# CS 760: Machine Learning
# **Reinforcement Learning**


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# Announcements

- **Logistics**:
  - No class on Wednesday, November 26th
  - HW4 due Monday
  - HW5 out Monday

# Outline

- **Intro to Reinforcement Learning**
  - Basic concepts, mathematical formulation, MDPs, policies
- **Valuing and Obtaining Policies**
  - Value functions, Bellman equation, value iteration, policy iteration

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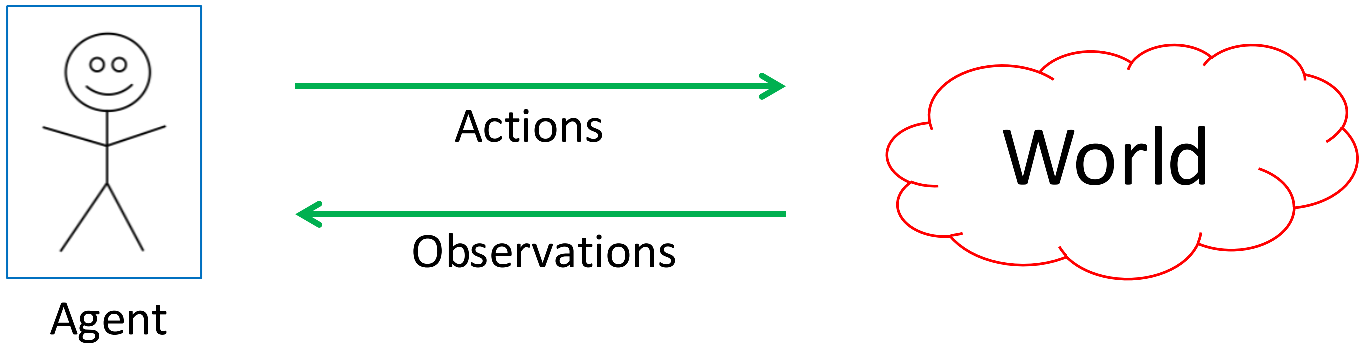
# A General Model

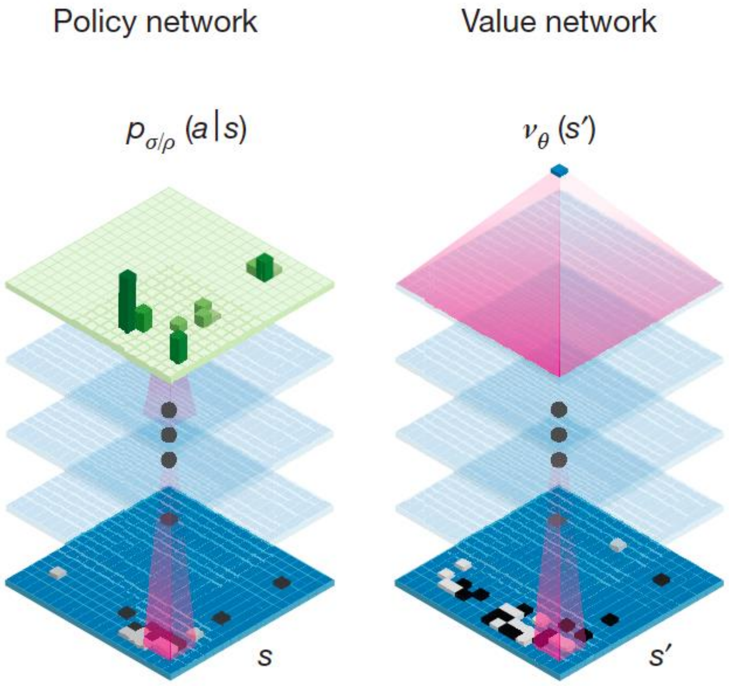
We have an **agent** **interacting** with the **world**




- Agent receives a reward based on state of the world
  - **Goal**: maximize reward / utility **($$$)**
  - Note: **data** consists of actions & observations
    - Compare to unsupervised learning and supervised learning

# Examples: Gameplay Agents

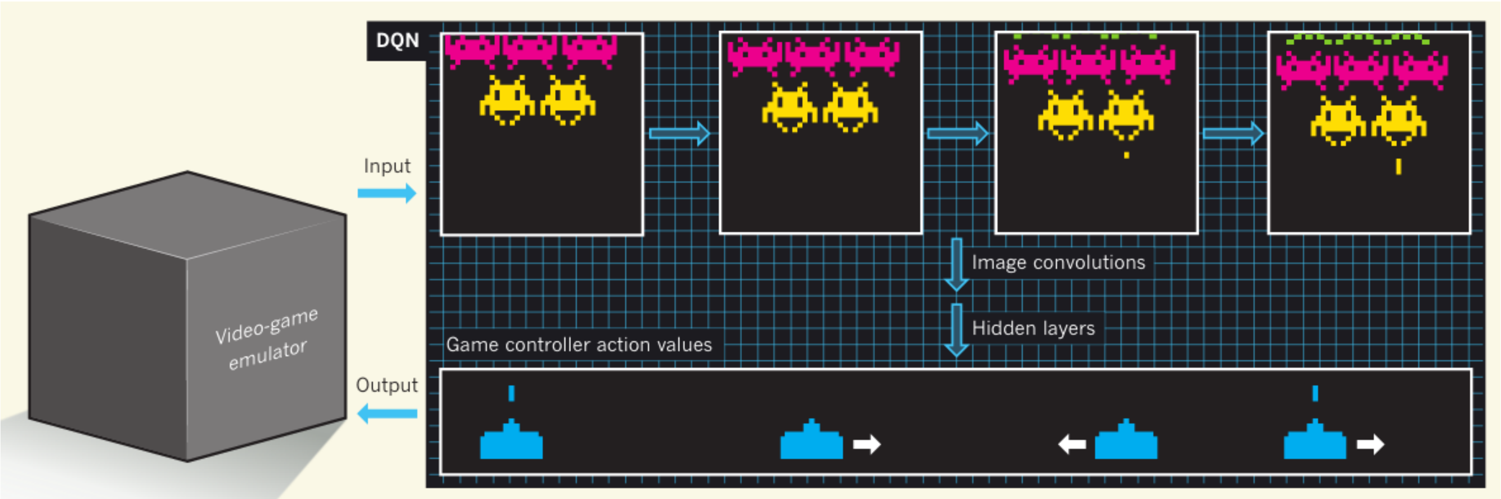
AlphaZero:

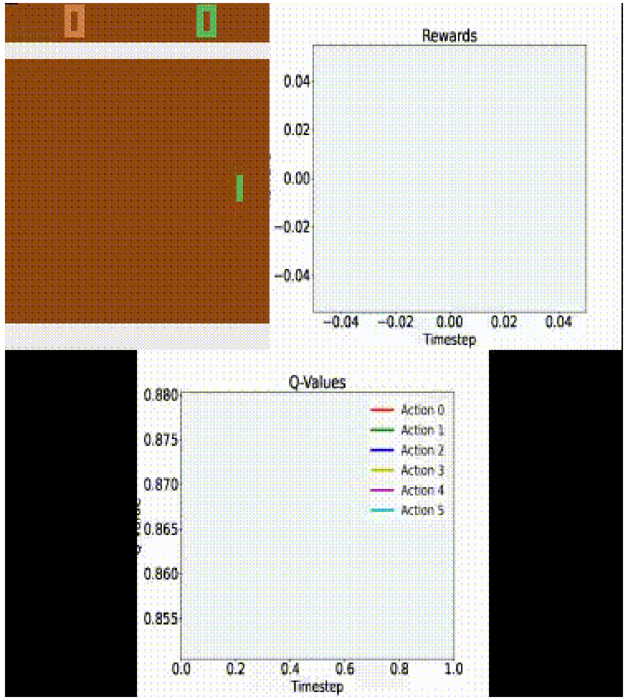

https://deepmind.com/research/alphago/

# Examples: Video Game Agents

## Pong, Atari



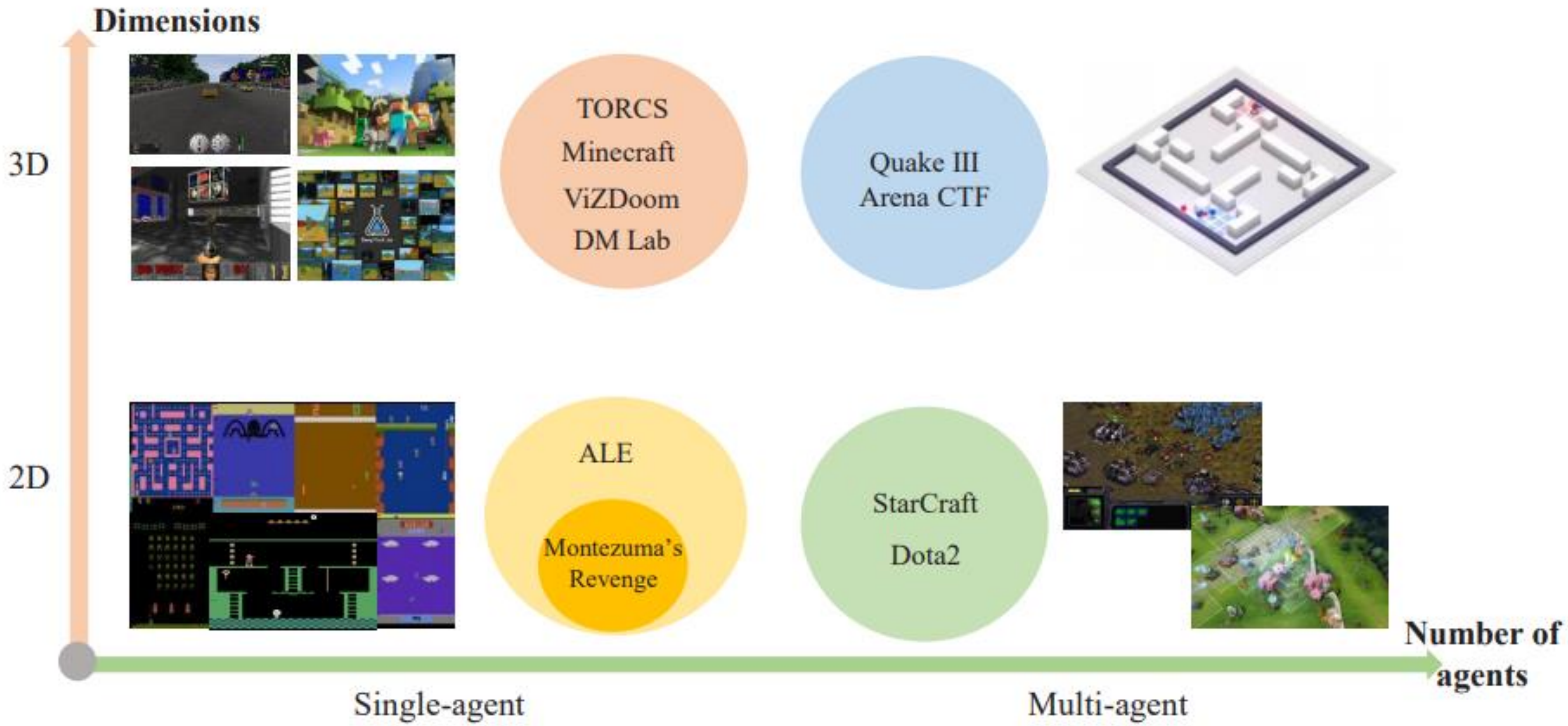

Mnih et al, “Human-level control through deep reinforcement learning”




A. Nielsen

# Examples: Video Game Agents

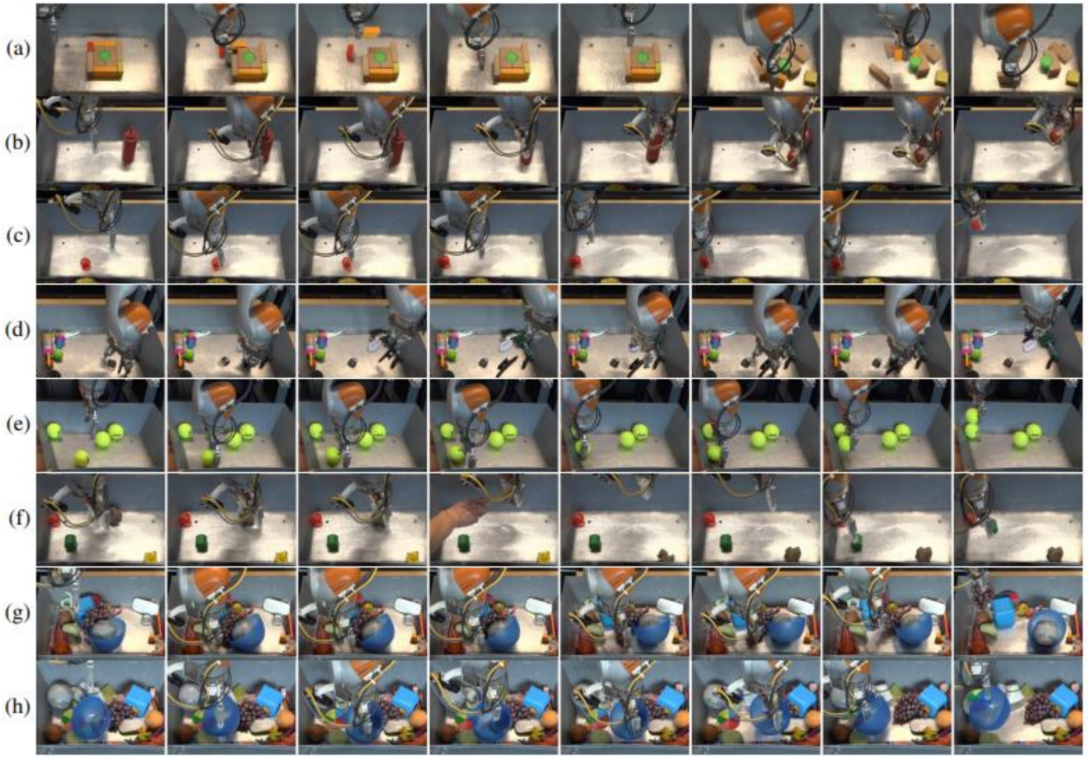
Minecraft, Quake, StarCraft, and more!




Shao et al, "A Survey of Deep Reinforcement Learning in Video Games"

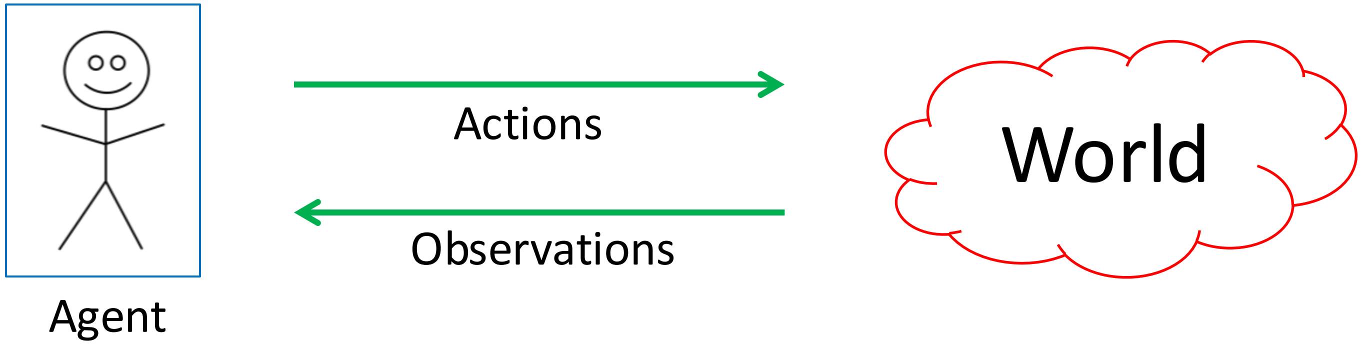
# Examples: Robotics

## Training robots to perform tasks (e.g. grasp!)


Ibarz et al, " How to Train Your Robot with Deep Reinforcement Learning – Lessons We’ve Learned "

# Building The Theoretical Model




Basic setup:

- Set of states S
- Set of actions A
- Information: at time *t*, observe state $s_t \in$ S. Get reward $r_t$
- Agent makes choice $a_t \in$ A. State changes to $s_{t+1,}$ continue

Goal: find a map from **states to actions** maximize rewards.

a "policy"

# Markov Decision Process (MDP)

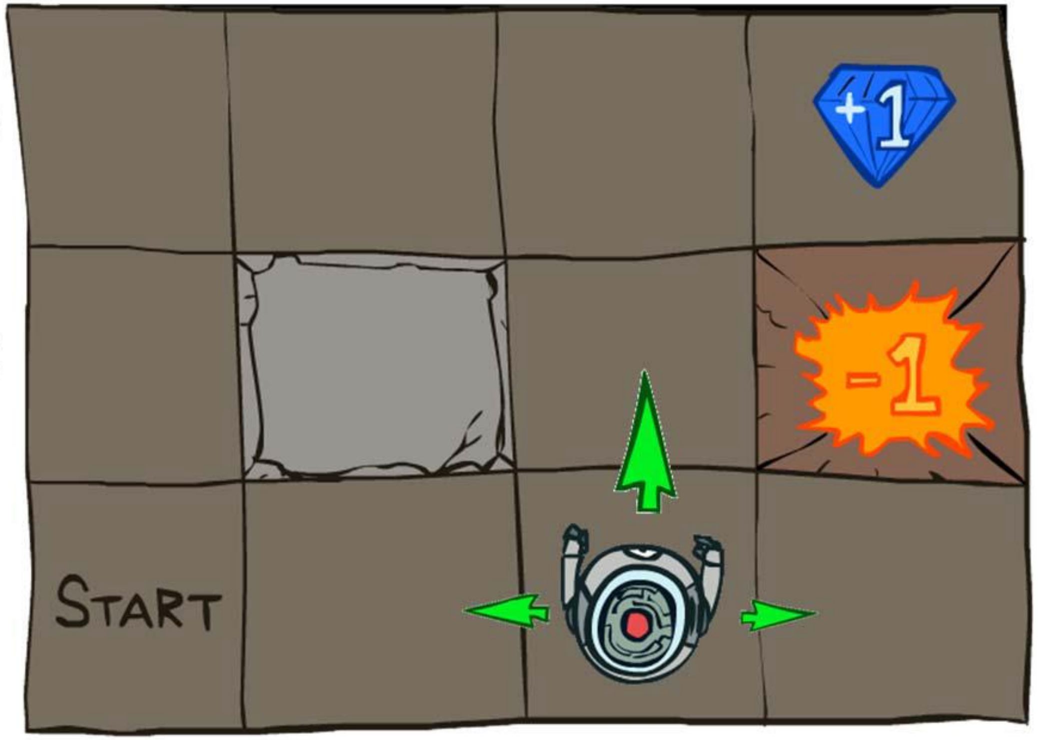
The formal mathematical model:

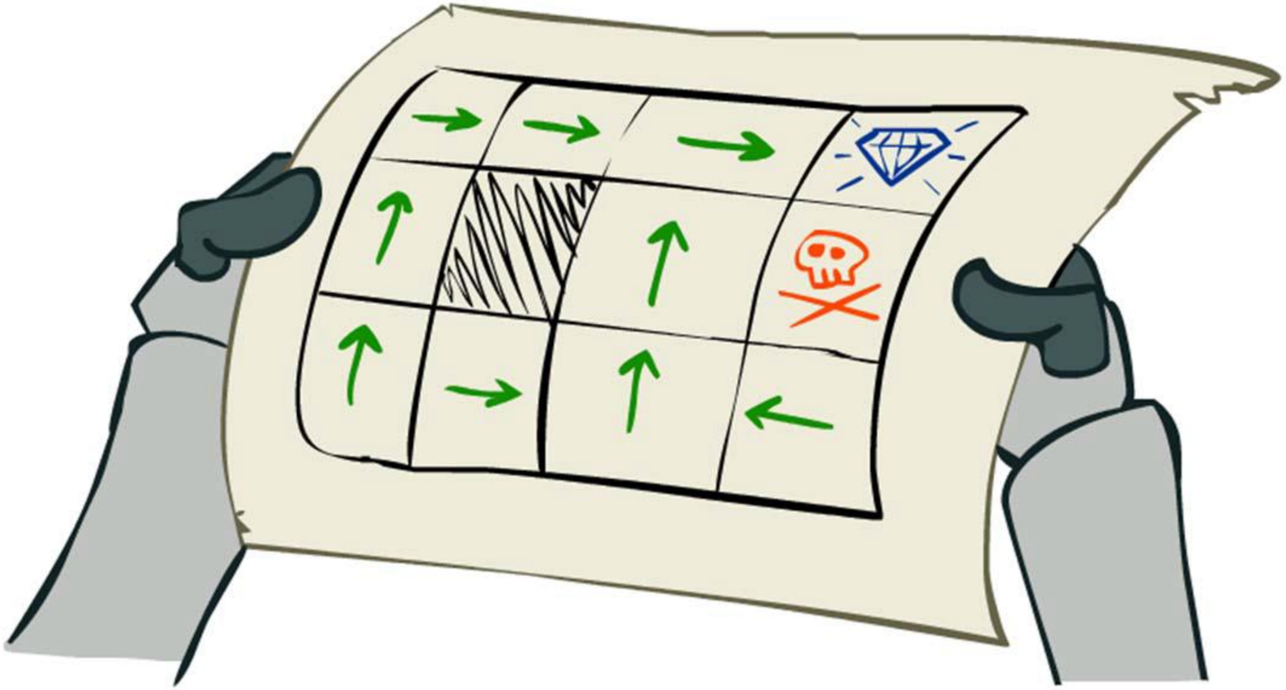
- **State set** $S$. Initial state $s_0$. **Action set** $A$
- **State transition model:** $P(s_{t+1}|s_t, a_t)$
  - Markov assumption: transition probability only depends on $s_t$ and $a_t$, and not previous actions or states.
- **Reward function:** $r(s_t)$
- **Policy:** $\pi(s) : S \rightarrow A$ action to take at a particular state.

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

# Example of MDP: Grid World

Robot on a grid; goal: find the best policy

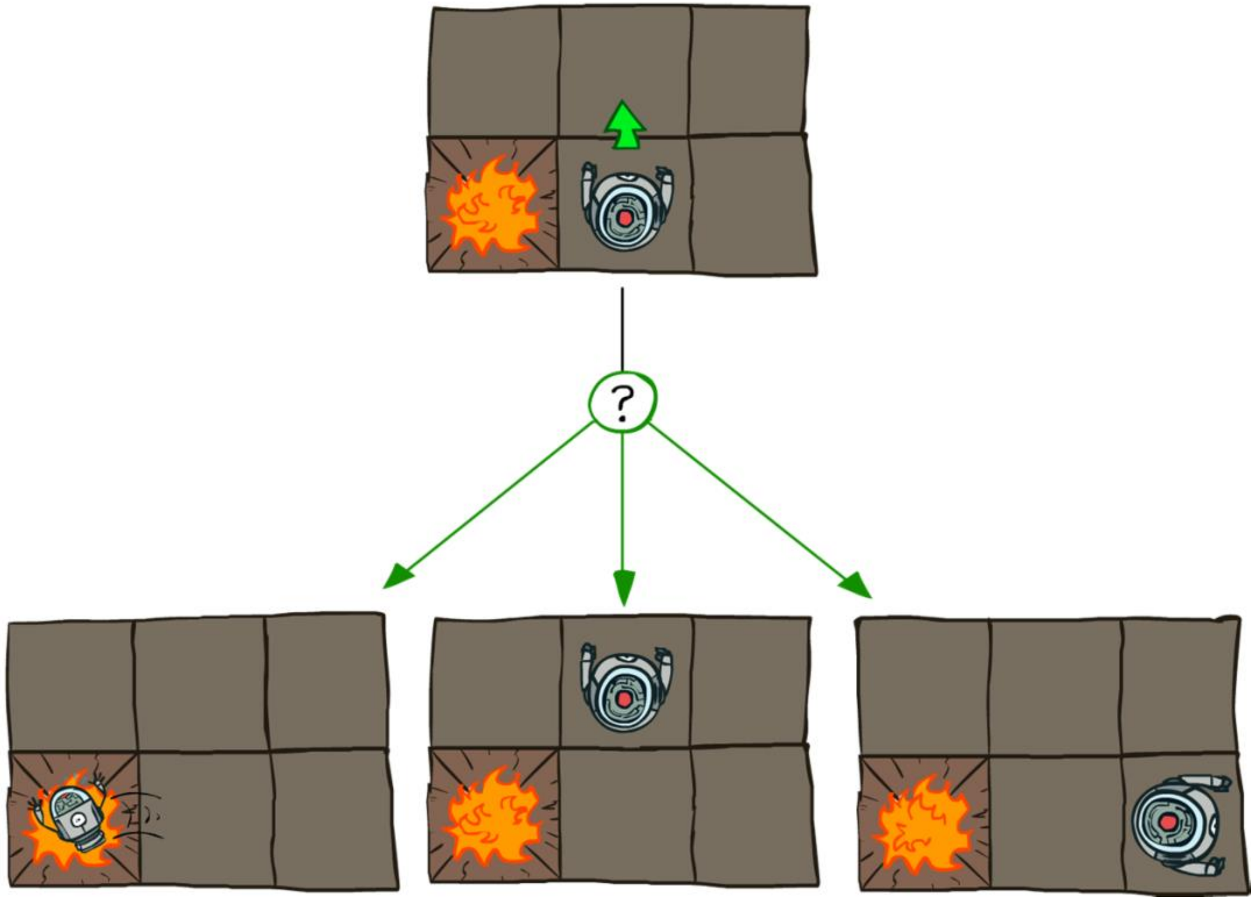


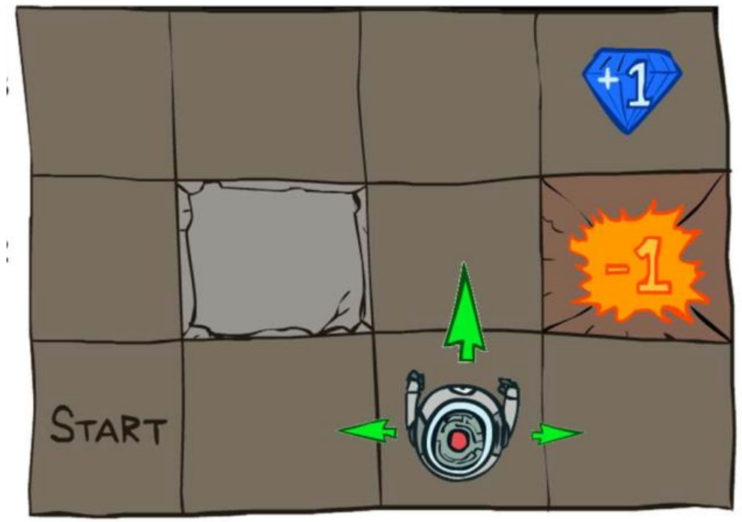



Source: P. Abbeel and D. Klein

# Example of MDP: Grid World

Note: (i) Robot is unreliable (ii) Reach target fast

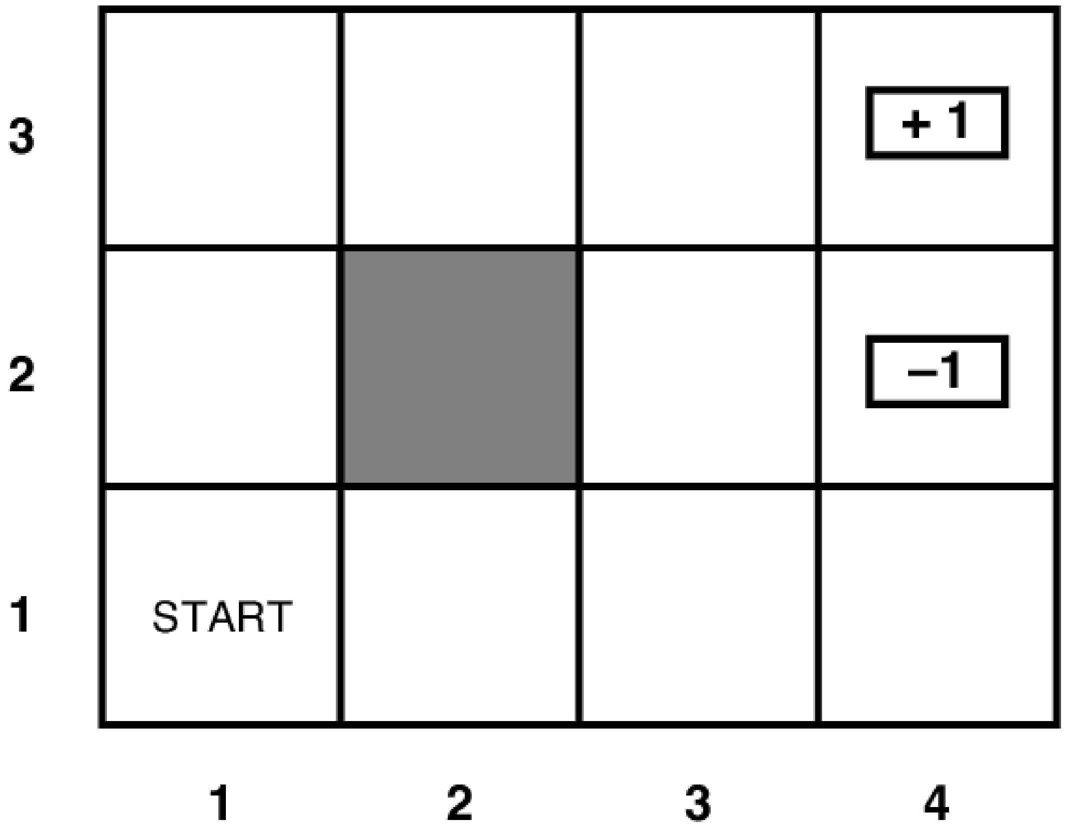


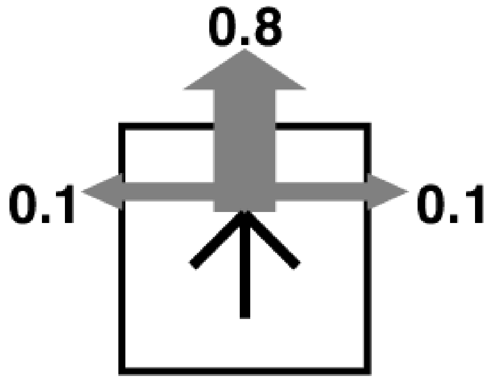




$r(s) = -0.04$ for every non-terminal state

# Grid World Abstraction

Note: (i) Robot is unreliable (ii) Reach target fast

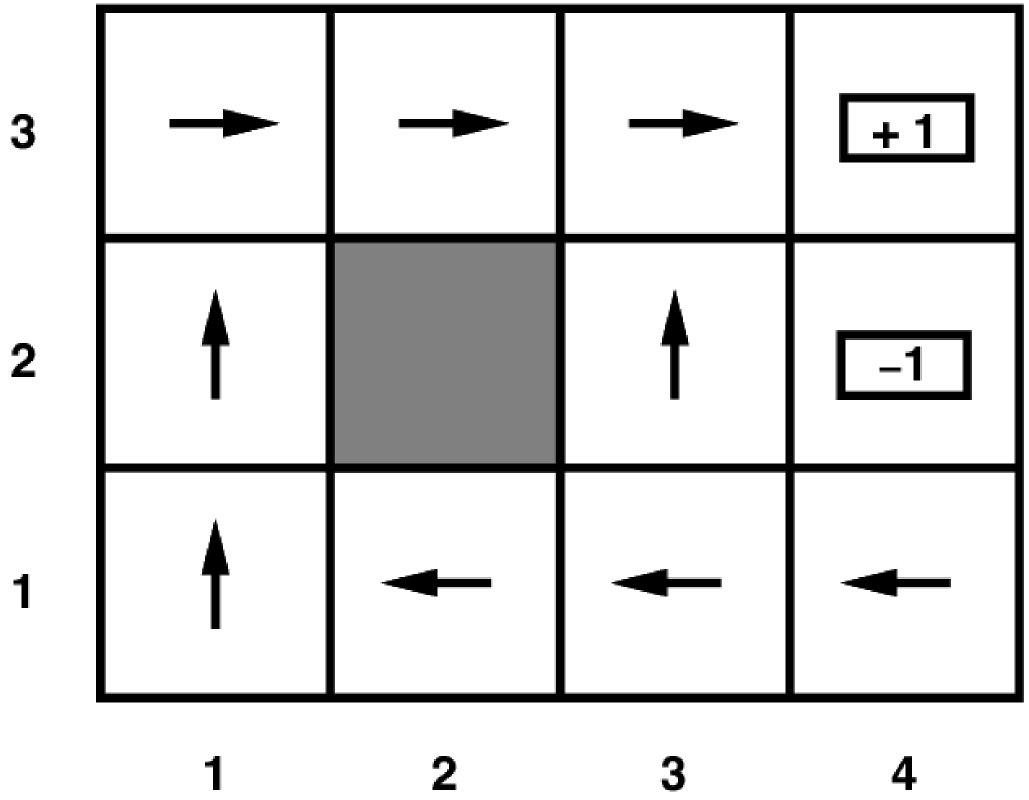


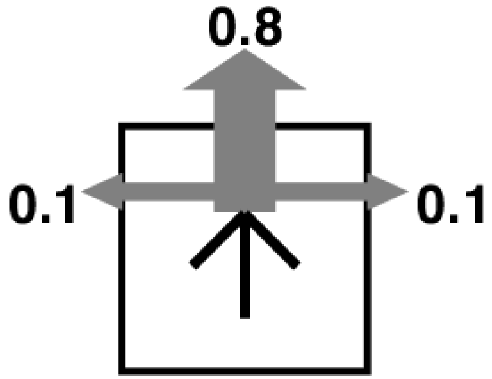




$r(s) = -0.04$ for every non-terminal state

# Grid World Optimal Policy

Note: (i) Robot is unreliable (ii) Reach target fast




$r(s) = -0.04$ for every non-terminal state

# Back to MDP Setup

The formal mathematical model:

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- **Reward function:** $r(s_t)$
- **Policy:** $\pi(s) : S \rightarrow A$ action to take at a particular state.

**How do we find the best policy?**

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

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# Defining the Optimal Policy

For policy π, **expected utility** over all possible state sequences from $s_0$ produced by following that policy:

$$V^{\pi}(s_0) = \sum_{\substack{\text{sequences} \\ \text{starting from } s_0}} P(\text{sequence})U(\text{sequence})$$

Called the **value function** (for π, $s_0$)

# Discounting Rewards

One issue: these are infinite series. **Convergence**?

- Solution

$$U(s_0, s_1 \dots) = r(s_0) + \gamma r(s_1) + \gamma^2 r(s_2) + \dots = \sum_{t \geq 0} \gamma^t r(s_t)$$

- Discount factor γ between 0 and 1
  - Set according to how important **present** is VS **future**
  - Note: has to be less than 1 for convergence

# From Value to Policy

Now that $V^{\pi}(s_0)$ is defined what *a* should we take?

- First, set V*(s) to be expected utility for **optimal** policy from s
- What's the expected utility of an action?
  - Specifically, action a in state s?

$$\sum_{s'} P(s'|s, a) V^*(s')$$

all the states we could go to

transition probability

expected rewards

# Obtaining the Optimal Policy

We know the expected utility of an action.

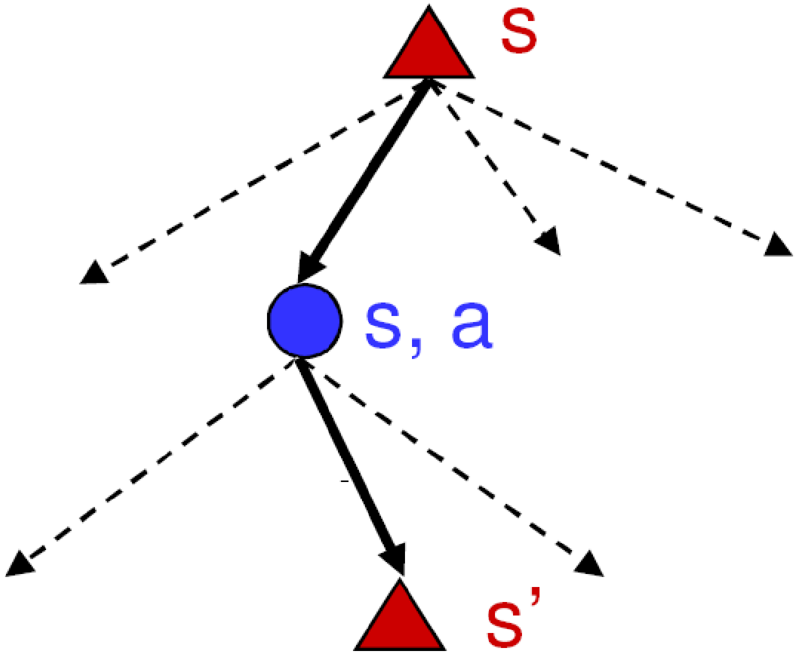
- So, to get the optimal policy, compute

$$\pi^*(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

all the states we could go to

transition probability

expected rewards




Credit L. Lazbenik

# Slight Problem…

Now we can get the optimal policy by doing

$$\pi^*(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

- So we need to know $V^*(s)$.
  - But it was defined in terms of the optimal policy!
  - So we need some other approach to get $V^*(s)$.
  - Need some other **property** of the value function!

# Bellman Equation

Let's walk over one step for the value function:

$$V^*(s) = r(s) + \gamma \max_a \sum_{s'} P(s'|s,a) V^*(s')$$

current state reward

discounted expected future **rewards**

- Bellman: inventor of dynamic programming

# Value Iteration

**Q**: how do we find $V^*(s)$?

- Why do we want it? Can use it to get the best policy
- Know: reward $r(s)$, transition probability $P(s'|s,a)$
- Also know $V^*(s)$ satisfies Bellman equation (recursion above)

**A**: Use the property. Start with $V_0(s)=0$. Then, update

$$V_{i+1}(s) = r(s) + \gamma \max_{a} \sum_{s'} P(s'|s,a) V_i(s')$$

# **Policy** Iteration

With value iteration, we estimate V*

- Then get policy (i.e., indirect estimate of policy)
- Could also try to get policies directly

- This is **policy iteration.** Basic idea:
  - Start with random policy $\pi$
  - Use it to compute value function $V^{\pi}$ (for that policy)
  - Improve the policy: obtain $\pi'$

# **Policy** Iteration: Algorithm

**What if don't know the transition probability? (next time)**

**Policy iteration.** Algorithm

- Start with random policy $\pi$
- Use it to compute value function $V^{\pi}$ : a set of linear equations

$$V^{\pi}(s) = r(s) + \gamma \sum_{s'} P(s'|s, a) V^{\pi}(s')$$

- Improve the policy: obtain $\pi'$

$$\pi'(s) = \arg\max_{a} r(s) + \gamma \sum_{s'} P(s'|s, a) V^{\pi}(s')$$

- Repeat

# Thanks Everyone!

Some of the slides in these lectures have been adapted/borrowed from materials developed by Mark Craven, David Page, Jude Shavlik, Tom Mitchell, Nina Balcan, Elad Hazan, Tom Dietterich, Pedro Domingos, Jerry Zhu, Yingyu Liang, Volodymyr Kuleshov, Fred Sala