right) - \mu \right)^2 \right]$$

But the data available to the agent is not i.i.d!

- The corruption is data-dependent.
- In fact, X_i, Z_i, Z_i' is not even jointly Gaussian.

PROOF OF STEP 1: UPPER BOUND

We show

$$\inf_{f_i, h_i} \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \right] \leq \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star \left(X_i, f_i^\star(X_i), A_i \right) - \mu \right)^2 \right]$$

PROOF OF STEP 1: UPPER BOUND

We show

$$\inf_{f_i, h_i} \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \right] \leq \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star \left(X_i, f_i^\star(X_i), A_i \right) - \mu \right)^2 \right]$$

$$= \mathbb{E}_{Z \sim \mathcal{N}(0,1)} \left[\left(\frac{(m-2) n_i^\star}{\left(\sigma^2 + \alpha^2 \left(\sigma^2 / n_i + \sigma^2 / n_i^\star \right) Z^2 \right)} + \frac{n_i + n_i^\star}{\sigma^{-2}} \right)^{-1} \right]$$

PROOF OF STEP 1: UPPER BOUND

We show

$$\inf_{f_i,h_i} \sup_{\mu\in\mathbb{R}} \mathbb{E}_\mu \left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right] \leq \sup_{\mu\in\mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star\left(X_i, f_i^\star(X_i), A_i\right) - \mu\right)^2\right]$$

$$= \mathbb{E}_{Z\sim\mathcal{N}(0,1)} \left[\left(\frac{(m-2)n_i^\star}{\left(\sigma^2 + \alpha^2\left(\sigma^2/n_i + \sigma^2/n_i^\star\right) Z^2\right)} + \frac{n_i + n_i^\star}{\sigma^{-2}}\right)^{-1}\right] =: R_\infty(n_i) \quad \text{(say)}$$

We show

$$\inf_{f_i,h_i}\sup_{\mu\in\mathbb{R}} \mathbb{E}_\mu\left[\left(h_i\left(X_i, f_i(X_i), A_i\right)-\mu\right)^2\right] \le \sup_{\mu\in\mathbb{R}} \mathbb{E}_\mu\left[\left(h_i^\star\left(X_i, f_i^\star(X_i), A_i\right)-\mu\right)^2\right]$$

$$= \mathbb{E}_{Z\sim\mathcal{N}(0,1)}\left[\left(\frac{(m-2)n_i^\star}{\left(\sigma^2+\alpha^2\left(\sigma^2/n_i+\sigma^2/n_i^\star\right)Z^2\right)}+\frac{n_i+n_i^\star}{\sigma^{-2}}\right)^{-1}\right] =: R_\infty(n_i) \quad \text{(say)}$$

Key ingredients

- When $f_i^\star =$ identity, first condition on X_i, Z_i, then $Z_i' \sim \mathcal{N}(0, \sigma^2+\eta^2)$.

PROOF OF STEP 1: UPPER BOUND

We show

$$\inf_{f_i, h_i} \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \right] \leq \sup_{\mu \in \mathbb{R}} \mathbb{E}_\mu \left[\left(h_i^\star \left(X_i, f_i^\star(X_i), A_i \right) - \mu \right)^2 \right]$$

$$= \mathbb{E}_{Z \sim \mathcal{N}(0,1)} \left[\left(\frac{(m-2)n_i^\star}{\left(\sigma^2 + \alpha^2 \left(\sigma^2/n_i + \sigma^2/n_i^\star \right) Z^2 \right)} + \frac{n_i + n_i^\star}{\sigma^{-2}} \right)^{-1} \right] =: R_\infty(n_i) \quad \text{(say)}$$

Key ingredients

- When $f_i^\star =$ identity, first condition on X_i, Z_i, then $Z_i' \sim \mathcal{N}(0, \sigma^2 + \eta^2)$.
- Properties of Gaussians
- Lots of algebra

PROOF OF STEP 1: LOWER BOUND

Choose prior $\Lambda = \mathcal{N}(0, \tau^2)$ for μ. Then for any f_i, h_i, we have

PROOF OF STEP 1: LOWER BOUND

Choose prior $\Lambda = \mathcal{N}(0,\tau^2)$ for μ. Then for any f_i, h_i, we have

$$\sup_{\mu \in \mathbb{R}} \mathbb{E}_{\text{data}\sim\mu} \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \right] \geq \mathbb{E}_{\mu\sim\Lambda} \left[\mathbb{E}_{\text{data}\sim\mu} \left[\left(h_i \left(X_i, f_i(X_i), A_i \right) - \mu \right)^2 \middle| \mu \right] \right]$$

← sup ≥ avg

PROOF OF STEP 1: LOWER BOUND

Choose prior $\Lambda = \mathcal{N}(0,\tau^2)$ for μ. Then for any f_i, h_i, we have

$$\sup_{\mu\in\mathbb{R}} \mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right] \geq \mathbb{E}_{\mu\sim\Lambda}\left[\mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \mu\right]\right]$$

← sup ≥ avg

$$= \mathbb{E}_{\text{data}\sim\mu}\left[\mathbb{E}_{\mu\sim\Lambda}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \text{data}\right]\right]$$

← Swap order of expectation

PROOF OF STEP 1: LOWER BOUND

Choose prior $\Lambda = \mathcal{N}(0,\tau^2)$ for μ. Then for any f_i, h_i, we have

$$\sup_{\mu\in\mathbb{R}} \mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right] \;\geq\; \mathbb{E}_{\mu\sim\Lambda}\left[\mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \mu\right]\right]$$

← sup ≥ avg

$$= \mathbb{E}_{\text{data}\sim\mu}\left[\mathbb{E}_{\mu\sim\Lambda}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \text{data}\right]\right]$$

← Swap order of expectation

$$\geq \mathbb{E}_{\text{data}}\left[\inf_{f_i}\left(|Z_i'|\left(\sigma^2 + \alpha^2\left(\frac{1}{|f_i(X_i)|}\sum_{y\in f_i(X_i)} y - \frac{1}{|Z_i|}\sum_{z\in Z_i} z\right)^2\right)^{-1} + \frac{|X_i| + |Z_i|}{\sigma^2} + \frac{1}{\tau^2}\right)^{-1}\right]$$

(X_i, Z_i, Z_i', μ) is not jointly Gaussian, but $Z_i', \mu \mid X_i, Z_i$ is Gaussian.
Minimized by choosing $h_i =$ posterior mean.

PROOF OF STEP 1: LOWER BOUND

Choose prior $\Lambda = \mathcal{N}(0,\tau^2)$ for μ. Then for any f_i, h_i, we have

$$\sup_{\mu\in\mathbb{R}} \mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right] \geq \mathbb{E}_{\mu\sim\Lambda}\left[\mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \mu\right]\right]$$

← sup ≥ avg

$$= \mathbb{E}_{\text{data}\sim\mu}\left[\mathbb{E}_{\mu\sim\Lambda}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \text{data}\right]\right]$$

← Swap order of expectation

$$\geq \mathbb{E}_{\text{data}}\left[\inf_{f_i}\left(|Z_i'|\left(\sigma^2 + \alpha^2\left(\frac{1}{|f_i(X_i)|}\sum_{y\in f_i(X_i)} y - \frac{1}{|Z_i|}\sum_{z\in Z_i} z\right)^2\right)^{-1} + \frac{|X_i| + |Z_i|}{\sigma^2} + \frac{1}{\tau^2}\right)^{-1}\right]$$

← (X_i, Z_i, Z_i', μ) is not jointly Gaussian, but $Z_i', \mu \,|\, X_i, Z_i$ is Gaussian. Minimized by choosing $h_i =$ posterior mean.

$$= \ldots = R_\tau(n_i) \quad \text{(say)}$$

← Minimized by applying the Hardy-Littlewood inequality and choosing $f_i(X_i) = \left\{\left(1 + \sigma^2/(|X|\ell^2)\right)^{-1} x, \forall x \in X_i\right\}$.

PROOF OF STEP 1: LOWER BOUND

Choose prior $\Lambda = \mathcal{N}(0,\tau^2)$ for μ. Then for any f_i, h_i, we have

$$\sup_{\mu\in\mathbb{R}} \mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2\right] \geq \mathbb{E}_{\mu\sim\Lambda}\left[\mathbb{E}_{\text{data}\sim\mu}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \mu\right]\right]$$

← sup ≥ avg

$$= \mathbb{E}_{\text{data}\sim\mu}\left[\mathbb{E}_{\mu\sim\Lambda}\left[\left(h_i\left(X_i, f_i(X_i), A_i\right) - \mu\right)^2 \middle| \text{data}\right]\right]$$

← Swap order of expectation

$$\geq \mathbb{E}_{\text{data}}\left[\inf_{f_i}\left(|Z_i'|\left(\sigma^2 + \alpha^2\left(\frac{1}{|f_i(X_i)|}\sum_{y\in f_i(X_i)} y - \frac{1}{|Z_i|}\sum_{z\in Z_i} z\right)^2\right)^{-1} + \frac{|X_i| + |Z_i|}{\sigma^2} + \frac{1}{\tau^2}\right)^{-1}\right]$$

← (X_i, Z_i, Z_i', μ) is not jointly Gaussian, but $Z_i', \mu \,|\, X_i, Z_i$ is Gaussian. Minimized by choosing h_i = posterior mean.

$$= \ldots = R_\tau(n_i) \quad \text{(say)}$$

← Minimized by applying the Hardy-Littlewood inequality and choosing $f_i(X_i) = \left\{\left(1 + \sigma^2/(|X|\ell^2)\right)^{-1} x, \forall x \in X_i\right\}$.

$$\to R_\infty(n_i)$$

Step 2: Then, we will show the agent's penalty is minimized when she collects n_i samples under $(f_i^\star, h_i^\star)$, i.e

$$p_i\left(M,\ ((n_i^\star, f_i^\star, h_i^\star), s_{-i}^\star)\right) \leq p_i\left(M,\ ((n_i, f_i, h_i), s_{-i}^\star)\right) \quad \text{for all } n_i \in \mathbb{N}$$

Step 2: Then, we will show the agent's penalty is minimized when she collects n_i samples under $(f_i^\star, h_i^\star)$, i.e

$$p_i\left(M, \left((n_i^\star, f_i^\star, h_i^\star), s_{-i}^\star\right)\right) \leq p_i\left(M, \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) \quad \text{for all } n_i \in \mathbb{N}$$

From Step 1 we have,

$$q_i(n_i) := \inf_{f_i, h_i} p_i\left(M, \left((n_i, f_i, h_i), s_{-i}^\star\right)\right)$$

Step 2: Then, we will show the agent's penalty is minimized when she collects n_i samples under $(f_i^\star, h_i^\star)$, i.e

$$p_i\left(M,\ \left((n_i^\star, f_i^\star, h_i^\star), s_{-i}^\star\right)\right) \le p_i\left(M,\ \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) \quad \text{for all } n_i \in \mathbb{N}$$

From Step 1 we have,

$$q_i(n_i) := \inf_{f_i, h_i} p_i\left(M,\ \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) = \mathbb{E}_{Z\sim\mathcal{N}(0,1)}\left[\left(\frac{(m-2)n_i^\star}{\left(\sigma^2 + \alpha^2\left(\sigma^2/n_i + \sigma^2/n_i^\star\right)Z^2\right)} + \frac{n_i + n_i^\star}{\sigma^{-2}}\right)^{-1}\right] + cn_i$$

Step 2: Then, we will show the agent's penalty is minimized when she collects n_i samples under $(f_i^\star, h_i^\star)$, i.e

$$p_i\left(M,\ \left((n_i^\star, f_i^\star, h_i^\star), s_{-i}^\star\right)\right) \le p_i\left(M,\ \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) \quad \text{for all } n_i \in \mathbb{N}$$

From Step 1 we have,

$$q_i(n_i) := \inf_{f_i, h_i} p_i\left(M,\ \left((n_i, f_i, h_i), s_{-i}^\star\right)\right) = \mathbb{E}_{Z\sim\mathcal{N}(0,1)}\left[\left(\frac{(m-2)n_i^\star}{\left(\sigma^2 + \alpha^2\left(\sigma^2/n_i + \sigma^2/n_i^\star\right)Z^2\right)} + \frac{n_i + n_i^\star}{\sigma^{-2}}\right)^{-1}\right] + cn_i$$

- The term inside $\mathbb{E}$ is convex. Hence so is $q(n_i)$.
- $q(n_i)$ is minimized at $n_i = n_i^\star$ (by our choice of α).

THIS PROOF REQUIRED LOTS OF ALGEBRA

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' subm
  - Set noise variance $\eta_i^2 = \alpha^2\left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$

THIS PROOF REQUIRED LOTS OF ALGEBRA

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' subm
  - Set noise variance $\eta_i^2 = \alpha^2\left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$

We set α to be the smallest number larger than $\sqrt{n_i^\star}$ such that $G(\alpha) = 0$, where,

THIS PROOF REQUIRED LOTS OF ALGEBRA

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' subm
  - Set noise variance $\eta_i^2 = \alpha^2 \left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$

We set α to be the smallest number larger than $\sqrt{n_i^\star}$ such that $G(\alpha) = 0$, where,

$$G(\alpha) := \left(\frac{m-4}{m-2}\frac{4\alpha^2}{\sigma/\sqrt{cm}} - 1\right)\frac{4\alpha}{\sqrt{\sigma}(m/c)^{1/4}} - \left(4(m+1)\frac{\alpha^2}{\sigma\sqrt{m/c}} - 1\right)\sqrt{2\pi}\exp\left(\frac{\sigma\sqrt{m/c}}{8\alpha^2}\right)\mathrm{Erfc}\left(\frac{\sqrt{\sigma}(m/c)^{1/4}}{2\sqrt{2}\alpha}\right)$$

THIS PROOF REQUIRED LOTS OF ALGEBRA

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' subm
  - Set noise variance $\eta_i^2 = \alpha^2 \left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$

We set α to be the smallest number larger than $\sqrt{n_i^\star}$ such that $G(\alpha) = 0$, where,

$$G(\alpha) := \left(\frac{m-4}{m-2}\frac{4\alpha^2}{\sigma/\sqrt{cm}} - 1\right)\frac{4\alpha}{\sqrt{\sigma}(m/c)^{1/4}} - \left(4(m+1)\frac{\alpha^2}{\sigma\sqrt{m/c}} - 1\right)\sqrt{2\pi}\exp\left(\frac{\sigma\sqrt{m/c}}{8\alpha^2}\right)\mathrm{Erfc}\left(\frac{\sqrt{\sigma}(m/c)^{1/4}}{2\sqrt{2}\alpha}\right)$$

- $\alpha^2 \geq n_i^\star$: step 1 of NIC (sufficiently penalise untruthful agents).
- "smallest number larger than": for efficiency (don't over-penalize truthful agents).
- $G(\alpha) = 0$: step 2 of NIC (collect a sufficient amount of data).

THIS PROOF REQUIRED LOTS OF ALGEBRA :)

- For each agent i:
  - $Z_i \leftarrow$ sample $n^\star = \sigma/\sqrt{cm}$ points from others' subm
  - Set noise variance $\eta_i^2 = \alpha^2\left(\text{mean}(Y_i) - \text{mean}(Z_i)\right)^2$

We set α to be the smallest number larger than $\sqrt{n_i^\star}$ such that $G(\alpha) = 0$, where,

$$G(\alpha) := \left(\frac{m-4}{m-2}\frac{4\alpha^2}{\sigma/\sqrt{cm}} - 1\right)\frac{4\alpha}{\sqrt{\sigma}(m/c)^{1/4}} - \left(4(m+1)\frac{\alpha^2}{\sigma\sqrt{m/c}} - 1\right)\sqrt{2\pi}\exp\left(\frac{\sigma\sqrt{m/c}}{8\alpha^2}\right)\text{Erfc}\left(\frac{\sqrt{\sigma}(m/c)^{1/4}}{2\sqrt{2}\alpha}\right)$$

- $\alpha^2 \geq n_i^\star$: step 1 of NIC (sufficiently penalise untruthful agents).
- "smallest number larger than": for efficiency (don't over-penalize truthful agents).
- $G(\alpha) = 0$: step 2 of NIC (collect a sufficient amount of data).

+

Wolfram Mathematica

OUTLINE

1. Mechanism design for collaborative normal mean estimation

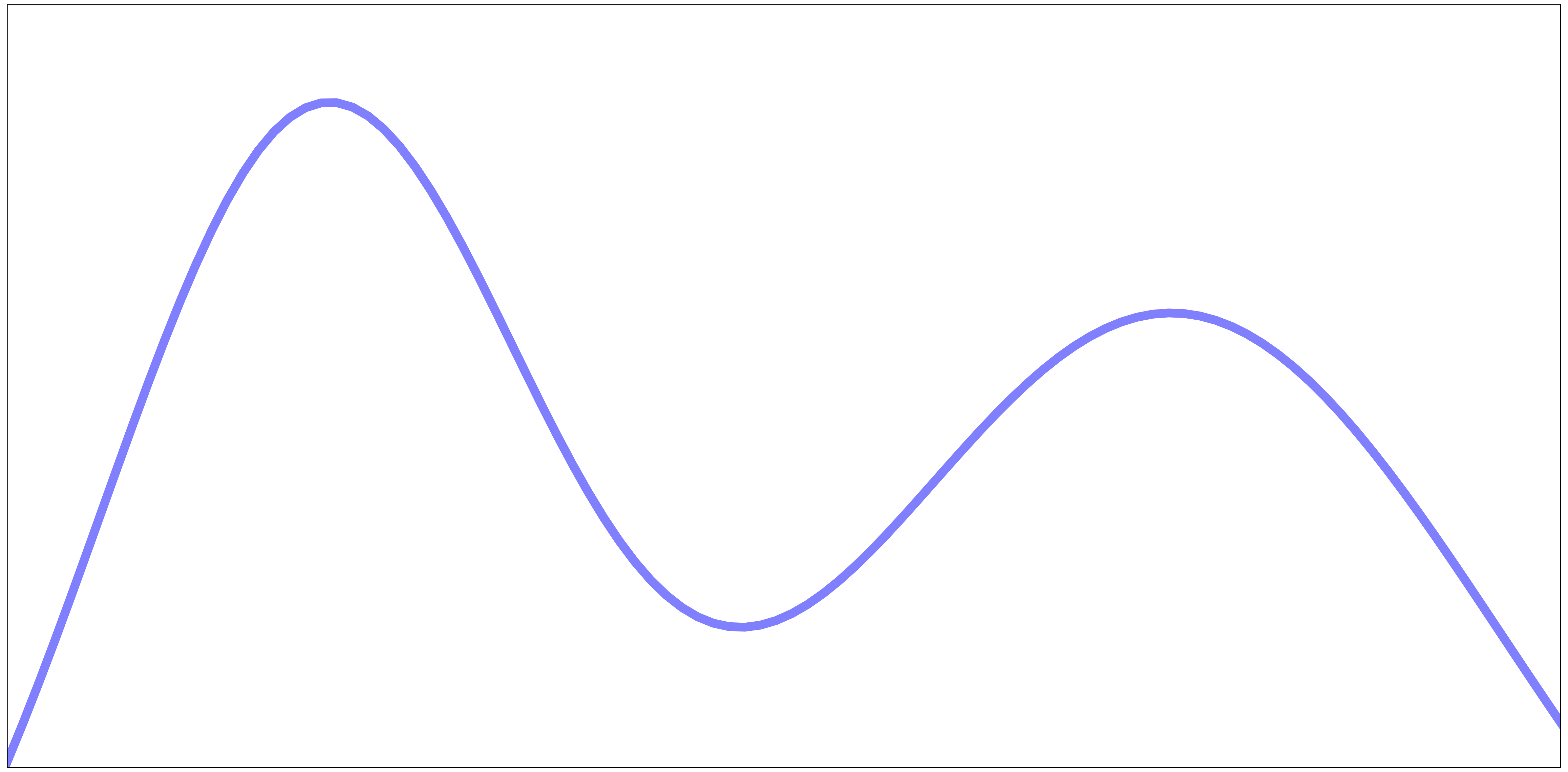
(Chen, Zhu, Kandasamy, *Neurips 2023*)

- Intuitions and Challenges
- Problem formalism
- Mechanism and theoretical analysis

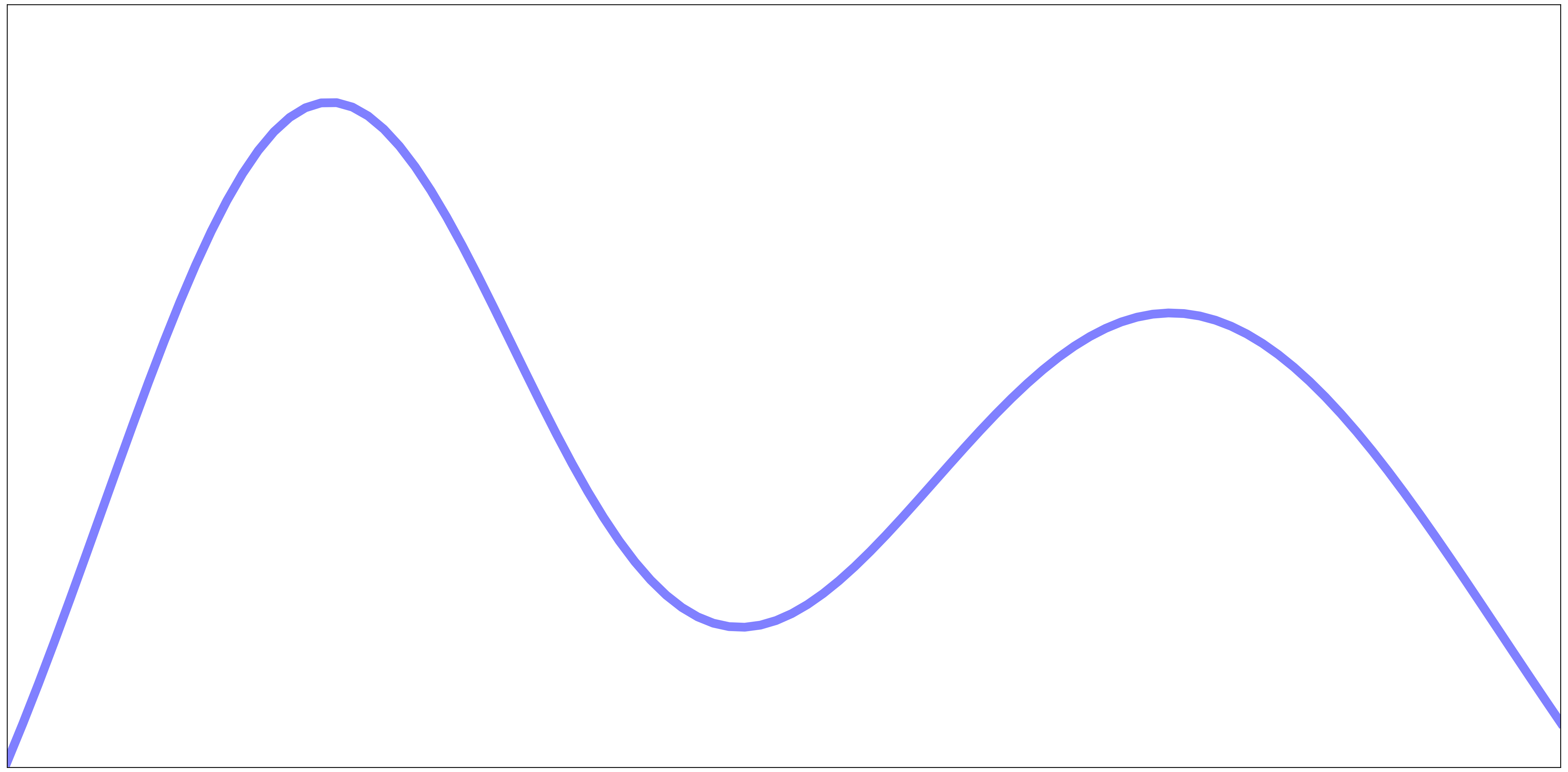
2. **Extensions & Future work**

- **Collaborative supervised learning, design of experiments**
- **Data marketplaces**

COLLABORATIVE SUPERVISED LEARNING AND EXPERIMENT DESIGN

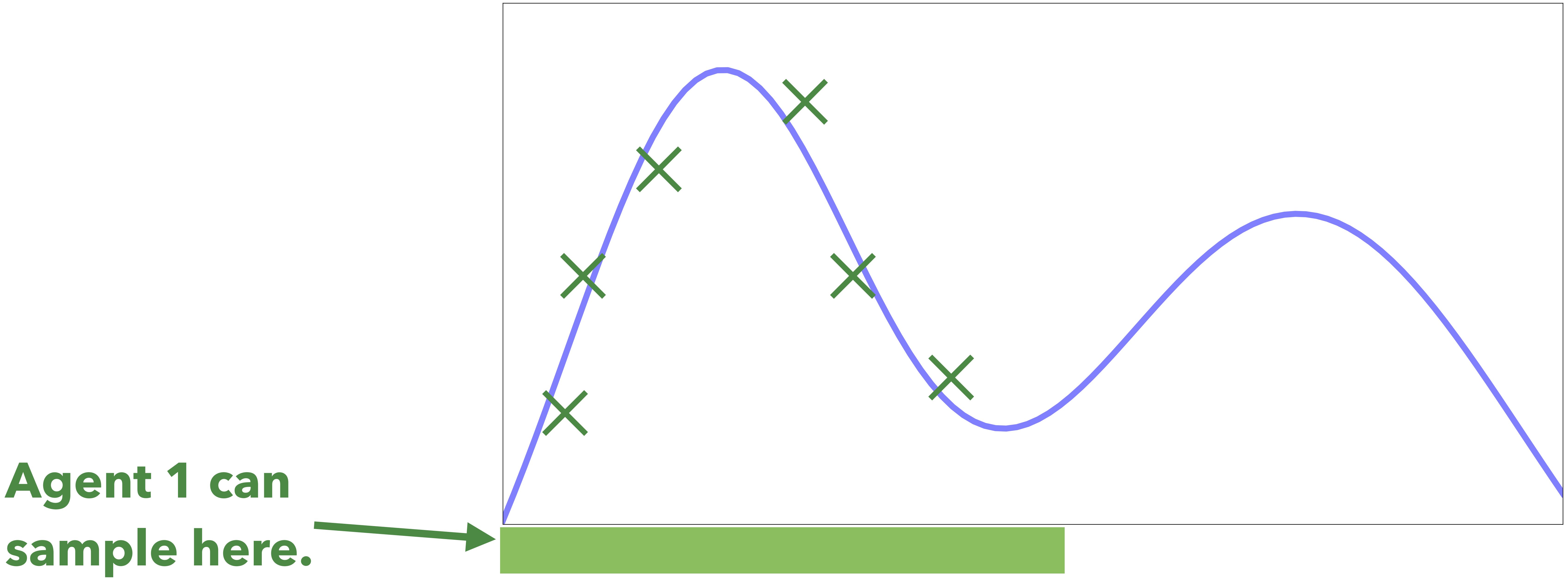


COLLABORATIVE SUPERVISED LEARNING AND EXPERIMENT DESIGN

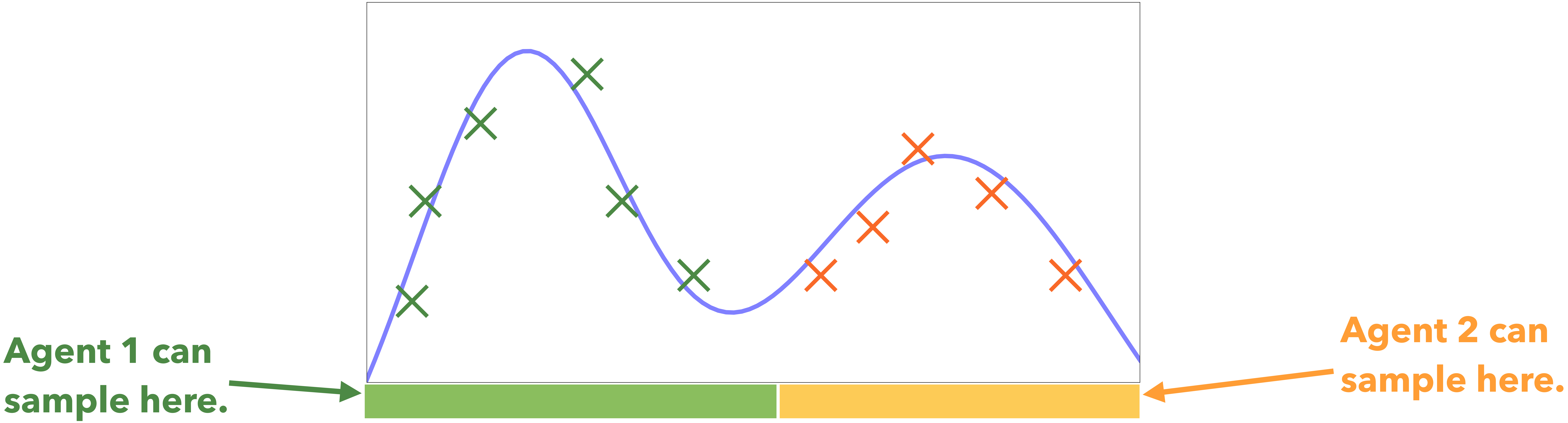


Data sharing when there is asymmetric data collection capabilities?

COLLABORATIVE SUPERVISED LEARNING AND EXPERIMENT DESIGN

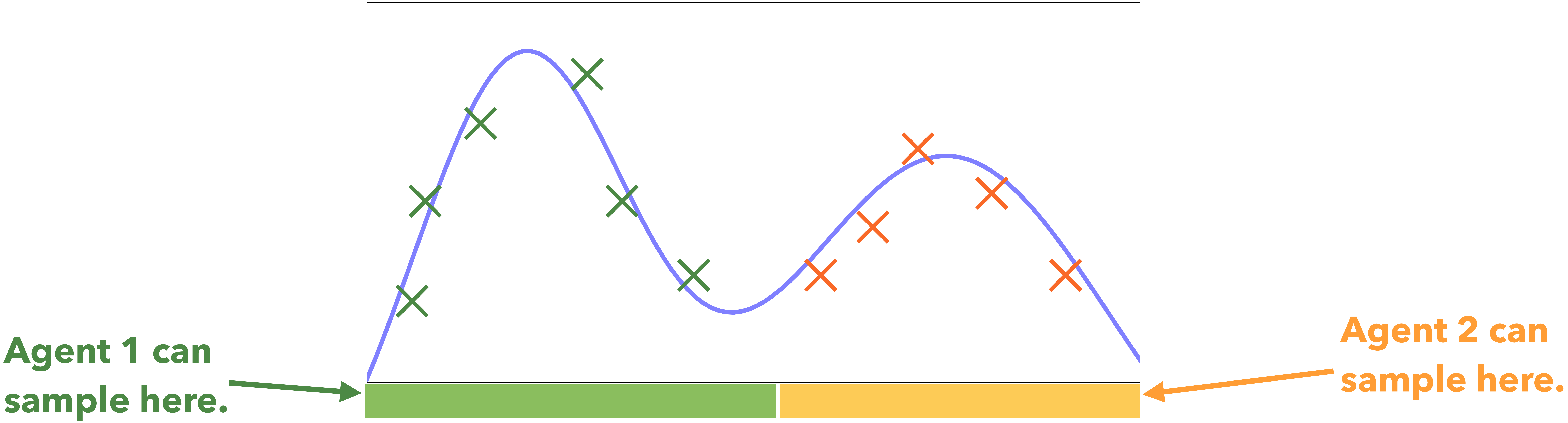


Data sharing when there is asymmetric data collection capabilities?



Data sharing when there is asymmetric data collection capabilities?

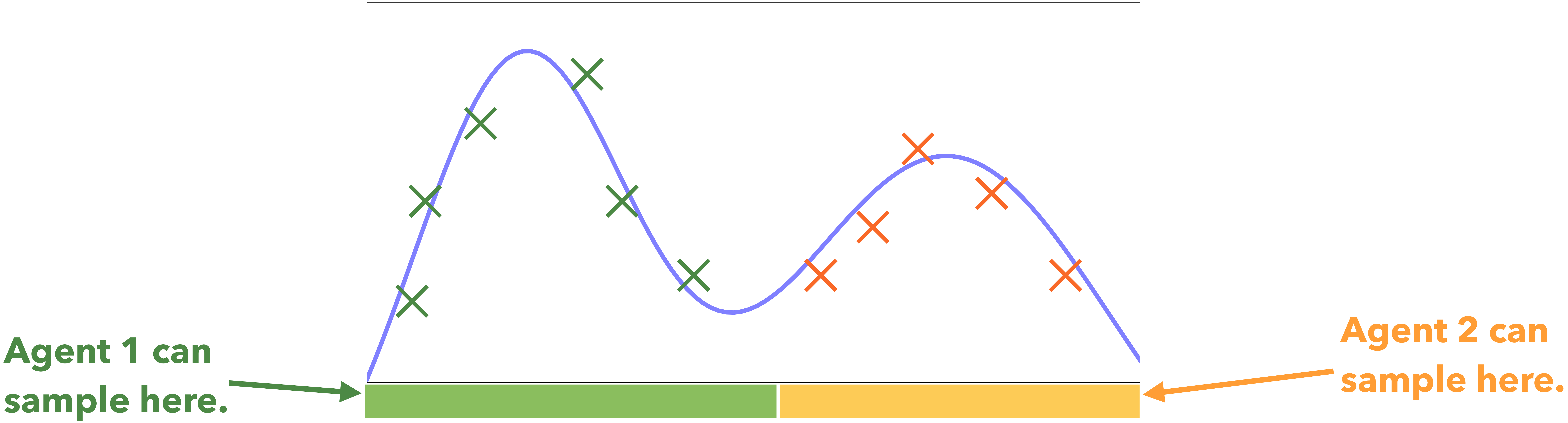
COLLABORATIVE SUPERVISED LEARNING AND EXPERIMENT DESIGN



Data sharing when there is asymmetric data collection capabilities?

+ Agents will be more willing to collaborate due to complementarity of data.

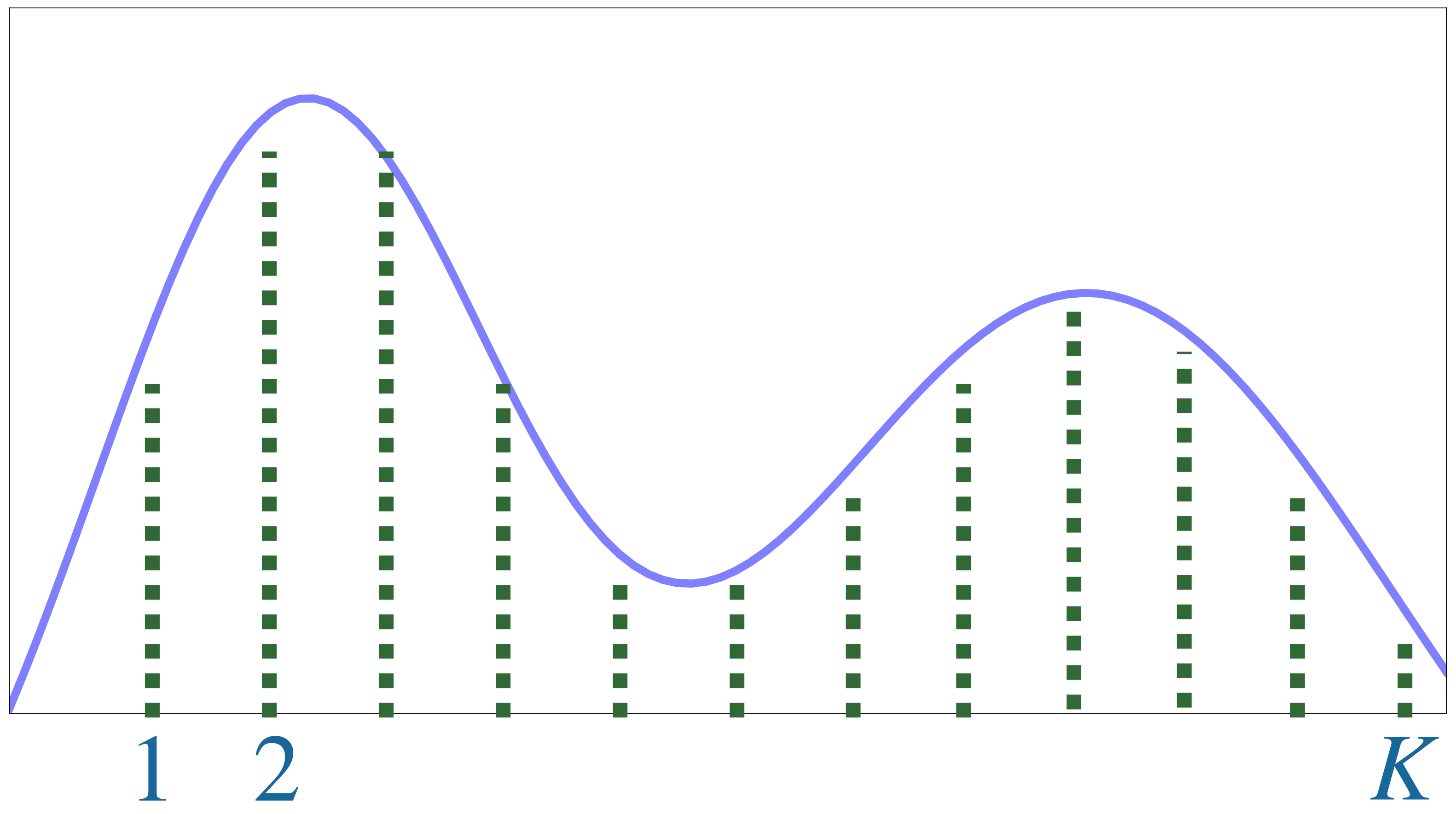
COLLABORATIVE SUPERVISED LEARNING AND EXPERIMENT DESIGN



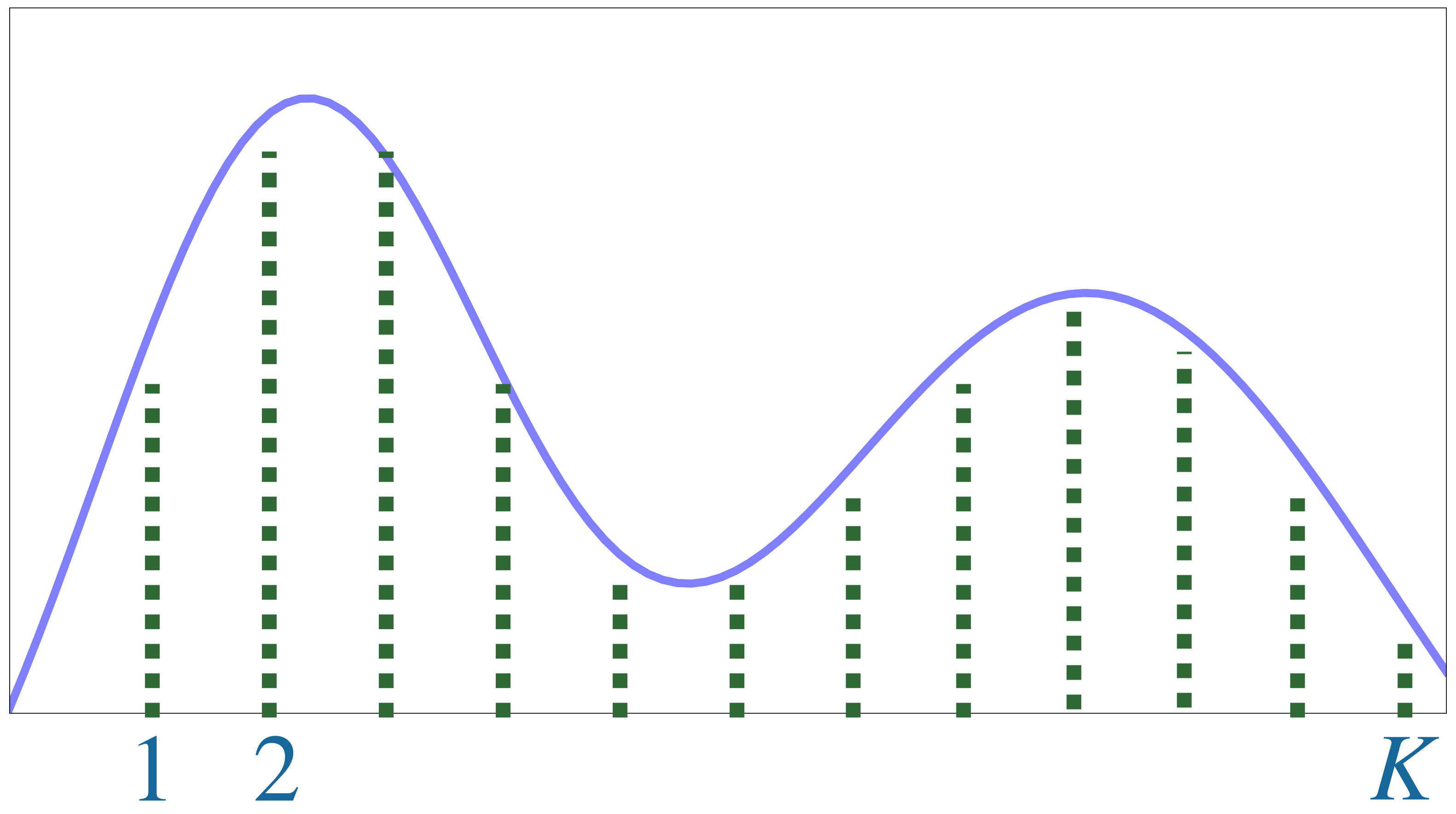
Data sharing when there is asymmetric data collection capabilities?

+ Agents will be more willing to collaborate due to complementarity of data.
– No way to validate an agent's data with other similar data.

Consider a K discretisation of the domain



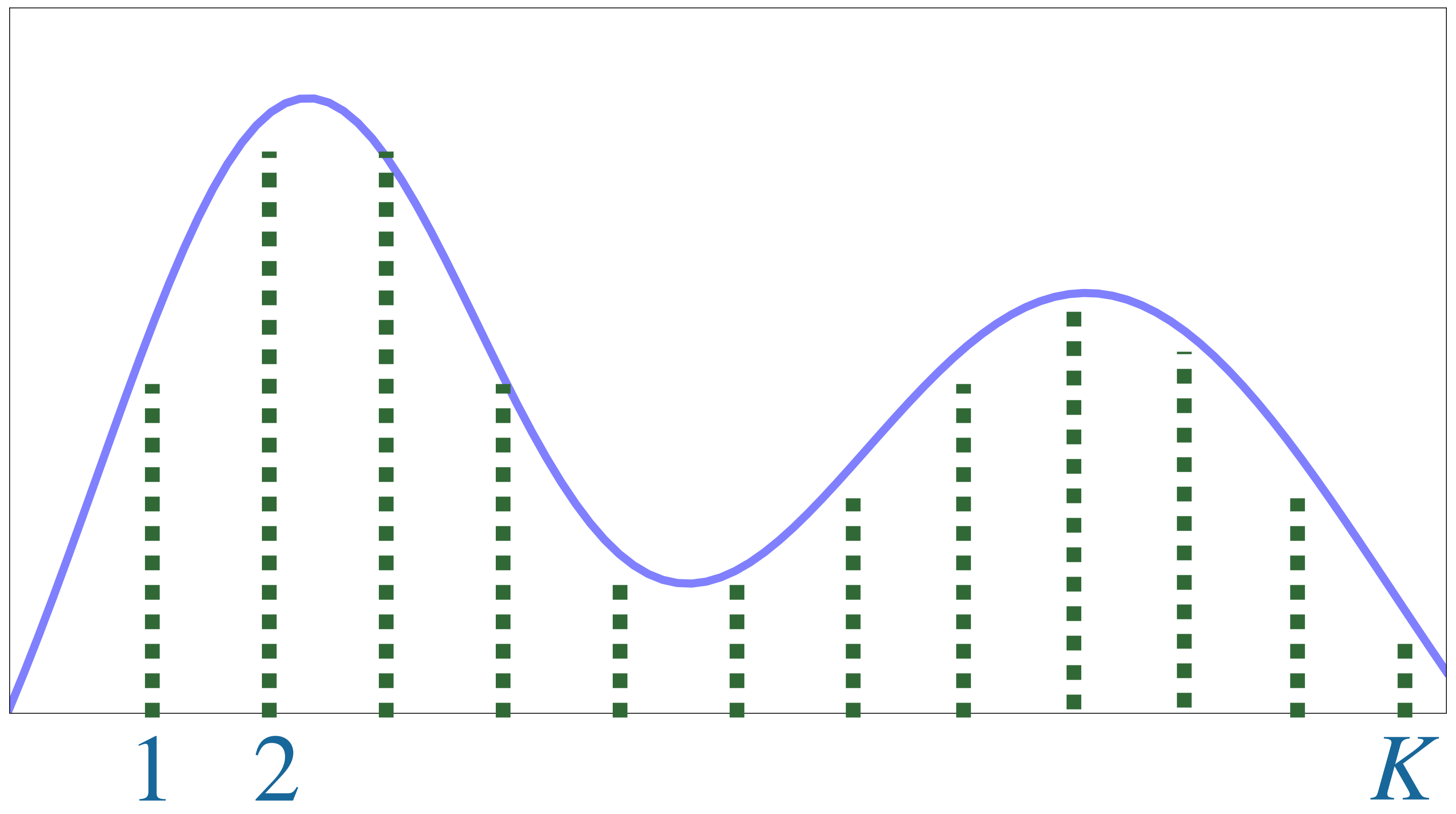
Consider a K discretisation of the domain



Agent i can sample from distribution k at cost $c_{i,k}$.

Penalty, $p_i = \sum_{k=1}^{K} \text{est-err}_k + \sum_{k=1}^{K} c_{i,k} n_{i,k}$

Consider a K discretisation of the domain



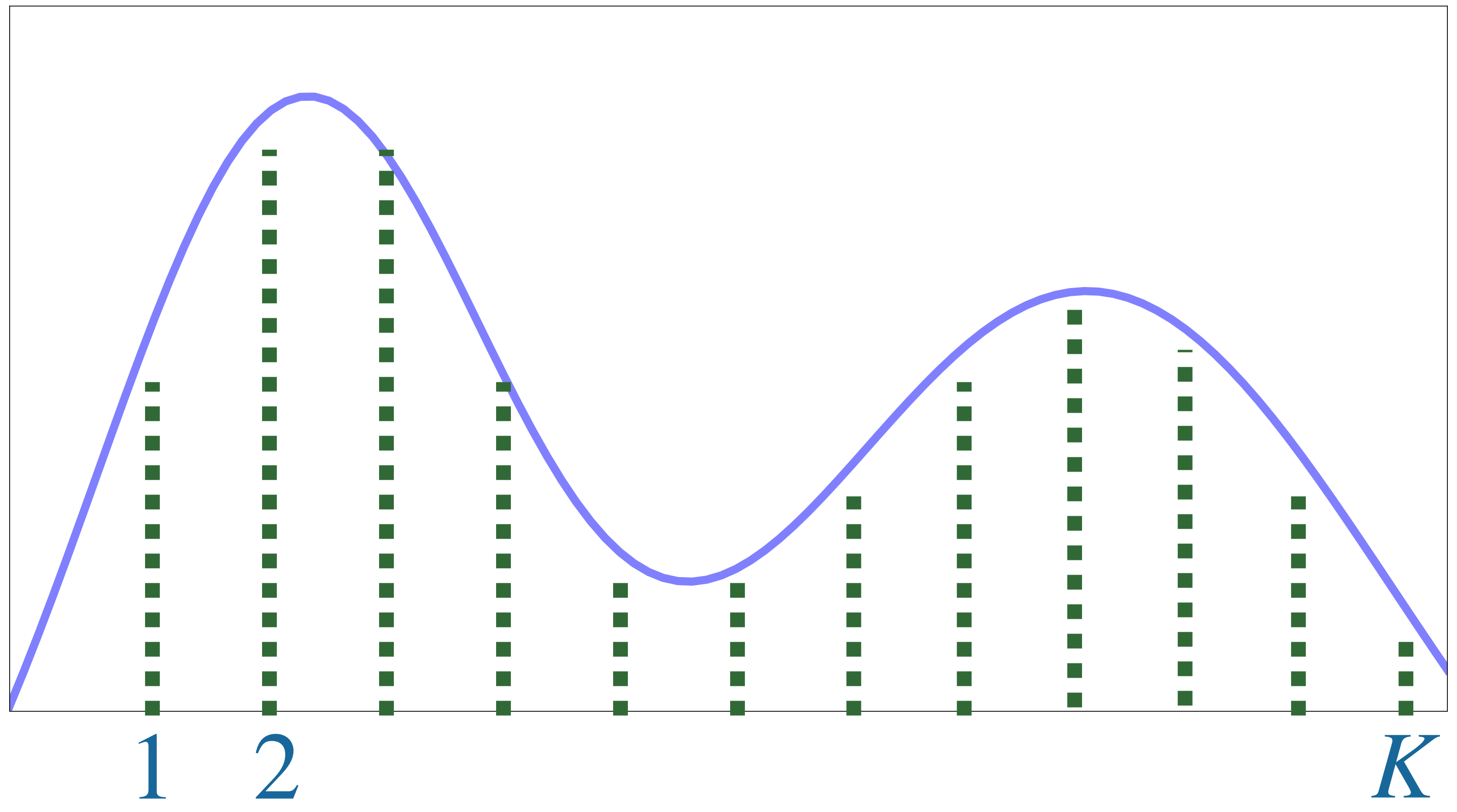
Agent i can sample from distribution k at cost $c_{i,k}$.

Penalty, $p_i = \sum_{k=1}^{K} \text{est-err}_k + \sum_{k=1}^{K} c_{i,k} n_{i,k}$

Theorem: There exists a NIC and IR mechanism for which,

$$P(M, s^\star) \leq 8\sqrt{m} \cdot \inf_{M,s} P(M, s)$$

Consider a K discretisation of the domain



Agent i can sample from distribution k at cost $c_{i,k}$.

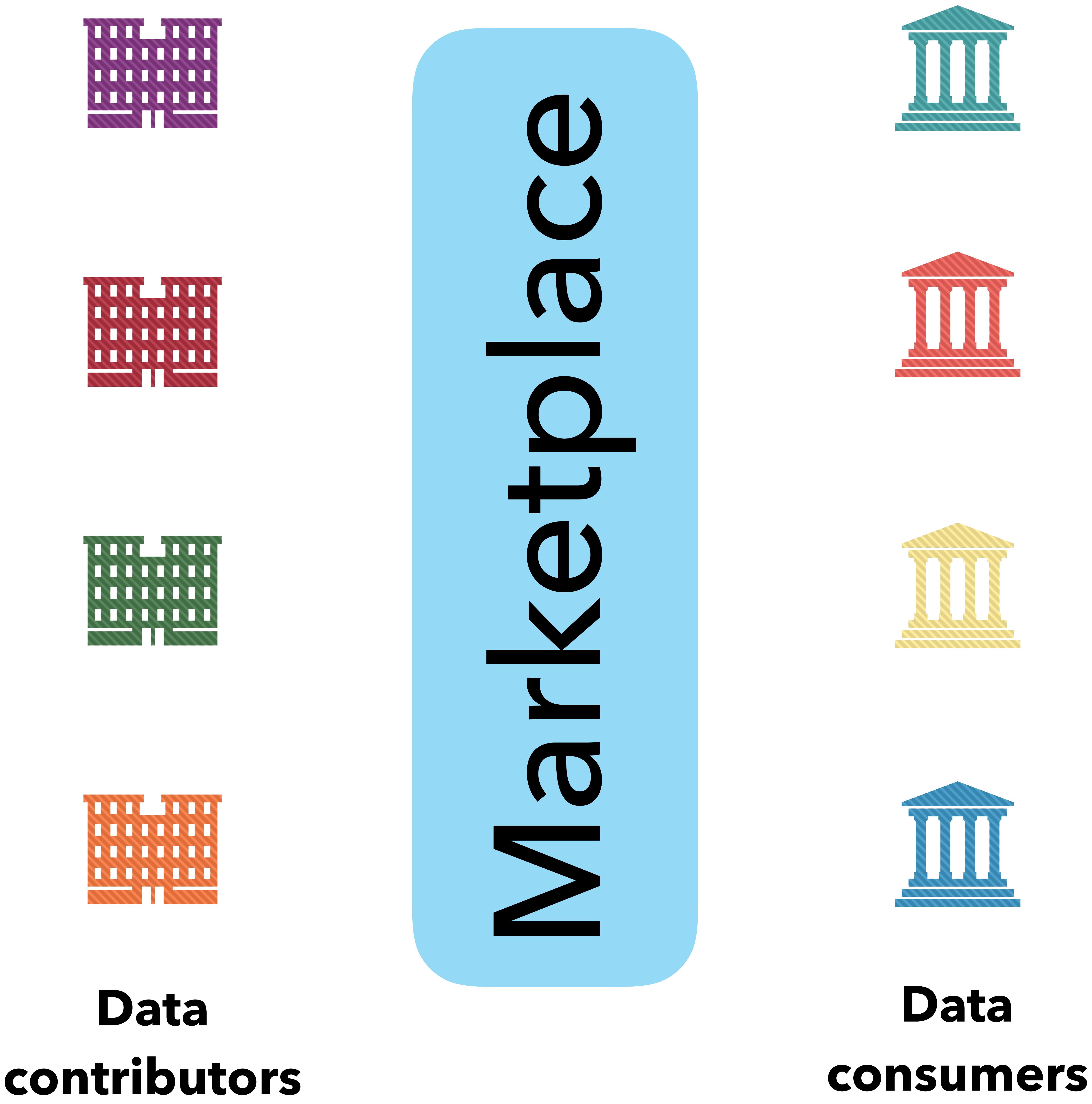
Penalty, $p_i = \sum_{k=1}^{K} \text{est-err}_k + \sum_{k=1}^{K} c_{i,k} n_{i,k}$

Theorem: There exists a NIC and IR mechanism for which,

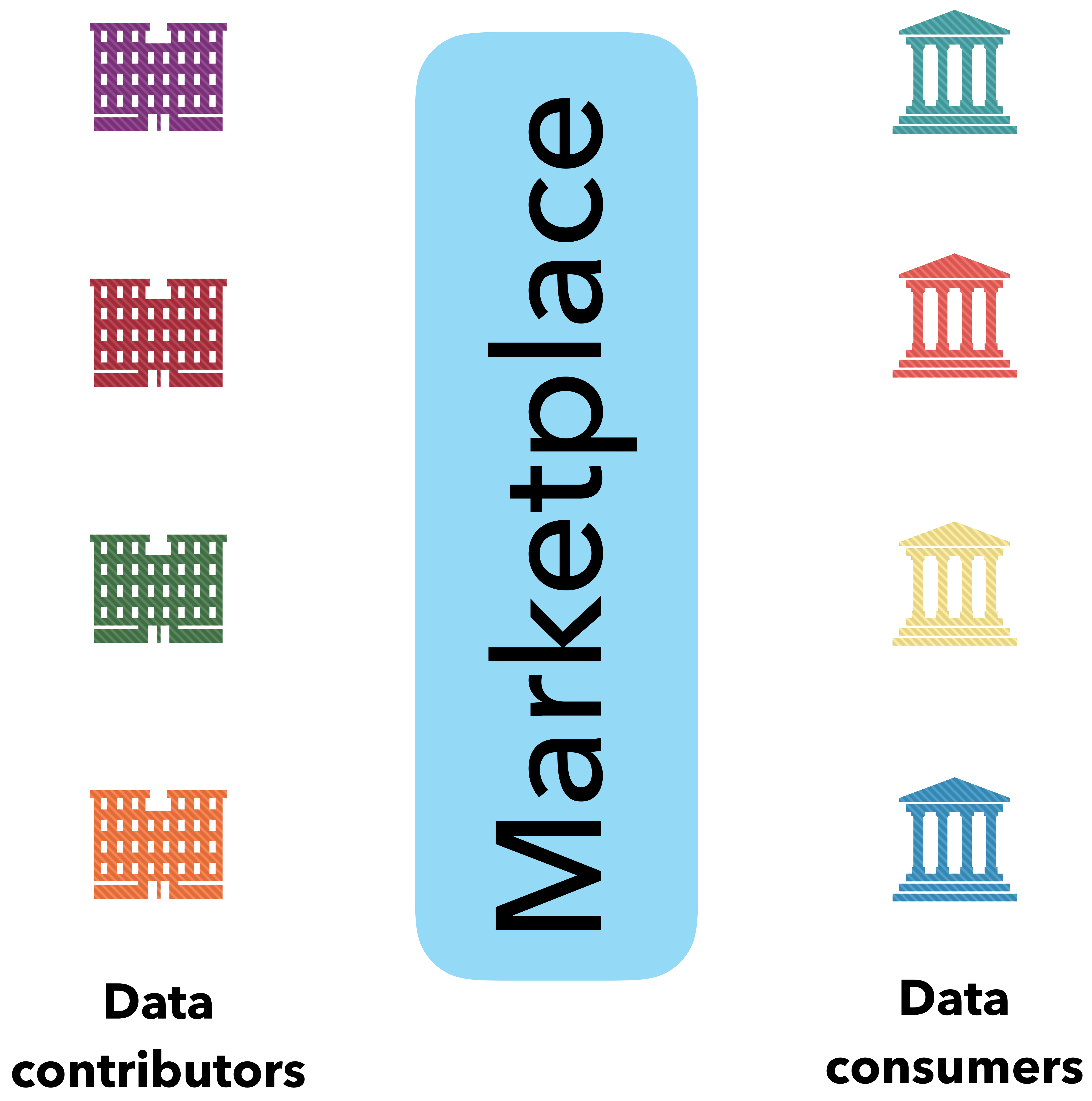
$$P(M, s^\star) \leq 8\sqrt{m} \cdot \inf_{M,s} P(M, s)$$

Theorem (hardness): There exists a set of costs $\{c_{i,k}\}_{i,k}$ such that for any NIC and IR mechanism, we have $P(M, s^\star) \in \Omega\left(\sqrt{m}\right) \cdot \inf_{M,s} P(M, s)$

DATA MARKETPLACES

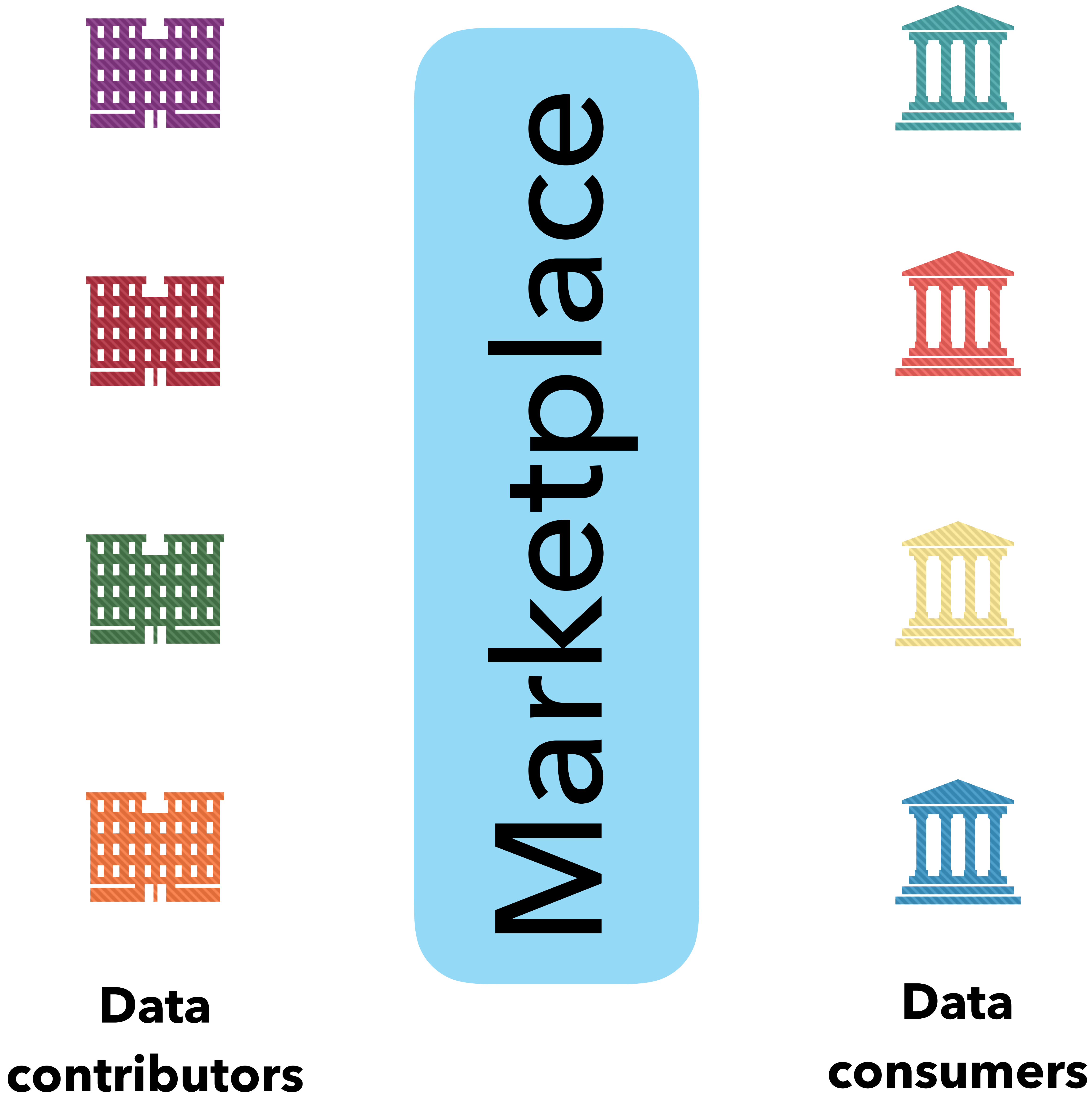


DATA MARKETPLACES



Consumers purchase data from contributors via a marketplace:

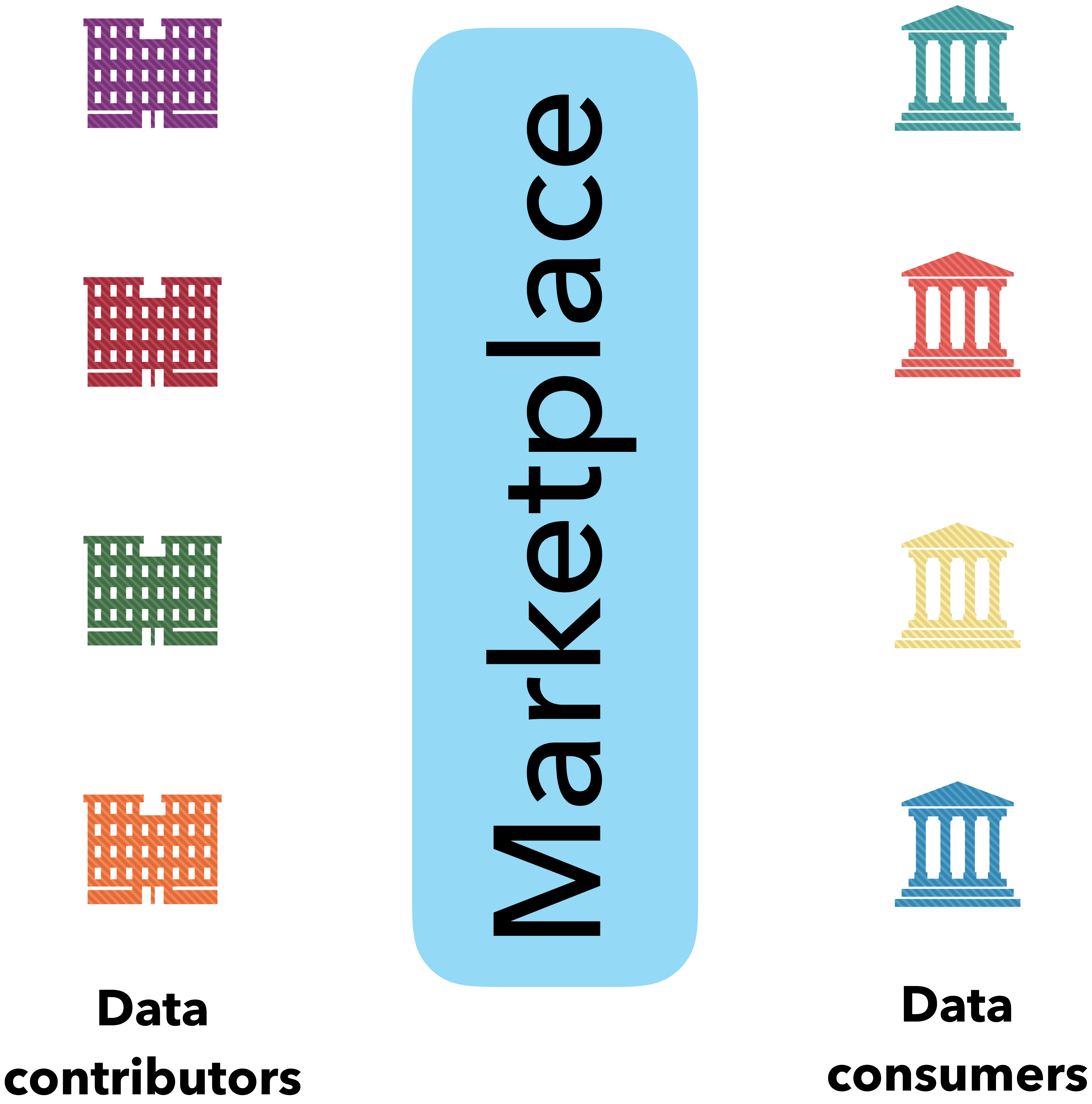
DATA MARKETPLACES



Consumers purchase data from contributors via a marketplace:

- Ensure contributors do not fabricate/ poison data.

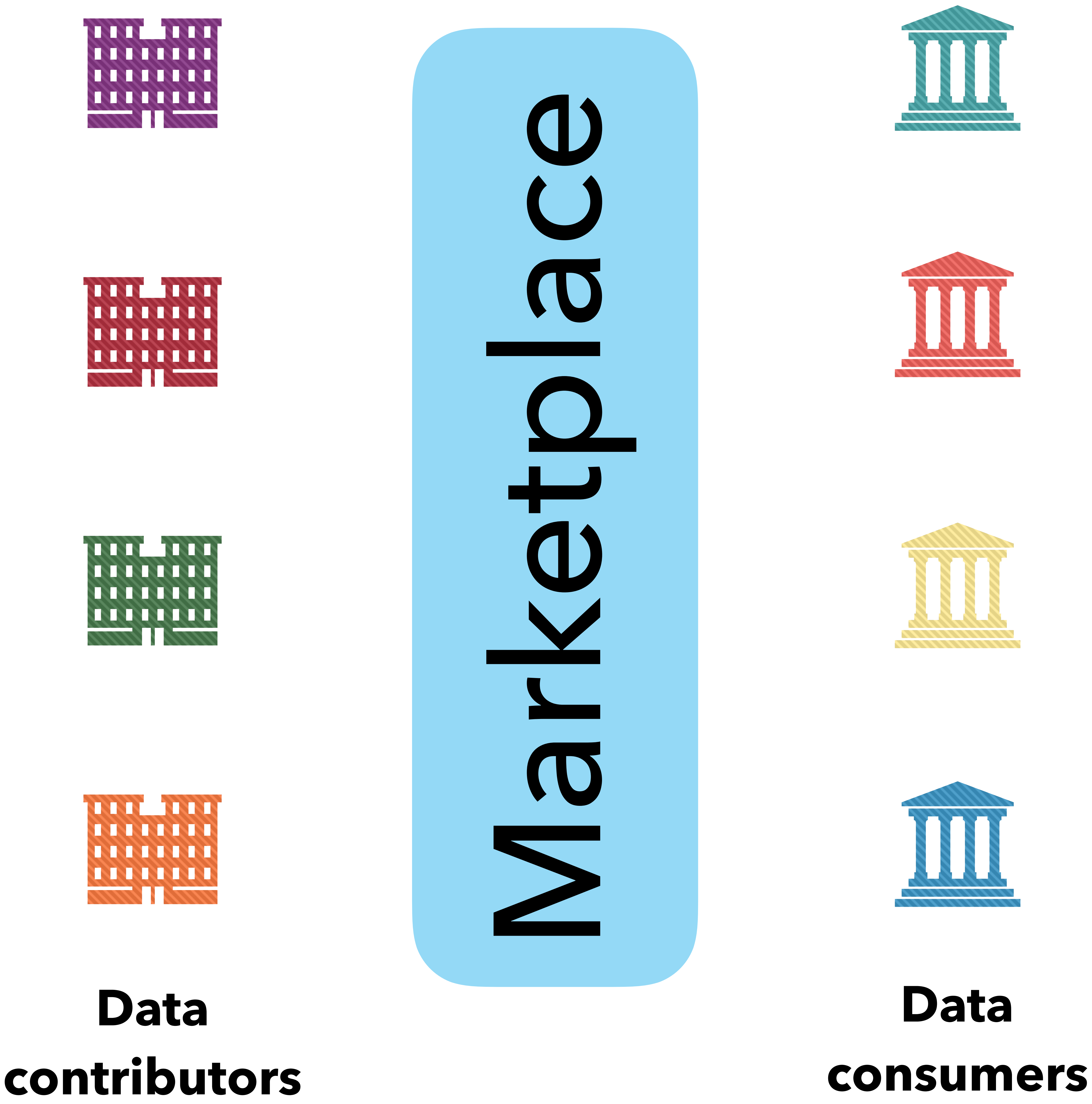
DATA MARKETPLACES



Consumers purchase data from contributors via a marketplace:

- Ensure contributors do not fabricate/ poison data.
- Pricing data that is being sold to consumers.

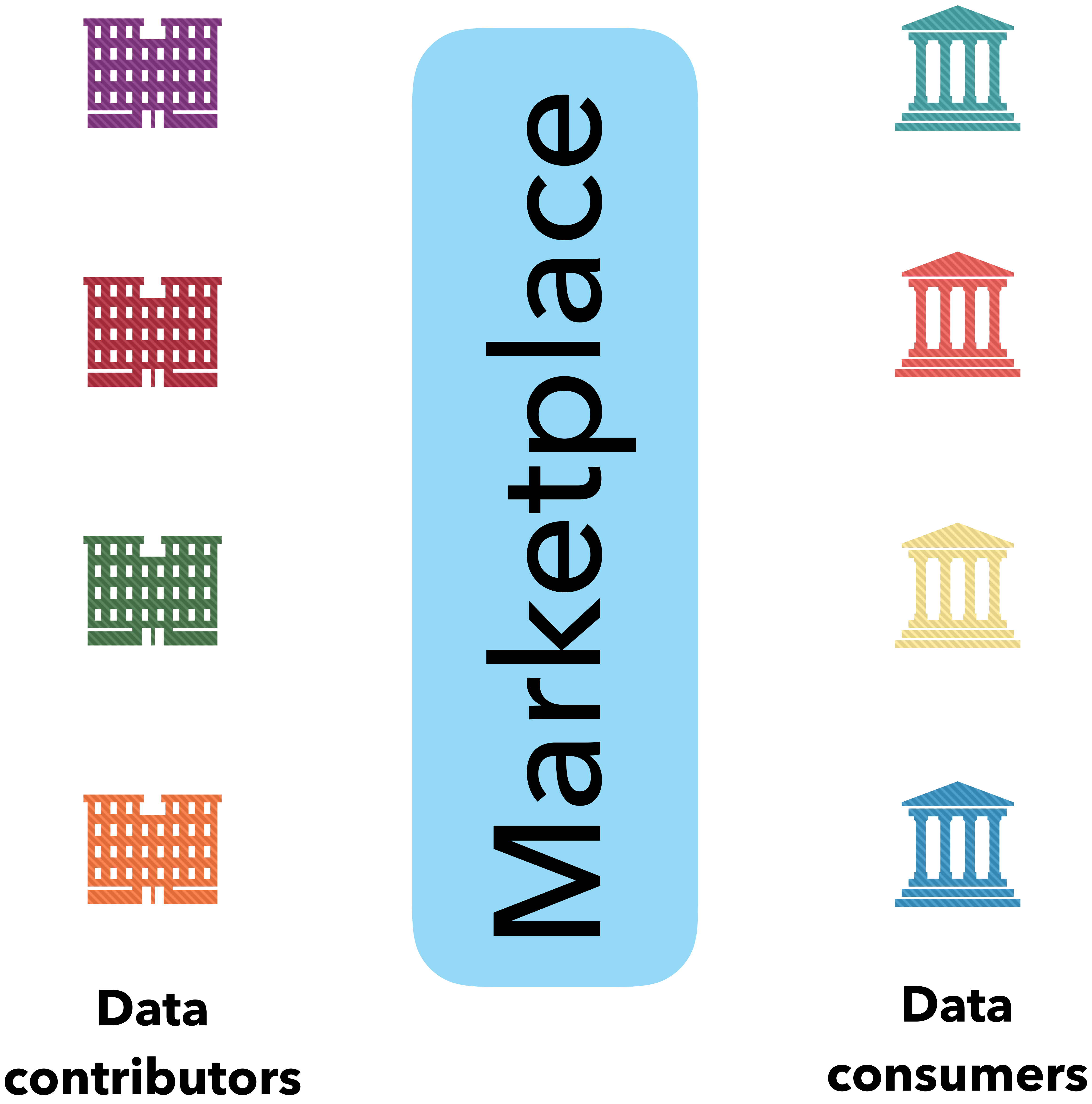
DATA MARKETPLACES



Consumers purchase data from contributors via a marketplace:

- Ensure contributors do not fabricate/ poison data.
- Pricing data that is being sold to consumers.
- Re-distributing the revenue back to the contributors.

DATA MARKETPLACES



Consumers purchase data from contributors via a marketplace:

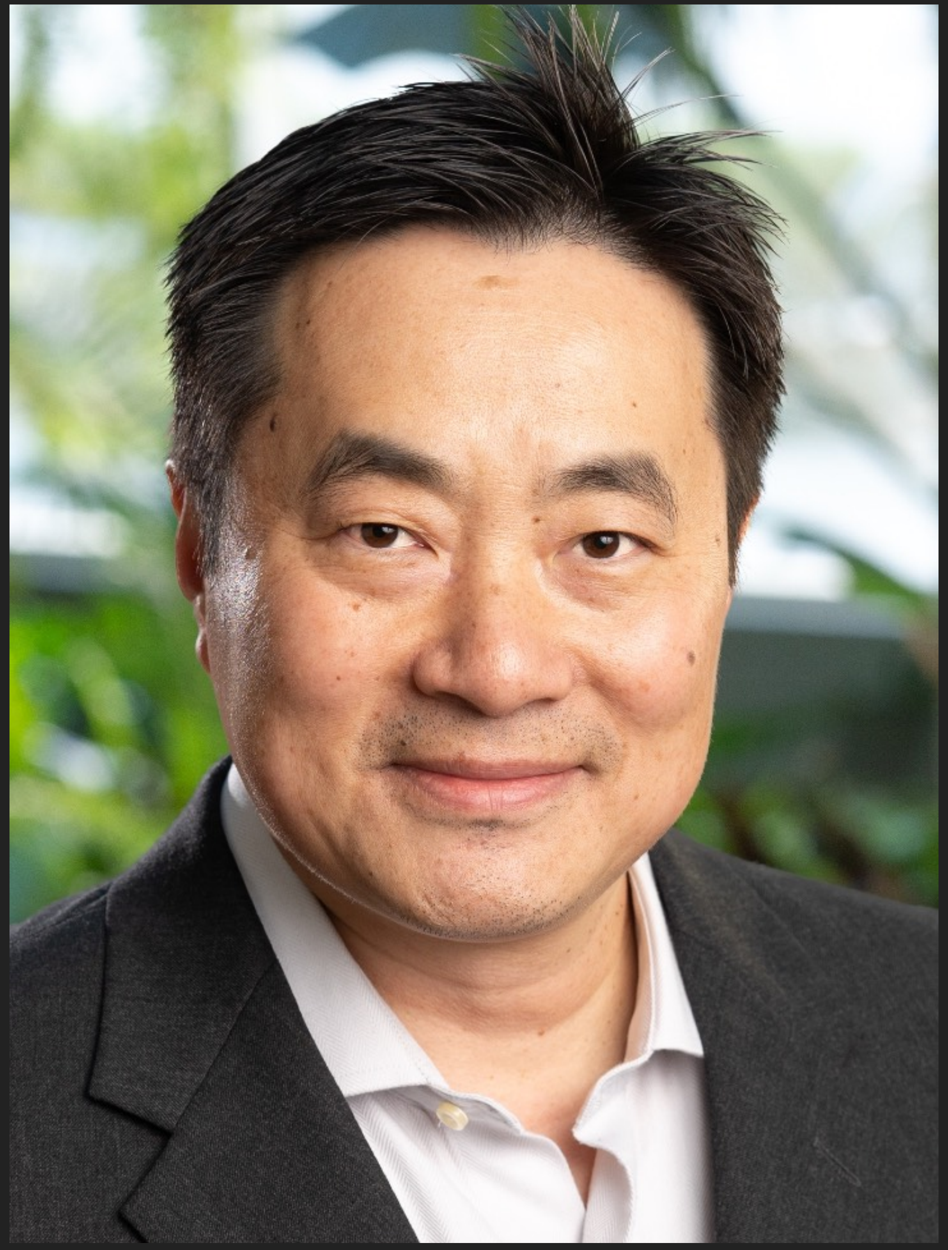
- Ensure contributors do not fabricate/ poison data.
- Pricing data that is being sold to consumers.
- Re-distributing the revenue back to the contributors.
- Learn consumer valuation of data via online feedback.

Keran Chen

Yiding Chen

Alex Clinton

Joon Suk Huh



Jerry Zhu

THANK YOU!

kandasamy@cs.wisc.edu

SUMMARY

- Data sharing has many benefits
  - Maximize the value created by data.
  - Democratize data

- But strategic agents can free-ride in naive mechanisms, either by not contributing data, or contributing fabricated datasets.

- Our mechanism is IR and NIC while achieving a factor 2 of the global minimum social penalty.